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Extension of the KDO turbulence/transition model to account for roughness

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Abstract Wall roughness significantly influences both laminar-turbulent transition process and fully developed turbulence. This work has developed a wall roughness extension for the KDO turbulence/transition model. The roughness effect is introduced via the modification of the k and v_t boundary conditions, i.e., the wall is considered to be raised at an extra height. The equivalent roughness height is linked to the actual roughness height, and the ratio between them is determined by reasoning. With such a roughness extension, the predictions of the KDO RANS model agree well with the measurements of turbulent boundary layer with a sand grain surface, while the KDO transition model yields accurate cross-flow transition predictions of flow past a 6:1 spheroid.

Key words transition model, turbulence model, skin friction, wall roughness.

1 Introduction

Computational Fluid Dynamics (CFD) is widely used as a predictive tool for fluid motions. Fluids in nature and engineering applications often interact with solid walls. The walls, either natural or man-made, are with roughness. Wall roughness significantly affects laminar-turbulence transition process. After transition, turbulence on a rough surface is enhanced compared with that on a smooth one, leading to higher skin frictions and heat transfer rates. Therefore, it is necessary to account for roughness in CFD simulations.

It is unrealistic to set up a graphical CAD model with every roughness-element for a CFD simulation. Thus, it is assumed that the roughness-element size in any direction is small compared with the boundary layer thickness so that, above the roughnesses, the flow is averaged over numerous roughness elements that exact location of which is not accounted for [1]. The wall presented in the computational domain is smooth, and the velocity on the wall is also zero. To account for the effect of roughness, the “equivalent sand grain” approach [2] is commonly employed. This approach links the real roughness height to an equivalent roughness height. According to Nikuradse’s experiments [2], there are empirical correlations between the height of the equivalent sand grain and the real roughness shape. The correlations proposed by Dirling [3] and Grabow and White [4] established the general paradigm of wall roughness modeling. The basic idea is to mimic roughness effect by increasing the turbulent eddy viscosity, v_t , or turbulent

energy, k , in the wall region. The increment is determined by the equivalent roughness height.

This equivalent sand grain approach is successful in predicting fully developed turbulence. However, to capture the laminar-turbulent transition process which is influenced by roughness effect, additional modeling technique must be supplemented. Xiang et al. [5] proposed a hypersonic cross-flow transition criterion considering surface roughness, yet no roughness modeling for fully developed turbulence was presented. Liu et al. [6] employed an additional transport equation of A_r , the roughness amplification factor, to reflect roughness effect in transition process. Liu et al. [6] also employed the Wilcox wall boundary condition for ω [7] to introduce roughness effect in turbulent region.

According to the previous work, it is evident that two roughness corrections are needed: one on the turbulent equations (k , ω), and one on the transition equations (γ). The roughness correction on the turbulent equations is a relatively mature one which, in fact, leads to enhanced turbulent viscosity. The enhanced turbulent viscosity itself is capable of reflecting the premature transition process. Thus, an ideal turbulence models with wall roughness corrections should be capable of capturing the influence of roughness on the transition process, without additional roughness corrections to transition equations, and this work aims to develop such a model.

2 Transition/turbulence model with roughness consideration

2.1 Baseline model

The **K**inetic energy **D**ependent **O**nly model (KDO) [8] adopts a physics-based formulation, with less empiricism, for Reynolds Averaged Navier–Stokes Equation closure. For transition predictions, KDO does not explicitly model a specific transition mechanism but can capture many types of transition phenomenon. Along with the evolution of the turbulent kinetic energy (k) equation, transition phenomena naturally appear, which is similar to what Navier–Stokes equations do. The key is that all the model elements for the RANS closure are flow-structure-adaptive. The model equations read,

$$k_{,i} + (U_j k)_{,j} = -\overline{u'_i u'_j} U_{i,j} + [(\nu + \nu_t) k_{,j}]_{,j} - \varepsilon \quad (1)$$

$$\varepsilon = \varepsilon_1 + \varepsilon_2 \quad (2)$$

$$\varepsilon_1 = 2\nu\sqrt{k}_{,j}\sqrt{k}_{,j} \quad (3)$$

For $\text{Re}_k < 10$,

$$\varepsilon_2 = A_\varepsilon k^{3/2} / d \quad (4)$$

$$A_\varepsilon = 1.34 (\text{Re}_k / 0.25)^{-0.8} \left(1 + (\text{Re}_k / 0.25)^{1.5}\right)^{0.45/1.5} \left(1 + (\text{Re}_k / 2.4)^{1.5}\right)^{-0.1/1.5} \quad (5)$$

