The Experimental Study on Interaction of Vibration and Dynamic Force in Precision Milling Process

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ABSTRACT

A good understanding of the dynamic characteristic in milling of aerospace aluminum, especially the coupling vibration caused by the interaction of the manufacturing process and the machine tool, helps promote the machining precision and surface quality of aerospace structural components. This paper is devoted to proposing the interaction theory of the vibration and dynamic force, which is verified in the milling of Al 7075-T651 by consideration both the machine tool load and machining process dynamic load. First, through detailed analysis of the interaction effect of vibration and the dynamic force, the dynamic milling process is simplified to theoretically model the dynamic interaction in the precision manufacturing process under non-chatter condition. Then, the dynamic process force, which is the key source of the interaction, is modeled and obtained based on wavelet packet transform preprocess; the Frequency Response Function (FRF) of machine tool is regarded as the interaction link between the dynamic force load and the vibration response; the machine tool non-cutting vibration is transformed as a special dynamic load superposed on the response. Finally, the interaction vibration is calculated applying interaction effect model, the predicated results obtained in interaction effect approach match well with the vibration signal directly obtained in the test.

Keywords

Milling; Vibration interaction; Dynamic force; PMI

1 INTRODUCTION

Precision milling process has been widely applied in the manufacturing of Al-based structural parts for aerospace applications, such as manned space stations, deep space explorers and high-performance satellites. However, the dimension accuracy and surface performance of the parts cannot be precisely controlled in many cases, and thus, the stable operation of aerospace equipment cannot be guaranteed. One of the most critical problems limiting the performance of precision milling parts is the coupled vibration caused by the dynamic load interaction of the manufacturing process and the machine tool. The complex dynamic characteristics in the milling process results in an in-depth understanding of the synthetically interaction of the vibrations induced by the manufacturing process and the machine tool. Previous studies in the literature have generally focused on the experimental-theoretical investigation of machine tool vibration and milling process dynamics without engaging in a thorough analysis of the Process and Machine Interaction (PMI) [1-2].

In some cases, most of the research works focus on the spindle and tool vibration. The vibration in precision milling would result in the periodic variation of the relative displacement between tool and workpiece and affect the surface generation in process [3]. Filiz et al. [4] developed analytical models for the dynamics of micro-scale cutting-tools and experimental models for an ultra-high-speed spindle. Zhang et al. [5-6] developed a specialized model for an aerostatic bearing spindle under impulsive excitation from intermittent cutting forces of ultra-precision raster milling to analyze the effects of the particular spindle vibration on surface generation. They found that the phase shift is a key factor influencing the surface patterns. To further determine the influence of spindle vibration on the machining surface, Ma et al. [7] investigated the effect of the geometric feature of a curved surface and the cutting direction along the tool path on the variation of force and vibration in high-speed milling. Jiang et al. [8] built a three-dimensional model of machined surface topography in simulation to analyze the influence of tool wear and vibration on the spatial trajectory of the knife tip. They concluded that axis amplitude is a key factor affecting surface residual height. These works show that the vibration load of machine tools has a significant impact on precision machining, but the research mainly focuses on the vibration of the machine tool.

As in the milling process, the vibrations include three parts: free vibration, forced vibration, and self-excited vibration (chatter). Generally, chatter occurs when the excitation frequency in cutting is equal or close to one of the natural frequencies of the machine tool, referred to as instable cutting [9]. Chatter behavior depends upon a number of different aspects including spindle speeds, material properties, tool geometry, and even the location of tool respect to the rest of machine [10]. However, when the depth of the cut is small in precision milling, the influence of chatter vibration can possibly be disregarded [11]. Therefore, the forced vibration of the cutting process is the main factor in stability precision milling. In some studies, the process force is limited in the static field to investigate deformation mechanism, surface integrity, and microscopic features [12-15]. A new basis for modeling dynamic forces from the static component and harmonic contributions is presented, as modeling the dynamic high speed milling force signal accounts for secondary harmonics [16]. Moradi et al. developed an extended dynamic model of a peripheral milling process including process damping and structural and cutting force nonlinearities using experimental coefficients [17].

