Rate-dependent Asymmetric Hysteresis Modeling and Robust Adaptive Trajectory Tracking for Piezoelectric Micropositioning Stages

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Rate-dependent asymmetric hysteresis modeling and robust adaptive trajectory tracking for piezoelectric micropositioning stages

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Abstract Hysteresis is an inherent characteristic of piezoelectric materials that can be determined by not only the historical input but also the input signal frequency. Hysteresis severely degrades the positioning precision of piezoelectric micropositioning stages. In this study, the hysteresis characteristics and the excitation frequency effects on the hysteresis behaviors of the piezoelectric micropositioning stage are investigated. Accordingly, a rate-dependent asymmetric hysteresis Prandtl-Ishlinskii (RDAPI) model is developed by introducing a dynamic envelope function into the play operators of the Prandtl-Ishlinskii (PI) model. The RDAPI model uses a relatively simple analytical structure with fewer parameters than other modified PI model to characterize the rate-dependent and asymmetric hysteresis behavior in piezoelectric micropositioning stages. Considering practical situations with the uncertainties and external disturbances associated with the piezoelectric micropositioning stages, the system dynamics are described using a second-order differential equation. On this basis, a corresponding adaptive robust control method that does not involve the construction of a complex hysteretic inverse model is developed. The Lyapunov analysis method proves the stability of the entire closed-loop control system. Experiments confirm that the proposed RDAPI model achieves a significantly improved accuracy compared with the PI model. Fur-

thermore, compared with the inverse RDAPI model-based feedforward compensation and the inverse RDAPI model-based proportional-integral-derivative control methods, the proposed robust adaptive control strategy exhibits improved tracking performance.

Keywords Piezoelectric micropositioning stages · Hysteresis modeling · Prandtl-Ishlinskii model · Precision trajectory tracking · Robust adaptive control

1 Introduction

Microtechnology and nanotechnology are required in many industries for precision control. The piezoelectric micropositioning stage is one of the most commonly utilized positioning devices and is widely employed in various fields, such as mechanical manufacturing, aerospace technology, and biological engineering [1, 2].

The piezoelectric micropositioning stage primarily comprises three parts: a piezoelectric ceramic actuator (PCA), flexure hinge machinery, and a strain gauge-type sensor [3]. The PCA is the core component of a piezoelectric micropositioning stage and affords micro/nanodisplacement. PCAs have the advantages of a fast response and small volume, and the resolution of a PCA is only restricted by the performance of the controller. When the engineering requires a low frequency and limited motion, PCAs can be reliably controlled using classical control methods. In [4], static linear piezoelectricity dynamics for PCAs were developed. Thus, classical control methods can be applied. However, with increasingly stringent requirements in terms of the driving-frequency and motion ranges in high-precision applications, PCAs exhibit complex nonlinear characteristics, particularly hysteresis, which can no longer be ignored. This strong nonlinearity significantly reduces the positioning accuracy or even re-

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Nomenclature

Symbols

b_s	Damping coefficient
C_A	Sum of the capacitance of the total piezoelectric ceramics
e_1	Tracking error
e_2	Intermediate variable
F	Fitness
F_A	Transduced force from the electrical side
$F_r[u](t)$	Classical PI play operator
$F_r^h[u](t)$	Proposed RDAPI play operator
H	Hysteresis effect
k_{amp}	Fixed gain of the voltage power amplifier
$k_i, i = 1, 2$	Positive design parameter
k_s	Moving mechanism stiffness
m	Moving mechanism mass
M	Population size
$p_i, i = 1, \dots, n$	Density parameter
$p, q, \text{ and } n$	Unknown system parameters
q	Total charge in the PCA
$\dot{q}$	Resulting current flowing through the circuit
q_c	Charge stored in the linear capacitance C_A
q_p	Transduced charge from the mechanical side due to the piezoelectric effect
r	Threshold of play operator
R_0	Equivalent internal resistance of driving circuit
T_{em}	Electro-mechanical transducer with transformer ratio
u	Input of the piezoelectric driver
v_A	Transduced voltage
v_h	Generated voltage due to H
w_1^t and w_1^r	Chromosome
x_1	Output displacement
$\dot{x}_2$	Time derivative of x_1
x_d	Reference trajectory

Greek letters

α, β	RDAPI model parameters
$\lambda, \gamma, \text{ and } \eta$	Adjusting parameters of adaptive update laws
ρ	Boundary of the disturbance $d(t), d(t) \leq \rho$
ς	Boundary of the nonlinear part $\alpha u^2(t) + \sum_{i=1}^n p_i F_r^h[u](t)$
ω	Unknown hysteresis nonlinearity

Abbreviations

AME	Absolute mean error
PCA	Piezoelectric ceramic actuator
PI	Prandtl-Ishlinskii model
RDAPI	Rate-dependent asymmetric PI model
RMSE	Root-mean-square error values

sults in system instability [5–7]. The hysteresis of PCAs is caused by the inherent ferroelectric phase-transition characteristics in piezoelectric ceramic materials; because of these characteristics, the output displacement is determined by not only the current input and system states but also the historical input. Additionally, the hysteresis behavior can vary with changes in the input signal frequency [8, 9]. To address this complex hysteresis nonlinearity, it is essential to establish a highly accurate model.

Hysteresis modeling has attracted significant attention, and several modeling approaches have been proposed. These approaches can be divided into two categories: differential-based and operator-based. Differential-based hysteresis mod-

els, such as the Bouc-Wen [10–12], Duhem [13, 14], and Dahl models [15], are constructed using differential equations. These models have a straightforward architecture with few parameters. However, the limited number of model parameters reduces the modeling accuracy. Operator-based hysteresis models include the Preisach [16–20], Krasnosel'skii-Pokrovskii [21, 22], and Prandtl-Ishlinskii (PI) models [23–27], which can be regarded as a series cumulation of weighted elemental hysteresis operators. The PI model is the most widely used because of its accurate analytical inverse. However, these models assume that hysteresis behavior is rate-independent. Therefore, efforts have been directed toward the development of a rate-dependent hysteresis model. At

present, the primary method for establishing rate-dependent hysteresis models is to introduce rate-dependent factors into the model [28, 29], e.g., by changing the density weights from static to dynamic [30] and designing rate-dependent operators [31]. These methods usually result in complex mathematical expressions and increase the difficulty of parameter identification. Thus, the rate-dependent hysteresis model requires further investigation.