For $\text{Re}_k \geq 10$,

$$\varepsilon_2 = \min(A_\varepsilon, 0.8) k^{3/2} \max(1 / L, (1 - e^{-\text{Re}_k / 1300}) / d) \quad (6)$$

$$A_\varepsilon = 1.4(\text{Re}_k / 4.3)^{-1.9} \left(1 + (\text{Re}_k / 4.3)^5\right)^{0.2/5} \left(1 + (\text{Re}_k / 28)^{4.9}\right)^{2.76/4.9} \quad (7)$$

$$\left(1 + (\text{Re}_k / 66)^{10}\right)^{-0.15/10} \left(1 + (\text{Re}_k / 110)^{10}\right)^{0.12/10} \left(1 + (\text{Re}_k / 175)^{10}\right)^{-0.4/10}$$

$$L = \sqrt{\Omega_i \cdot \Omega_i} / \sqrt{\nabla_j \Omega_i \cdot \nabla_j \Omega_i}, \quad \Omega_i = \varepsilon_{ijk} U_{k,j} \quad (8)$$

By extending Bradshaw's assumption down to the wall, the Reynolds stress constitutive relation is established as,

$$R_b = \tau_{12} / k \quad (9)$$

$$-\overline{u'_i u'_j} = R_b k \frac{2S_{ij}}{S}, \quad S = \sqrt{2S_{ij}S_{ij}} \quad (10)$$

τ_{12} is the principal Reynolds shear stress, $-\overline{u'_i u'_j}$ is the Reynolds stress tensor, k is the turbulent kinetic energy, R_b is Bradshaw's coefficient, and S_{ij} is the mean strain rate tensor.

For the KDO turbulence model [8], the Bradshaw's coefficient reads,

$$R_b = \min \left[0.018(\text{Re}_k / 1)^{0.56} \left(1 + (\text{Re}_k / 120)^{2.5}\right)^{-0.56/2.5} \left(1 + (\text{Re}_k / 225)^{10}\right)^{0.05/10}, 0.283 \right] \quad (11)$$

in which, $\text{Re}_k = \rho \sqrt{k} d / \mu$ is turbulent Reynolds number. For the KDO transition model [9],

the Bradshaw's coefficient reads,

$$R_b = \min \left[0.1165(r/1)^{0.37} \left(1 + (r/1)^{1.15}\right)^{-0.157/1.15} \left(1 + (r/72)^2\right)^{-0.213/2}, 0.283 \right] \quad (12)$$

in which, $r = \mu_t / \mu$ is eddy viscosity ratio. The eddy viscosity ratio, a measurement of the intensity of turbulence, is a transport variable. Due to the transport properties of r , R_b is capable of capturing transition phenomena, such as bypass transition, natural transition, separation induced transition and cross flow transition. To conclude, the KDO model is one turbulence model that could predict both fully turbulent flows and laminar-turbulent flow transitions, by solving only the k equation. The information of turbulence and laminar-turbulent flow transition are both included in the k equation, and there is no such distinction as turbulent equation or intermittency

equation. Therefore, a typical roughness correction imposed on the KDO model could potentially reflect the roughness effects on the transition process.

