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Quantized State Feedback-Based $\mathcal{H}_\infty$ Control for Nonlinear Parabolic PDE Systems via Finite-Time Interval

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Abstract In this paper, the finite-time $\mathcal{H}_\infty$ control problem of nonlinear parabolic partial differential equation (PDE) systems with parametric uncertainties is studied. Firstly, based on the definition of the quantiser, static state feedback controller and dynamic state feedback controller with quantization are presented, respectively. The finite-time $\mathcal{H}_\infty$ control design strategies are subsequently proposed to analyze the nonlinear parabolic PDE systems with respect to the effect of quantization. And by constructing appropriate Lyapunov functionals for the studied systems, sufficient conditions for the existence of the feedback control gains and the quantizer's adjusting parameters which guarantee the prescribed attenuation level of $\mathcal{H}_\infty$ performance are expressed as nonlinear matrix inequalities. Then, by using some inequalities and decomposition technic, the nonlinear matrix inequalities are transformed to standard linear matrix inequalities (LMIs). Furthermore, the optimal $\mathcal{H}_\infty$ control performances are pursued by solving optimization problems subject to the LMIs. Finally, to illustrate the feasibility and effectiveness of the finite-time $\mathcal{H}_\infty$ control design strategies, an application to the catalytic rod in a reactor is explored.

Keywords Parabolic PDE · Quantization · Finite-time · LMIs · $\mathcal{H}_\infty$ control

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1 Introduction

Many processes in engineering fields and multidimensional dynamical systems depend not only on time variable but also on spatial variable inherently, which means that the system performances are decided by both spatial position and time variable, such as fluid heat exchangers, thermal diffusion processes, and dissipative dynamical systems, et al. [1,2]. And these distributed parameter systems or processes can be frequently described by partial differential equations (PDEs). During the past several decades, an immense amount of interesting findings on the control synthesis and stability analysis of PDE systems have been made by scholars, such as [3–6]. These research achievements are meaningful for the applications and further studies.

As the constructions of industrial distributed parameter systems governed by PDEs are more and more complicated, higher demands on safety and reliability are usually confronted. However, unexpected deviation or uncertainties generated by external and internal disturbances always occur inherently. These uncertainties may lead to system instability and performance degradation if they are not appropriately taken account in the controller design. Thus, the issue of control design for PDE systems with uncertain parameters is of practical and theoretical value. Fortunately, valuable researches of controller design and stability analysis for parabolic PDE systems with parametric uncertainties have been made by many scholars. For example, with the consideration of uncertain variables presented in the system structure, robust output-based controller design methods for quasi-linear parabolic PDE systems and parabolic PDE systems with time-dependent spatial domains can be seen in [7] and [8], respectively. In order to deal with both the parameter variations and boundary uncertainties, Cheng et al. [9] presented a boundary control law for a class of parabolic PDE system by using the Volterra integral transformation technic. On the basis of Galerkin method, the nonlinear parabolic PDE system was described by finite dimensional ordinary differential equation (ODE) model, then predictive control strategy was provided [10] by employing multilayer neural network to parameterize the unknown nonlinearities. And two formulations were developed [11] to investigate the sparse solutions for the optimal control problems subject to PDEs with uncertain coefficients. More recently, adaptive feedback control design strategy was proposed [12] to stabilize a class of coefficient matrix uncertain ODE system which is cascaded with uncertain constant and coefficient matrix parabolic PDE. More researches of uncertain PDE systems can be referred to [13–16] and references therein. On the other hand, associated with the control issues, the properties of these complicated systems are usually required to be considered in a finite-time interval. Thus, the finite-time control is also worth investigating. Fruitful achievements of finite-time control have been made [17–21] for network system. Preciously, the control strategies of semilinear PDE systems in the finite-time interval were also investigated by Song et al. [22,23].

Recently, networked control systems have attracted more and more attention of scholars due to the development of computer technology and control theory [24]. The application of a multipurpose networked control system usually brings flexible architectures and reduces the budget and installation. Thus, networked control systems have been widely applied in many practical systems [25–30]. To assimilate the advantages of networked control technology, some interesting results have been proposed for the stability analysis and control synthesis of parabolic PDE systems. Such as, Demetriou [31] considered a mobile network collocated sensors and actuators for a class of diffusion PDE systems. By utilizing the distributed in space measurements, a network-based $\mathcal{H}_\infty$ filter for parabolic PDEs systems with the objective of enlarging the sampling time intervals to reduce the amount of communications while preserving a prescribed error system performance was designed in [32]. Other more recent researches can be referred in [33, 34].

However, due to the limited capability of communication channel, the signal information (e.g., system state, measurement output, and control signal) is usually required to be quantized before it is transmitted into the network. Signal quantization has been widely adopted and studied in recent literatures [35–40]. Nevertheless, accompanied with the signal quantization, quantization errors, which may yield system instability and performance degradation, and also bring difficulties for the system analysis and control, inevitably occur. Thus, the control synthesis for nonlinear systems with the effect of quantization is significant and necessary. Generally, the quantizer applied in the existing literatures can be divided into two categories: static quantizer and dynamic one. For static signal quantization, there are some interesting results have been reported. Such as, event triggered $\mathcal{H}_\infty$ control problem of parabolic systems with Markovian jumping sensor faults was studied under the effect of time delays in [41]. By employing a static logarithmic quantizer for the transmitted measurement output, distributed event-triggered networked control strategy for parabolic systems subjected to diffusion PDEs was proposed in [42]. Considering the measured output signal was quantized by a static quantizer, finite-time based fuzzy bounded control strategy for semilinear PDE systems with markov jump actuator failures [22], and $\mathcal{H}_\infty$ asynchronous control strategy for nonlinear markov jump distributed parameter systems [23], were provided, respectively. Meanwhile, the $\mathcal{H}_\infty$ performance for the systems was also widely studied in the mentioned researches [1, 4, 7, 8, 32, 41, 23].

The above researches are of abyssal significance in the field of control synthesis and attract more and more scholars to further studies. Although there are already various achievements for the controller design of parabolic PDE systems in the existing literatures, there are still few researches on the quantized state feedback control for parameter uncertain nonlinear parabolic PDE systems via finite-time interval, which is worthy of further study and application. Moreover, to the best of the authors' knowledge, it's realized that these results of the $\mathcal{H}_\infty$ controller design for parabolic PDE systems are mostly by static feedback control, the finite-time $\mathcal{H}_\infty$ control of parameter uncertain nonlinear parabolic PDE systems under dynamic feedback controller has not been studied yet, which motivates the present research. Thus, the objective of this paper is to study the finite-time $\mathcal{H}_\infty$ control problem for the parameter uncertain nonlinear parabolic PDE systems via quantized state feedback. Meanwhile, to ensure the prescribed $\mathcal{H}_\infty$ performance index of the systems, sufficient conditions of the designed controller and quantizer's adjusting parameters are developed in forms of matrix inequalities. The contributions and novelties can be summarized as follows:

1) Quantized static/dynamic state feedback control strategies for parameter uncertain nonlinear parabolic PDE systems via finite-time interval are studied. Sufficient conditions to obtain the prescribed $\mathcal{H}_\infty$ performance are provided in terms of nonlinear matrix inequalities.

2) With the consideration of the system state is transmitted via the digital communication channel, dynamic quantizer which is regarded to be more general and more advantageous than that of a static one is adopted in this paper. Moreover, the quantization errors generated by the quantizer are well treated.

3) By using some inequalities and decomposition technic, the nonlinear matrix inequalities are transformed to standard LMIs. Furthermore, the optimal $\mathcal{H}_\infty$ performances are pursued by solving optimization problems.

The rest sections of this paper are organized as follows: Section II addresses the studied parabolic PDE systems and some lemmas which are necessary for the results presented in this paper; Section III gives the static and dynamic state feedback controllers with quantization, and provides the sufficient conditions for the finite-time $\mathcal{H}_\infty$ controller design; Application to the catalytic rod in a reactor is explored to illustrate the feasibility and efficiency of the designed methods in Section IV; Section V draws some conclusions.

