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Study on Web Pillar Stability in Open-Pit Highwall Mining

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Abstract: When highwall mining technology is applied to recover large amounts of residual coal left under the highwall of a big open-pit mine, reasonable coal pillar width is the premise for maintaining the stability of web pillars. By adopting the numerical simulation method, the characteristics of the abutment stress distributions in the web pillars under different slope angles and mining depths are studied, and the function of the stress distribution in the web pillar is established. The relationship between the abutment stress and the ultimate strength of the web pillar under different widths is also analyzed and used in combination with the failure characteristics of the pillar yield zone to explore the instability mechanism of the web pillar. The retaining widths of the web pillars are determined. Based on the modeling results, a mechanical bearing model of the web pillar is established, a cusp catastrophe model of pillar-overburden is constructed, and the formula for the web pillar instability criterion is obtained. By analyzing and calculating the ultimate strength of the web pillar, the formula for calculating the yield zone width at both sides of the pillar is achieved. Using the instability criterion of web pillars in highwall mining, a reasonable pillar width can be deduced theoretically, which provides significant guidance on the application of highwall mining technology.

Keywords: open-cut mine; highwall mining; instability criterion; web pillars

1. Introduction

China's coal resources are abundant. The demonstrated coal reserves account for approximately 11.1% of global reserves[1]. Open-pit mining is one of the important mining methods for coal resources, and has the outstanding advantages of safe operation, large output, environmental efficiency, good working conditions, high mining intensity, and so on. Recently, open-pit mining has developed rapidly and its application is becoming increasingly widespread. In China, the proportion of open-pit coal mining has grown from 4% in the early 1990s to 16.2% in 2010[2]. Although the advantages of open-pit mining have been highlighted in actual engineering practice, a large amount of residual coal is often left behind and cannot be recovered after open-pit mining. The advent of the LDC100 highwall miner, independently developed in China, provides an effective approach toward achieving the recovery of residual coal in the highwall.

The highwall miner has been implemented as a new open-pit highwall mining technology that combines the advantages of open-pit mining and underground mining to achieve highwall mining in soft and thick coal seams. The mining and transporting process is fully automatic and remotely controlled, i.e. carrying out unmanned intelligent mining and transportation operations in the coal chamber. In the highwall mining process, web pillars are set up between the chambers to support the overlying strata and prevent landslides, burial of equipment, and other disasters. When the web pillars become partially unstable, it may induce the chain instability of the pillar group. This results in slope instability and landslides, which endanger the safety of mining equipment and cause substantial resource loss. Therefore, the stability of web pillars should be extensively and systematically investigated, and a reasonable web pillar size should be designed.

Globally, current researches on web pillar stability in open-pit mining are mainly based on the empirical formula of coal pillar strength (Bunting, 1911; Greenwald, 1939; Holland and Gaddy, 1957; Holland, 1964; Salamon and Munro, 1967; Bieniawski, 1968; Sheorey, 1992; Galvin and Salamon, 1999)[3-10] to determine the width of the web pillar. John et al. adopted hybrid empirical and numerical modeling techniques for web pillar design in India, and proposed the introduction of a correction factor into the empirical pillar strength equation for slender pillars with a width to height ratio of less than unity to achieve a more reasonable design of the coal pillar parameters[11]. Ďuriš and Richard used rockbolt support to address the issue of coal pillar stability and stabilization, and sufficiently reinforced the existing pillars [12]. Alexandros et al. investigated approximate solutions using two-dimensional (2D) numerical simulation techniques and three-dimensional (3D) numerical analyses to evaluate the geomechanical responses of the lignite pillar formed by the room and pillar mining method[13]. They pointed out that 2D approximation techniques adequately approximate to the actual 3D problem. Hikaru et al. considered the recovery ratio and used numerical analysis to develop an appropriate design for the width of the web coal pillar[14]. Zhang et al. used the FLAC^{3D} (Fast Lagrangian Analysis of Continua in Three Dimensions; Itasca Consulting Group Inc. Minneapolis, Minnesota, USA) numerical simulation software to systematically investigate the laws governing the stress increase coefficient of the coal pillar and the pillar's stability[15]. They obtained a function describing the relationship among the stress increase coefficient of the upper and lower pillars, the property of interstratified rock, and the spatial relationship.

However, the results obtained by the above mentioned studies are not necessarily suitable to the engineering geological conditions and the occurrence of coal and rock in China. Notably, the quality of coal in most parts of China is soft. Moreover, these studies have not provided a detailed description of the instability mechanism and have not proposed a reasonable instability criterion for the web pillar, based on which the designed width of the web pillar is lack of reliability. Therefore, taking an open-pit coal mine in Inner Mongolia, China as the engineering geological background, this paper systematically studies the instability mechanism and the web pillar width by means of experimental test analysis, numerical simulation, and cusp catastrophe theory. It is expected to provide a basis for the design and safety implementation of highwall mining schemes and provide guidance for similar projects.

