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Fast satellite selection algorithm for GNSS multisystems based on the Sherman-Morrison formula

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Abstract The usage efficiency of GNSS multisystem observation data can be greatly improved by applying rational satellite selection algorithms. Such algorithms can also improve the real-time reliability and accuracy of navigation. By combining the Sherman-Morrison formula and singular value decomposition (SVD), a smaller geometric dilution of precision (GDOP) value method with increasing number of visible satellites is proposed. Moreover, by combining this smaller GDOP value method with the maximum volume of tetrahedron method, a new rapid satellite selection algorithm based on the Sherman-Morrison formula for GNSS multisystems is proposed. The basic idea of the algorithm is as follows: first, the maximum volume of tetrahedron method is used to obtain four initial reference satellites; then, the visible satellites are co-selected by using the smaller GDOP value method to reduce the GDOP value and improve the accuracy of the overall algorithm. By setting a reasonable precise threshold, the satellite selection algorithm can be autonomously run without intervention. The experimental results based on measured data indicate that (1) the GDOP values in most epochs over the whole period obtained with the satellite selection algorithm based on the Sherman-Morrison formula are less than 2. Furthermore, compared with the optimal estimation results of the GDOP for all visible satellites, the results of this algorithm can meet the requirements of high-precision navigation and positioning when the corresponding number of selected satellites reaches 13. Moreover, as the number of selected satellites continues to increase, the calculation time increases, but the decrease in the GDOP value is not obvious. (2) The algorithm includes an adaptive

function based on the end indicator of the satellite selection calculation and the reasonable threshold. When the reasonable precise threshold is set to 0.01, the selected number of satellites in most epochs is less than 13. Furthermore, when the number of selected satellites reaches 13, the GDOP value is less than 2, and the corresponding probability is 93.54%. These findings verify that the proposed satellite selection algorithm based on the Sherman-Morrison formula provides autonomous functionality and high-accuracy results.

Keywords: Beidou navigation satellite system (BDS); global navigation satellite system (GNSS); satellite selection

Introduction

With the full operation of the Beidou navigation satellite system (BDS), the number of in-orbit satellites in the global navigation satellite system (GNSS) has reached hundreds (Liu and Cao 2020). The navigation and positioning chip developed in China can receive multisystem and multifrequency GNSS data. If GNSS data from all visible satellites are used in navigation and positioning, theoretically, its accuracy is optimized, but the computational efficiency is reduced, which could seriously affect the real-time performance of navigation and positioning operations. Conversely, if only a single system is selected through a satellite selection method, observation data will be wasted.

Appropriate satellite selection can ensure the accuracy of calculations and improve the timeliness and reliability of navigation and positioning operations, thus enhancing GNSS applications, such as mobile device (Yu et al. 2017), precise point positioning and real-time kinematic difference applications (Dabove et al. 2020; Ge et al. 2018; Olivart i Llop et al. 2020). The traditional satellite selection algorithms mainly include the elevation angle and azimuth angle determination method (Wang et al. 2008) and maximum volume method (Tian et al. 2014; Blanco-Delgado et al. 2017; Kong et al. 2014). The algorithms of the elevation angle and azimuth angle determination method mainly focus on the selection of 4-6 satellites based on the selected satellites distribution in the space. And the maximum volume methods mainly focus on the selection of 4 satellites for a single system. However, with the increase in the number of visible satellites, the number of iterations has sharply increased and the real-time performance of navigation has been seriously affected. Thus five satellites selection method (Teng et al. 2015), and three-dimensional convex hull satellites selection algorithm for multi-GNSS constellations are proposed to improve GNSS chip performance. In recent years, many scholars have combined artificial intelligence with satellite selection algorithms and proposed various types of intelligent satellite selection schemes (Wu et al. 2010; Doong 2009). Wang and Jia et al. (2019) and Wang and Yang et al. (2019) proposed an intelligent particle swarm optimization algorithm by using intelligent matching for particle and satellite positions. In terms of the efficiency and accuracy of satellite selection, this type of algorithm is difficult to implement and requires high equipment performance, so it is not convenient for use in most models. Based on the initial satellite selection scheme, a satellite constellation method based on the NSGA-II genetic model (Xu et al. 2017;Huang et al. 2018) and a genetic algorithm based on a crossover operator (Song et al. 2016) were proposed. However, genetic algorithms have notable requirements for initial sample quality and quantity (Chen et al. 2015; Huo and Zhang 2015). Constellation selection algorithms based on matrix theory provide alternatives to intelligent algorithms. A determinant method was used to improve the traditional satellite selection algorithm by analyzing the corresponding observation matrix (Fu et al. 2019). Cong et al. (2006) proposed a satellite selection algorithm based on SVD matrix decomposition. Methods of this type with selection satellites algorithms have low computational complexity, can be quickly run and are convenient for selecting satellites through mathematical analysis, equivalent substitution and weighting methods based on matrix theory. However, most of the current satellite selection algorithms are verified with simulation data and not experimental data.