Another challenge is eliminating the hysteresis nonlinearity of the piezoelectric micropositioning stage to achieve high-precision tracking control is another challenging topic. The feedforward compensation control method based on the hysteresis inverse model is the most widely used approach. In this method, the hysteresis nonlinearity is compensated for by constructing a corresponding inverse hysteresis model of the established hysteresis model [32–35]. In [36], a feedforward compensation controller was designed using the inverse Bouc-Wen model for a piezoelectric micropositioning stage with multiple degrees of freedom. The compensator combines the inverse multiplication structure with the Bouc-Wen model, thereby avoiding additional calculation of parameters. This type of feedforward compensation controller has a simple structure. However, its performance depends entirely on the accuracy of the established inverse model, and it is difficult to obtain a corresponding inverse model for some hysteresis models. Additionally, the controller cannot cope with environmental perturbations (i.e., temperature changes or external disturbances). For inverse model feedback control, the hysteresis nonlinearity is compensated for by the inverse model compensator, and the feedback loop is then used for ensuring the stability of the system in the presence of disturbances [37–40]. In [41], an inverse Preisach model was used as a feedforward compensation controller. Furthermore, a proportional-integral-derivative (PID) feedback controller was designed to improve the control accuracy, and it enabled effective control. Nonetheless, the control precision of the inverse model feedback method relies on the accuracy of the inverse model. The non-inverse model feedback control method directly incorporates the system hysteresis model into the generated control signal. A nonlinear controller is designed to drive the controlled object and adjusts the control parameters online based on the tracking error to improve the control accuracy [42–46]. This method omits the cumbersome inverse-model establishment process; thus, its performance is unaffected by the inverse-model accuracy [47]. Another advantage is that numerous excellent nonlinear control theories can be applied to nonlinear hysteretic system control. In [48], a dynamic backlash-like hysteresis model was defined without the need to construct an inverse hysteresis model to alleviate the influence of the hysteresis. Moreover, a robust adaptive controller that uses the properties of the hysteresis model was proposed, and this controller achieves a high tracking precision. In [49], an

inversion-free predictive controller based on a dynamic linearized multilayer feedforward neural network model was proposed, which can obtain an explicit-form control law without an inverse model. However, it is uncertain whether these schemes can handle external disturbances. Moreover, for piezoelectric micropositioning stages, the classical hysteresis model cannot describe the rate-dependent and asymmetric hysteresis characteristics.

Thus, the effects of the excitation frequency on the hysteresis behaviors of the piezoelectric micropositioning stage were investigated in the present study. A novel rate-dependent asymmetric hysteresis Prandtl-Ishlinskii (RDAPI) model is proposed to describe the asymmetric and rate-dependent hysteresis behaviors that the conventional PI model cannot describe. First, a dynamic envelope function is introduced into the play operators, while the density weights remain static. In this manner, the RDAPI model can describe the hysteresis loops that are affected by not only the historical input but also the input voltage frequency. Second, the proportional input function is replaced with a polynomial input function to describe the asymmetry. On this basis, a robust adaptive control method is presented for the piezoelectric micropositioning stages. Unknown parameters and external disturbances are considered in the dynamic nonlinear system. In contrast to the commonly used approach of constructing a complex hysteretic inverse model, the proposed control method directly uses the established RDAPI model to describe the hysteresis behavior, and the adaptive control laws are designed to eliminate hysteresis nonlinearity and uncertainties online. The main contributions of this study are as follows:

- A hysteresis model that can describe rate-dependent and asymmetry hysteresis behaviors is presented for piezoelectric micropositioning stages.
- The proposed RDAPI model has few parameters and a simple analytical structure, which makes parameter identification relatively straightforward.
- The proposed control method directly uses the established RDAPI model to describe the hysteresis behavior without constructing a complex hysteretic inverse model, and the robust adaptive method enhances the control accuracy and robustness.

The remainder of this paper is organized as follows. Section 2 describes an experiment that was performed to evaluate the hysteresis nonlinearity characteristics of the piezoelectric micropositioning stage. Section 3 presents the electromechanical model of the piezoelectric micropositioning stage and the proposed RDAPI hysteresis model. In Section 4, details regarding the robust adaptive controller design process and stability analysis are provided. Experimental studies and comparisons are presented in Section 5. Finally, Section 6 concludes the paper.

2 Characterization of piezoelectric micropositioning stage

2.1 Experimental setup of piezoelectric micropositioning stage

Fig. 1 Experimental device structure of the piezoelectric micropositioning stage control system.

Fig. 2 Experimental environment of the piezoelectric micropositioning stage control system.

The experimental environment and device structure are shown in Figs. 1 and 2, respectively. The experimental device consists of a piezoelectric micropositioning stage, an

integrated positioning controller, a data-acquisition card, and a host computer.

- MPT-2MRL102A-the piezoelectric micropositioning stage-is produced by Suzhou Boshi Robot Co., Ltd. It is equipped with a strain-gauge displacement sensor to measure the output displacement in real time. It can convert the measured displacement signal ($0-60 \mu\text{m}$) into an analog voltage signal ($0-10 \text{ V}$), with a repeated positioning accuracy of up to 12 nm . The sampling frequency is 0.0001 Hz .
- PCI-1716 is a peripheral component interconnect (PCI) bus multifunction data-acquisition card produced by Advantech. It is used for digital-to-analog and analog-to-digital conversion.
- An integrated positioning controller, which contains the drive-power module of the piezoelectric micropositioning stage, is used to generate the real-time control signal via the Real-Time Workshop (RTW) in MATLAB/Simulink and output the control signal to the piezoelectric micropositioning stage.
- The host computer displays the input and output signals online.

The entire control process of the piezoelectric micropositioning stage experimental system is as follows. First, the control block is set up in MATLAB/Simulink, and the control signal is generated through the RTW real-time working environment. The control signal is then translated into an analog voltage signal via a data-acquisition card and drives the piezoelectric micropositioning stage such that the stage produces a small displacement. Subsequently, the acquisition card transmits the collected displacement signals back to the host computer to complete the closed loop of the experimental system.

2.2 Characterization of hysteresis nonlinearities

Because of energy dissipation, hysteresis occurs between the input voltage and the output displacement in piezoelectric micropositioning stages. The hysteresis characteristics of the piezoelectric micropositioning stage are characterized by applying different input signals. First, the input voltage signal is selected as a sinusoidal signal. Fig. 3 presents the voltage-to-displacement hysteresis nonlinearity. It can be observed that the hysteresis causes that the same system input (u_1, u_2) to correspond to different system outputs (y_1, y_2) when the system is in an increasing state $\dot{u}(t) > 0$ or a decreasing state $\dot{u}(t) < 0$. It is thus characterized as “multi-valued”. Therefore, the system output is related to not only the current system input but also the historical output, that is, the memory properties. Fig. 3 also shows the increasing and decreasing edges of the hysteresis loop are not centrally symmetric. Obviously, the hysteresis nonlinearity has

Fig. 3 Multi-valued mapping and asymmetry of hysteresis.