Notations: $\mathcal{R}^{m\times n}$, $\mathcal{R}^n$, $\mathcal{R}^+$, and $\mathcal{R}$ represent the $m\times n$ dimensional real matrices, n-dimensional real vectors, positive real numbers, and all real numbers. Matrices $\mathcal{X}^T$ and $\mathcal{X}^{-1}$ stand for the transpose and inverse of $\mathcal{X}$. $\mathbb{H}^n \triangleq \mathcal{L}_2([0,l];\mathcal{R}^n)$ denotes a well defined Hilbert space, with $\|f(\cdot)\|_2 = \sqrt{\int_0^l f^T(s)f(s)ds}$, and $f(s):[0,l]\to\mathcal{R}^n$ is a vector function which is square integrable. Matrix I with appropriate dimension means the identity matrix. A symmetric negative (positive, semipositive) definite matrix A is expressed as $A<(>,\geq)0$. The minimum and maximum eigenvalues of a square matrix B are denoted by $\lambda_{min}(B)$ and $\lambda_{max}(B)$. The symmetry part of a matrix is replaced with the symbol "$*$", e.g.,

$$\begin{bmatrix} E+[F+G+*] & H \\ * & J \end{bmatrix} \triangleq \begin{bmatrix} E+[F+G+F^T+G^T] & H \\ H^T & J \end{bmatrix}.$$

2 Problem formulation and preliminaries

The mathematical model of nonlinear parabolic PDE systems with parametric uncertainties can be expressed as:

$$\begin{cases} \dfrac{\partial x(s,t)}{\partial t} = \big(A + \Delta A\big)\dfrac{\partial^2 x(s,t)}{\partial s^2} + f(x(s,t)) \\ \quad + (B_u + \Delta B_u)u(t) + (B_\omega + \Delta B_\omega)\omega(t), \\ y(t) = \displaystyle\int_\Omega Cx(s,t)ds, \end{cases} \tag{1}$$

where $x(\cdot,t) \triangleq [x_1(\cdot,t)\ x_2(\cdot,t) \cdots x_n(\cdot,t)]^T \subseteq \mathcal{D}$ is the state vector, the given local domain which contains the equilibrium region $x(\cdot,t) = 0$ is denoted as $\mathcal{D} \triangleq \{x(\cdot,t) \in \mathbb{H}^n | \wp_{i,min} \leq x_i(\cdot,t) \leq \wp_{i,max}, i \in 1,2,...,n\}$ with known scalars $\wp_{i,min} \leq 0$ and $\wp_{i,max} \geq 0$, $s \in \Omega = [0,l] \subseteq \mathcal{R}^+$ and $t \in \mathcal{R}^+$ are the spatial position and time variable, respectively. $\omega(t)$ is the external disturbance and is assumed to satisfy $\int_\Omega \int_0^{\mathcal{T}} \omega^T(t)\omega(t)dtds \leq \bar{\omega}$ with the finite time $\mathcal{T}$ and positive scalar $\bar{\omega}$. A, B_u, B_ω are constant matrixes. $\Delta A, \Delta B_u, \Delta B_\omega$ are parametric uncertainties caused by inaccurate modeling, poor design or manufacturing, or other external physical disturbances. In this study, the parametric uncertainties are assumed to satisfy $[\Delta A\ \Delta B_u\ \Delta B_\omega] = M\$[N_1\ N_2\ N_3]$. $f(x(s,t))$ is a nonlinear function which is sufficiently smooth, $u(t)$ denotes the input, and $y(t)$ is the measured output.

The following initial condition and boundary conditions are considered for the system (1):

$$\begin{aligned} &x(s,0) = x_0(s), \\ &x(0,t) = x(l,t) = 0, \quad t > 0. \end{aligned} \tag{2}$$

An assumption of function $f(x(s,t))$, the definition of finite-time stablizable, and some lemmas are used in this paper.

Assumption 1. For all $x \in \mathcal{R}^n \neq 0$. Then an operator $f : \mathcal{R}^n \to \mathcal{R}^n$ is said to be Lipschitz constraint if there is a scalar $\kappa > 0$ satisfying:

$$\|f(x(s,t))\| \leq \kappa \|x(s,t)\|. \tag{3}$$

Definition 1. (Finite-time stabilizable) Given a positive definite matrix R and three positive constants c_1, c_2, $\mathcal{T}$, with $c_1 < c_2$, if there exists a controller $u(t)$ for system (1), $\forall t \in [0,\mathcal{T}]$ such that:

$$\int_\Omega x^T(s,0)Rx(s,0)ds < c_1 \Rightarrow \int_\Omega x^T(s,t)Rx(s,t)ds < c_2, \tag{4}$$

then the closed-loop system (1) is said to be finite-time stabilizable with respect to $(c_1, c_2, \mathcal{T})$.

Lemma 1. [43] If there is a well defined matrix $0 < \Im \in \mathcal{R}^{n\times n}$ and a vector function $z(\cdot)$ which is differentiable with $z(0) = 0$ or $z(l) = 0$, then:

$$\int_0^l \dot{z}^T(s)\Im\dot{z}(s)ds \geq \frac{\pi^2}{4l^2}\int_0^l z^T(s)\Im z(s)ds. \tag{5}$$

Lemma 2. [44] For real scalar $t \in \mathcal{R}^+$, vector function $z : [0,t] \to \mathcal{R}^n$, if a well defined matrix $\mathbb{F}$ satisfy $0 < \mathbb{F} \in \mathcal{R}^{n\times n}$, then it can be derived that:

$$\frac{1}{t}\Big[\int_0^t z(s)ds\Big]^T\mathbb{F}\Big[\int_0^t z(s)ds\Big] \leq \int_0^t z^T(s)\mathbb{F}z(s)ds. \tag{6}$$

Lemma 3.[45] If $\mathbb{N} + [\mathbb{S}\mathbb{T} + *] < 0$ holds with appropriate dimensions matrices $\mathbb{N}, \mathbb{S}$ and $\mathbb{T}$, then for a given scalar ε, there exists a matrix $\mathbb{U}$ such that:

$$\begin{bmatrix} \mathbb{N} & * \\ \varepsilon\mathbb{S}^T + \mathbb{U}\mathbb{T} & -\varepsilon[\mathbb{U} + *] \end{bmatrix} < 0. \tag{7}$$

Lemma 4.[45] For symmetric matrix $\mathbb{M}$, real matrices $\mathbb{X}, \mathbb{Y}$ with proper dimensions, and $\varDelta$ satisfies $\varDelta^T\varDelta \leq I$, then:

$$\mathbb{M} + [\mathbb{X}\varDelta\mathbb{Y} + *] < 0 \tag{8}$$

holds if and only if there is a positive scalar ϕ such that:

$$\mathbb{M} + \frac{1}{\phi}\mathbb{X}\mathbb{X}^T + \phi\mathbb{Y}^T\mathbb{Y} < 0. \tag{9}$$

The definition of quantizer $q(t)$ is employed [46]. If there is real numbers $H > 0$ and $\varDelta_q > 0$ that satisfying:

$$\begin{aligned} &\|q(t) - t\| \leq \varDelta, if\|t\| \leq H, \\ &\|q(t)\| > H - \varDelta, if\|t\| > H, \end{aligned} \tag{10}$$

where the range and error bound of the quantizer are represented by H and $\varDelta_q$, respectively. And a kind of dynamic quantizer is considered as:

$$q_\mu(t) = \mu q(\frac{t}{\mu}), \tag{11}$$

with the dynamic parameter μ.

3 Main results

In the section, static/dynamic state feedback control with quantization for the parabolic PDE system (1) will be presented. Fig. 1 shows the diagram of state feedback control strategy.

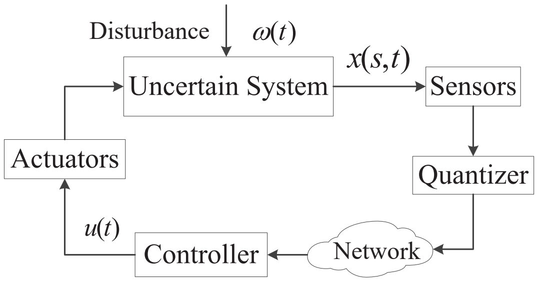

Fig. 1 Block diagram of state feedback control strategy.

3.1 Static state feedback controller with quantization

The static state feedback controller with quantization is considered as:

$$u(t) = \int_{\Omega} K q_{\mu}(x(s,t))ds, \tag{12}$$

with the undetermined gain matrix K. By the definition of quantizer described in (10) and (11), the above controller can be written as:

$$u(s,t) = \int_{\Omega} K\mu(s,t)q\Big(\frac{x(s,t)}{\mu(s,t)}\Big)ds = \int_{\Omega} K[\vartheta(s,t) + x(s,t)]ds, \tag{13}$$

where $\vartheta(s,t) = \mu(s,t)\Big[q\Big(\frac{x(s,t)}{\mu(s,t)}\Big) - \frac{x(s,t)}{\mu(s,t)}\Big]$.

By substituting the controller into (1) in the presence of parametric uncertainties $\Delta A, \Delta B_u, \Delta B_\omega$, the quantized state feedback control system is derived:

$$\begin{cases} \dfrac{\partial x(s,t)}{\partial t} = \big(A + \Delta A\big)\dfrac{\partial^2 x(s,t)}{\partial s^2} + f(x(s,t)) \\ \quad + (B_u + \Delta B_u)K \displaystyle\int_{\Omega} [\vartheta(s,t) + x(s,t)]ds \\ \quad + (B_\omega + \Delta B_\omega)\omega(t), \\ y(t) = \displaystyle\int_{\Omega} Cx(s,t)ds. \end{cases} \tag{14}$$

The following theorem provides a sufficient design condi- tion for controller (12).