Considering the problems above, based on the Sherman-Morrison formula and SVD matrix decomposition, the smaller GDOP value method is presented. Furthermore, through an iterative process, a novel fast satellite selection algorithm based on the Sherman-Morrison formula and maximum volume of tetrahedron method is proposed in this paper. The algorithm is verified by using measured data from the UB4B0 GNSS chip, which was produced by Unicore Communications Incorporated. The results for 6-17 selected satellites and all visible satellite data are compared. Moreover, a reasonable threshold is set to test the autonomous functionality of the satellite selection algorithm based on the Sherman-Morrison formula.

Fast satellite selection algorithm based on the Sherman-Morrison formula

Geometric dilution of precision (GDOP) is used in GNSSs to specify the additional multiplicative effect of satellite geometry on GNSS precision. Theoretically, all visible satellites have minimal GDOP values, and the maximum number of calculations must be considered in GNSS navigation. To improve the timeliness and ensure the accuracy of GNSS navigation, we propose a novel method based on the Sherman-Morrison formula to select the visible satellites with the lowest GDOPs as follows.

GDOP accuracy index for satellite selection

The accuracy of a satellite selection algorithm is mainly evaluated based on the spatial geometry of the receiver and visible satellites (Kihara and Okada 1984). The most important parameter used to evaluate the geometric configuration is the GDOP value, which is usually expressed as the product of the user equivalent range error (UERE) and the GDOP. The navigation parameter error can be described as:

$$m = \delta_{\mathrm{UERE}} \cdot \mathrm{GDOP} \tag{1.}$$

where GDOP is the geometric precision factor and δ_{UERE} is the precision of pseudorange positioning. The navigation accuracy is further improved by reducing the GDOP value when δ_{UERE} is determined. In addition, GDOP can be defined as follows:

$$\mathrm{GDOP} = \sqrt{\mathrm{trace}\left(\boldsymbol{A}^{\mathrm{T}}\boldsymbol{A}\right)^{-1}} \tag{2.}$$

where trace (·) represents the trace of the matrix and

$$\boldsymbol{A}^{\mathrm{T}}\boldsymbol{A} = \begin{bmatrix} \Sigma kk & \Sigma kl & \Sigma km & \Sigma k \\ \Sigma lk & \Sigma ll & \Sigma lm & \Sigma l \\ \Sigma mk & \Sigma ml & \Sigma mm & \Sigma m \\ \Sigma k & \Sigma l & \Sigma m & n \end{bmatrix} \quad (3.)$$

where k, l, and m represent the partial derivatives of the connection between a station and a satellite in the x-axis, y-axis and z-axis directions, respectively; n represents the number of satellites; and Σ represents the cumulative sum.

Smaller GDOP value method based on the Sherman-Morrison formula

We assume that $\boldsymbol{A}_j$ is the observation coefficient matrix for j in-view satellites from multiple constellations with intersystem time off-sets considered. By removing the ith (i=1,…, j) satellite from j satellites, we can obtain the observation coefficient matrix $\boldsymbol{A}_{j-1}^{i}$ of j-1 satellites. The relationship between two observation coefficient matrices can be expressed as follows:

$$\boldsymbol{A}_j^{\mathrm{T}}\boldsymbol{A}_j = \boldsymbol{A}_{j-1}^{i}{}^{\mathrm{T}}\boldsymbol{A}_{j-1}^{i} + \boldsymbol{a}_i^{\mathrm{T}}\boldsymbol{a}_i \quad (4)$$