Fig. 5 Hysteresis loops under different input frequencies.

Fig. 4 Major hysteresis loop of the piezoelectric micropositioning stage.

an asymmetric characteristic. Second, the input voltage signal is selected as a sinusoidal signal with decreasing amplitude. The major hysteresis loops of the piezoelectric micropositioning stage are presented in Fig. 4. When the amplitude of the input voltage changes, the hysteresis loops change accordingly. As the input amplitude increases, the hysteresis loop widens. Additionally, the same input voltage (u) corresponds to five output displacements (y_1, y_2, y_3, y_4, y_5).

Finally, a sinusoidal signal with different frequencies is selected as input voltage. As shown in Fig. 5, when the frequency of the input signal increases, the hysteresis loop widens, and the maximum output displacement decreases. Thus, the hysteresis behaviors of the piezoelectric micropositioning stage are also affected by the frequency of the input voltage. A satisfactory hysteresis model must describe all the aforementioned characteristics simultaneously.

3 Description of piezoelectric micropositioning stage

3.1 Model of the piezoelectric micropositioning stage

Fig. 6 shows the general model of the piezoelectric micropositioning stage, which includes (a) the electrical part and (b) the mechanical part [3, 50]. The notation for the piezoelectric micropositioning stage is presented in the Nomenclature section.

According to the aforementioned general model of the piezoelectric micropositioning stage, assuming that $R_0 = 0$, the complete electrical model can be expressed as follows:

$$q(t) - T_{em}x(t) = C_A k_{amp} \left[u(t) - \frac{H(q)}{k_{amp}} \right] \quad (1)$$

where $x(t)$ represents the output displacement of the mechanical part, and $u(t)$ represents the control input of the

Fig. 6 General model of the piezoelectric positioning stage: (a) electrical part; (b) mechanical part.

piezoelectric driver. According to its physical characteristics, the mechanical dynamics of the piezoelectric micropositioning platform can be represented by a second-order motion model:

$$m\ddot{x}(t) + b_s\dot{x}(t) + \bar{k}_s x(t) = \frac{T_{em}}{C_A} q(t) \quad (2)$$

where $\bar{k}_s = k_s + \frac{T_{em}^2}{C_A}$. Equations (1) and (2) represent an analytical model of the piezoelectric micropositioning stage. By substituting (1) into (2), the overall model of the piezoelectric micropositioning stage can be obtained:

$$\ddot{x} + a_0\dot{x} + a_1x = K\omega(t) + d(t) \quad (3)$$

where $a_0 = \frac{b_s}{m}$, $a_1 = \frac{\bar{k}_s}{m} - \frac{T_{em}^2}{mC_A}$, $K = \frac{k_{amp}T_{em}}{m}$, and $d(t)$ represent the unknown external disturbances, and $\omega \in R$ represents the unknown hysteresis nonlinearities of the piezoelectric micropositioning stage, which satisfy

$$\omega(t) = H[u](t) \quad (4)$$

where u represents the input voltage of the piezoelectric micropositioning stage, and $H[u](t)$ represents the hysteresis behavior, which is introduced below.

The control objectives are to design a robust adaptive controller for piezoelectric micropositioning stages with uncertain parameters and external disturbances; to eliminate the effects of hysteresis behavior, such that the output displacement $x(t)$ can accurately track the reference displacement signal $x_d(t)$; and to ensure the stability of the closed-loop system. The following assumptions are made.

Assumption 1: The external disturbance $d(t)$ satisfies $|d(t)| \leq \rho$, where ρ is a positive constant that denotes the upper $d(t)$ boundary.

Assumption 2: The uncertainty parameters in (3) satisfy $a_{i\min} \leq a_i \leq a_{i\max}$, $i = 0, 1$ and $K_{\min} \leq K \leq K_{\max}$, where $a_{i\min}$, $a_{i\max}$, $K_{\min}$, and $K_{\max}$ are known constants.

Assumption 3: The reference displacement signal x_d is a smooth and bounded function. The first and second derivatives exist and satisfy $\dot{x}_d^2 + \ddot{x}_d^2 \leq \chi$ for a positive real number χ .

Remark 1: Assumptions 1 and 3 are commonly used in controller design. The range of the system parameters is specified by Assumption 2, which is reasonable according to prior knowledge regarding piezoelectric micropositioning stages. Assumption 2 is often used in nonlinear controller design.

3.2 Hysteresis model for the piezoelectric micropositioning stage

In this study, the PI model was used to describe the hysteresis behavior of the piezoelectric micropositioning stage [21]. The classical PI model is expressed as

$$H[u](t) = p_0 u(t) + \int_0^R p(r) F_r[u](t) dr \quad (5)$$

where R represents the upper integration limit, $F_r[u](t)$ represents the classical play operator, r represents the threshold of play operator, $p(r)$ represents the density function satisfying $p(r) > 0$, and p_0 is a constant determined by the density function $p(r)$. The play operator is a continuous rate-independent hysteresis operator that satisfies

$$\begin{cases} F_r[u](0) = f_r(u(0), 0) \\ F_r[u](t) = f_r(u(t), F_r[u](t_i)) \end{cases} \quad (6)$$

where $t_i < t_{i+1}$ ($0 \leq i \leq N-1$), and $0 = t_0 < t_1 < \dots < t_i < \dots < t_N$ is a partition of $[0, t_N]$ (the interval is T), the input voltage signal u is a monotonic function on each subinterval $(t_i, t_{i+1}]$, and $f_r : R \rightarrow R$ is defined as follows:

$$f_r(u, \omega) = \max(u - r, \min(u + r, \omega)) \quad (7)$$

The classical play operator has rate-independent characteristics. Assume that $\eta(t)$ is a continuous function of t on $[0, t_N]$; then, $\eta(t)$ and the play operator have the following relationship:

$$F_r[u \cdot \eta] = F_r[u] \cdot \eta \quad (8)$$

Hence, the classical play operator cannot cope with the rate-dependent hysteresis characteristics of piezoelectric micropositioning stages. In this study, a dynamic envelope function was introduced into the classical play operator to explain the influence of frequency.