Furthermore, combining the dynamic force and vibration response, Jalili et al. [18] studied the influences of the axial depth of cut, cutting tool diameter, cutting tool length, and number of cutter

teeth on the frequency response of the tool tip vibrations using a 3-D nonlinear dynamic model of the milling process. Jiang et al. [19] found a larger tool radius and a smaller cutting depth can control the vibration of curved thin-walled part. Wang et al. [20] proposed a cutting force prediction algorithm considering the influence of cutter vibrations and cutter run-out. Albertelli et al. [21] designed an in-process model-based estimator of cutting forces and tool-tip vibration based on both a machine dynamic model and indirect measurements coming from multiple sensors placed in the machine. Grossi et al. [22-23] found that dynamic cutting force coefficients change appreciably with spindle speed as mechanics of cutting change, and then carried out a deep investigation of cutting force coefficients to estimate the cutting force and tool-tip vibrational behavior.

The process vibration is the result of the coupling of machine-process vibration. However, limited studies in the field have focused on the interaction effect of the machine tool dynamic load and the process dynamic force. Generally in the interaction analysis, the cutting force would cause related vibration of machine, and the vibration also reacts on the cutting force due to the cutting parameters fluctuation by vibration.

In this paper, the interaction effect of milling vibration and the dynamic force is explored during the milling of Al 7075-T651. First, the dynamic milling process is simplified to theoretically study the vibration interaction model in the precision milling process. Whatever the vibration interaction sources are, they can be transformed as the dynamic load in the study. Then, the dynamic process force in process is measured and preprocess based on wavelet packet transform as the coupled force which reflects the cutting mechanism as well as the process vibration; the machine tool non-cutting vibration is regarded as another dynamic load; the Frequency Response Function (FRF) of dynamic performance is regards the interaction link between the dynamic force and the vibration response. Finally, the interaction vibration is calculated applying interaction effect model based on forced vibration and linear system assumption to verify the interaction effect model.

2 THEORETICAL ANALYSIS

2.1 Interaction Effect Analysis Approach

Based on PMI analysis as shown in Fig. 1, the process and the machine tool loads result in the unpredictable effects of interaction. The interaction of vibration to the dynamic forces of milling can include dynamic instabilities in the system, such as chatter vibrations or dynamic deflections. However,

The dynamic milling process considering the dynamic milling force is simplified in the vibration interaction theoretical modeling. As shown in Fig. 2, the dynamic milling process is simplified as a three-degrees-of-freedom system [24-25], which is expressed in three directions (X-feed direction, Y-perpendicular to feed direction, and Z-axial direction), the governing equation can be described as follows:

$$[M]\ddot{S}(t) + [C]\dot{S}(t) + [K]S(t) = F(t) \quad (1)$$

where

$$F(t) = [F_x(t), F_y(t), F_z(t)]^T \quad (2)$$

$F(t)$ is the milling forces vector; M , C , and K are the modal mass, damping, and stiffness of tool and workpiece, respectively; and S is the vibration displacement vector. With the subscript 'm' and 'w', M_m, C_m, K_m, S_m stand for the milling tool and M_w, C_w, K_w, S_w stand for the workpiece.

$$S_m(t) = [x_m(t), y_m(t), z_m(t)]^T \quad (3)$$

$$S_w(t) = [x_w(t), y_w(t), z_w(t)]^T \quad (4)$$

The vibration leads to dynamic change of cutting depth. Fig. 3 demonstrates the dynamic changes of cutting thickness produced by the single cutter tooth, which can be summarized as:

$$\phi_j = (j - 1)\phi_p + \phi \quad (5)$$

$$\phi_p = 2\pi/N \quad (6)$$

Fig. 2 Schematic diagram of milling dynamics

In which ϕ_j is the instantaneous radial contact angle of the j th cutter tooth, ϕ_p is the average circumferential angle of each cutter tooth. The total cutting thickness can be expressed by the sum of static cutting thickness and dynamic cutting thickness:

$$h(\phi_j) = \{f_t \sin \phi_j + \Delta f_v\} g(\phi_j) \quad (7)$$

Where $f_t \sin \phi_j$ is static cutting thickness, f_t is the feed rate per tooth, Δf_v is dynamic change of cutting depth, $g(\phi_j)$ is milling cutter teeth participation parameter.

Fig. 3 Dynamic changes of cutting thickness

In the study of dynamic force, only its dynamic change is considered,

$$h(\phi_j) = \Delta x \sin \phi_j + \Delta y \cos \phi_j \quad (8)$$

where $\Delta x = \Delta x_m - \Delta x_w$, $\Delta y = \Delta y_m - \Delta y_w$, $\Delta z = \Delta z_m - \Delta z_w$ are the dynamic changes in the directions.