where $\boldsymbol{a}_i = \begin{bmatrix} k_i & l_i & m_i & -1 \end{bmatrix}$ is the partial derivative of satellite i in the direction of the coordinate axis. Based on the Sherman-Morrison formula (Bartlett 1951), we can obtain the following equation:

$$\boldsymbol{G}_{j-1}^{i} = \left(\boldsymbol{A}_{j-1}^{i}{}^{\mathrm{T}}\boldsymbol{A}_{j-1}^{i}\right)^{-1} = \left(\boldsymbol{A}_j^{\mathrm{T}}\boldsymbol{A}_j - \boldsymbol{a}_i^{\mathrm{T}}\boldsymbol{a}_i\right)^{-1} = \boldsymbol{G}_j + \frac{\boldsymbol{G}_j\boldsymbol{a}_i^{\mathrm{T}}\boldsymbol{a}_i\boldsymbol{G}_j}{1-\boldsymbol{a}_i\boldsymbol{G}_j\boldsymbol{a}_i^{\mathrm{T}}} \quad (5)$$

where $(1-\boldsymbol{a}_i\boldsymbol{G}_j\boldsymbol{a}_i^{\mathrm{T}})$ is a scalar, denoted as S_i; therefore, the square of the GDOP value of j-1 satellites is

$$\begin{aligned} GDOP_{j-1}^{i}{}^{2} &= \mathrm{trace}\boldsymbol{G}_{j-1}^{i} \\ &= \mathrm{trace}\boldsymbol{G}_j + \mathrm{trace}\left|\frac{\boldsymbol{G}_j\boldsymbol{a}_i^{\mathrm{T}}\boldsymbol{a}_i\boldsymbol{G}_j}{S_i}\right| \\ &= GDOP_j^2 + \mathrm{trace}\left|\frac{\boldsymbol{G}_j\boldsymbol{a}_i^{\mathrm{T}}\boldsymbol{a}_i\boldsymbol{G}_j}{S_i}\right| \end{aligned} \quad (6)$$

After removing satellite *i*, Eq. (6) indicates that $GDOP_{j-1}^{i}{}^{2}$ includes one more item than does $GDOP_j^2$.

Now, we need establish the rules for $\mathrm{trace}\left|\frac{\boldsymbol{G}_j\boldsymbol{a}_i^{\mathrm{T}}\boldsymbol{a}_i\boldsymbol{G}_j}{S_i}\right|$. The SVD of $\boldsymbol{A}_{j-1}^{i}{}^{\mathrm{T}}\boldsymbol{A}_{j-1}^{i}$ can be given as follows:

$$\boldsymbol{A}_{j-1}^{i}{}^{\mathrm{T}}\boldsymbol{A}_{j-1}^{i} = \boldsymbol{U}\boldsymbol{Z}\boldsymbol{U}^{\mathrm{T}} \tag{7}$$

According to (3), $\boldsymbol{U}$ is a (4×4) orthogonal matrix, and $\boldsymbol{Z}$ is a (4×4) diagonal matrix, where $\boldsymbol{Z} = \mathrm{diag}(z_1 \quad z_2 \quad z_3 \quad z_4)$.

By substituting (7) into (4), the orthogonal transformation of both sides of the equation can be obtained, furthermore, based on the Sherman-Morrison formula, we can obtain the following formula:

$$\begin{aligned}
&\left\{\boldsymbol{U}^{\mathrm{T}}\left(\boldsymbol{A}_j^{\mathrm{T}}\boldsymbol{A}_j\right)\boldsymbol{U}\right\}^{-1} = \left\{\boldsymbol{U}^{\mathrm{T}}\left(\boldsymbol{U}\boldsymbol{Z}\boldsymbol{U}^{\mathrm{T}}\right)\boldsymbol{U} + \boldsymbol{U}^{\mathrm{T}}\boldsymbol{a}_i^{\mathrm{T}}\boldsymbol{a}_i\boldsymbol{U}\right\}^{-1} \\
&= \left(\boldsymbol{Z} + \boldsymbol{U}^{\mathrm{T}}\boldsymbol{a}_i^{\mathrm{T}}\boldsymbol{a}_i\boldsymbol{U}\right)^{-1} \\
&= \left(\boldsymbol{Z} + \boldsymbol{u}\boldsymbol{u}^{\mathrm{T}}\right)^{-1} \\
&= \boldsymbol{Z}^{-1} - \frac{1}{1+\boldsymbol{u}^{\mathrm{T}}\boldsymbol{Z}^{-1}\boldsymbol{u}} \cdot \boldsymbol{Z}^{-1}\boldsymbol{u}\boldsymbol{u}^{\mathrm{T}}\boldsymbol{Z}^{-1}
\end{aligned} \tag{8}$$