The rate-dependent play operator is expressed as

$$\begin{cases} F_r^h[u](0) = f_r^h(u(0), y(0)) \\ F_r^h[u](t) = f_r^h(u(t), F_r^h[u](t_i)) \end{cases} \quad (9)$$

$$f_r^h(u, \omega) = \max(h_l(u, \dot{u}) - r, \min(h_r(u, \dot{u}), \omega)) \quad (10)$$

where the dynamic envelope functions $h_l(u, \dot{u})$ and $h_r(u, \dot{u})$ can be selected as follows:

$$\begin{cases} h_l(u(t), \dot{u}(t)) = a|\dot{u}(t)| + bu(t) \\ h_r(u(t), \dot{u}(t)) = c|\dot{u}(t)| + du(t) \end{cases} \quad (11)$$

Here, a, b, c , and d are constants to be identified, which satisfy $a \neq c, b \neq d$.

According to the hysteresis behavior, the hysteresis loop contains two parts: the rising edge and the falling edge. Therefore, two different dynamic input functions, $h_l(u, \dot{u})$ and $h_r(u, \dot{u})$, are selected to express the rate-dependent play operator. To express the asymmetric hysteresis, a polynomial input function $\alpha u^2(t) + \beta u(t)$ is introduced to replace $p_0 u(t)$. Therefore, an improved RDAPI model can be obtained:

$$H[u](t) = \alpha u^2(t) + \beta u(t) + \int_0^R p(r) F_r^h[u](t) dr \quad (12)$$

where α and β are constants to be identified, $u(t)$ represents the system input voltage, $p(r)$ represents the density function of the rate-dependent PI model, and $F_r^h[u](t)$ represents the rate-dependent play operator.

The integral form of (12) is not easy to realize in practical applications. Therefore, (12) is transformed into a superposition of the n play operators. Subsequently, a discrete RDAPI model expression is obtained via the discrete superposition principle:

$$H[u](t) = \alpha u^2(t) + \beta u(t) + \sum_{i=1}^n p_i F_r^h[u](t) \quad (13)$$

Compared with previously reported models [28–30], the proposed RDAPI model has a simpler analytical format with fewer identification parameters to describe the rate-dependent and asymmetric hysteresis behaviors of piezoelectric micropositioning stages, as explained in Section 2.3. According to the system model of the piezoelectric micropositioning stage (3) and the RDAPI model (13), the dynamic equation can be written as follows:

$$\ddot{x} + a_0 \dot{x} + a_1 x = K \left(\alpha u^2(t) + \beta u(t) + \sum_{i=1}^n p_i F_r^h[u](t) \right) + d(t) \quad (14)$$

Based on (14), the RDAPI model contains two parts: a linear part $\beta u(t)$ and a nonlinear part $\alpha u^2(t) + \sum_{i=1}^n p_i F_r^h[u](t)$. For the nonlinear part, the input voltage of the piezoelectric micropositioning stage u is limited, and $\dot{u}$ exists; therefore, the nonlinear part is bounded. Accordingly, $\varsigma > 0$ exists, and it satisfies

$$\left| \alpha u^2(t) + \sum_{i=1}^n p_i F_r^h[u](t) \right| \leq \varsigma \quad (15)$$

Now, let $x = x_1, \dot{x}_1 = x_2$; then, (14) can be written as

$$\begin{cases} \dot{x}_1 = x_2 \\ p\dot{x}_2 = \beta u(t) + \alpha u^2(t) + \sum_{i=1}^n p_i F_r^h[u](t) + pd(t) - qx_2 - nx_1 \\ y = x_1 \end{cases} \quad (16)$$

where $p = \frac{1}{K}$, $q = \frac{a_0}{K}$, and $n = \frac{a_1}{K}$.

3.3 Identification of model parameters

The parameters to be identified in the RDAPI model include a, b, c , and d in the dynamic input function (11), α and β in the polynomial input function, and the weight parameters $p_i (i = 1, 2, \dots, n)$ of the play operators. We denote $Z = \{a, b, c, d, p_1, p_2, p_3, \dots, p_n, \alpha, \beta\}$ as the parameter vector to be identified in the RDAPI model.

The RDAPI model threshold is defined follows:

$$r_i = \frac{i}{n-1} \|u(t)\|_\infty \quad (17)$$

where $i = 1, 2, \dots, n$, with n being the number of play operators. The choice of n determines the computational burden and the accuracy of the RDAPI model. A larger n generally corresponds to a more accurate hysteresis model. However, the computational burden of parameter identification increase with n . In this study, n was selected as 8 through repeated numerical simulations. Table 1 presents the numbers of identification parameters for different rate-dependent PI models [28–30] with $n = 8$, thereby highlighting the advantage of the proposed RDAPI model.

Table 1 Comparison of identification parameters in different models

Model	Number of parameters
Yang [28]	
N = 8	21
Janaideh [29]	
N = 8	17
Xie [30]	
N = 8, m = 8	24
Proposed	
N = 8	14

In the identification process, the objective function is selected as

$$\phi = \sum_{j=1}^J \sum_{n=1}^{N_j} (y_d(n) - y(n))^2 \quad (18)$$

where J represents number of the different frequency input signals, N_j represents the number of data points of the input signals at different frequencies, and $y_d(n)$ and $y(n)$ represent the actual output displacement of the piezoelectric micropositioning stage and the model output displacement under input voltage signals of different frequencies, respectively.

An improved genetic algorithm [51] was used to identify the RDAPI model parameters.

First, the cumulative probability $Q(k)$ of the k th individual is defined:

$$Q(k) = \frac{\sum_{i=1}^k F_i}{\sum_{j=1}^M F_j} \quad (19)$$

where M represents the population size, and F_k represents the fitness of the k th individual.

Then, a two-point arithmetic crossover operator is selected. The two chromosome expressions are

$$w_1^t = (w_1^1, w_2^1, \dots, w_n^1), w_2^t = (w_1^2, w_2^2, \dots, w_n^2) \quad (20)$$

The offspring resulting from a two-point arithmetic crossover from point i to j is

$$w_1^{t+1} = (w_1^1, \dots, w_{i-1}^1, w_i', \dots, w_j', w_{j+1}^1, \dots, w_n^1) \quad (21)$$

$$w_2^{t+1} = (w_1^2, \dots, w_{i-1}^2, w_i'', \dots, w_j'', w_{j+1}^2, \dots, w_n^2) \quad (22)$$

where w_k' and w_k'' are generated as follows:

$$w_k' = \vartheta \alpha w_k^1 + (1 - \vartheta) w_k^2 \quad (23)$$

$$w_k'' = \vartheta w_k^2 + (1 - \vartheta) w_k^1 \quad (24)$$

Here, $\vartheta \in [0, 1]$ is a random number.