2.2 Roughness extension

The “equivalent sand grain approach” is employed here. The basic idea is that, according to experimental data [2], the log-law still holds in a turbulent boundary layer with wall roughness. The difference lies in that, the $y^+ - U^+$ profile moves upwards, leading to a modified log-law,

$$u^+ = \frac{1}{\kappa} \ln \frac{y^+}{h_s^+} + C \quad (13)$$

h_s is the sand grain height. The position of the wall can be considered to be raised from y to $y+d_0$. d_0 is the equivalent sand grain height, which is less than h_s due to that there are spaces among sand elements. On the other hand, roughness enhances the turbulent viscosity to a value that is much larger than the molecular viscosity on the wall, and we have,

$$u_\tau^2 = \nu_t \frac{\partial U}{\partial y} = u_\tau \kappa (y + d_0) \frac{\partial U}{\partial y} \quad (14)$$

Note that equation (14) assumes that the boundary layer is fully turbulent, so equation (14) might not be valid for transitional flows. The solution of equation (14) is,

$$u^+ = \frac{1}{\kappa} [\ln(y + d_0) - \ln(d_0)] \quad (15)$$

By substituting equation (13) to equation (15), it is easy to obtain the relationship between d_0 and h_s .

$$d_0 = \exp(-C\kappa)h_s \quad (16)$$

Aupoix and Spalart [1] set C to 8.5 which is valid for very rough surfaces. In such a roughness model, d_0 equals $0.03h_s$, and this is inconsistent with the intuition that d_0 and h_s be in the same

magnitude. On the other hand, Chedevergne and Aupoix [10] stated that C can range from 5.5 to 9.7, indicating that even if the log-law still holds, the universality is compromised.

The present work employs the idea of “equivalent sand grain approach”, but the empirical coefficient is determined by reasoning instead of the modified log-law. This work defines an equivalent roughness height, also called d_0 ,

$$d_0 = C_r h_s \quad (17)$$

The original wall distance, d , in the KDO turbulence model is replayed by,

$$\tilde{d} = d + d_0 = d + C_r h_s \quad (18)$$

For an surface that experienced polishing treatment, the rough elements are uniformly distributed on the surface, so C_r is around 0.5. For a surface uniformly covered with sand grain, considering that the sand grain is with spherical shape, C_r is set to 0.35. The two constants are assessed in the following sections via CFD simulations.

The roughness effect is introduced via boundary conditions. The turbulent kinetic energy, k , on a smooth wall is zero. But for a rough wall,

$$k_w = k^+ \big|_w u_\tau^2 = k^+ \big|_w (\nu + \nu_t) \frac{\partial U}{\partial y} \big|_w \quad (19)$$

k^+ can be expressed by y^+ , and a model is calibrated by flat plate boundary layer at $Re_\theta=4060$ [11],

$$k^+ = f(y^+) = 0.131 \left(y^+ / 1 \right)^2 \left(1 + \left(y^+ / 3 \right)^{1.6} \right)^{-0.5/1.6} \left(1 + \left(y^+ / 8 \right)^{3.9} \right)^{-1.38/3.9} \left(1 + \left(y^+ / 19 \right)^{5.4} \right)^{-0.46/5.4} \left(1 + \left(y^+ / 40 \right)^4 \right)^{0.18/4} \quad (20)$$

As seen in figure 1, equation (20) is valid in the range $0 < y^+ < 10^3$ which covers the viscous layer, the buffer layer, and the log layer. Along with the increment of y^+ , the flow undergoes

laminar state, laminar-turbulent transition, and turbulent state, indicating that equation (20) could potentially capture all the flow states. For a rough wall, $k^+|_w$ in equation (19) is calculated by,

$$k^+|_w = 0.131 \left(d_0^+ / 1 \right)^2 \left(1 + \left(d_0^+ / 3 \right)^{1.6} \right)^{-0.5/1.6} \left(1 + \left(d_0^+ / 8 \right)^{3.9} \right)^{-1.38/3.9} \left(1 + \left(d_0^+ / 19 \right)^{5.4} \right)^{-0.46/5.4} \left(1 + \left(d_0^+ / 40 \right)^4 \right)^{0.18/4} \quad (21)$$

Fig. 1 $y^+ - k^+$ profile calibration

On a smooth wall $v_t|_w$ is set to 0. As to $v_t|_w$ in equation (19), a Neumann boundary condition is employed,

$$\frac{\partial v_t}{\partial n} \Big|_w = \frac{v_t}{d} \quad (22)$$

The KDO RANS model [8] with this roughness correction is termed as KDOR. The KDO transition model [9] with this roughness correction is termed as KDOR-tran.