where $\boldsymbol{u} = \boldsymbol{U}^{\mathrm{T}}\boldsymbol{a}_i^{\mathrm{T}} = \left[u_1 \quad u_2 \quad u_3 \quad u_4\right]^{\mathrm{T}}$.

Orthogonal transformation does not change the trace of the matrix, so the following relations can be obtained:

$$\begin{aligned}
&\mathrm{trace}((\boldsymbol{A}_{j-1}^{i}{}^{\mathrm{T}}\boldsymbol{A}_{j-1}^{i})^{-1}) = \mathrm{trace}(\boldsymbol{Z}^{-1}) \\
&= \mathrm{trace}((\boldsymbol{U}^{\mathrm{T}}(\boldsymbol{A}_j^{\mathrm{T}}\boldsymbol{A}_j)\boldsymbol{U})^{-1}) + \mathrm{trace}(k\cdot\boldsymbol{u}_z\boldsymbol{u}_z^{\mathrm{T}}) \\
&= \mathrm{trace}((\boldsymbol{A}_j^{\mathrm{T}}\boldsymbol{A}_j)^{-1}) + k\cdot\mathrm{trace}(\boldsymbol{u}_z\boldsymbol{u}_z^{\mathrm{T}})
\end{aligned} \tag{9}$$

where $\boldsymbol{u}_z = \boldsymbol{Z}^{-1}\boldsymbol{u}$, $\mathrm{trace}(\boldsymbol{u}_z\boldsymbol{u}_z^{\mathrm{T}}) = \sum_{i=1}^{4}\left(\frac{u_i}{z_i}\right)^2 = q > 0$ $\quad \mathrm{k} = 1\Big/(1+\sum_{i=1}^{4}\frac{u_i^2}{z_i}) > 0$. Therefore, the following formula can be obtained:

$$\mathrm{GDOP}_{j-1}^{i}{}^{2} = \mathrm{GDOP}_j^2 + k\cdot q > \mathrm{GDOP}_j^2 \tag{10}$$

Eq.(10) indicates that: (1) optimal GDOP value is achieved when all visible satellites are used for computing the three-dimensional position; (2) with increasing number of satellites, GDOP value will be smaller. Here we defined the satellite selection method about Eq.(10) as smaller GDOP value method based on the Sherman-Morrison formula.

In order to maximize the efficiency of satellite data use and reduce the number of calculations required, how to select four or more satellites from all visible satellites firstly, and then to add satellite to improve GDOP value according to Eq. (10), are the questions that we will discuss next section.

Combining the smaller GDOP value method and the maximum volume of tetrahedron method

The researches (Kaplan and Hegarty 2006; Kihara and Okada 1984; Newson 1899-1990; Blanco-Delgado et al. 2017) have been approved minimizing GDOP by selecting a set of satellites that maximizes the volume spanned by the user-to-satellite unit vectors when four satellites are used for computing the three-dimensional position. The relation between GDOP and the volume of tetrahedron can be expressed as follows:

$$\mathrm{GDOP} = \sqrt{\mathrm{trace}(\boldsymbol{A}^{\mathrm{T}}\boldsymbol{A})^{-1}} = \frac{\sqrt{\mathrm{trace}\left(\mathrm{adj}(\boldsymbol{A}^{\mathrm{T}}\boldsymbol{A})\right)}}{\left|\det\left(\boldsymbol{A}\right)\right|} = \frac{\sqrt{\mathrm{trace}\left(\mathrm{adj}(\boldsymbol{A}^{\mathrm{T}}\boldsymbol{A})\right)}}{6V} \tag{11}$$

Where V stands for the volume of tetrahedron spanned by the user-to-satellite unit vectors. When V reaches a maximum, the GDOP is considered as an optimal value.