To accelerate the For the convergence speed speed of population, the mutation operator in the improved genetic algorithm is as follows: define the individual code as $w = (w_1, w_2, \dots, w_k, \dots, w_n)$, where $w_k \in [U_{\min}^k, U_{\max}^k]$ represents the mutant gene. Then, w_k mutates to

$$w_k' = \begin{cases} w_k + \left| r_2 (U_{\max}^k - w_k) w_k \right| & \text{if } r_1 < 0.5 \\ w_k - \left| r_2 (w_k - U_{\min}^k) w_k \right| & \text{if } r_1 \geq 0.5 \end{cases} \quad (25)$$

Table 2 RDAPI model parameter identification

a	$-2.065e^{-4}$	q_1	0.1899	q_5	0.0340	α	-0.0902
b	1.0203	q_2	0.1377	q_6	0.0726	β	0.6715
c	$2.996e^{-4}$	q_3	0.0865	q_7	0.0183		
d	1.0001	q_4	0.0400	q_8	0.2264		

where $r_1 \in [0, 1]$ and $r_2 \in [0, 1]$ are random numbers. If the variation in w_k' exceeds the value range,

$$\begin{cases} \text{if } w_k' < U_{\min}^k, \text{ then } w_k' = U_{\min}^k \\ \text{if } w_k' > U_{\max}^k, \text{ then } w_k' = U_{\max}^k \end{cases} \quad (26)$$

The actual input and output data of the piezoelectric micropositioning stage are used as the training set. The process of identifying the RDAPI model parameters related to the improved genetic algorithm is as follows.

Step 1: Select the fitness function:

$$Fitness(i) = \frac{1}{\phi(i)} \quad (27)$$

where $\phi(i)$ represents the objective-function value of the i^{th} individual.

Step 2: Generate the initial population. Randomly generate an initial population composed of M initial individuals, where each individual code string length l represents the number of parameters to be identified. The iteration starts with the initial population. The initial evolution generation t is set to 0, and the maximum number of iterations is G .

Step 3: According to (27), calculate the fitness value of each individual in the population.

Step 4: Perform the roulette selection according to (19).

Step 5: Perform the crossover operation in accordance with (20)–(22) to update the population.

Step 6: According to (25), apply the mutation operator to the population to obtain the next-generation population.

Step 7: Check whether the identification is complete. If the number of iterations satisfies $t < G$, return to Step 3 and continue the identification; otherwise, stop the operation and output the optimal solution.

In this study, we selected the piezoelectric micropositioning stage input and output data at frequencies of 1, 10, 20, and 50 Hz as the input for identification. The sampling time was set as $T = 0.0001$ s. The length of each individual coding string was set as $l = 12$, which is equal to the number of parameters to be identified. The population number was set as $M = 200$, and the maximum number of iterations as $G = 1400$. The identification parameters are presented in Table 2. The experimental results for the identified RDAPI model are presented in Section 5.1. **Note** : To facilitate identification, in the process of identifying parameters, the control input signal and actual output displacement are normalized to the range of 0–1. In actual applications, denormalization is performed on the data.

Fig. 7 Robust adaptive control structure diagram.

4 Robust adaptive controller design

In this section, a robust adaptive controller without constructing a complex hysteretic inverse model is proposed for the piezoelectric micropositioning stages. The proposed control method directly uses the established RDAPI model to describe the hysteresis behavior and design adaptive control laws to eliminate hysteresis nonlinearity and uncertainties online. The robust adaptive control structure diagram is presented in Fig. 7.

The robust adaptive control scheme was designed as follows. First, let e_1 be the tracking error of the controlled systems, which is defined as

$$e_1 = x_1 - x_d \quad (28)$$

Define an intermediate variable e_2 as,

$$e_2 = x_2 + k_1 e_1 - \dot{x}_d \quad (29)$$

where k_1 is a positive design parameter. The time derivative of e_1 is

$$\dot{e}_1 = x_2 - \dot{x}_d \quad (30)$$

Considering (28), $e_2 = \dot{e}_1 + k_1 e_1$. Then,

$$\dot{e}_1 = e_2 - k_1 e_1 \quad (31)$$

The time derivative of e_2 is

$$\begin{aligned} \dot{e}_2 = \frac{1}{p} \left(\beta u(t) + \alpha u^2(t) + \sum_{i=1}^n p_i F_r^h[u](t) \right) \\ + p d(t) - q x_2 - n x_1 \\ + k_1 e_2 - k_1^2 e_1 - \ddot{x}_d \end{aligned} \quad (32)$$

Then, we obtain

$$\begin{aligned} p \dot{e}_2 = \beta u(t) + \alpha u^2(t) + \sum_{i=1}^n p_i F_r^h[u](t) + p d(t) \\ - q x_2 - n x_1 + p(k_1 e_2 - k_1^2 e_1 - \ddot{x}_d) \end{aligned} \quad (33)$$

The control law is defined as

$$\begin{cases} u = \frac{1}{\beta} [-k_2 e_2 - e_1 + \hat{q} x_2 + \hat{n} x_1 + \hat{p} u_{eq} - \zeta \text{sign}(e_2)] \\ u_{eq} = -k_1 e_2 + k_1^2 e_1 + \ddot{x}_d - \rho \text{sign}(e_2) \end{cases} \quad (34)$$

where k_2 is a positive design parameter; $\hat{p}, \hat{q}$, and $\hat{n}$ represent the estimates of p, q , and n , respectively; the estimation errors are defined as $\tilde{p} = \hat{p} - p, \tilde{q} = \hat{q} - q$, and $\tilde{n} = \hat{n} - n$, respectively; ρ represents the bound of the disturbance $d(t)$; and ζ represents the bound of the nonlinear part $\alpha u^2(t) + \sum_{i=1}^n p_i F_r^h[u](t)$. The symbolic function $\text{sign}(e_2)$ can be expressed as,

$$\text{sign}(e_2) = \begin{cases} 1 & \text{if } e_2 > 0 \\ 0 & \text{if } e_2 = 0 \\ -1 & \text{if } e_2 < 0 \end{cases} \quad (35)$$

The adaptive update laws are designed as follows:

$$\dot{\hat{p}} = -\lambda u_{eq} e_2 \quad (36)$$

$$\dot{\hat{n}} = -\gamma x_1 e_2 \quad (37)$$

$$\dot{\hat{q}} = -\eta x_2 e_2 \quad (38)$$

where λ, γ , and η are the adjusting parameters of the adaptive update laws.