2. Engineering background

This paper is based on the geological background of an open-pit mine located in Inner Mongolia Province, China. The strike length of the mine is 5.1km. Two main coal seams numbered 19[#] and 21[#] are lignite of high-quality. The expected advance length of the open cut operations is 360m per year, and it is required to cut the coal seam 21[#] along the seam floor for timely internal coal transporting. The tracking distance of the internal transporting is 50m. The strata at the west end-slope of the open-pit mine are almost horizontally distributed and there is no obvious change in lithology along the strike direction. The slope strata consist of surface soil, gritstone, siltstone, coal and base sandstone from top to bottom (Figure 1).

The LDC100 highwall miner [16] is utilized to extract the coal left under the west highwall of the open-pit mine. It is an unmanned mining machine with a production capacity of 60t/h and a suited coal-rock hardness factor ($f \leq 2$), which is applicable for the recovery of lignite open-pit mines. The recovery chamber has a rectangular cross-section being 2m wide and 2.5m high. The maximum mining depth is 100m and the mining process can be completed once every 3 days. The mechanical properties of the rocks and soil mass at the highwall of the open-pit mine are shown in Table 1.

Figure 1 Schematic diagram of lithology distribution of slope strata

Table 1 Mechanical properties of the rocks and soil mass

Strata formation	Compressive strength/MPa	Tensile strength/MPa	Cohesion /MPa	Friction angle/(°)	Elasticity modulus/GPa	poisson ratio	Density/(kg·m ⁻³)
sandstone	45.19	3.40	2.18	23	15.16	0.22	2010
21#coal	17.66	2.06	0.30	17.6	7.41	0.29	1270
siltstone	25.76	3.02	2.35	25	14.58	0.21	2090
gritstone	51.43	4.90	2.12	22	15.62	0.21	1990
soil	9.49	1.34	0.24	20	5.69	0.3	2671

2 Numerical Modeling on Instability Mechanism of Web Pillars

2.1 Model Construction and Experiment Schemes

The failure criteria used in the Mohr-coulumb model are the Mohr-coulumb criterion and the maximum tensile stress criterion. The three principal stresses are $\sigma_1 \leq \sigma_2 \leq \sigma_3$, the failure criterion in the plane of (σ_1, σ_3) is represented in Figure 2.

Taking the west highwall of an open-pit coal mine in Inner Mongolia as the engineering background, the numerical simulation models of the web pillars under four slope angles (i.e. 20°, 30°, 40° and 50°) are established separately by using the finite difference modeling software FLAC3D, which is widely used in the engineering field, to study the instability mechanism of web pillars under different slope angles and mining depths. The model represents a geological condition around the target area shown in Figure 1. To eliminate the boundary effect, coal pillars being 60m wide are retained at each side of the chambers based on the theory of elastic-plastic mechanics. Since element size has a great impact on the modeling results, meshing elements in the target area should be highly precisely arranged to minimize the influence. The element width is set to be 1m along the pillar strike direction. 10 elements are set in each horizontal and vertical direction of the pillar cross-section. A total of 100 elements are divided in the pillar area. Horizontal constraints are imposed on both sides of the model, that is, the horizontal displacement is set to be zero, and the base boundary of the model is fixed, which means the horizontal and vertical displacements of the base boundary are both zero. The top surface and the slope face of the model are free boundaries. The loading stress is gravity, and the Mohr-Coulomb elastic-plastic constitutive model is used for calculation and analysis. The failure criteria used in the Mohr-coulumb model are the Mohr-coulumb failure criterion and the maximum tensile stress criterion. The relationship between the three principal stresses is $\sigma_1 \leq \sigma_2 \leq \sigma_3$, and the failure criterion in the plane of (σ_1, σ_3) is represented in Figure 2.

Figure 2 Mohr-coulomb Failure criterion

According to the specifications of the LDC100 highwall miner, the chamber is 2.5m in height ($h=2.5m$) and 2m in width ($a=2m$), the maximum mining depth is 100m ($l=100m$), and one mining process can be completed in 3 days. In order to analyze the abutment stress, the yield zone distribution and the instability mechanism of the web pillar at different slope angles and mining depths, four modeling schemes are designed. Tables 2, 3, 4, and 5 present the modeling schemes under the slope angles of 20°, 30°, 40° and 50°, respectively. For each scheme, set four kinds of mining depths, i.e. $l=50m$, $l=65m$, $l=80m$ and $l=100m$, arrange four chambers for each mining depth and the chamber is 2.5m in height ($h=2.5m$) and 2m in width ($a=2m$), retain three web pillars between the chambers, and design four sizes of pillar width for each mining depth, thus a total of 16 simulation models in each scheme. Details of the web pillar sizing are listed in Tables 2 to 5.