After having selected four satellites based on the maximum volume of tetrahedron method, we know that small GDOP values imply good accuracies, we add herein another visible satellite based on the smaller GDOP method above to minimize GDOP value.

Let GDOP_i denote the GDOP value after add i satellites, GDOP_{i+1} denote the GDOP value after add $i+1$ satellites. And when $i=0$, GDOP_i is the minimizing GDOP related to the maximum volume of tetrahedron method in Eq. (11). According to Eq. (10), the relation between GDOP_i and GDOP_{i+1} can be expressed as follows:

$$\mathrm{GDOP}_{i+1}{}^{2} = \mathrm{GDOP}_i^2 - k_{i+1}\cdot q_{i+1} \qquad (i = 0,1,2\ldots\ i \le n-4) \tag{12}$$

Where n is the number of all visible satellites; $k_{i+1}\cdot q_{i+1}$ can be defined as GDOP smaller efficiency indicator.

In selecting the satellites, adding one satellite every time, minimum GDOP_{i+1} value will be achieved with the $k_{i+1} \cdot q_{i+1}$ maximization according to Eq.(12). Thus GDOP value will be smaller and smaller with the number of the selected satellites increasing. However, when the number of the selected satellites reaches a certain value, the rate of GDOP decrease tend to approach zero. Therefore, in order to make the satellite selection method with higher computational efficiency, reasonable threshold can be set to stop adding more satellite to minimize GDOP at this time. The relation between the end indicator of the satellite selection calculation and the reasonable threshold can be expressed as follows:

$$\Delta\mathrm{GDOP}_{i+1,i}{}^{2} = \mathrm{GDOP}^{2}_{i,\mathrm{Max}(k_i \cdot q_i)} - \mathrm{GDOP}^{2}_{i+1,\mathrm{Max}(k_{i+1} \cdot q_{i+1})} \leq TH \quad (i = 0,1,2\ldots \ i \leq n-4) \tag{13}$$

Where the $\Delta\mathrm{GDOP}_{i+1,i}{}^{2}$ stands for the end indicator of the satellite selection calculation; $\mathrm{GDOP}_{i,\mathrm{Max}(k_i \cdot q_i)}$ stands for the GDOP value after having added i satellites, moreover, with the $k_i \cdot q_i$ maximization; $\mathrm{GDOP}_{i+1,\mathrm{Max}(k_{i+1} \cdot q_{i+1})}$ stands for the GDOP value after having added $i+1$ satellites, moreover, with the $k_{i+1} \cdot q_{i+1}$ maximization; TH stands for the reasonable threshold.

Thus the characteristics of the maximum volume of tetrahedron satellite selection method are combined with the advantages of the Sherman-Morrison formula in the proposed approach, as shown in Fig. 1.

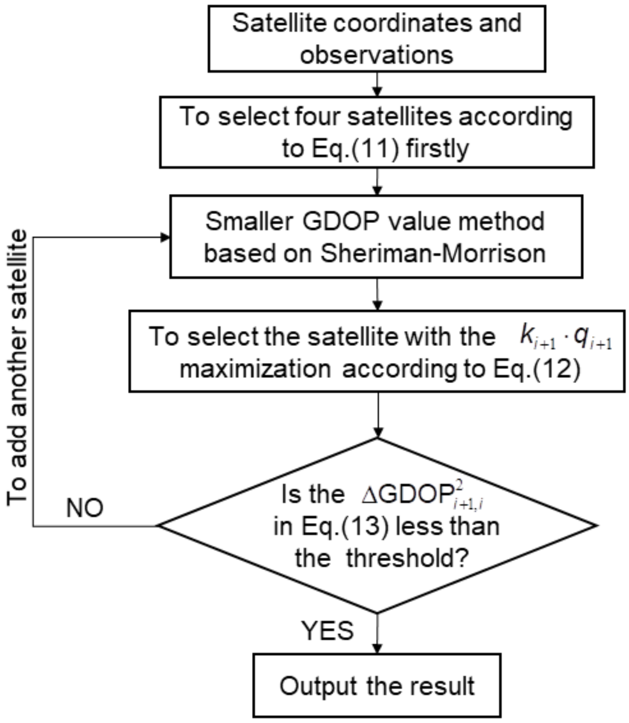

Fig. 1 Flow chart of the satellite selection algorithm based on the Sherman-Morrison formula