The key conclusion from the foregoing statement is presented in Theorem 1.

Theorem 1: Consider the entire control system, which is composed of the piezoelectric positioning stage system (14), the updated laws (37, 38), the actual controller (34), and the Lyapunov function (39). If Assumptions 1–3 are satisfied, all the signals in the closed-loop systems are uniformly ultimately bounded, and the tracking error e_1 exhibits asymptotic convergence under properly selected design parameters $k_1, k_2, \lambda, \gamma$, and η . Additionally, the output displacement $x(t)$ can accurately track the reference displacement signal $x_d(t)$ for all $t \geq 0$.

Proof of Theorem 1: Define the following Lyapunov function:

$$V = \frac{1}{2}e_1^2 + \frac{1}{2}pe_2^2 + \frac{1}{2}\left[\frac{\tilde{p}\tilde{p}^T}{\lambda} + \frac{\tilde{q}\tilde{q}^T}{\eta} + \frac{\tilde{n}\tilde{n}^T}{\gamma}\right] \quad (39)$$

The time derivative of V is

$$\dot{V} = e_1\dot{e}_1 + pe_2\dot{e}_2 + \frac{\tilde{p}\dot{\tilde{p}}}{\lambda} + \frac{\tilde{q}\dot{\tilde{q}}}{\eta} + \frac{\tilde{n}\dot{\tilde{n}}}{\gamma} \quad (40)$$

Substituting (31) and (33) into (40) yields

$$\begin{aligned} \dot{V} = & -k_1e_1^2 + e_2 \left(e_1 + \beta u(t) + \alpha u^2(t) + \sum_{i=1}^n p_i F_r^h[u](t) \right. \\ & \left. + pd(t) - qx_2 - nx_1 - p\ddot{x}_d + pk_1e_2 - pk_1^2e_1 \right) \\ & + \frac{\tilde{p}\dot{\tilde{p}}}{\lambda} + \frac{\tilde{q}\dot{\tilde{q}}}{\eta} + \frac{\tilde{n}\dot{\tilde{n}}}{\gamma} \end{aligned} \quad (41)$$

By substituting the updated laws (37) and (38) into (41), we obtain:

$$\dot{V} = -k_1e_1^2 + e_2 \left(\begin{aligned} & e_1 + \beta u(t) + \alpha u^2(t) + \alpha u^2(t) \\ & + \sum_{i=1}^n p_i F_r^h[u](t) - \hat{q}x_2 - \hat{n}x_1 + pd(t) \\ & - p\ddot{x}_d + pk_1e_2 - pk_1^2e_1 - \tilde{p}u_{eq} \end{aligned} \right) \quad (42)$$

Substituting the control law (34) into (42) yields

$$\dot{V} = -k_1e_1^2 + e_2 \left[\begin{aligned} & -k_2e_2 + p(-\rho \text{sign}(e_2) + d(t)) \\ & -\zeta \text{sign}(e_2) + \alpha u^2(t) + \sum_{i=1}^n p_i F_r^h[u](t) \end{aligned} \right] \quad (43)$$

Considering $|e_2| = e_2 * \text{sign}(e_2)$ and $|d(t)| \leq \rho$, it follows that,

$$\dot{V} \leq -k_1e_1^2 - k_2e_2^2 + \left| \alpha u^2(t) + \sum_{i=1}^n p_i F_r^h[u](t) - \varsigma \right| |e_2| \quad (44)$$

The nonlinear part of the RDAPI model satisfies $|\alpha u^2(t) + \sum_{i=1}^n p_i F_r^h[u](t)r| \leq \varsigma$; therefore,

$$\dot{V} \leq -k_1e_1^2 - k_2e_2^2 \quad (45)$$

where k_1 and k_2 are positive design parameters. Thus, $\dot{V} \leq 0$ for all $t > 0$. Considering that $V \geq 0$ and $\dot{V} \leq 0$, it can be concluded that the system is stable. Thus, the closed-loop stability of the designed robust adaptive controller is verified.

Equation (45) is solved for any $t \geq 0$:

$$\int_0^t k_1e_1^2(\tau) d\tau + \int_0^t k_2e_2^2(\tau) d\tau \leq V(0) - V(t) \quad (46)$$

According to $\dot{V} \leq 0$, $V(t) \leq V(0)$. Then, (46) can be written as follows: $\int_0^t k_1e_1^2(\tau) d\tau + \int_0^t k_2e_2^2(\tau) d\tau \leq V(0)$. Clearly, e_1 and e_2 are bounded. We define the following non-negative continuous function:

$$V_1(t) = V(t) - \int_0^t (\dot{V}(\tau) + k_1e_1^2(\tau) + k_2e_2^2(\tau)) d\tau \quad (47)$$

The derivative of (47) can be obtained as follows:

$$\dot{V}_1 = -k_1e_1^2(t) - k_2e_2^2(t) \quad (48)$$

Since e_1 and e_2 are bounded, $\dot{e}_1$ and $\dot{e}_2$ exist. Thus $\dot{V}_1$ is uniformly continuous on $[0, t]$. According to Barbalat's lemma, $\dot{V}_1 \rightarrow 0$ when $t \rightarrow \infty$; thus, when $t \rightarrow \infty$, the tracking error e_1 is asymptotically convergent. This completes the proof.

Remark 2: From (46), e_1 and e_2 are bounded. Because the range of output displacement for this piezoelectric micropositioning stage is 0–60 μm , the desired output displacement x_d is bounded. Additionally, from $e_1 = x_1 - x_d$, x_1 is bounded. According to (29), e_1 , e_2 , and $\dot{x}_d$ are bounded; hence, x_2 is bounded. Thus, the control law u is bounded.

5 Experimental results

The proposed RDAPI model and the robust adaptive control strategy were tested on piezoelectric micropositioning stages to evaluate their efficiency.

5.1 Experimental results for RDAPI model

The conventional PI model was selected for comparison to validate the proposed RDAPI model. The input voltage signal was selected as a sinusoidal signal with different frequencies. Fig. 8 presents the modeling performance of the proposed RDAPI model and the classical PI model with input voltage signals having frequencies of 1, 10, 20, and 50 Hz, respectively. When the input signal frequency was 1 Hz, the modeling error of the RDAPI model was close to that of the classical PI model. When the input voltage signal frequency was 50 Hz, the root-mean-square error (RMSE) was reduced by 47.02%. The maximum errors were 1.2195 and 0.4965 μm for the RDAPI model and the classical PI model, respectively. The results indicate that at a high frequency, the modeling error of the classical PI model increases more than that of the RDAPI model does. In this section, the conventional PI model is selected for comparison to verify the validity of the proposed RDAPI model. The input voltage signal was selected as a sinusoidal signal with diverse frequencies.