The tangential, the radial dynamic cutting forces and the axial cutting force can be expressed as:

$$F_{tj} = K_{tc} a_p h(\phi_j) \quad (9)$$

$$F_{rj} = K_r F_{tj} \quad (10)$$

$$F_{zj} = F_{aj} = K_{ac} \Delta a_{pj} f_t \quad (11)$$

a_p is the axial cutting depth; $K_r = \frac{K_{rc}}{K_{tc}}$, K_{rc} is the coefficient of radial force, K_{tc} is the tangential force coefficient. Then

$$F_{xj} = -F_{tj} \cos \phi_j - F_{rj} \sin \phi_j \quad (12)$$

$$F_{yj} = +F_{tj}\sin\phi_j - F_{rj}\cos\phi_j \quad (13)$$

The total cutting force is:

$$F_x = \sum_{j=0}^{N-1} F_{xj}(\phi_j) \quad (14)$$

$$F_y = \sum_{j=0}^{N-1} F_{yj}(\phi_j) \quad (15)$$

$$F_z = \sum_{j=0}^{N-1} F_{zj} \quad (16)$$

Therefore, the cutting force can be expressed as:

$$\begin{pmatrix} F_x \\ F_y \\ F_z \end{pmatrix} = \frac{1}{2} a_p K_{tc} \begin{bmatrix} \alpha_{xx} & \alpha_{xy} & 0 \\ \alpha_{yx} & \alpha_{yy} & 0 \\ 0 & 0 & \frac{2K_{ac}\Delta a_{pj}}{a_p K_{tc}} \end{bmatrix} \begin{pmatrix} \Delta_x \\ \Delta_y \\ \Delta_z \end{pmatrix} \quad (17)$$

$$\alpha_{xx} = \sum_{j=0}^{N-1} -g_j [\sin 2\phi_j + K_r(1 - \cos 2\phi_j)] \quad (18)$$

$$\alpha_{xy} = \sum_{j=0}^{N-1} -g_j [(1 + \cos 2\phi_j) + K_r \sin 2\phi_j] \quad (19)$$

$$\alpha_{yx} = \sum_{j=0}^{N-1} g_j [(1 - \cos 2\phi_j) + K_r \sin 2\phi_j] \quad (20)$$

$$\alpha_{yy} = \sum_{j=0}^{N-1} g_j [\sin 2\phi_j - K_r(1 + \cos 2\phi_j)] \quad (21)$$

By means of discretization, Eq. (17) can be expressed in matrix form in time domain:

$$[F(\tau)] = \frac{1}{2} a_p K_{tc} [A(\tau)] [\Delta(\tau)] \quad (22)$$

Where $A(\tau)$ is the periodic function with angular frequency ω , $[\Delta(\tau)]$ is the dynamic displacement vector.

In frequency domain, the periodic function in milling force can be expanded as follows:

$$[A(\tau)] = \sum_{r=-\infty}^{\infty} [A_r] e^{ir\omega\tau} \quad (23)$$

$$[A_r] = \frac{1}{T} \int_0^T [A(\tau)] e^{-ir\omega\tau} d\tau \quad (24)$$

For both the milling machine tool and the workpiece, the dynamic milling force would excite the continuously vibration. Taking one direction as an example, the response of milling force excitation can be calculated through the Duhamel integral as follows [26]:

$$x(t) = \frac{1}{m\omega_d} \int_0^t F(\tau) e^{-\varepsilon\omega_n(t-\tau)} \sin\omega_d(t-\tau) d\tau + e^{-\varepsilon\omega_n t} (x_0 \cos\omega_d t + \frac{\dot{x}_0 + \varepsilon\omega_n x_0}{\omega_d} \sin\omega_d t) \quad (25)$$

When the initial displacement x_0 and the initial velocity $\dot{x}_0$ is zero, Eq. (25) can be omitted:

$$x(t) = \frac{1}{m\omega_d} \int_0^t F(\tau) e^{-\varepsilon\omega_n(t-\tau)} \sin\omega_d(t-\tau) d\tau = \int_0^t h_{xx}(t-\tau) F(\tau) d\tau \quad (26)$$