Table 2 Simulated web pillar widths under different mining depths at slope angle 20°

Mining depth /m	Pillar width size I/m	pillar width size II/m	pillar width size III/m	pillar width size IV/m
50	3.7	3.9	4.1	4.3
65	3.8	4.0	4.2	4.4
80	3.9	4.1	4.3	4.5
100	4.1	4.3	4.5	4.7

Table 3 Simulated web pillar widths under different mining depths at slope angle 30°

Mining depth /m	Pillar width size I/m	pillar width size II/m	pillar width size III/m	pillar width size IV/m
50	3.9	4.1	4.3	4.5
65	4.1	4.3	4.5	4.7
80	4.3	4.5	4.7	4.9
100	4.6	4.8	5.0	5.2

Table 4 Simulated web pillar widths under different mining depths at slope angle 40°

Mining depth /m	Pillar width size I/m	pillar width size II/m	pillar width size III/m	pillar width size IV/m
50	4.1	4.3	4.5	4.7
65	4.4	4.6	4.8	5.0
80	4.7	4.9	5.1	5.3
100	5.1	5.3	5.5	5.7

Table 5 Simulated web pillar widths under different mining depths at slope angle 50°

Mining depth /m	Pillar width size I/m	pillar width size II/m	pillar width size III/m	pillar width size IV/m
50	4.3	4.5	4.7	4.9
65	4.7	4.9	5.1	5.3
80	5.1	5.3	5.5	5.7
100	5.6	5.8	6.0	6.2

2.2 Analysis of Abutment Stress Distributions of Web Pillars

By improving shear resistance of web pillars, the abutment stress distribution before pillar failure is obtained. Though analyzing the characteristics of the abutment stress distributions in the web pillar along both the strike direction and the dip direction, the position with the maximum abutment stress under the condition of “triangular loading” is achieved. Also, variation of the loading sustained in this position at different slope angles, mining depths and pillar widths is revealed.

Due to space limitation, this paper only exhibits the modeling results of the abutment stress distributions of the web pillars with different widths and mining depths at the slope angle of 40° (Figure 3). The engineering positions with the maximum abutment stress under varying pillar widths, mining depths, and slope angles are recorded in Table 6. It is observed that there is an “end effect” in highwall mining. This is because the stiffness of the web pillar is smaller than the stiffness of the solid slope barrier, which shares some overburden load upon the web pillar. All the maximum abutment stresses occur somewhere ahead of the web pillars towards the end of chamber excavations. Additionally, the locations of the maximum abutment stresses are not related to the pillar width but are closely associated with the maximum mining depth and the slope angle (burial depth). Notably, the most risky position of the web pillar is at the site where the coal pillar bears the maximum abutment stress. If instability occurs in this position, it may produce a chain reaction and lead to instability of the entire pillar group. The positions with the maximum abutment stress (P_d) are fitted in Figure 4 and Equation (1) is derived to demonstrate the relationship among P_d , the slope angle and the mining depth.

$$P_d = 2.54 - 0.15\theta + 0.9689L \quad (1)$$

where P_d is the position bearing the maximum abutment stress [m], θ is the slope angle [°], and L is the mining depth [m].

(a) Mining depth=50m

(b) Mining depth=65m

(c)Mining depth=80m

(d)Mining depth=100m

Figure 3 Abutment stress distributions under different mining widths and pillar sizes at the slope angle of 40°

Table 6 Maximum abutment stress positions at different slope angles and mining depths

Mining depth/m Slope angle/°	50	65	80	100
20	47	62	78	97
30	46	61	76	95
40	45	60	74	93
50	44	59	72	91

Figure 4 Position fitting of the maximum abutment stress at different slope angles and mining depths

Based on the positions with the maximum abutment stress at different slope angles and mining depths, the stress distributions along the dip direction and the variation of the loading sustained in these positions are studied. In this paper, here only presents the results of the stress distributions in the most risky positions under different pillar widths and mining depths at the slope angle of 40° (Figures 5 to 8). The results show that the abutment stress is distributed symmetrically along the center of the pillar. As the coal pillar does not break, the stress concentration factor is relatively larger at the two sides but smaller in the center of the pillar. The stress distribution is approximately in a bowl shape. The maximum and minimum abutment stresses of the web pillars with different widths and mining depths at the slope angles of 20°, 30°, 40° and 50° are recorded in Tables 7, 8, 9, and 10, respectively.

(a)Pillar width=4.1m

(b)Pillar width=4.3m

(c)Pillar width=4.5m

(d)Pillar width=4.7m

Figure 5 Side abutment stress distributions for different pillar widths at slope angle 40° and depth 50m

(a)Pillar width=4.4m

(b)Pillar width=4.6m

(c)Pillar width=4.8m

(d)Pillar width=5.0m

Figure 6 Side abutment stress distributions for different pillar widths at slope angle 40° and depth 65m

(a)Pillar width=4.7m

(b)Pillar width=4.9m

(c)Pillar width=5.1m

(d)Pillar width=5.3m

Figure 7 Side abutment stress distributions for different pillar widths at slope angle 40° and depth 80m

(a)Pillar width=5.1m

(b)Pillar width=5.3m

(c)Pillar width=5.5m

(d)Pillar width=5.7m

Figure 8 Side abutment stress distributions for different pillar widths at slope angle 40° and depth 100m

Table 7 Maximum and minimum bearing stress values of different pillars widths at slope angle 20°