Table 3 presents the RMSE and the absolute mean error (AME) values of the actual output and the model output with different input voltage signal frequencies. The RMSE

Fig. 8 Hysteresis loops of the proposed RDAPI and classical PI models with different input frequencies.

Table 3 Modeling errors of the RDAPI and classic PI models with diverse input frequencies.

Input frequency	PI model (RMSE (μm) / AME (μm))	RDAPI model
1 Hz	0.3716/0.2996	0.3194/0.2594
10 Hz	0.4621/0.3787	0.2327/0.1832
20 Hz	0.3293/0.2583	0.2907/0.2309
50 Hz	0.5515/0.4671	0.3751/0.3395

of the RDAPI model was within $0.3715 \mu\text{m}$, whereas that of the classical PI model increased to $0.5515 \mu\text{m}$ at 50 Hz. The results indicate that a clear improvement in the modeling accuracy was achieved by using the RDAPI modeling method, particularly at high frequencies. To analyze the modeling error more intuitively, the RMSE and AME were plotted as bar graphs, as shown in Fig. 9. The experimental results indicate that compared with the classical PI model, the RDAPI model can more accurately describe the nonlinear hysteretic characteristics of the piezoelectric micropositioning stage, particularly at high frequencies.

5.2 Experimental results for robust adaptive control

To validate the proposed robust adaptive control strategy, a series of experiments was conducted. The inverse RDAPI model-based feedforward compensation control (IRDAPI-based IFC) and inverse RDAPI model-based PID control (IRDAPI-based PID) methods were used for comparison. In the experiments, different trajectory forms were selected as reference signals, with frequencies ranging from 1 to 50 Hz. The parameters of the controller were selected via a trial-and-error approach. First, the adaptive adjustment parameters λ , η , and γ should be determined. A trade off exists between the tracking error and oscillation. The parameters can be set to small values initially and then gradually increased until a suitable value is reached and selected. Fig. 10 presents the tracking performance with different values of λ , η , and γ . As shown, when the selected adaptive adjustment parameters are $\lambda = 1e-10$, $\eta = 1e-7$, and $\gamma = 0.005$, the tracking error is small and does not introduce sharp oscillation into the system. Then, by weighing the convergence speed and oscillation, the control gain parameters k_1 and k_2

Fig. 9 Comparison of modeling errors between the classical PI and proposed RDAPI models with different input frequencies.

Fig. 10 Tracking errors for different selected adaptive parameters.

can be tuned (gradually increased) to a suitable value. The control gain parameters are selected as $k_1 = 1000$ and $k_2 = 0.005$. According to the RDAPI model parameters identified in Section 3.3, $\beta = 0.6714$. Calculations indicate that the boundary of the nonlinear part $\varsigma = \alpha u^2(t) + \sum_{i=1}^n p_i F_r^h[u](t)$ does not exceed 0.45. Therefore, $\varsigma = 0.45$. According to the

Fig. 11 Comparison results for the tracking performance under sinusoidal signals with a frequency of 1 Hz. (a) reference and control output displacements; (b) tracking error; (c) hysteresis reduction for different controllers.

Fig. 12 Comparison results for the tracking performance under sinusoidal signals with a frequency of 10 Hz. (a) reference and control output displacements; (b) tracking error; (c) hysteresis reduction for different controllers.

performance of the experimental stage and environment, the disturbance (environmental disturbance, temperature and humidity changes, etc.) does not exceed $1 \mu\text{m}$; therefore, the limit of the disturbance was set as $\rho = 1$.

5.2.1 Displacement tracking control under sinusoidal reference signals

The desired trajectory is selected as $x_d(t) = 20\sin(2\pi ft + 3/2\pi) + 20$, where f represents one of the input voltage signal frequencies (1, 10, 20, or 50 Hz). The tracking performance is shown in Figs. 11-14. The tracking results indicate that the proposed robust adaptive control approach can achieve a high tracking accuracy even when the frequency

Fig. 13 Comparison results for the tracking performance under sinusoidal signals with a frequency of 20 Hz. (a) reference and control output displacements; (b) tracking error; (c) hysteresis reduction for different controllers.

Fig. 14 Comparison results for the tracking performance under sinusoidal signals with a frequency of 50 Hz. (a) reference and control output displacements; (b) tracking error; (c) hysteresis reduction for different controllers.

reaches 50 Hz. Compared with the IRD-API-based IFC and IRD-API-based PID controllers, the proposed robust adaptive controller can significantly reduce the average tracking error. Regarding the maximum error of motion, the proposed approach is relatively ineffective. The RMSE and AME (Table 4) were used to quantify the performance of the different control approaches. For an intuitive comparison, the RMSE and AME were also plotted as bar graphs, as shown in Fig. 15. The RMSE of the proposed robust adaptive controller remained within $0.4039 \mu\text{m}$, and the AME remained within $0.2383 \mu\text{m}$. The RMSEs of the IRD-API-based IFC and IRD-API-based PID controllers reached 0.6010 and $0.7139 \mu\text{m}$, respectively, at 50 Hz. Additionally, the AMEs reached

Fig. 15 Comparison of tracking errors for different controllers under the sinusoidal signals.

Table 4 Comparison of tracking errors among the proposed, IRD-API-based PID, and IRD-API-based IFC methods under sinusoidal trajectories with different input frequencies based on the RD-API model.

Input frequency	Proposed	IRD-API-PID (RMSE (μm) / AME (μm))	IRD-API-IFC
1 Hz	0.0789/0.0225	0.2147/0.1901	0.3206/0.2698
10 Hz	0.3314/0.2253	0.3985/0.3182	0.4710/0.3936
20 Hz	0.3614/0.2423	0.5248/0.3978	0.5673/0.4868
50 Hz	0.4039/0.2383	0.6010/0.5102	0.7139/0.6440

0.5102 and $0.6440 \mu\text{m}$, respectively. Figs. 11(c), 12(c), 13(c), and 14(c) show the input–output relationship of the closed-loop system with different controllers, indicating that the proposed robust adaptive controller can restrain the hysteresis nonlinearity better than IRD-API-based IFC and IRD-API-based PID controllers can.

Fig. 16 Comparison results for the tracking performance under triangular signals with a frequency of 1 Hz. (a) reference and control output displacements; (b) tracking error; (c) hysteresis reduction for different controllers.