Theorem 1. For the state feedback system (14), if there exist matrixes $P > 0$, U and V with proper dimensions, scalars $r_1 > 0$, $r_2 > 0$, and $\rho_1 = \rho_u r_1$ such that the following conditions are fulfilled under the prescribed scalar $\gamma > 0$, quantizer's range H and error Δ_q, given positive matrix R and positive constants δ, c_1, c_2, $\mathcal{T}$, with $c_1 < c_2$:

$$\big[P\big(A + \Delta A\big) + *\big] > 0, \tag{15}$$

$$\Pi_1 = \begin{bmatrix} [\Theta_1 + *] + \frac{C^TC}{l} & 0 & 0 & 0 & \Theta_1 \\ * & \mathbb{A}_1 & P & \mathbb{B}_1 & 0 \\ * & * & -r_2 I & 0 & 0 \\ * & * & * & -\frac{\gamma^2}{l} I & 0 \\ * & * & * & * & -r_1 I \end{bmatrix} < 0, \tag{16}$$

$$c_2 > \frac{\lambda_2 c_1 + \bar{\omega}}{\lambda_1} e^{\delta \mathcal{T}}, \tag{17}$$

where $\Theta_1 = \frac{1}{l} P(B_u + \Delta B_u)K$, $\mathbb{B}_1 = P\big(B_\omega + \Delta B_\omega\big)$, $\mathbb{A}_1 = -\frac{\pi^2}{4l^2}[P\big(A + \Delta A\big) + *] + (r_2\kappa^2 + \frac{4r_1 l^2 \Delta_q^2 \rho_u^2}{M^2})I - \delta P$, $\lambda_1 = \lambda_{min}(R^{-\frac{1}{2}} P R^{-\frac{1}{2}})$, $\lambda_2 = \lambda_{max}(R^{-\frac{1}{2}} P R^{-\frac{1}{2}})$, and the dynamic parameter $\mu(s,t)$ is adjusted on-line as:

$$\frac{\rho_u}{H} \|x(s,t)\| \leq \mu(s,t) \leq \frac{2\rho_u}{H} \|x(s,t)\|, \tag{18}$$

where $\rho_u \geq 1$. Then the quantized state feedback controller (12) subject to the adjusting rule (18), guarantees the prescribed $\mathcal{H}_\infty$ performance γ for system (14) in the sense of finite-time stable with respect to $(c_1, c_2, \mathcal{T})$.

Proof. For the state feedback system (14), select the Lyapunov function as:

$$V(t) = \int_\Omega x^T(s,t) P x(s,t) ds, \tag{19}$$

where P is a positive matrix to be determined. By the time derivative of $V(t)$,

$$\begin{aligned} \dot{V}(t) = &\Big[\int_\Omega x^T(s,t) P(A + \Delta A) d\big[\frac{\partial x(s,t)}{\partial s}\big] + *\Big] \\ &+ \Big[\int_\Omega x^T(s,t) P f(x(s,t)) ds + *\Big] \\ &+ \Big[\int_\Omega x^T(s,t) ds P(B_u + \Delta B_u) K \int_\Omega \vartheta(s,t) ds + *\Big] \\ &+ \Big[\int_\Omega x^T(s,t) ds P(B_u + \Delta B_u) K \int_\Omega x(s,t) ds + *\Big] \\ &+ \Big[\int_\Omega x^T(s,t) P\big(B_\omega + \Delta B_\omega\big)\omega(t) ds + *\Big]. \end{aligned} \tag{20}$$

Integrating by part with the consideration of the boundary conditions (2) gives:

$$\begin{aligned} &\int_\Omega x^T(s,t) P(A + \Delta A) d\big[\frac{\partial x(s,t)}{\partial s}\big] \\ &= x^T(s,t) P(A + \Delta A) \frac{\partial x(s,t)}{\partial s}\Big|_{s=0}^{s=l} \\ &- \int_\Omega \frac{\partial x^T(s,t)}{\partial s} P(A + \Delta A) \frac{\partial x(s,t)}{\partial s} ds \\ &= -\int_\Omega \frac{\partial x^T(s,t)}{\partial s} P(A + \Delta A) \frac{\partial x(s,t)}{\partial s} ds. \end{aligned} \tag{21}$$

Then, by (16) that $\big[P(A+\Delta A)+*\big]>0$, with Lemma 1, substituting (21) into (20), one can obtain:

$$\begin{aligned}\dot{V}(t) \leq &-\frac{\pi^2}{4l^2}\int_{\Omega} x^T(s,t)\big[P(A+\Delta A)+*\big]x(s,t)ds\\ &+\Big[\int_{\Omega} x^T(s,t)Pf(x(s,t))ds+*\Big]\\ &+\Big[\int_{\Omega} x^T(s,t)P(B_u+\Delta B_u)Kds\int_{\Omega}\vartheta(s,t)ds+*\Big]\\ &+\Big[\int_{\Omega} x^T(s,t)P(B_u+\Delta B_u)Kds\int_{\Omega}x(s,t)ds+*\Big]\\ &+\Big[\int_{\Omega} x^T(s,t)P(B_\omega+\Delta B_\omega)\omega(t)ds+*\Big].\end{aligned} \tag{22}$$

By the on-line adjusting rule (18), we have:

$$\Big\|\frac{x(s,t)}{\mu(s,t)}\Big\| \leq H, \tag{23}$$

according to (10),

$$\Big\|q\Big(\frac{x(s,t)}{\mu(s,t)}\Big)-\frac{x(s,t)}{\mu(s,t)}\Big\| < \Delta_q. \tag{24}$$

Consider the homogeneity property of Euclidean norm to $\vartheta(s,t)$, we arrive at:

$$\|\vartheta(s,t)\| \leq \frac{2\Delta_q}{H}\|x(s,t)\|. \tag{25}$$

Let the Assumption 1 satisfied, then the following inequalities stand:

$$\begin{aligned}&\vartheta^T(s,t)\vartheta(s,t) \leq \frac{4\Delta_q^2\rho_u^2}{H^2}x^T(s,t)x(s,t),\\ &f^T(x(s,t))f(x(s,t)) \leq \kappa^2 x^T(s,t)x(s,t).\end{aligned} \tag{26}$$

By Lemma 2, it's obvious that,

$$\int_{\Omega}\vartheta^T(s,t)ds\int_{\Omega}\vartheta(s,t)ds \leq l\int_{\Omega}\vartheta^T(s,t)\vartheta(s,t)ds. \tag{27}$$

By adding and subtracting:

$$r_1 l\int_{\Omega}\vartheta^T(s,t)ds\int_{\Omega}\vartheta(s,t)ds + r_2\int_{\Omega}f^T(x(s,t))f(x(s,t))ds$$

to the right side of (22),

$$\begin{aligned}
\dot{V}(t) \leq \int_{\Omega} \Big\{ & x^T(s,t)\mathbb{A}_1 x(s,t)ds \\
& + \int_{\Omega} x^T(s,t)ds \frac{[P(B_u + \Delta B_u)K + *]}{l} \int_{\Omega} x(s,t)ds \\
& + \Big[x^T(s,t)Pf(x(s,t)) + *\Big] \\
& + \Big[\int_{\Omega} x^T(s,t)\frac{P(B_u + \Delta B_u)K}{l}ds \int_{\Omega} \vartheta(s,t)ds + *\Big] \\
& + \Big[\int_{\Omega} x^T(s,t)P(B_\omega + \Delta B_\omega)\omega(t)ds + *\Big] \\
& - r_1 \int_{\Omega} \vartheta^T(s,t)ds \int_{\Omega} \vartheta(s,t)ds - r_2 f^T(x(s,t))f(x(s,t))\Big\} ds.
\end{aligned} \tag{28}$$

Consider the system (14) with $\mathcal{H}_\infty$ performance index γ, and choose:

$$J_{\mathcal{T}} = \int_0^{\mathcal{T}} \|y(t)\|^2 dt - \gamma^2 \int_0^{\mathcal{T}} \|w(t)\|^2 dt. \tag{29}$$

Combine (28) to (29) gives:

$$\dot{V}(t) - \delta V(t) + \|y(t)\|^2 - \gamma^2\|w(t)\|^2 = \int_0^{\mathcal{T}} \int_0^l X^T(s,t)\Pi_1 X(s,t)dsdt, \tag{30}$$

where $X(s,t) = [\int_\Omega x^T(s,t)ds \; x^T(s,t) \; f^T(x(s,t)) \; \omega^T(t) \; \int_\Omega \vartheta^T(t)ds]^T$.