Depth	3.7m		3.9m		4.1m		4.3m		3.8m		4.0m		4.2m		4.4m	
	$\sigma_{\min}$	$\sigma_{\max}$	$\sigma_{\min}$	$\sigma_{\max}$	$\sigma_{\min}$	$\sigma_{\max}$	$\sigma_{\min}$	$\sigma_{\max}$	$\sigma_{\min}$	$\sigma_{\max}$	$\sigma_{\min}$	$\sigma_{\max}$	$\sigma_{\min}$	$\sigma_{\max}$	$\sigma_{\min}$	$\sigma_{\max}$
50m	1.179	1.406	1.151	1.389	1.121	1.376	1.101	1.363								
65m									1.235	1.466	1.208	1.452	1.181	1.439	1.169	1.428
			3.9m	4.1m	4.3m	4.5m					4.1m	4.3m	4.5m	4.7m		
80m			$\sigma_{\min}$	$\sigma_{\max}$	$\sigma_{\min}$	$\sigma_{\max}$	$\sigma_{\min}$	$\sigma_{\max}$	$\sigma_{\min}$	$\sigma_{\max}$	$\sigma_{\min}$	$\sigma_{\max}$	$\sigma_{\min}$	$\sigma_{\max}$	$\sigma_{\min}$	$\sigma_{\max}$
100m			1.262	1.515	1.239	1.501	1.211	1.489	1.193	1.476						
									1.377	1.605	1.343	1.591	1.312	1.580	1.282	1.569

Table 8 Maximum and minimum bearing stress values of different pillars widths at slope angle 30°

Depth	3.9m		4.1m		4.3m		4.5m		4.1m		4.3m		4.5m		4.7m	
	$\sigma_{\min}$	$\sigma_{\max}$	$\sigma_{\min}$	$\sigma_{\max}$	$\sigma_{\min}$	$\sigma_{\max}$	$\sigma_{\min}$	$\sigma_{\max}$	$\sigma_{\min}$	$\sigma_{\max}$	$\sigma_{\min}$	$\sigma_{\max}$	$\sigma_{\min}$	$\sigma_{\max}$	$\sigma_{\min}$	$\sigma_{\max}$
50m	1.266	1.513	1.239	1.501	1.217	1.486	1.183	1.469								
65m									1.386	1.625	1.366	1.611	1.332	1.596	1.315	1.582
			4.3m	4.5m	4.7m	4.9m					4.6m	4.8m	5.0m	5.2m		
80m			$\sigma_{\min}$	$\sigma_{\max}$	$\sigma_{\min}$	$\sigma_{\max}$	$\sigma_{\min}$	$\sigma_{\max}$	$\sigma_{\min}$	$\sigma_{\max}$	$\sigma_{\min}$	$\sigma_{\max}$	$\sigma_{\min}$	$\sigma_{\max}$	$\sigma_{\min}$	$\sigma_{\max}$
100m			1.498	1.728	1.475	1.715	1.449	1.701	1.429	1.686						
									1.638	1.869	1.613	1.851	1.573	1.835	1.533	1.811

Table 9 Maximum and minimum bearing stress values of different pillars widths at slope angle 40°

Depth	4.1m		4.3m		4.5m		4.7m		4.4m		4.6m		4.8m		5.0m	
	$\sigma_{\min}$	$\sigma_{\max}$	$\sigma_{\min}$	$\sigma_{\max}$	$\sigma_{\min}$	$\sigma_{\max}$	$\sigma_{\min}$	$\sigma_{\max}$	$\sigma_{\min}$	$\sigma_{\max}$	$\sigma_{\min}$	$\sigma_{\max}$	$\sigma_{\min}$	$\sigma_{\max}$	$\sigma_{\min}$	$\sigma_{\max}$
50m	1.375	1.622	1.351	1.608	1.322	1.591	1.292	1.575								
65m									1.508	1.775	1.502	1.761	1.476	1.749	1.459	1.736
			4.7m	4.9m	5.1m	5.3m					5.1m	5.3m	5.5m	5.7m		
80m			$\sigma_{\min}$	$\sigma_{\max}$	$\sigma_{\min}$	$\sigma_{\max}$	$\sigma_{\min}$	$\sigma_{\max}$	$\sigma_{\min}$	$\sigma_{\max}$	$\sigma_{\min}$	$\sigma_{\max}$	$\sigma_{\min}$	$\sigma_{\max}$	$\sigma_{\min}$	$\sigma_{\max}$
100m			1.688	1.931	1.669	1.915	1.635	1.899	1.612	1.866						
									1.931	2.162	1.901	2.141	1.867	2.119	1.845	2.101

Table 10 Maximum and minimum bearing stress values of different pillars widths at slope angle 50°