3 Computational results

3.1 Turbulent boundary layer

Blanchard [12] conducted experiments over various surfaces. The turbulence on a sand grain paper the average height of which is 0.425 mm is often studied as a benchmark test case. The case corresponds to a zero pressure gradient flow, with an external velocity of 45m/s. Since the experiment focused on the roughness effects on fully developed turbulence, the KDOR model is

employed. Figure 1 shows the skin friction distributions on the wall. The results of KDOR agree well with the experimental data, with $C_r=0.35$ which corresponds to sand grain surface. The classic results of the roughness-extended SA model [1], termed as Boeing, is also shown as a reference. The KDO model yields much lower skin friction which corresponds to a turbulent boundary layer on a smooth wall. The velocity profile at $x=1.2\text{m}$ is shown in figure 2. The KDOR model yields a shifted log-law velocity profile which is a typical velocity profile with roughness.

Fig. 2 Skin friction on the surface

Fig. 3 Stream-wise velocity profile

3.2 Cross-flow transition on a spheroid

DVLR (now DLR) conducted experiments [13] on flow past a 6:1 spheroid at various Reynolds number and attack angle. The long diameter was 2.4m and the short diameter was 0.6m. The roughness height, h_s , was about $3.3\mu\text{m}$. The attack angle is set to 15° and the Reynolds number based on the long diameter is set to 6.5×10^6 . The flow was nearly incompressible and the Mach number is set to 0.2. This test condition was extensively studied by various transition models. The inflow turbulence intensity, Tu , is a controversial one. In the literature, Tu can range from 0.1% to 1%. A discussion with the researchers from DLR confirmed us that Tu was about 0.2% which is employed here. Since the surface of the spheroid was polished rather than covered with sand grain, it is preferred that C_r be 0.50. However, to explore the influence of C_r on the transition pattern, values 0.35 and 0.00 are tested. $C_r=0.00$ corresponds to a spheroid with a perfectly smooth surface. Since it is a cross-flow transition case, the KDOR-tran model is employed. The cross-flow transition pattern on the spheroid is illustrated by the skin friction contours. The experimental result is shown in figure 4. Figure 5, 6, and 7 show the results of KDOR-tran with C_r equals 0.50, 0.35, and 0.00, respectively. It is clear that the cross-flow transition pattern agrees with the measurements very well when C_r equals 0.50. However, the transition onset location is slightly delayed compared with the measurements. A slightly increased Tu could optimize the predictions, but this work insists the value of 0.2% provided by DLR. When C_r reduces to 0.35, the cross-flow transition pattern begins to deviate from the true pattern. When C_r reduces to 0.00, the transition onset locations are greatly delayed and the transition pattern differs a lot from the true pattern. It is necessary to point out that the $C_r=0.00$ and $Tu=0.2\%$ condition yields laminar flows on the surface, and the results in figure 7 corresponds to the $C_r=0.00$ and $Tu=0.6\%$ condition. According to the simulations, wall roughness does play an important role on the laminar

-turbulence transition process, and it necessary to take into account the roughness effects in turbulence modeling.

Fig. 4 measured skin friction contours

Fig. 5 predicted skin friction on the spheroid with $Cr=0.50$ and $Tu=0.2\%$

Fig. 6 predicted skin friction on the spheroid with $Cr=0.35$ and $Tu=0.2\%$

Fig. 7 predicted skin friction on the spheroid with $Cr=0.00$ and $Tu=0.6\%$

4 Conclusions

An extension of the KDO turbulence/transition model has been derived. It assumes non-zero viscosity and turbulent kinetic energy at the wall and change the definition of the wall distance, d . Thus, CFD code developers need only to alter the boundary conditions. The governing equations remains unchanged. Unlike the classic roughness extensions which utilize the altered log-law to calibrate empirical coefficients, this extension use reasoning as the empiricism. The ratios between effective roughness height and “sand grain rough height” are 0.35 and 0.50 for sand grain paper and polished surface, respectively. The ratios indicate that the two roughness heights are of the same order, so the ratios are reasonable. In addition, the ratios are both verified by the CFD simulations.