By (16), we know that $\Pi_1 < 0$, which means:

$$e^{-\delta t}[\dot{V}(t) - \delta V(t)] < e^{-\delta t}[\gamma^2\|w(t)\|^2 - \|y(t)\|^2]. \tag{31}$$

Then under the initial value condition, integrating (31) from 0 to $\mathcal{T}$ arrives:

$$0 < e^{-\delta \mathcal{T}} V(\mathcal{T}) - V(0) < \int_0^{\mathcal{T}} e^{-\delta t}[\gamma^2\|w(t)\|^2 - \|y(t)\|^2]dt. \tag{32}$$

Inequality (32) further gives:

$$\|y(t)\|^2 < \gamma^2\|w(t)\|^2, \tag{33}$$

and

$$\begin{aligned}
V(\mathcal{T}) &< e^{\delta\mathcal{T}}\big[V(0) + \int_0^{\mathcal{T}} e^{-\delta t}[\gamma^2\|w(t)\|^2 dt\big] \\
&< e^{\delta\mathcal{T}}\big[\int_\Omega x^T(s,0)Px(s,0)ds + \bar{\omega}\big] \\
&= e^{\delta\mathcal{T}}\big[\int_\Omega x^T(s,0)R^{\frac{1}{2}}(R^{-\frac{1}{2}}PR^{-\frac{1}{2}})R^{\frac{1}{2}}x(s,0)ds + \bar{\omega}\big] \\
&< e^{\delta\mathcal{T}}\lambda_2 \int_\Omega x^T(s,0)Rx(s,0)ds + e^{\delta\mathcal{T}}\bar{\omega} \\
&< e^{\delta\mathcal{T}}\lambda_2 l c_1 + e^{\delta\mathcal{T}}\bar{\omega}.
\end{aligned} \tag{34}$$

On the other hand,

$$\begin{aligned} V(\mathcal{T}) &= \int_{\Omega} x^T(s,\mathcal{T})Px(s,\mathcal{T})ds \\ &= \int_{\Omega} x^T(s,\mathcal{T})R^{\frac{1}{2}}(R^{-\frac{1}{2}}PR^{-\frac{1}{2}})R^{\frac{1}{2}}x(s,\mathcal{T})ds \\ &> \lambda_1 \int_{\Omega} x^T(s,\mathcal{T})Rx(s,\mathcal{T})ds. \end{aligned} \tag{35}$$

Thus,

$$\int_{\Omega} x^T(s,t)Rx(s,t)ds < \frac{\lambda_2 c_1 + \bar{\omega}}{\lambda_1} e^{\delta\mathcal{T}}, \forall t \in [0,\mathcal{T}]. \tag{36}$$

By (17), one can derive $\int_{\Omega} x^T(s,t)Rx(s,t)ds < c_2$.

Then, according to Definition 1, the prescribed $\mathcal{H}_\infty$ performance in the finite time interval $[0,\mathcal{T}]$ for system (14) under the quantized state feedback controller (12) could be obtained. This ends the proof. □

Remark 1. To guarantee the prescribed $\mathcal{H}_\infty$ performance for system (1), the terms which reflect the effect of quantization errors and uncertainties as $\Theta_1, \mathbb{A}_1$, and ΔB_ω exist in Theorem 1. The quantization errors for the network system also appeared in [46]. Here, in Theorem 1, both the quantization errors and system uncertainties which may yield the deterioration of system performances and even system instability are considered.

Due to the uncertain terms $\Delta A, \Delta B_u, \Delta B_\omega$ and coupled terms PB_uK, the conditions presented in Theorem 1 can not be applied directly to design the quantized output controller (12). In order to cope with this problem, the following method is provided to eliminate these terms, such that the finite-time $\mathcal{H}_\infty$ control design conditions are given in terms of standard LMIs which can be solved by Matlab matrix toolbox.

Since $[\Delta A \ \ \Delta B_u \ \ \Delta B_\omega] = M\$[N_1 \ \ N_2 \ \ N_3]$, then the matrix Π_{11} can be rewritten as:

$$\Pi_1 = \Pi_2 + [\Lambda_{11}\$\Lambda_{12} + \Lambda_{21}\$\Lambda_{22} + \Lambda_{31}\$\Lambda_{32} + *], \tag{37}$$

where

$$\Pi_2 = \begin{bmatrix} \Theta_2 + \frac{C^TC}{l} & 0 & 0 & 0 & \Theta_2 \\ * & \mathbb{A}_2 & P & PB_\omega & 0 \\ * & * & -r_2I & 0 & 0 \\ * & * & * & -\frac{\gamma^2}{l}I & 0 \\ * & * & * & * & -r_1I \end{bmatrix},$$

$$\begin{aligned} &\Lambda_{11} = \begin{bmatrix} 0 & -\frac{\pi^2}{4l^2}(PM)^T & 0 & 0 & 0 \end{bmatrix}^T, \ \Lambda_{12} = \begin{bmatrix} 0 & N_1 & 0 & 0 & 0 \end{bmatrix}, \\ &\Lambda_{21} = \begin{bmatrix} \frac{1}{l}(PM)^T & 0 & 0 & 0 & 0 \end{bmatrix}^T, \ \Lambda_{22} = \begin{bmatrix} N_2K & 0 & 0 & 0 & N_2K \end{bmatrix}, \\ &\Lambda_{31} = \begin{bmatrix} 0 & (PM)^T & 0 & 0 & 0 \end{bmatrix}^T, \ \Lambda_{32} = \begin{bmatrix} 0 & 0 & 0 & N_3 & 0 \end{bmatrix}, \\ &\Theta_2 = \frac{1}{l}[PB_uK + *], \ \mathbb{A}_2 = -\frac{\pi^2}{4l^2}[PA + *] + (r_2\kappa^2 + \frac{4r_1l^2\Delta_q^2\rho_u^2}{H^2})I - \delta P. \end{aligned}$$

By Lemma 4 and Schur complement, for $\varepsilon_A, \varepsilon_u, \varepsilon_\omega > 0$, (37) is equal to:

$$\Pi_3 = \begin{bmatrix} \Pi_2 & [\Lambda_{11}\ \varepsilon_A \Lambda_{12}^T\ \Lambda_{21}\ \varepsilon_u \Lambda_{22}^T\ \Lambda_{31}\ \varepsilon_\omega \Lambda_{32}^T] \\ * & -diag\{\varepsilon_A I, \varepsilon_A I, \varepsilon_u I, \varepsilon_u I, \varepsilon_\omega I, \varepsilon_\omega I\} \end{bmatrix}. \tag{38}$$

Let $K = U^{-1}V$, then the matrix (38) can be expressed as:

$$\Pi_3 = \Pi_4 + [\Lambda_{41} \Upsilon \Lambda_{42} + *], \tag{39}$$

where
$\Pi_4 = \begin{bmatrix} \bar{\Pi}_2 & [\Lambda_{11}\ \varepsilon_A \Lambda_{12}^T\ \Lambda_{21}\ 0\ \Lambda_{31}\ \varepsilon_\omega \Lambda_{32}^T] \\ * & -diag\{\varepsilon_A I, \varepsilon_A I, \varepsilon_u I, \varepsilon_u I, \varepsilon_\omega I, \varepsilon_\omega I\} \end{bmatrix}$,
$\bar{\Pi}_2 = \begin{bmatrix} [\frac{1}{l} B_u V + *] + \frac{C^T C}{l} & 0 & 0 & 0 & \Theta \\ * & \mathbb{A}_2 & P & PB_\omega & 0 \\ * & * & -r_2 I & 0 & 0 \\ * & * & * & -\frac{\gamma^2}{l} I & 0 \\ * & * & * & * & -r_1 I \end{bmatrix}$,
$\Lambda_{41} = \left[\ \frac{1}{l}(PB_u - B_u U)^T\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ \varepsilon_u N_2^T\ 0\ 0 \right]^T$,
$\Lambda_{42} = \left[\ I\ 0\ 0\ 0\ I\ 0\ 0\ 0\ 0\ 0\ 0 \right]$.
Since $\left[P(A + \Delta A) + *\right] = [PA + *] + [PM\$N_1 + *]$, by Lemma 4, for a positive scalar ε_1, the following inequality stands:

$$\begin{bmatrix} -[PA + *] + \varepsilon_1 N_1^T N_1 & PM \\ * & -\varepsilon_1 I \end{bmatrix} < 0. \tag{40}$$

By using Lemma 3 and Schur complement to (39), if $\Pi_3 < 0$, then the following result can be derived:

Theorem 2. For the state feedback system (14), if there exist matrixes $P > 0$, U and V with proper dimensions, scalars $r_1, r_2, \varepsilon_A, \varepsilon_u, \varepsilon_\omega, \varepsilon_1 > 0$, and $\rho_{u1} = \rho_u r_1$ such that the following conditions are fulfilled under the prescribed scalar $\gamma > 0$, quantizer's range H and error Δ_q with a given scalar ε_k, given positive matrix R and positive constants δ, c_1, c_2, $\mathcal{T}$, with $c_1 < c_2$:

$$\begin{bmatrix} -[PA + *] + \varepsilon_1 N_1^T N_1 & PM \\ * & -\varepsilon_1 I \end{bmatrix} < 0, \tag{41}$$

$$\begin{bmatrix} \Omega_{11} & \Omega_{12} & \Omega_{13} & \Omega_{14} & \Omega_{15} & \Omega_{16} \\ * & \Omega_{22} & 0 & 0 & \Omega_{25} & \Omega_{26} \\ * & * & \Omega_{33} & 0 & 0 & 0 \\ * & * & * & \Omega_{44} & 0 & \Omega_{46} \\ * & * & * & * & \Omega_{55} & 0 \\ * & * & * & * & * & \Omega_{66} \end{bmatrix} < 0, \tag{42}$$

$$c_2 > \frac{\lambda_2 c_1 + \bar{\omega}}{\lambda_1} e^{\delta \mathcal{T}}, \tag{43}$$

where
$\Omega_{11} = diag\{\frac{1}{l}[B_u V + *] + \frac{C^T C}{l}, r_2 \kappa^2 I - \frac{\pi^2}{4l^2}[PA + *] - \delta P\}$,

$\Omega_{12} = \begin{bmatrix} 0 & 0 & \frac{1}{l}B_uV \\ P & PB_\omega(s) & 0 \end{bmatrix}^T$, $\Omega_{13} = \begin{bmatrix} 0 & 0 \\ -\frac{\pi^2}{4l^2}PM & \varepsilon_A N_1 \end{bmatrix}$,
$\Omega_{14} = \begin{bmatrix} \frac{1}{l}PM & 0 \\ 0 & 0 \end{bmatrix}$, $\Omega_{15} = \begin{bmatrix} 0 & 0 \\ PM & 0 \end{bmatrix}$,
$\Omega_{16} = diag\{\frac{\varepsilon_k}{l}[PB_u - B_uU], \frac{\rho_{u1}\Delta_q}{H}\}$, $\Omega_{22} = -diag\{r_2 I, \frac{\gamma^2}{l}I, r_1 I\}$,
$\Omega_{25} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \varepsilon_\omega N_3^T & 0 \end{bmatrix}^T$, $\Omega_{26} = \begin{bmatrix} 0 & 0 & V \\ 0 & 0 & 0 \end{bmatrix}^T$,
$\Omega_{33} = -diag\{\varepsilon_A I, \varepsilon_A I\}$, $\Omega_{44} = -diag\{\varepsilon_u I, \varepsilon_u I\}$,
$\Omega_{46} = \begin{bmatrix} 0 & 0 \\ \varepsilon_k \varepsilon_\omega N_2^T & 0 \end{bmatrix}$, $\Omega_{55} = -diag\{\varepsilon_\omega I, \varepsilon_\omega I\}$,
$\Omega_{66} = -diag\{\varepsilon_k[U + *], \frac{r_1}{4l^2}I\}$, and the dynamic parameter $\mu(s,t)$ is adjusted on-line as (18). Then the prescribed $\mathcal{H}_\infty$ performance for system (14) under the state feedback controller (12) with quantization could be obtained in the finite-time internal $[0, \mathcal{T}]$. Moreover, the control gain is given by $K = U^{-1}V$.

3.2 Dynamic state feedback controller with quantization

The dynamic state feedback controller with quantization is considered as:

$$\begin{cases} \dfrac{\partial \phi(s,t)}{\partial t} = A_d\phi(s,t) + B_d q_\mu(x(s,t)), \\ u(t) = \displaystyle\int_\Omega [C_d\phi(s,t) + D_d q_\mu(x(s,t))]ds, \end{cases} \tag{44}$$

with the undetermined gain matrices A_d, B_d, C_d and D_d. By the definition of quantizer described in (10) and (11), the above controller can be written as:

$$\begin{cases} \dfrac{\partial \phi(s,t)}{\partial t} = A_d\phi(s,t) + B_d[(x(s,t)) + \vartheta(s,t)], \\ u(t) = \displaystyle\int_\Omega \big\{C_d\phi(s,t) + D_d[x(s,t) + \vartheta(s,t)]\big\}ds. \end{cases} \tag{45}$$

By substituting the controller into (1), the quantized state feedback control system is derived:

$$\begin{cases} \dfrac{\partial x(s,t)}{\partial t} = \big(A + \Delta A\big)\dfrac{\partial^2 x(s,t)}{\partial s^2} + f(x(s,t)) \\ \quad + (B_u + \Delta B_u)\displaystyle\int_\Omega C_d\phi(s,t)ds + (B_\omega + \Delta B_\omega)\omega(t) \\ \quad + (B_u + \Delta B_u)\displaystyle\int_\Omega D_d[x(s,t) + \vartheta(s,t)]ds, \\ y(t) = \displaystyle\int_\Omega Cx(s,t)ds. \end{cases} \tag{46}$$

Assume that the initial state of the dynamic state feedback controller (44) is zero, i.e., $\phi(s,0) = 0$, then the following result provides the $\mathcal{H}_\infty$ control design for the closed-loop system (46) in the finite time interval $[0, \mathcal{T}]$.

Theorem 3. For the dynamic state feedback system (46), if there exist matrixes $P_1, P_2 > 0$, A_d, B_d, C_d and D_d with proper dimensions, scalars $r_1, r_2, r_3, r_4 > 0$, such that the following conditions are fulfilled under the prescribed scalar $\gamma > 0$, quantizer's range H and error Δ_q, given positive matrix R, and positive constants δ, c_1, c_2, $\mathcal{T}$, with $c_1 < c_2$:

$$\big[P_1\big(A + \Delta A\big) + *\big] > 0, \tag{47}$$

$$\Psi_{11} = \begin{bmatrix} \mathbb{C}_1 & 0 & 0 & 0 & \mathbb{D}_1 & \mathbb{D}_1 & 0 & 0 \\ * & \mathbb{E}_1 & P_1 & [P_2 B_d]^T & 0 & 0 & \mathcal{F}_1 & 0 \\ * & * & -r_2 I & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \mathbb{W}_1 & 0 & 0 & 0 & P_2 B_d \\ * & * & * & * & -r_1 I & 0 & 0 & 0 \\ * & * & * & * & * & -r_3 I & 0 & 0 \\ * & * & * & * & * & * & -\frac{\gamma^2}{l} I & 0 \\ * & * & * & * & * & * & * & -r_4 I \end{bmatrix} < 0, \tag{48}$$

$$c_2 > \frac{\bar{\lambda}_2 c_1 + \bar{\omega}}{\bar{\lambda}_1} e^{\delta \mathcal{T}}, \tag{49}$$

where $\mathbb{C}_1 = \frac{[P_1(B_u + \Delta B_u)D_d + *] + C^T C}{l}$, $\mathbb{D}_1 = \frac{P_1(B_u + \Delta B_u)D_d}{l}$, $\mathbb{E}_1 = -\frac{\pi^2}{4l^2}[P_1(A + \Delta A) + *] + (r_1 l^2 + r_4)\frac{4\Delta_q^2 \rho_u^2}{H^2} I + r_2 \kappa^2 I - \delta P_1$, $\mathcal{F}_1 = P_1(B_\omega + \Delta B_\omega)$, $\mathbb{W}_1 = [P_2 A_d + *] + r_3 l^2 I - \delta P_2$, $\bar{\lambda}_1 = \lambda_{min}(R^{-\frac{1}{2}} P_1 R^{-\frac{1}{2}})$, $\bar{\lambda}_2 = \lambda_{max}(R^{-\frac{1}{2}} P_1 R^{-\frac{1}{2}})$, and the dynamic parameter $\mu(s,t)$ is adjusted on-line as (18). Then under the dynamic state feedback controller (44), closed-loop system (46) is finite-time stable with respect to $(c_1, c_2, \mathcal{T})$, and satisfies the $\mathcal{H}_\infty$ performance with index γ.