Depth	4.3m		4.5m		4.7m		4.9m		4.7m		4.9m		5.1m		5.3m	
	$\sigma_{\min}$	$\sigma_{\max}$	$\sigma_{\min}$	$\sigma_{\max}$	$\sigma_{\min}$	$\sigma_{\max}$	$\sigma_{\min}$	$\sigma_{\max}$	$\sigma_{\min}$	$\sigma_{\max}$	$\sigma_{\min}$	$\sigma_{\max}$	$\sigma_{\min}$	$\sigma_{\max}$	$\sigma_{\min}$	$\sigma_{\max}$
50m	1.492	1.729	1.466	1.713	1.432	1.698	1.413	1.685								
65m									1.689	1.925	1.662	1.908	1.635	1.892	1.603	1.877
			5.1m	5.3m	5.5m	5.7m					5.6m	5.8m	6.0m	6.2m		
80m			$\sigma_{\min}$	$\sigma_{\max}$	$\sigma_{\min}$	$\sigma_{\max}$	$\sigma_{\min}$	$\sigma_{\max}$	$\sigma_{\min}$	$\sigma_{\max}$	$\sigma_{\min}$	$\sigma_{\max}$	$\sigma_{\min}$	$\sigma_{\max}$	$\sigma_{\min}$	$\sigma_{\max}$
100m			1.939	2.158	1.902	2.146	1.871	2.132	1.846	2.119						
									2.292	2.539	2.271	2.522	2.239	2.501	2.210	2.479

2.3 Analysis of Instability Mechanism and Ultimate Strength of Web Pillars

Based on the distributions of the positions with the maximum abutment stress, namely, the positions at which the web pillars are most vulnerable to instability, the distribution characteristics of the side abutment stress in these positions under varying pillar widths and mining depths at the slope angles of 20° , 30° , 40° , and 50° are analyzed. Taking the case of the slope angle at 40° for detailed analysis (Figures 9 to 12), it is evident that as the pillar width decreases gradually, the curve shape of the stress changes from a saddle shape to an approximate platform shape, and finally to an arch shape. Correspondingly, the web pillar undergoes an evolution of “stable state-critical equivalent state-ultimate failure state”. This is because with the narrowing of the pillar width, the pillar’s ultimate strength decreases gradually while the abutment stress grows gradually. The abutment stress of the web pillar is firstly smaller than the pillar’s ultimate strength (Figure 9a) then equal to the pillar strength (Figure 9b), implying the web pillar is in a stable state, and finally larger than the ultimate strength of the coal pillar (Figure 9c and 9d), suggesting the web pillar is damaged. According to the limit equilibrium theory [17], the ultimate strengths of the web pillars with different widths can be obtained. Under conditions of different slope angles and mining depths, the web pillars with the same width almost have the same ultimate strength, indicating that the web pillar ultimate strength has nothing to do with the slope angle and the mining depth, but is positively correlated with the pillar width, namely, the larger the pillar width, the bigger the pillar’s ultimate strength, as illustrated in Figure 13.

Figure 9 Side abutment stress distributions for different pillar widths at slope angle 40° and depth 50m

(c) pillar width=4.6m

(d) pillar width=4.4m

Figure 10 Side abutment stress distributions for different pillar widths at slope angle 40° and depth 65m

(a) Pillar width=5.3m

(b) Pillar width=5.1m

(c) Pillar width=4.9m

(d) Pillar width=4.7m

Figure 11 Side abutment stress distributions for different pillar widths at slope angle 40° and depth 80m

(a) Pillar width=5.7m

(b) Pillar width=5.5m

(c)Pillar width=5.3m

(d)Pillar width=5.1m

Figure 12 Side abutment stress distributions for different pillar widths at slope angle 40° and depth 100m

Figure 13 Ultimate strengths of coal pillars with different widths

2.4 Analysis of Yield Zone Distribution

The modeling results of the yield zone distributions in the web pillars with different mining depths and widths at the slope angle of 40° are taken for detailed analysis (Figures 14 to 17). The results show that the failure pattern of the web pillars is shear failure. If the web pillar has a small width, failure occurs when the yield zones at both sides of the web pillar merge with each other (Figure 14a and 14b). With the increase of the pillar width, a certain proportion of elastic core zone (Figure 14c and 14d) exists in the center of the web pillar, and also the yield zones are not collected, implying the web pillar is in a stable state. The modeling results of the web pillar stability are consistent with the previous analysis of the state identification of web pillars based on the abutment stress distributions. By combining the relationship between the abutment stress and the ultimate strength of the web pillar with the failure characteristics of the yield zones, the instability mechanism of the web pillar in highwall mining is disclosed, that is, when the abutment stress of the web pillar exceeds its ultimate strength, shear failure occurs. In view of improving the recovery rate, appropriate retaining widths of the web pillars under different slope angles and mining depths are identified (Table 11).