As shown in Fig. 1, after obtaining the satellite coordinates and observation data, the maximum volume of tetrahedron satellite selection method is used to select the four initial visible satellites, and the satellite selection algorithm based on the Sherman-Morrison formula is then used to co-select the visible satellites. The specific steps of the smaller GDOP value method in the co-selection process related to the visible satellites can be described as follows. First, determine whether the maximum $k_i \cdot q_i$ maximization, and the indicator $\Delta\mathrm{GDOP}_{i+1,i}{}^2$ is less than the threshold value TH. If the threshold requirement is met, stop adding more satellite, and end the calculation cycle. Finally output the satellite selection result.

Experimental verification and analysis

To verify the feasibility of the satellite selection algorithm based on the Sherman-Morrison formula, measured data are used. The GNSS data collected with a UB4B0 chip produced by Unicore Communications Incorporated were analyzed with a routine written in the MATLAB

2017b programming platform. Moreover, a processing program for the satellite selection algorithm based on the Sherman-Morrison formula was used to test the empirical validity of the results.

Results and analysis of the proposed algorithm in multiepoch, multisatellite experiments

Here, a total of 2400 ephemeris files were collected with the UB4B0 chip, and the epoch interval was 10 seconds. The observations and ephemeris files from BDS/GPS/GLONASS were used to further assess the experimental results. According to the design principle of the satellite selection algorithm based on the Sherman-Morrison formula, 6-17 visible satellites were selected without setting a threshold, and the experimental results are shown in Fig. 2. Additionally, the optimal estimation results of the GDOP for all visible satellites are given in Fig. 2.

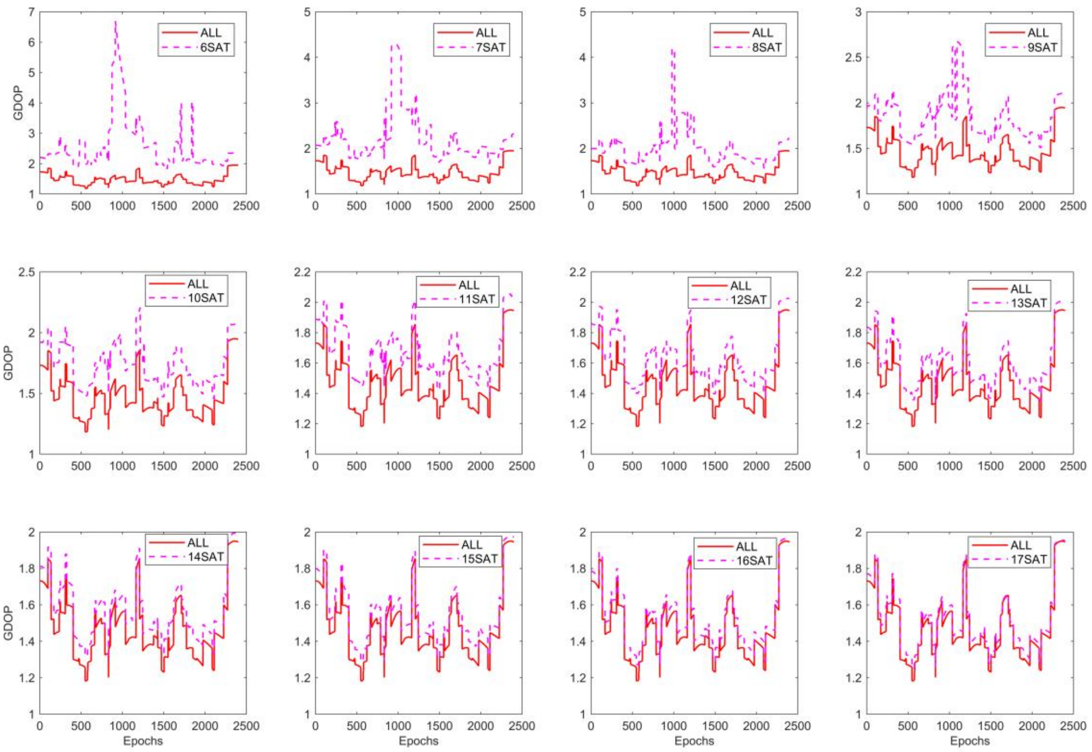

Fig. 2 Comparison of the results for all visible satellites and those of the satellite selection algorithm based on the Sherman-Morrison formula

In Fig. 2, ALL represents the optimal GDOP estimation results for all visible satellites. Moreover, 6 SAT-17 SAT represent the results for the corresponding number of selected satellites (i.e., 6, 7, 8,...,17).