Therefore, the response can be calculated under the linear system assumption as follows:

$$\begin{bmatrix} x(t) \\ y(t) \\ z(t) \end{bmatrix} = \begin{bmatrix} \int_0^t h_{xx}(t-\tau)F_x(\tau) d\tau + \int_0^t h_{xy}(t-\tau)F_y(\tau) d\tau + \int_0^t h_{xz}(t-\tau)F_z(\tau) d\tau \\ \int_0^t h_{yx}(t-\tau)F_x(\tau) d\tau + \int_0^t h_{yy}(t-\tau)F_y(\tau) d\tau + \int_0^t h_{yz}(t-\tau)F_z(\tau) d\tau \\ \int_0^t h_{zx}(t-\tau)F_x(\tau) d\tau + \int_0^t h_{zy}(t-\tau)F_y(\tau) d\tau + \int_0^t h_{zz}(t-\tau)F_z(\tau) d\tau \end{bmatrix} \quad (27)$$

The vibration analysis above is based on the cutting force excited vibration in milling under non-chatter condition. Furthermore, the machine tool vibration in the system should also be considered, thus Eq. (27) can be extended to the study of milling vibration with the mechanical vibration superposition principle.

$$\begin{bmatrix} x_m(t) \\ y_m(t) \\ z_m(t) \end{bmatrix} = \begin{bmatrix} \int_0^t h_{mxx}(t-\tau)F_x(\tau) d\tau + \int_0^t h_{mxy}(t-\tau)F_y(\tau) d\tau + \int_0^t h_{mxz}(t-\tau)F_z(\tau) d\tau \\ \int_0^t h_{myx}(t-\tau)F_x(\tau) d\tau + \int_0^t h_{myy}(t-\tau)F_y(\tau) d\tau + \int_0^t h_{myz}(t-\tau)F_z(\tau) d\tau \\ \int_0^t h_{mzx}(t-\tau)F_x(\tau) d\tau + \int_0^t h_{mzy}(t-\tau)F_y(\tau) d\tau + \int_0^t h_{mzz}(t-\tau)F_z(\tau) d\tau \end{bmatrix} + \begin{bmatrix} x_{m0}(t) \\ y_{m0}(t) \\ z_{m0}(t) \end{bmatrix} \quad (28)$$

For workpiece vibration,

$$\begin{bmatrix} x_w(t) \\ y_w(t) \\ z_w(t) \end{bmatrix} = \begin{bmatrix} \int_0^t h_{wxx}(t-\tau)F_x(\tau) d\tau + \int_0^t h_{wxy}(t-\tau)F_y(\tau) d\tau + \int_0^t h_{wxz}(t-\tau)F_z(\tau) d\tau \\ \int_0^t h_{wyx}(t-\tau)F_x(\tau) d\tau + \int_0^t h_{wyy}(t-\tau)F_y(\tau) d\tau + \int_0^t h_{wyz}(t-\tau)F_z(\tau) d\tau \\ \int_0^t h_{wzx}(t-\tau)F_x(\tau) d\tau + \int_0^t h_{wzy}(t-\tau)F_y(\tau) d\tau + \int_0^t h_{wzz}(t-\tau)F_z(\tau) d\tau \end{bmatrix} \quad (29)$$

3 EXPERIMENTAL SETUP

To verify the theoretical model based on interaction effect analysis, the milling experiments are conducted on the milling machine (Model XR1000), as shown in Fig. 4. The milling forces are measured by a Kistler Force Dynamometer (Type 9139AA) mounted at the machine bed, and the vibration accelerations of the machine tool and workpiece in X and Y directions are obtained by two Kistler Annular Ceramic Shear Tri-axial Accelerometers (Type 8763B). In the test, cutting vibration along the Z direction is regarded having little effect on this direction due to the rigidity of the system. Therefore, this paper mainly studies the vibration of the spindle and workpiece along the X and Y directions. The signals of forces and vibrations are recorded before (free-run), during and after steady-state milling with a Kistler Instrument (Type 5165 and 5167). The impact testing is conducted with a PCB hammer (model 086C04) to analysis the frequency response functions.

Fig. 4 Experimental setup

In the experiments, the aluminum alloy (7075-T651) workpieces with a size of $125 \times 75 \times 10$ mm is applied. The particular alloy is widely used in the aerospace industry and has the following chemical composition: Al 89.655%, Cu 1.533%, Cr 0.199%, Fe 0.397%, Mg 2.333%, Mn 0.083%, Si 0.114%, Ti 0.037%, and Zn 5.649%. The side and face milling cutter tool in the test has a tooth number of 3, a diameter of 8.0 mm, a rake angle of 15° , a clearance angle of 6° , and a helix angle of 55° . Table 1 shows the cutting parameters of the orthogonal experiments. A straight groove is machined for each milling process, and the Y direction of the machine tool is selected as the feed direction.