Figure 14 Yield zone distributions of web pillars with different widths at slope angle 40° and depth 50m

(c)Pillar width=4.8m

(d)Pillar width=5.0m

Figure 15 Yield zone distributions of web pillars with different widths at slope angle 40° and depth 65m

(a)Pillar width=4.7m

(b)Pillar width=4.9m

(c)Pillar width=5.1m

(d)Pillar width=5.3m

Figure 16 Yield zone distributions of web pillars with different widths at slope angle 40° and depth 80m

(a)Pillar width=5.1m

(b)Pillar width=5.3m

(c)Pillar width=5.5m

(d)Pillar width=5.7m

Figure 17 Yield zone distributions of web pillars with different widths at slope angle 40° and depth 100m

Table 11 Retaining widths of web pillars at different slope angles and mining depths

Depth/m	Slope angle/°			
	20	30	40	50
50	4.1	4.3	4.5	4.7
65	4.2	4.5	4.8	5.1
80	4.3	4.7	5.1	5.5
100	4.5	5.0	5.5	6.0

3 Instability Criterion of Web Pillars

3.1 A Cusp Catastrophe Model of Web Pillar

According to the maximum abutment stress positions of web pillars under the effect of “triangular loading” different from that under uniform distribution of the overburden rock and the modeling results of the abutment stress distributions, it is found that the abutment stress in the most risky position along the dip direction has a bowl shape distribution rather than an equal uniform distribution described by the effective region theory. Accordingly, a bearing model of the web pillar is established (Figure 18). The arch-shaped solid curve in the model illustrates the actual distribution of the abutment stress, which is symmetrically distributed along the center of the web pillar. Provided that the actual stress curve is the two broken lines in the figure, hence the overburden load of the web pillar (P) can be calculated by Equation (2).

Figure 18 Bearing model of web pillar

$$P = \frac{\sigma_{min} + \sigma_{max}}{2} w_s \quad (2)$$

where, σ_{min} is the minimum abutment stress, σ_{max} is the maximum abutment stress, w_s is the pillar width.

By substituting the maximum and minimum abutment stress values in Tables 7, 8, 9 and 10 into Equation (2), the overburden load of the web pillar in different situations of pillar widths, mining depths, and slope angles can be obtained. Though fitting, the bearing stress of the web pillar changing with slope angle, mining depth and pillar width can be expressed by Equation (3).

$$P = 0.0638\theta + 0.0023L^2 + 0.195W_s^2 \quad (3)$$

where, θ is the slope angle [$^\circ$], L is the mining depth [m], and w_s is the pillar width [m].

Mining activities induce stress redistribution and overburden load concentrating in the coal pillar, forming symmetrical yield zones at both sides of the coal pillar. The yield zone width and the chamber width are represented by x_p and w_m , respectively. The constitutive relation curve of the elastic core zone is different from that of the yield zone [18] (Figure 19), and it shows a linear relation in the elastic core zone. The coal pillar in the elastic core zone has high strength, conforming to the elasticity theory, and has elastic or strain hardening property. However, in the yield zone, the constitutive relation curve is nonlinear and has strain softening property. Once the coal pillar reaches the peak strength, it will release stress quickly and the pillar strength will drop rapidly, resulting in decreasing ability to resist deformation with the growth of the deformation value.

Figure 19 Constitutive relation curves of the coal pillar

The relationship among the stress of the coal pillar (σ), the strain (ϵ) and the damage parameter (D) can be expressed as follows[19]:

$$\sigma = E\epsilon(1-D) \quad (4)$$

where, $D = 1 - \exp\left(-\frac{\epsilon}{\epsilon_0}\right)$, ϵ_0 is the strain variable of the coal pillar under certain load [m], and E is the elasticity modulus of the coal pillar.

In the yield zones, $2x_p$ is the total width, and h is the coal seam thickness. Hence the relation between the bearing stress in the yield zones (P_s) and the deformation value in the zones (u) can be described by Equation (5).

$$P_s = \frac{2x_p Eu}{h} \exp\left(-\frac{u}{u_0}\right) \quad (5)$$

In the equation, u_0 is the deformation value of the coal pillar under the maximum bearing stress, $u_0 = \sigma_c h / E$, [m].

The width of the elastic core zone is $w_s - 2x_p$, which conforms to the elasticity principle, and the corresponding bearing stress in this zone can be deduced by the following equation:

$$P_e = \frac{Eu}{h} (w_s - 2x_p) \quad (6)$$

Then the strain energy (v_s) and the elastic potential energy (v_e) of the coal pillar in the yield zones and the elastic core zone can be achieved by Equation (7) and (8), respectively.

$$V_s = \frac{2Ex_p}{h} \int_0^u \exp\left(-\frac{u}{u_0}\right) du \quad (7)$$

$$V_e = \frac{2E(w_s - 2x_p)}{h} \int_0^u u du \quad (8)$$

The potential energy of the overburden (V_p) is expressed as follows:

$$V_p = (0.0638\theta + 0.0023L^2 + 0.195W_s^2)u \quad (9)$$

Then the total potential energy function in the mechanical model shown in Figure 18 can be calculated by Equation (10).

$$V = \frac{2Ex_p}{h} \int_0^u u \left(-\frac{u}{u_0}\right) du + \frac{2E(w_s - 2x_p)}{h} \int_0^u u du - (0.0638\theta + 0.0023L^2 + 0.195W_s^2)u \quad (10)$$

Here, u is considered as the state variable to carry out analysis on the basis of the cusp catastrophe theory. Take the first derivative of V and set it equal to zero ($V'=0$), then the equation for the equilibrium surface (M) is derived below:

$$V' = \frac{2Ex_p}{h} u \exp\left(-\frac{u}{u_0}\right) + \frac{E(w_s - 2x_p)}{h} u - 0.0638\theta - 0.0023L^2 - 0.195W_s^2 = 0 \quad (11)$$