Proof. Select the following Lyapunov functional for the state feedback system (46):

$$\mathcal{V}(t) = \int_\Omega x^T(s,t) P_1 x(s,t) ds + \int_\Omega \phi^T(s,t) P_2 \phi(s,t) ds, \tag{50}$$

where $P_1, P_2 > 0$ are undetermined constant matrices. By (47) that $\big[P_1(A + \Delta A) + *\big] > 0$, the following relationship can be found according to (20) and

(21):

$$
\begin{aligned}
\dot{\mathcal{V}}(t) \leq & -\frac{\pi^2}{4l^2}\int_{\Omega} x^T(s,t)\big[P_1(A+\Delta A)+*\big]x(s,t)ds \\
& + \Big[\int_{\Omega} x^T(s,t)P_1 f(x(s,t))ds + *\Big] \\
& + \Big[\int_{\Omega} x^T(s,t)P_1(B_u+\Delta B_u)ds\int_{\Omega} C_d\phi(s,t)ds + *\Big] \\
& + \Big[\int_{\Omega} x^T(s,t)P_1(B_u+\Delta B_u)ds\int_{\Omega} D_d x(s,t)ds + *\Big] \\
& + \Big[\int_{\Omega} x^T(s,t)P_1(B_u+\Delta B_u)ds\int_{\Omega} D_d\vartheta(s,t)ds + *\Big] \\
& + \Big[\int_{\Omega} x^T(s,t)P_1(B_\omega+\Delta B_\omega)\omega(t)ds + *\Big] \\
& + \int_{\Omega} \phi^T(s,t)[P_2A_d + *]\phi(s,t)ds \\
& + \Big[\int_{\Omega} \phi^T(s,t)P_2B_d\vartheta(s,t)ds + \int_{\Omega} \phi^T(s,t)P_2B_d x(s,t)ds + *\Big].
\end{aligned} \tag{51}
$$

Then with the properties of (26), by adding and subtracting:

$$
\begin{aligned}
& r_1 l\int_{\Omega} \vartheta^T(s,t)\vartheta(s,t)ds + r_2\int_{\Omega} f^T(x(s,t))f(x(s,t))ds \\
& \quad + r_3 l\int_{\Omega} \phi^T(s,t)ds\int_{\Omega} \phi(s,t)ds + r_4\int_{\Omega} \vartheta^T(s,t)\vartheta(s,t)ds
\end{aligned} \tag{52}
$$

to the right side of (51), gives:

$$
\begin{aligned}
\dot{\mathcal{V}}(t) \leq \int_{\Omega} \Big\{ & x^T(s,t)\mathbb{E}_1 x(s,t)ds + \Big[x^T(s,t)P_1 f(x(s,t)) + *\Big] \\
& + \Big[\int_{\Omega} x^T(s,t)P_1 f(x(s,t))ds + *\Big] \\
& + \Big[\int_{\Omega} x^T(s,t)dsP_1(B_u + \Delta B_u)C_d \int_{\Omega} \phi(s,t)ds + *\Big] \\
& + \Big[\int_{\Omega} x^T(s,t)dsP_1(B_u + \Delta B_u)D_d \int_{\Omega} x(s,t)ds + *\Big] \\
& + \Big[\int_{\Omega} x^T(s,t)dsP_1(B_u + \Delta B_u)D_d \int_{\Omega} \vartheta(s,t)ds + *\Big] \\
& + \Big[\int_{\Omega} x^T(s,t)P_1(B_\omega + \Delta B_\omega)\omega(t)ds + *\Big] \\
& + \int_{\Omega} \phi^T(s,t)[P_2 A_d + *]\phi(s,t)ds + \Big[\int_{\Omega} \phi^T(s,t)P_2 B_d \vartheta(s,t)ds + *\Big] \\
& + \Big[\int_{\Omega} \phi^T(s,t)P_2 B_d x(s,t)ds + *\Big] - r_1 l \int_{\Omega} \vartheta^T(s,t)\vartheta(s,t)ds \\
& - r_2 \int_{\Omega} f^T(x(s,t))f(x(s,t))ds - r_3 l \int_{\Omega} \phi^T(s,t)ds \int_{\Omega} \phi(s,t)ds \\
& - r_4 \int_{\Omega} \vartheta^T(s,t)\vartheta(s,t)ds \Big\} ds.
\end{aligned} \tag{53}
$$

Consider the system (46) with $\mathcal{H}_\infty$ performance index γ as (29), Combine (28) to (29) gives:

$$\dot{\mathcal{V}}(t) - \delta\mathcal{V}(t) + \|y(t)\|^2 - \gamma^2\|w(t)\|^2 = \int_0^{\mathcal{T}} \int_0^l \bar{X}^T(s,t)\Psi_{11}\bar{X}(s,t)dsdt, \tag{54}$$

where $\bar{X}(s,t) = [\int_{\Omega} x^T(s,t)ds \ x^T(s,t) \ f^T(x(s,t)) \ \phi^T(s,t) \ \int_{\Omega} \vartheta^T(s,t)ds \ \int_{\Omega} \phi^T(s,t)ds \ \omega(t) \ \vartheta(s,t)]^T$.

By (48) that $\Psi_{11} < 0$, which means:

$$e^{-\delta t}[\dot{\mathcal{V}}(t) - \delta\mathcal{V}(t)] < e^{-\delta t}[\gamma^2\|w(t)\|^2 - \|y(t)\|^2]. \tag{55}$$

Then under the initial value condition, integrating (55) from 0 to $\mathcal{T}$ arrives:

$$0 < e^{-\delta\mathcal{T}}\mathcal{V}(\mathcal{T}) - \mathcal{V}(0) < \int_0^{\mathcal{T}} e^{-\delta t}[\gamma^2\|w(t)\|^2 - \|y(t)\|^2]dt. \tag{56}$$

Since the initial state of the controller (44) is zero, the inequality (56) further means:

$$\|y(t)\|^2 < \gamma^2\|w(t)\|^2, \tag{57}$$

and

$$\begin{aligned}\mathcal{V}(\mathcal{T}) &< e^{\delta\mathcal{T}}\big[\mathcal{V}(0) + \int_0^{\mathcal{T}} e^{-\delta t}[\gamma^2\|w(t)\|^2 dt\big] \\ &< e^{\delta\mathcal{T}}\big[\int_\Omega x^T(s,0)Px(s,0)ds + \int_\Omega \phi^T(s,0)P\phi(s,0)ds + \bar{\omega}\big] \\ &< e^{\delta\mathcal{T}}\big[\int_\Omega x^T(s,0)R^{\frac{1}{2}}(R^{-\frac{1}{2}}PR^{-\frac{1}{2}})R^{\frac{1}{2}}x(s,0)ds + \bar{\omega}\big] \\ &< e^{\delta\mathcal{T}}\bar{\lambda}_2\int_\Omega x^T(s,0)Rx(s,0)ds + e^{\delta\mathcal{T}}\bar{\omega} \\ &< e^{\delta\mathcal{T}}\bar{\lambda}_2 lc_1 + e^{\delta\mathcal{T}}\bar{\omega}.\end{aligned} \tag{58}$$

On the other hand,

$$\begin{aligned}\mathcal{V}(\mathcal{T}) &= \int_\Omega x^T(s,\mathcal{T})P_1x(s,\mathcal{T})ds + \int_\Omega \phi^T(s,\mathcal{T})P_2\phi(s,\mathcal{T})ds \\ &> \int_\Omega x^T(s,\mathcal{T})R^{\frac{1}{2}}(R^{-\frac{1}{2}}P_1R^{-\frac{1}{2}})R^{\frac{1}{2}}x(s,\mathcal{T})ds \\ &> \bar{\lambda}_1\int_\Omega x^T(s,\mathcal{T})Rx(s,\mathcal{T})ds.\end{aligned} \tag{59}$$

Thus,

$$\int_\Omega x^T(s,t)Rx(s,t)ds < \frac{\bar{\lambda}_2 c_1 + \bar{\omega}}{\bar{\lambda}_1}e^{\delta\mathcal{T}}, \forall t \in [0,\mathcal{T}]. \tag{60}$$

By (49), one can derive $\int_\Omega x^T(s,t)Rx(s,t)ds < c_2$.

Then, according to Definition 1, the system (46) under the dynamic state feedback controller (44) with quantization is finite-time stable in the interval $[0,\mathcal{T}]$, and the prescribed $\mathcal{H}_\infty$ performance could be satisfied. This ends the proof. □

Since $[\Delta A\ \Delta B_u\ \Delta B_\omega] = M\$[N_1\ N_2\ N_3]$, similar to the proof of Theorem 2, the following theorem provides conditions in term of standard LMIs.

Theorem 4. If there exist matrixes $P_1, P_2 > 0$, P_{2A}, P_{2B}, A_d, B_d, U_1, U_2, V_1 and V_2 with proper dimensions, scalars $\varepsilon_\Delta, \varepsilon_1, r_1, r_2, r_3, r_4 > 0$, and $\rho_{ur} = \rho_u(r_1l^2 + r_4)$ such that the following conditions are fulfilled under the prescribed scalar $\gamma > 0$, quantizer's range H and error Δ_q with a given positive matrix R and positive constants ε_k, δ, c_1, c_2, $\mathcal{T}$, with $c_1 < c_2$:

$$\begin{bmatrix} -[P_1A + *] + \varepsilon_1 N_1^T N_1 & P_1M \\ * & -\varepsilon_1 I \end{bmatrix} < 0, \tag{61}$$

$$\begin{bmatrix} \bar{\Omega}_{11} & \bar{\Omega}_{12} & \bar{\Omega}_{13} & \bar{\Omega}_{14} & \bar{\Omega}_{15} & \bar{\Omega}_{16} & \bar{\Omega}_{17} \\ * & \bar{\Omega}_{22} & 0 & \bar{\Omega}_{24} & 0 & 0 & 0 \\ * & * & \bar{\Omega}_{33} & 0 & 0 & 0 & \bar{\Omega}_{37} \\ * & * & * & \bar{\Omega}_{44} & 0 & \bar{\Omega}_{46} & 0 \\ * & * & * & * & \bar{\Omega}_{55} & 0 & 0 \\ * & * & * & * & * & \bar{\Omega}_{66} & \bar{\Omega}_{67} \\ * & * & * & * & * & * & \bar{\Omega}_{77} \end{bmatrix} < 0, \tag{62}$$

$$c_2 > \frac{\bar{\lambda}_2 c_1 + \bar{\omega}}{\bar{\lambda}_1} e^{\delta \mathcal{T}}, \tag{63}$$

where

$\bar{\Omega}_{11} = diag\{\frac{[B_u V_2 + *] + C^T C}{l}, -\frac{\pi^2}{4l^2}[P_1 A + *] + r_2 \kappa^2 I - \delta P_1\}$,