Table 1 Milling experiments parameters

Exp. no.	Factors		
	Spindle speed n (r/min)	Feed rate f (m/min)	Axial cutting depth a_p (mm)
1	8000	0.4	0.3
2	8000	0.6	0.4
3	8000	0.8	0.5
4	10000	0.4	0.4
5	10000	0.6	0.5
6	10000	0.8	0.3
7	12000	0.4	0.5
8	12000	0.6	0.3
9	12000	0.8	0.4

4 RESULTS AND DISCUSSIONS

4.1 Frequency Response Functions of the Spindle and Workpiece

Prior to the cutting experiments, the FRFs of the spindle and workpiece in X, Y, and Z directions are obtained by impacting the tool point and the workpiece cutting position, respectively. The analysis bandwidth is 1250 Hz, and the frequency resolution is 1 Hz during experiments. The results are shown in Fig. 5, which illustrates the vibration resistance ability in the cutting process. The FRFs are regarded as the interaction relation function between cutting excitation force and response vibration based on linear system assumption, and the dynamic exciting force is the main resource of forced vibration while there is also small self-excited vibration in process.

It can be found that the first-order natural frequencies of the tool in X, Y, and Z directions are about 910 Hz, 900 Hz, and 490 Hz, respectively, with the corresponding amplitude-frequency characteristic values 1.06 g/N, 0.36 g/N, and 0.06 g/N. The first-order natural frequencies in X, Y, and Z directions of the workpiece are 253 Hz, 487 Hz, and 476 Hz, respectively, with the corresponding amplitude-frequency characteristic values 0.03 g/N, 0.07 g/N, and 0.1 g/N. It is confirmed that the cutting parameters in Table 1 avoid the chatter during machining.

Fig. 5 The FRFs of the spindle and the workpiece in the impact tests

4.2 Dynamic Milling Force and Vibration

4.2.1 Force and Vibration Signals in the Milling Process

Take the ninth experiment as an example, the original force and vibration signals produced during the milling process are demonstrated in Fig. 6, which show the components of the cutting forces and vibrations in different directions. The forces in three directions ranges from -30 N to 30 N due to the alternative cutter tooth. The vibrations of spindle are larger than those of the workpiece, which indicates the workpiece receives smaller impacts under good clamping stiffness.

Based on the aforementioned analysis, the original signals contain not only the forces and vibrations produced by the milling process, but also affected by the spindle and the machine tool. Therefore, these milling signals require preprocessing to distinguish the forces and vibrations generated by the different causes.

Fig. 6 The forces and vibrations produced in the milling process in different directions

4.2.1 Preprocessing of the Milling Force Signal

The frequency spectrums of the milling force signals are shown in Fig. 7, dynamic forces generated during the milling process consist of two parts: one is the periodic component produced by the periodic rotary motion of the spindle, which has extremely high energy and will cause the periodic interaction vibration of machine tool; the other is the random fluctuating component of cutting force. Therefore, in order to extract the periodic components and get accurate interaction effect analysis [27], wavelet packet transform is applied to preprocess the force signals.

Fig. 7 The spectrums of the actual milling force signal in different directions

The milling force signal is decomposed into four layers using the dmey wavelet. The optimal wavelet packet decomposition tree is shown in Fig. 8. The decomposition result of the wavelet packet is called the wavelet coefficient and designated as $S_4^0, S_4^1, S_4^1, \dots$. The wavelet coefficients of each node in the fourth layer are reconstructed using the wavelet packet reconstruction algorithm, so that the original force signal is represented as follows:

$$S = S_4^0 + S_4^1 + \dots + S_4^{15} \quad (30)$$

Fig. 8 Four-level decomposition structure of wavelet packet

The entropy of each band is acquired as $E(4, 0), E(4, 1), \dots, E(4, 15)$, and these entropies are normalized using Eq. (31). The energy distribution of each frequency band of the milling force signal in the X, Y, and Z directions are shown in Fig. 9.

$$H_{(4,j)} = \frac{E_{(4,j)}}{\sqrt{\sum_{j=0}^{15} E_{(4,j)}^2}} \quad (31)$$

Fig. 9 The energy distribution in each band of the wavelet packet reconstructed signal

In order to analyze the differences in working state in different frequency bands and identify the corresponding relationship between energy distribution and working state, the reconstructed signals of all nodes in the fourth layer are analyzed by fast Fourier transform in the frequency domain. Taking the milling force signal in the Y direction (the feed direction) as an example, spectrums of each node in the fourth layer are shown in Fig. 10.