Equation (11) is the equilibrium condition of the mechanical model. Make the cusp catastrophe model successfully established, take one more derivative of the equilibrium surface equation, and make the second derivative of the equation ($V''=0$) equal to zero, $V''=0$.

$$V'' = 2(w_s - x_p)x_p \frac{E}{u_0} \left(2 - \frac{u}{u_0}\right) + \left(-4w_s x_p + 2w_s x_p \frac{u}{u_0} + 4x_p^2 - 2x_p^2 \frac{u}{u_0}\right) \left(\frac{1}{u_0}\right) e^{-\frac{u}{u_0}} = 0 \quad (12)$$

The meaningful solution of $V''=0$ is obtained, i.e. $u=u_1=2u_0$. The equation of the equilibrium surface (12) is expanded at $u=u_1=2u_0$ according to the Taylor's Formula, and is simplified by taking the cubic term:

$$\frac{4x_p Eu_1 e^{-2}}{3h} \left\{ \left(\frac{u-u_1}{u_1}\right)^3 + \frac{3(u-u_1)}{2u} \left[\frac{(w_s - 2x_p)e^2}{2x_p} - 1 \right] + \frac{3}{2} \left[1 + \frac{(w_s - 2x_p)e^2}{2x_p} - \frac{Phe^2}{2x_p Eu_1} \right] \right\} = 0 \quad (13)$$

Make the dimensionless quantity z be the state variable, and let p and q be the control variables.

$$z = \frac{u-u_1}{u_1}, \quad p = \frac{3}{2}(k_0 - 1), \quad q = \frac{3}{2}(1 + k_0 - t) \quad (14)$$

$$k_0 = \frac{k_e}{k_s} = \frac{E(w_s - 2x_p)/h}{2x_p E e^{-2}/h} = \frac{(w_s - 2x_p)e^2}{2x_p}, \quad t = \frac{he^2}{2x_p Eu_1} (0.0638\theta + 0.0023L^2 + 0.195W_s^2) \quad (15)$$

where, k_e and k_s are the material stiffness in the elastic core zone and in the yield zone respectively. t is the parameters related to highwall mining conditions, that is, it is relevant to mining height, retaining width, mining depth, overburden bulk density, coal deformation parameter and other factors.

According to Equations (13), (14) and (15), the standard equilibrium equation of the cusp catastrophe model is obtained:

$$z^3 + pz + q = 0 \quad (16)$$

Take the derivative of Equation (16) to obtain the singular point equation of the system, as shown below:

$$3z^2 + p = 0 \quad (17)$$

By combining Equation (16) with Equation (17), the bifurcation set equation of the system can be obtained:

$$\Delta = 8p^3 + 27q^2 = 0 \quad (18)$$

By substituting Equation (14) into Equation (18), a simplified expression is achieved:

$$\Delta = 2(k_0 - 1)^3 + 9(1 + k_0 - t)^2 = 0 \quad (19)$$

:

Then Equation (15) and $u_0 = \sigma_c h / E$ are substituted into Equation (19), hence a fitted expression is derived:

$$\Delta = 2 \left[\frac{(w_s - 2x_p)e^2}{2x_p} - 1 \right]^3 + 9 \left[1 + \frac{(w_s - 2x_p)e^2}{2x_p} - \frac{e}{4x_p\sigma_c} (0.0638\theta + 0.0023L^2 + 0.195w_s^2) \right]^2 = 0 \quad (20)$$

$\Delta > 0$ means the system is in a stable state, and $\Delta = 0$ implies the system is in the critical equilibrium state. Only when $\Delta < 0$ can the system across the bifurcation set and fail instantly. Therefore, the sufficient and necessary conditions for the instability catastrophe of coal pillars are shown below:

$$2 \left[\frac{(w_s - 2x_p)e^2}{2x_p} - 1 \right]^3 + 9 \left[1 + \frac{(w_s - 2x_p)e^2}{2x_p} - \frac{e^2}{4x_p\sigma_c} (0.0638\theta + 0.0023L^2 + 0.195w_s^2) \right]^2 < 0 \quad (21)$$

3.2 Yield Zone Width Calculation

In view of the problems existing in the classical elastic-plastic analysis of rock mechanics and based on the limit equilibrium theory, it is further assumed that the coal seam roof and floor have the same lithology and have larger strength than the coal strength [20]. A mechanical model [21-22] of chamber and surrounding rocks is established without considering body force (Figure 20).