$\bar{\Omega}_{12} = \begin{bmatrix} 0 & 0 \\ P_1 & P_{2B}^T \end{bmatrix}$, $\bar{\Omega}_{13} = \begin{bmatrix} \frac{B_u V_2}{l} & \frac{B_u V_1}{l} \\ 0 & 0 \end{bmatrix}$, $\bar{\Omega}_{14} = \begin{bmatrix} 0 & 0 \\ P_1 B_\omega & 0 \end{bmatrix}$,

$\bar{\Omega}_{15} = diag\{\frac{P_1 M}{l}, P_1 M\}$, $\bar{\Omega}_{16} = \begin{bmatrix} 0 & 0 \\ 0 & -\frac{\pi^2}{4l^2} N_1^T \end{bmatrix}$,

$\bar{\Omega}_{17} = \begin{bmatrix} \frac{\varepsilon_k}{l}[P_1 B_u - B_u U_2] + V_2^T & \frac{\varepsilon_k [P_1 B_u - B_u U_1]}{l} & \frac{\Delta_q \rho_{ur}}{H} \\ 0 & 0 & 0 \end{bmatrix}$,

$\bar{\Omega}_{22} = -diag\{r_2 I, [P_{2A} + *] + r_3 l^2 I - \delta P_2\}$, $\bar{\Omega}_{24} = -diag\{0, P_{2B}\}$,

$\bar{\Omega}_{33} = -diag\{r_1 I, r_3 I\}$, $\bar{\Omega}_{37} = \begin{bmatrix} V_2^T & 0 & 0 \\ 0 & V_1^T & 0 \end{bmatrix}$, $\bar{\Omega}_{44} = -diag\{\frac{\gamma^2}{l} I, r_4 I\}$,

$\bar{\Omega}_{46} = \begin{bmatrix} 0 & N_3^T \\ 0 & 0 \end{bmatrix}$, $\bar{\Omega}_{55} = -diag\{\varepsilon_\Delta I, \varepsilon_\Delta I\}$, $\bar{\Omega}_{66} = -diag\{\varepsilon_\Delta I, \varepsilon_\Delta I\}$,

$\bar{\Omega}_{67} = \begin{bmatrix} \varepsilon_k \varepsilon_\Delta N_2 & \varepsilon_k \varepsilon_\Delta N_2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$, $\bar{\Omega}_{77} = -diag\{\varepsilon_k [U_2 + *], \varepsilon_k [U_1 + *], \frac{r_1 l^2 + r_4}{4} I\}$,

and the dynamic parameter $\mu(s,t)$ is adjusted on-line as (18), then the closed-loop system (46) is finite-time stable, and the prescribed $\mathcal{H}_\infty$ performance of (29) with index γ could be obtained under the dynamic state feedback controller (44). Moreover, the control gain matrices are given by $A_d = P_{2A} P_2^{-1}$, $B_d = P_{2B} P_2^{-1}$, $C_d = U_1^{-1} V_1$, $D_d = U_2^{-1} V_2$.

Remark 2. The conditions in Theorem 2 are standard LMIs which can be solved by Matlab LMI toolbox. Thus, the controllers can be frequently derived by solving (41)-(42) in Theorem 2 and (61)-(62) in Theorem 4. To obtain the best robust control effect, the following optimization problem is employed:

$$\begin{aligned} &min \quad \gamma^2, \\ &subject\ to\ LMIs\ in\ Thorem\ 2\ (or\ Theorem\ 4). \end{aligned} \tag{64}$$

Remark 3. When the candidate parameters ε_1, ε_A, ε_u, ε_ω, ε_k and r_2 in theorems are associated with the spatial variable s, then the results also stand and the conditions could be relaxed.

Remark 4. If the Dirichlet boundary conditions (2) are replaced with the mixed boundary conditions $x(s,t)|_{s=0} = \frac{\partial x(s,t)}{\partial s}|_{s=l} = 0$, Theorem 1-4 can also be derived. Moreover, when the system structure B_ω and uncertain parameter ΔB_ω are related to spatial variable, all the results proposed in this paper also stand with parameters $B_\omega(s)$ and $N_3(s)$.

Remark 5. In this paper, the dynamic parameter $\mu(s,t)$ which is transmitted to the system are adjusting by the rule presented in [46].

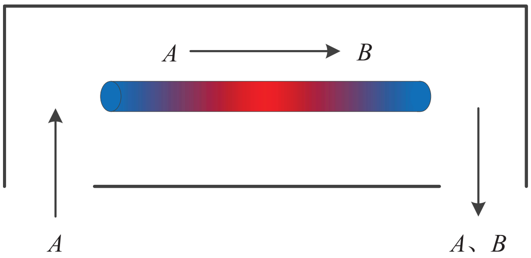

Fig. 2 Catalytic rod in a reactor.

The adjusting rule:

$$\mu(s,t)=\begin{cases} floor\big(\frac{2\rho_u}{H}|x(s,t)|\times 10^{\ell}\big)\times 10^{-\ell}, 0\le \frac{2\rho_u}{H}|x(s,t)|<\frac{1}{2},\\ 1,\ \ \frac{1}{2}\le\frac{\rho_u}{H}|x(s,t)|<1,\\ floor\big(\frac{\rho_u}{H}|x(s,t)|\big),\ \ 1<\frac{\rho_u}{H}|x(s,t)|, \end{cases} \tag{65}$$

where $\ell = min\{\ell \in \mathcal{N}^+ | \frac{2\rho_u}{H}|x(s,t)|\times 10^{\ell} > 1\}$ and the function $floor(\zeta)$ denotes the largest integer of ζ, but less than ζ.

4 Application to catalytic rod in a reactor

Consider a furnace which is filled with specie A and a catalytic reaction of the form $A\to B$ takes place on a long thin rod shown in Fig. 2. This rod acts as a cooling medium since the reaction in the furnace is exothermic. The process model of temperature on the rod which is also studied in [47] can be illustrated by:

$$\begin{cases} \frac{\partial T(s,t)}{\partial t}=(A+\Delta A)\frac{\partial^2 T(s,t)}{\partial s^2}+\beta_1(e^{-\frac{\alpha}{1+T(s,t)}}-e^{-\alpha})\\ \qquad -\beta_2 T(s,t)+(B_u+\Delta B_u)u(t)+(B_\omega+\Delta B_\omega)\omega(t),\\ y(t)=\int_0^l CT(s,t)ds, \end{cases} \tag{66}$$

where $T(s,t)\in\mathcal{R}$ is the dimensionless temperature of the catalytic rod, β_1 is the heat in the reaction, α represents the activation energy, β_2 denotes the transfer coefficient of heat, $y(t)\in\mathcal{R}$ denotes the measured output, $u(t)$ and $\omega(t)$ denote the control input and the related disturbance. B_u is the distribution matrix of control actuators, B_ω is a known constant matrix, and C is the matrix of the point sensor. $l=2$ is the length of the catalytic rod.

For simulation purposes, the initial temperature and the surface temperature of the battery are assumed to be relative level, which means the following conditions are meeted:

$$T(s,t)|_{s=0} = T(s,t)|_{s=l} = 0, \quad t > 0,$$
$$T_0(s) = 0.2\sin(\pi s). \tag{67}$$

Here, the coefficients are given by:

$$A = 1.0, B_u = 22.7975, B_\omega = 16.3175, \beta_1 = 6.5, \beta_2 = 1.3, \alpha = 2.0. \tag{68}$$

The parametric uncertainties $\Delta A_1, \Delta B_u$ and ΔB_ω are assumed to be:

$$[\Delta A \; \Delta B_u \; \Delta B_\omega] = M\$[N_1 \; N_2 \; N_3], \tag{69}$$

where $M = 0.5406$, $\$ = 0.5$, $N_1 = 0.0101$, $N_2 = -0.1569$, $N_3 = 0.2778$. Then the state of open-loop system (67) can be obtained and shown in Fig. 3 with the disturbance $\omega(t) = 0.3e^{-0.2t} + e^{-0.2t}\cos(2\pi t) + e^{-0.2t}\sin(3\pi t)$ and $\bar{\omega} = 7.5$. By the simulation results, the Assumption 1 can be fulfilled with $\kappa = 0.55$. Set the range $H = 100$ and error $\Delta = 0.1$ of the quantizer. And give the matrix and scalars as $R = I$, $\varepsilon_k = 2.5$, $c_1 = 0.4$, $c_2 = 150$, $\mathcal{T} = 20$, and $\delta = 0.001$:

1) By solving the static control design conditions presented in Theorem 2, the variable of optimal $\mathcal{H}_\infty$ performance $\gamma = 0.6006$, the gain of the controller $K = -8.5980$, the adjusting parameter $\rho_u = 1.0001$, and the other used parameters $r_1 = 0.0276$, $r_2 = 1.4692 \times 10^{-4}$, $\varepsilon_A = 0.0027$, $\varepsilon_u = 0.0021$, $\varepsilon_\omega = 0.0213$, $\varepsilon_1 = 0.7922$ can be found by solving the optimization problem (64). The state profile of closed-loop system under static state feedback control (12) is depicted in Fig. 4.