Fig. 10 The spectrums of the force signal in the Y direction applying wavelet packet decomposition

With the help of fast Fourier transformation, frequency bands with larger energy distribution can be selected to construct the milling force equation. According to the principle of the wavelet packet energy spectrum, several frequency bands of the force in Y direction with energy ratios greater than 1% (the Reference line in Fig. 9) are selected, while those with energy ratios close to 0 are abandoned. The sum of the cosine signals of these selected frequency bands is applied to express the milling force

equation $F(\tau)$, and the required main frequency, amplitude, and phase information of the milling force signal in the Y direction are shown in Table 2.

Table 2 The wavelet packet decomposition information of the milling force signal in the Y direction

Nodes	(4,2)	(4,3)	(4,4)	(4,5)	(4,7)	(4,8)	(4,15)
Frequency band (Hz)	156-233	234-311	312-389	390-467	546-623	624-701	1173-1250
Energy distribution (%)	54.07	1.20	7.22	28.49	1.92	1.02	2.49
Main frequency (Hz)	200	298	381	400	600	650	1202
Amplitude (N)	14.27	0.79	1.94	10.29	1.99	1.29	2.49
Phase (rad)	-3.05	3.08	1.07	2.80	2.25	0.91	2.48

Therefore, the milling force equation calculated in the Y direction according to the original force signal is as follows:

$$\begin{aligned}
 F_y = & 14.27 \cos(2\pi * 200t - 3.05) + 0.79 \cos(2\pi * 298t + 3.08) \\
 & + 1.94 \cos(2\pi * 381t + 1.07) \\
 & + 10.29 \cos(2\pi * 400t + 2.80) + 1.99 \cos(2\pi * 600t + 2.25) \\
 & + 1.29 \cos(2\pi * 650t + 0.91) \\
 & + 2.49 \cos(2\pi * 1202t + 2.48)
 \end{aligned} \tag{32}$$

Similarly, the milling force equation in the X direction can be obtained as follows by using the same processing method:

$$\begin{aligned}
 F_x = & 0.17 \cos(2\pi * 79t - 1.80) + 0.97 \cos(2\pi * 101t + 1.89) \\
 & + 19.16 \cos(2\pi * 200t + 1.89) \\
 & + 0.68 \cos(2\pi * 301t + 0.84) + 1.68 \cos(2\pi * 381t + 3.00) \\
 & + 9.21 \cos(2\pi * 400t + 1.01) \\
 & + 1.45 \cos(2\pi * 600t + 1.06) + 1.34 \cos(2\pi * 800t + 1.79)
 \end{aligned} \tag{33}$$

Then the milling force equation in the Z direction can be obtained as follows:

$$\begin{aligned}
 F_z = & 0.18 \cos(2\pi * 70t - 0.72) + 0.63 \cos(2\pi * 99t - 0.54) + 8.84 \cos(2\pi * 200t - 0.76) \\
 & + 0.58 \cos(2\pi * 301t - 3.12) + 0.35 \cos(2\pi * 326t + 0.94) \\
 & + 1.44 \cos(2\pi * 400t - 1.68) \\
 & + 1.07 \cos(2\pi * 498t + 1.68) + 0.85 \cos(2\pi * 600t + 0.76) \\
 & + 0.56 \cos(2\pi * 650t + 0.36) \\
 & + 0.74 \cos(2\pi * 999t - 0.81)
 \end{aligned} \tag{34}$$

The force equations can be applied in predicting the milling force signal. The predicted signal not only eliminates noise while retaining feature information, but also avoids the influence of aperiodic truncation on the signal. The predicted force signals based on the Eqs. (32) - (34) are shown in Fig. 11 which contain characteristic components in milling and match well with the dynamic force model.

Fig. 11 The predicted milling force signal in different directions

4.2.3 Preprocessing of the Spindle Free-run Signal

As shown in Eq. (28) in the modelling, the spindle free-run signal is superposed on the forced vibration signal. The measured spindle free-run vibration signal and its spectrum are shown in Fig. 12.