Test on a flat plate boundary layer with sand grain surface shows that the KDOR model can well predict full developed turbulence. Test on a spheroid with polished surface shows that the KDOR-tran model is capable of capturing roughness-influenced cross-flow transition. The two models both employ the new roughness extension. The key formula of the roughness extension employs a y^+-k^+ distribution which is obtained from the DNS data of a smooth flat plate. Surprisingly, the formula works well for rough walls, which exhibits some universality. Therefore, this work has successfully developed a roughness extension for the KDO turbulence/transition model. With such an extension, the KDO model is capable of capturing not only fully developed turbulence, but also the influence of roughness on the transition process, without additional roughness corrections to transition equations.

Abbreviations

KDO: turbulent kinetic energy dependent only; CFD: Computational Fluid Dynamics; CAD: Computer-Aided Design

Declarations

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Availability of data and materials

All data generated or analyzed during this study are included in this published article.

Authors' contributions

The contribution of the authors to the work is equivalent. All authors read and approved the final manuscript.

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interactions.

Competing interests

The authors declare that they have no competing interests.

References

- [1] Aupoix B, Spalart PR. Extensions of the Spalart–Allmaras turbulence model to account for wall roughness. *International Journal of Heat and Fluid Flow*. 2003 Aug 1;24(4):454-62.
- [2] Nikuradse, J., 1933. Stromungsgesetze in rauhen Rohren. Tech. Rept 361, VDI-Forschungsheft.
- [3] DIRLING, JR R. A method for computing roughwall heat transfer rates on reentry nosetips. In 8th Thermophysics Conference 1973 (p. 763).
- [4] Grabow RM, White CO. Surface roughness effects nosetip ablation characteristics. *AIAA Journal*. 1975 May;13(5):605-9.
- [5] XIANG Xinghao¹, ZHANG Yifeng¹, YUAN Xianxu¹, TU Guohua¹, WAN Bingbing¹, CHEN Jianqiang^{1,2*}. Study on C- γ -Re θ Model for Hypersonic Three-Dimensional Boundary Layer Transition Prediction
- [6] Liu Z, Zhao Y, Chen S, Yan C, Cai F. Predicting distributed roughness induced transition with a four-equation laminar kinetic energy transition model. *Aerospace Science and Technology*. 2020 Apr 1;99:105736.
- [7] Wilcox DC. Reassessment of the scale-determining equation for advanced turbulence models. *AIAA journal*. 1988 Nov;26(11):1299-310.
- [8] Xu J, Zhang Y, Bai J. One-equation turbulence model based on extended Bradshaw assumption. *AIAA Journal*. 2015 Jun;53(6):1433-41.
- [9] Xu J, Xu D, Zhang Y, Bai J. Capturing transition with flow–structure–adaptive KDO RANS model. *Aerospace Science and Technology*. 2019 Feb 1;85:150-7.
- [10] Chedevigne F, Aupoix B. Accounting for wall roughness effects in turbulence models: a wall function approach. In 7th EUCASS conference 2017 (Vol. 300).
- [11] Schlatter P, Örlü R. Assessment of direct numerical simulation data of turbulent boundary layers. *Journal of Fluid Mechanics*. 2010 Sep;659:116-26.
- [12] Blanchard A. Analyse expérimentale et théorique de la structure de la turbulence d'une couche limite sur paroi rugueuse (Doctoral dissertation).
- [13] Kreplin HP, Vollmers H, Meier HU. Wall shear stress measurements on an inclined prolate spheroid in the DFVLR 3 m x 3 m low speed wind tunnel. Data Report, DFVLR IB. 1985:222-84.