5.2.2 Displacement tracking control under triangular reference signals

A triangular-wave signal with an amplitude of $40 \mu\text{m}$ was selected as the desired trajectory, and the input voltage signal frequencies of 1, 10, 20, and 50 Hz were selected.

The tracking performance is shown in Figs. 16–19. The experimental results indicate that the proposed control strategy can eliminate the effects of hysteresis behavior at different frequencies more effectively than the IRDAPI-based IFC and IRDAPI-based PID controllers can. The RMSE and AME values are presented in Table 5. The bar graphs are shown in Fig. 20. The RMSE of the proposed method was maintained within $0.3124 \mu\text{m}$, whereas those of the IRDAPI-based IFC and IRDAPI-based PID methods were 0.8225 and $0.6097 \mu\text{m}$, respectively. The AME of the proposed method was within $0.1811 \mu\text{m}$, whereas those of the IRDAPI-based IFC and IRDAPI-based PID methods were 0.6976 and $0.4862 \mu\text{m}$, respectively. As indicated by the results, compared with the IRDAPI-based PID and IRDAPI-based IFC controllers, the proposed control method does not reduce the maximum tracking error; however, its RMSE and AME were significantly smaller. For trajectory-tracking problems, the most important aspect of the tracking performance is the average error. Thus, the proposed control strategy achieved better tracking performance than the other methods did.

5.2.3 Displacement tracking control under other reference signals

To further verify the effectiveness of the proposed control method, additional types of reference signals were selected as the desired signals. First, a triangular wave with a de-

Fig. 17 Comparison results for the tracking performance under triangular signals with a frequency of 10 Hz. (a) reference and control output displacements; (b) tracking error; (c) hysteresis reduction for different controllers.

Fig. 18 Comparison results for the tracking performance under triangular signals with a frequency of 20 Hz. (a) reference and control output displacements; (b) tracking error; (c) hysteresis reduction for different controllers.

Table 5 Comparison of tracking errors among the proposed, IRDAPI-based PID, and IRDAPI-based IFC methods under triangular trajectories with different input frequencies based on the RDAPI model.

Input frequency	Proposed	IRDAPI-PID (RMSE (μm) / AME (μm))	IRDAPI-IFC
1 Hz	0.0779/0.0962	0.3371/0.2656	0.3069/0.2520
10 Hz	0.2173/0.1197	0.2656/0.6105	0.4403/0.3762
20 Hz	0.2705/0.1811	0.5814/0.5175	0.4828/0.4007
50 Hz	0.3124/0.1664	0.8225/0.6976	0.6097/0.4862

Fig. 19 Comparison results for the tracking performance under triangular signals with a frequency of 50 Hz. (a) reference and control output displacements; (b) tracking error; (c) hysteresis reduction for different controllers.

Fig. 20 Comparison of tracking errors for different controllers under the triangular signals.

Fig. 21 Comparison results for different controllers. (a) reference and control output displacements; (b) tracking error; (c) hysteresis reduction for different controllers.

Table 6 Comparison of tracking errors among the proposed, IRDAP-based PID, and IRDAP-based IFC methods under different trajectories based on the RDAP model.

Reference signals	Proposed	IRDAP-PID (RMSE (μm) / AME (μm))	IRDAP-IFC
Variable amplitude	0.1662/0.0626	0.5248/0.5813	0.4227/0.03995
Stairstep	0.1667/0.0158	0.2241/0.3109	0.5123/0.6408
Complex harmonic	0.0229/0.0195	0.7033/0.5725	0.4685/0.3728

creasing amplitude was selected as the reference displacement. The results of the displacement tracking experiment are shown in Fig. 21. When tracking the first triangular waveform, the proposed controller exhibited a large tracking error. After the robust adaptive controller was adjusted online, the system became stable at the second triangular waveform. Then, a stairstep signal was selected as the reference trajectory. The experimental results are presented in Fig. 22. When the desired displacement changed sharp, the IRDAP-based PID controller exhibited a large amount of chatter, and the IRDAP-based IFC controller required a long settling time. In contrast, the proposed controller achieved satisfactory control performance within a shorter settling time. Although the proposed controller exhibited an overshoot. However, compared with that of the IRDAP-based PID controller, the overshoot was smaller, and compared with that of the IRDAP-based IFC controller, the convergence was faster. Additionally, the proposed method had the smallest tracking error.

A complex sine-wave signal with different frequencies and amplitudes was selected as the reference trajectory. The experimental results are presented in Fig. 23. The tracking errors for the three types of reference signals are presented in Table 6.

Fig. 22 Comparison results for different controllers; (a) reference and control output displacements; (b) tracking error.

Fig. 23 Comparison results for different controllers; (a) reference and control output displacements; (b) tracking error; (c) hysteresis reduction for different controllers.

In the foregoing experiments, the proposed robust adaptive controller outperformed both the IRDAPI-based IFC and IRDAPI-based PID control methods with regard to the RMSE. In particular, when the reference trajectory was complex, e.g., harmonics and triangular waves compounded by multiple frequencies, the proposed control method offered significant advantages over the other methods. The experimental results indicate that the online parameter adjustment of the robust adaptive controller plays an important role in ensuring strong robustness and high control accuracy. The results also indicate that the proposed robust adaptive controller is more effective than previously reported controllers in terms of suppressing hysteresis behavior in piezoelectric micropo-

sitioning stages under different types of reference displacements.

6 Conclusion

An RDAPI model was proposed herein for describing the rate-dependent asymmetric hysteresis behaviors of piezoelectric micropositioning stages. The model introduces dynamic envelope functions into play operators to describe hysteresis behavior and replaces the proportional input function with the polynomial input function to describe asymmetry. The model uses few parameters and a simple analytical structure to describe the hysteresis. On this basis, a robust adaptive controller was designed to improve the tracking performance. The proposed control method uses the RDAPI model to directly represent the hysteresis behaviors of piezoelectric micropositioning stages and thereby avoids the construction of a complex hysteretic inverse model. Experimental results indicated that compared with that of the classical PI model, the RMSE of the proposed RDAPI model can be reduced by 47.02% at 50 Hz. This suggests that the RDAPI model can describe the actual hysteresis loop at high frequencies more accurately. Sinusoidal, triangular, stairstep, and complex harmonic reference signals were utilized to evaluate the robust adaptive controller. The experimental results indicated that the robust adaptive controller can achieve satisfactory performance for piezoelectric micropositioning stages.

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Declarations

Data statement

The datasets generated during the current study are available from the corresponding author on reasonable request.

Conflict of interest

The authors declare that they have no conflict of interest.

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