Fig. 12 The spindle free-run vibration signal and its frequency spectrum in the X and Y directions

The previous preprocessing method is also applied to extract the characteristic frequency information of the measured spindle free-run signal. After being decomposed into four layers by dmey

wavelet, the energy distribution of each frequency band of the spindle free-run signal in the X and Y directions are shown in Fig. 13.

Fig. 13 The energy distribution in each band of the wavelet packet reconstructed signal

Taking the spindle free-run vibration signal in the X direction as an example, spectrums of each node in the fourth layer are shown in Fig. 14.

Fig. 14 The spectrums of spindle free-run vibration signal in the X direction applying wavelet packet decomposition

According to the principle of wavelet packet energy detection, main frequency bands (energy ratio greater than 1%, evaluated by the Reference line in Fig. 13) are selected to construct the free-run vibration equation $a(\tau)$ of the spindle. The required main frequency, amplitude, and phase information of the spindle free-run vibration signal in the X direction are shown in Table 3.

Table 3 The wavelet packet decomposition of the spindle free-run vibration signal in the X direction

Selected nodes	(4,1)	(4,2)	(4,3)	(4,5)	(4,9)	(4,10)
Frequency band (Hz)	78-155	156-233	234-311	390-467	702-779	780-857
Energy distribution (%)	1.67	41.71	1.09	1.08	5.18	46.20
Main frequency (Hz)	86	200	234	400	762	800
Amplitude (g)	0.004	0.130	0.007	0.020	0.022	0.132
Phase (rad)	1.46	-1.90	0.56	-1.84	-0.08	0.88

The spindle free-run vibration equations can be expressed as the sum of multiple cosine signals:

$$\begin{aligned}
 a_x = & 0.004 \cos(2\pi * 86t + 1.46) + 0.130 \cos(2\pi * 200t - 1.90) \\
 & + 0.007 \cos(2\pi * 234t + 0.56) \\
 & + 0.020 \cos(2\pi * 400t - 1.84) + 0.022 \cos(2\pi * 762t - 0.08) \\
 & + 0.132 \cos(2\pi * 800t + 0.88)
 \end{aligned} \tag{35}$$

$$\begin{aligned}
 a_y = & 0.006 \cos(2\pi * 127t + 2.68) \\
 & + 0.117 \cos(2\pi * 200t + 0.92) + 0.016 \cos(2\pi * 234t + 0.56) \\
 & + 0.013 \cos(2\pi * 400t + 0.70) + 0.020 \cos(2\pi * 566t + 1.10) \\
 & + 0.009 \cos(2\pi * 634t + 1.20) \\
 & + 0.019 \cos(2\pi * 800t - 2.35) + 0.013 \cos(2\pi * 1034t - 1.86)
 \end{aligned} \tag{36}$$

For the same time series t , the preprocessed milling force signal and the preprocessed spindle free-run vibration signal can be ensured to be synchronized in the time domain. The predicted spindle free-run vibration signals based on the Eqs. (35) - (36) are shown in Fig. 15.

(a) Predicted spindle free-run vibration of X-direction (b) Predicted spindle free-run vibration of Y-direction

Fig. 15 The predicted spindle free-run vibration signals in different directions

4.3 Comparison of Interaction Analysis Result

The manufacturing process and machine interaction shows that the process cutting force would result in machine vibration. The interaction effect based on forced vibration theory in equations is investigates applying the frequency-domain dynamic force and frequency response function. In interaction analysis, the predicted vibration is calculated in the proposition of process interaction by applying Eqs. (28) - (29). The time-domain signals of the predicted vibration and the actual measured vibration are shown in Fig. 16. It can be seen that the predicated results obtained in interaction effect approach matches with the vibration signal direct tested, whether they come from the measurement point of the spindle or the workpiece.

Fig. 16 Comparison between the predicted vibration and the measured vibration (Exp. no. 9)

In order to compare the amplitude of the calculated vibration and the actual measured vibration in the frequency domain, the spectrums of two vibration signals are shown in Fig. 17. It shows that the main frequencies of the vibration signal spectrograms in the X and Y directions of the spindle are close at 200 Hz, 400 Hz, and 600 Hz. 200 Hz is the rotational frequency of the spindle, 400 Hz is the double frequency of the rotational frequency, and 600 Hz (the tooth-passing frequency) is the three-times frequency of the rotational frequency. Therefore, the amplitudes of the characteristic frequency at the low frequency match well. However, at the high frequency, there is some differences due to the extraction method of the signal and some other unpredictable effects.