2) By solving the dynamic control design conditions presented in Theorem 4, the variable of optimal $\mathcal{H}_\infty$ performance $\gamma = 7.7378$, the gain parameters of the controller $A_d = -8.5980$, $B_d = 7.3277 \times 10^{-11}$, $C_d = -3.2142 \times 10^4$, $D_d = -2.4922$, the adjusting parameter $\rho_u = 1.0982$, and the other used parameters $r_1 = 0.2371$, $r_2 = 0.0139$, $r_3 = 8.2321 \times 10^7$, $r_4 = 0.3647$, $\varepsilon_1 = 75.1742$, $\varepsilon_\Delta = 0.1245$ can be found by solving the optimization problem (64). Fig. 5 shows the state profile of closed-loop system under dynamic state feedback control (44). The difference $e(s,t)$ between the state of the system under static and dynamic state feedback is depicted in Fig. 6.

The simulations of static state feedback controller and dynamic state feedback controller are described in Fig. 7 with blue dashed line and green dashed line respectively, and the red line denotes the difference between the static and dynamic controller. The evolution of the norm $\|x(s,t)\|_2$ under the static controller and the dynamic controller are depicted in Fig. 8.

Through simulation results presented above, it is obviously that the design methods for the nonlinear parabolic PDE systems with parametric uncertainties can guarantee the prescribed $\mathcal{H}_\infty$ performance in the finite-time interval $t \in [0, 20]$.

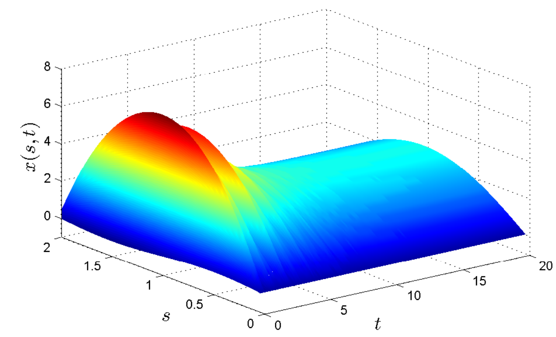

Fig. 3 Open-loop state profile of system (67).

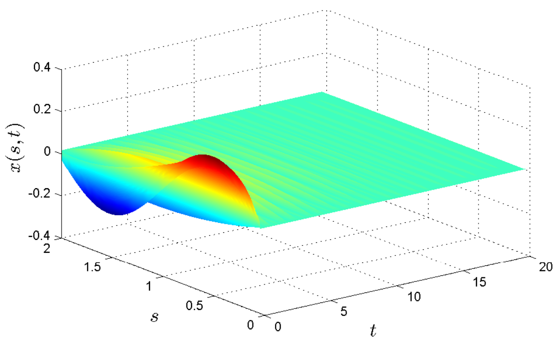

Fig. 4 Close-loop state profile of system (67) with controller (12).

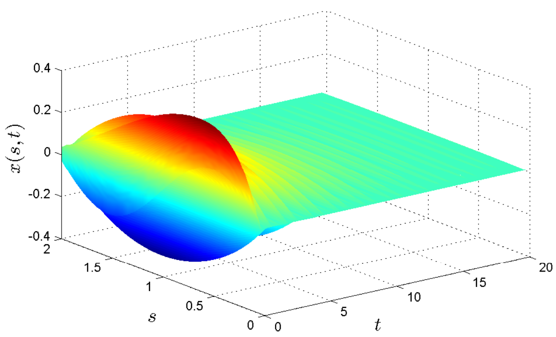

Fig. 5 Close-loop state profile of system (67) with controller (44).

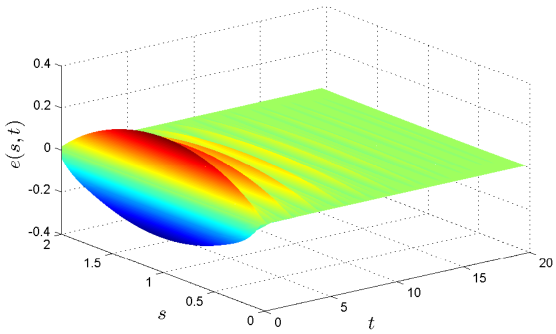

Fig. 6 Difference between the static and dynamic lose-loop state profile of system (67).

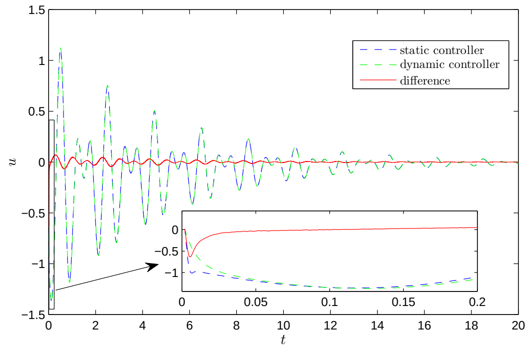

Fig. 7 Quantized state feedback controller (12) and (44).

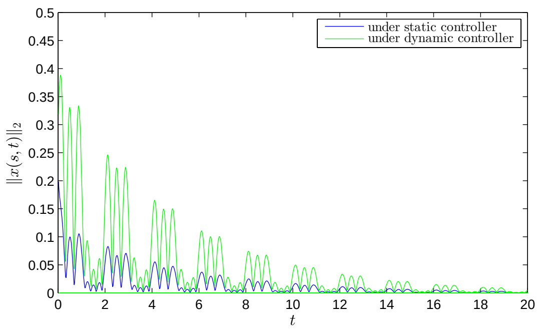

Fig. 8 $\|x(s,t)_2\|$ under the the controller (12) and (44).

Comparative Explanations: The developed quantized state feedback control design strategies in this paper provide efficient methods for the finite-time $\mathcal{H}_\infty$ control of the nonlinear parabolic PDE system with parametric uncertainties. Compared with the existing results, some innovations and advantages of the presented feedback control strategies could be identified in the following aspects:

1) Different from [7–10, 13–15], the system state information is considered to be transmitted via the digital communication channel which is used frequently in the sensor side. Then static state feedback controller is studied for the parameter uncertain nonlinear parabolic PDE system via finite-time interval. And quantized dynamic state feedback controller is also investigated, which has not been studied yet in the existing literatures. In addition, by the presented simulation results, it can be observed that the static state feedback controller (12) has a better control effect and a lower optimal index of $\mathcal{H}_\infty$ performance compared with the dynamic one (44).

2) In comparison with [22, 23, 41, 42], a kind of dynamic quantizer which is regarded to be more general and more advantageous than that of a static one for the control input signal is adopted in this paper.

3) Compared with [9, 13], new finite-time $\mathcal{H}_\infty$ control design conditions for nonlinear parabolic PDE system under the static and dynamic state feedback controllers are provided in terms of LMIs. And the $\mathcal{H}_\infty$ performances is optimized by solving the optimization problems subject to the LMIs.

5 Conclusion

This paper has studied the finite-time $\mathcal{H}_\infty$ control problem of parametric uncertain nonlinear parabolic PDE systems via static/dynamic state feedback. Considering that the system state information is transmitted through digital communication channel, a dynamic quantizer is adopted to deal with the limited capability of the communication channel. Moreover, the quantization errors generated by the quantizer are well treated. And finite-time stability conditions of the existence of the designed controllers and adjusting parameters are presented in terms of nonlinear matrix inequalities by constructing appropriate Lyapunov functionals. Then standard LMIs are derived by using some inequalities and decomposition technic. In addition, the $\mathcal{H}_\infty$ performances can be optimized by solving optimization problems subject to these standard LMIs. At last, an exploration to a catalytic rod in a reactor is presented to verify the effectiveness of the proposed strategies. In the following research, under the effect of quantization and actuator faults, the control problem for nonlinear parabolic PDE systems with space-varying parametric uncertainties, and the stabilization problem for nonlinear PDE systems with time-varying delays will be investigated.

Conflict of interest

The authors declare that they have no conflict of interest.

Data availability

The datasets generated during and/or analysed during the current study are available from the corresponding author on reasonable request.

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