Fig. 2 shows that the GDOP value decreases as the number of selected satellites increases. The results of the fast satellite selection algorithm based on the Sherman-Morrison formula indicate that the algorithm results reflect the variational GDOP trend. Additionally, the results in Fig. 2 can be summarized as follows: when the number of selected satellites ranges from 6 to 8, the GDOP value decreases at a rapid rate; however, compared with the results for all visible satellites, these results display relatively large differences in GDOP values. When the number of selected satellites ranges from 9 to 13, the GDOP value decreases significantly; compared with the results for all visible satellites, these results display smaller differences in GDOP values. When the number of selected satellites ranges from 13 to 17, the decrease in the GDOP is not obvious; compared with the results for all visible satellites, these results display very small GDOP differences. Moreover, the GDOP is less than 2 in all cases, which indicates that the positioning accuracy is high. In summary, compared with the optimal GDOP estimation results based on all visible satellites, the results when the number of selected satellites reaches 13 yield a GDOP value of less than 2, which indicates that the algorithm has high accuracy and stability. Therefore, 13 satellites can be regarded as the optimal number of selected satellites.

To verify the computational efficiency of the satellite selection algorithm based on the Sherman-Morrison formula, Table 1 gives the calculation time results of experiments over 2400 epochs.

Table 1 Calculation time over 2400 epochs for the satellite selection algorithm based on the Sherman-Morrison formula

Satellite number	6	7	8	9	10	11
Time (s)	815.254	821.957	828.785	836.621	844.747	857.624

Satellite number	12	13	14	15	16	17
Time (s)	864.962	871.653	882.498	898.352	912.415	931.564

Considering the satellite selection principle, the calculation time of the algorithm based on the Sherman-Morrison formula increases with increasing number of selected satellites. When the number of selected satellites is greater than or equal to 13, the calculation time of the algorithm increases rapidly. However, the average calculation time is less than 0.5 s in all epochs when the number of selected satellites is between 6 and 17. Furthermore, the results suggest that the satellite selection algorithm based on the Sherman-Morrison formula provides good real-time performance.

Results of automatic satellite selection when setting the threshold value

The calculation results of the proposed satellite selection algorithm are shown in Fig. 3 for threshold values is set as 0.01, and the end indicator $\Delta \mathrm{GDOP}_{i+1,i}{}^{2}$ less than it. The GDOP values are distributed between 1.3 and 2.6, and most of them are less than 2. This finding suggests that the selection algorithm based on the Sherman-Morrison formula has high accuracy and is autonomous. The statistics for the algorithm results over 2400 epochs are shown in Tab. 2.

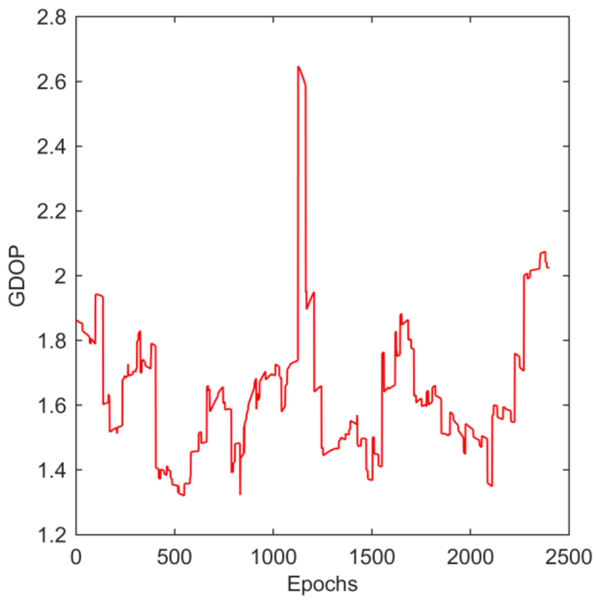

Fig. 3 Results of the algorithm based on the Sherman-Morrison formula with a threshold