Fig. 17 Spectrum of the predicted vibration and the measured vibration (Exp. no. 9)

In order to determine the magnitude error of the predicted vibration, the root mean square (RMS) of the vibration signal is calculated using Eq. (37) in which measured vibration signal and the calculated vibration are μ_a and $\mu_{\hat{a}}$, respectively, and the relative error of the RMS between the two can be expressed as:

$$Vibration_{rms} = \sqrt{\frac{1}{N} \sum_{i=1}^N a(i)^2} \quad (37)$$

$$\varepsilon_{rms} = \frac{|\mu_{\hat{a}} - \mu_a|}{\mu_a} \times 100\% \quad (38)$$

Table 4 lists the RMS error of the predicted vibrations of the spindle and the workpiece under all precision milling parameter combinations. After statistical calculation, the overall percentage of the errors of vibrations from the spindle and the workpiece in X and Y directions are around 20.8%, 21.8%, 17.4% and 17.6% respectively. The prediction error of the spindle vibration is larger than that of the workpiece vibration, which can be attribute to the complex structure of the spindle. The vibration of the spindle consists of the vibrations from various components, such as bearings and sleeves, and is prone to producing vibrations in different dominant frequencies. On the other hand, the vibration of

the workpiece is mainly composed by the self-excited vibrations from workpiece and the tool. The spindle speed and the number of cutter tooth influence the dominant frequencies significantly, which is easier to be identified through spectral analysis and can be predicted better.

Table 4 The relative RMS error of the vibrations

Exp. no.	Factors			RMS error of spindle vibration (%)		RMS error of workpiece vibration (%)	
	n (r/min)	v_f (m/min)	a_p (mm)	X-direction	Y-direction	X-direction	Y-direction
1	8000	0.4	0.3	25.1	27.3	22.7	18.3
2	8000	0.6	0.4	22.4	21.1	19.5	17.7
3	8000	0.8	0.5	19.2	25.2	13.8	18.2
4	10000	0.4	0.4	24.7	19.8	15.2	22.3
5	10000	0.6	0.5	23.1	23.3	23.4	19.6
6	10000	0.8	0.3	17.5	23.6	11.6	13.7
7	12000	0.4	0.5	17.2	19.2	15.2	13.4
8	12000	0.6	0.3	21.4	18.5	20.3	17.1
9	12000	0.8	0.4	16.2	18.1	14.7	18.0

5 CONCLUSIONS

This paper investigated the interaction effect between the dynamic force and the vibration response in the precision milling of 7075-T651 based on forced-vibration transformation approach. By both theoretical modelling and experimental verification, the following conclusions can be drawn from the above investigation:

- The interaction vibration is excited by both the machine tool load and machining process dynamic load, the dynamic excitation loads and the vibration response can be superimposed under the linear system assumption. The spindle-tool structural dynamic characteristics is the interaction link between the dynamic force load and the vibration response, even in non-chatter condition, it mainly determines the accuracy level in precision milling applications.
- The forces and vibrations can be expressed by the linear combinations of various frequency components. These frequency components are selected by the entropies in the wavelet packets due to the large quantities of the energies being found in the dominant frequencies. The proposed forces and vibrations prediction method shows high consistencies with the measured values.
- The overall percentage of the errors of predicted vibrations from the spindle and the workpiece in X and Y directions are controlled at around 20.8%, 21.8%, 17.4% and 17.6% respectively. The

lower errors of workpiece vibrations indicates that the self-excited vibrations generated by the workpiece and the tool can be identified and predicted better than spindle vibrations.

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Conflicts of interest/Competing interests

The authors declare that they have no conflict of interest or competing interests.

Availability of data and materials

The data and materials that support the findings of this study are available from the corresponding author upon reasonable request.

Code availability

Not applicable.

Ethical Approval

Compliance with ethical standards.

Consent to Participate and Publish

The authors consent to participate and publish.

Authors Contributions

Weicheng Guo: Data collection, validation, writing original manuscript.

Miaoxian Guo: Conceptualization, methodology.

Ye Yi: Graphic plotting.

Chongjun Wu: Manuscript revision.

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