Tab. 2 Statistics for the algorithm results over 2400 epochs

GDOP	Less than 1.5	Less than 1.6	Less than 1.7	Less than 1.8	Less than 1.9	Less than 2.0
Sat_all method	60.88%	82.79%	87.00%	91.46%	94.63%	100.00%
Sherman-Morrison	26.33%	48.75%	71.13%	83.46%	89.71%	93.54%

According to Tab. 2, 93.54% of the GDOP values obtained with the algorithm based on the Sherman-Morrison formula over 2245 epochs are less than 2. Thus, under normal conditions, the GDOP value is less than 2, indicating that the approach can meet the requirements of high-precision navigation. Therefore, the proposed algorithm is characterized by high accuracy and broad applicability.

The number of selected satellites and the probability statistics for the algorithm based on the Sherman-Morrison formula are shown in Fig. 4 and Tab. 3.

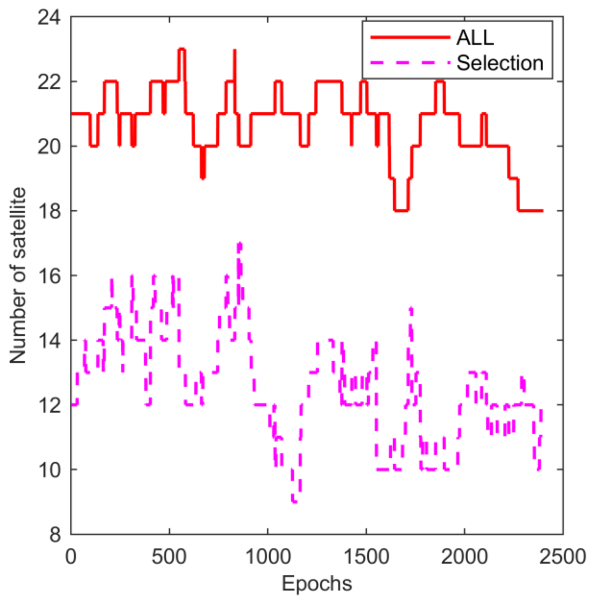

Fig. 4 Comparison of number of satellites selected in the all-visible-satellite method and the satellite selection method based on the Sherman-Morrison formula

Tab. 3 The probability statistics for the algorithm based on the Sherman-Morrison formula

Satellite number	13 or less	More than 13
Probability	85.79%	14.21%

Tab. 3 indicates that the probability is 85.79% when the number of selected satellites is less than or equal to 13. Additionally, the probability is 14.21% when the number of selected satellites is greater than 13. Combined with the results in Fig. 1 and Tab. 1, it can be seen that after setting a reasonable threshold for the end indicator of the satellite selection calculation, the algorithm based on the Sherman-Morrison formula exhibits high precision and supports autonomous functionality.

Conclusions

Based on theoretical and experimental verifications, the main conclusions of this study can be summarized as follows.

(1) The satellite selection algorithm based on the Sherman-Morrison formula reflections the variations in the GDOP. That is, the GDOP value decreases gradually with increasing number of selected satellites. When the number of selected satellites is greater than or equal to 13, the GDOP remains basically stable. The average calculation time in each epoch is less than 0.5 s, which indicates that the proposed method can meet the real-time requirements of navigation and positioning.

(2) The satellite selection algorithm based on the Sherman-Morrison formula provides autonomous function by setting a precision threshold. When the reasonable precise threshold is set to 0.01, the GDOP values are distributed between 1.3 and 2.6. In addition, 93.54% of the values are less than 2. This finding further verifies that the algorithm can maintain high positioning and navigation accuracy under autonomous conditions.

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Data Availability the processed GNSS data are not publicly available but can be obtained from the corresponding author Kezhao Li (lkznwpu@126.com). The GNSS data were collected by using a UB4B0 chip, which was produced by Unicore Communications Incorporated, China. The communication software was written by Junpeng Shi using the MATLAB 2017b programming platform.

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