Figure 20 Mechanical model of chamber and surrounding rocks

Since the yield zone is the limit equilibrium zone, the shear stress τ and the vertical stress σ_y at the interface between the coal seam roof and floor satisfy the following conditions:

$$\tau = c_0 + \sigma_y \tan \varphi_0 \quad (22)$$

where, c_0 and φ_0 are the internal friction angle and the cohesion of the coal seam interface. The equilibrium equation in the x -direction is established.

$$h\sigma_x + 2\pi dx - h(\sigma_x + \frac{\partial \sigma_x}{\partial x} dx) = 0 \quad (23)$$

According to the Mohr-Coulomb failure criterion, the yield zone of coal body is in the limit equilibrium state, then

$$\sigma_1 = \frac{1 + \sin \varphi}{1 - \sin \varphi} \sigma_3 + \frac{2c \cos \varphi}{1 - \sin \varphi} \quad (24)$$

Due to the symmetry principle of problem, the vertical and horizontal stresses in the x -axis are both the principle stresses. In the equation, $\sigma_1 = \sigma_y$, $\sigma_3 = \sigma_x$, c is the internal friction angle and φ is the cohesion of coal body. Take the differential of Equation (24), the following expression is obtained.

$$\frac{\partial \sigma_y}{\partial x} = \frac{1 + \sin \varphi}{1 - \sin \varphi} \frac{\partial \sigma_x}{\partial x} \quad (25)$$

If $A = (1 + \sin \varphi) / (1 - \sin \varphi)$, then

$$\frac{\partial \sigma_y}{\partial x} = A \frac{\partial \sigma_x}{\partial x} \quad (26)$$

Equation (26) is substituted into Equation (24) to derive the following expression:

$$\frac{\partial \sigma_y}{\partial x} - \frac{2A \tan \varphi_0}{h} \sigma_y = \frac{2c_0 A}{h} \quad (27)$$

By substituting the boundary condition $\sigma_x=0$, and making $N = (2c \cos \varphi) / (1 + \sin \varphi)$, the vertical stress and the horizontal stress in the x -axis of the yield zone can be deduced by Equation (28):

$$\begin{cases} \sigma_y^p = (NA + \frac{c_0}{\tan \varphi_0}) e^{\frac{2A \tan \varphi_0}{h} x} - \frac{c_0}{\tan \varphi_0} \\ \sigma_x^p = (N + \frac{c_0}{A \tan \varphi_0}) (e^{\frac{2A \tan \varphi_0}{h} x} - 1) \end{cases} \quad (0 \leq x \leq x_p) \quad (28)$$

In combination with the stress balance theory, after the chamber excavated, the load imposed on the original coal body along the length of the chamber is transferred to the adjacent coal body. Provided that the vertical stress and the horizontal stress in the x -axis are equal to the average stress along the height of the coal seam, according to the symmetry principle of problem, the following equation can be achieved:

$$\frac{w_m \gamma H}{2} = \int_0^{x_p} (\sigma_y^p - \gamma H) dx + \int_{x_p}^{+\infty} (\sigma_y^e - \gamma H) dx \quad (29)$$

Integrating both sides of Equation (29), if $X = 2A \tan \varphi_0 x_p h$, then $x_p = X / (2A \tan \varphi_0)$, and

$$\frac{w_m \gamma H A \tan \varphi_0}{h} = \left\{ (NA + \frac{c_0}{A \tan \varphi_0}) (X + 1) e^X - 2(\gamma H + \frac{c_0}{\tan \varphi_0}) X - (NA + \frac{c_0}{\tan \varphi_0}) \right\} \quad (30)$$

Through fitting, then

$$x_p = \ln \left[\frac{\frac{2(\gamma H \tan \varphi_0 + c_0)}{(NA \tan \varphi_0 + c_0)} X + \frac{w_m \gamma H \tan^2 \varphi_0}{h(NA \tan \varphi_0 + c_0)} + 1}{(X + 1)} \right] \quad (31)$$

4 Case Study

According to the engineering practice of the open-pit coal mine in Inner Mongolia and the specifications of the highwall miner, the burial depth of the 21[#] coal seam is $H=100\text{m}$, the average bulk density of the overlying strata

is $\gamma=23.6\text{kN/m}^3$, the chamber width is $w_m=2.0\text{m}$, the chamber height is $h=2.5\text{m}$, the mining depth is 100m , the cohesion is $c=c_0=0.30\text{MPa}$, the internal friction angle is $\varphi=\varphi_0=17.6^\circ$, and the compressive strength of the web pillar is $\sigma_c=17.66\text{MPa}$. Substituting these parameters and the depths and retaining pillar widths provided in Table 11 into Equation (31) and Equation (21), the yield zone widths at each side of the web pillars and the instability criteria of the web pillars with different widths under different slope angles and mining depths are derived, as displayed in Table 12. It can be seen that the instability criteria for the retaining pillar widths are all greater than zero ($\Delta>0$), implying the web pillars are in a stable state. Therefore, the designed web pillar widths are proved to be reasonable.

Table 12 Instability criteria for coal pillar widths under different slope angles and mining depths

Slope angle / $^\circ$	Mining depth /m	Retaining pillar width /m	Burial depth /m	Yield zone width x_p /m	Instability criterion Δ
20	50	4.1	18.00	1.11	1143
20	65	4.2	23.40	1.16	1244
20	80	4.3	28.80	1.25	1312
20	100	4.5	36.00	1.37	1530
30	50	4.3	29.00	1.26	1401
30	65	4.5	37.70	1.39	1636
30	80	4.7	46.40	1.50	1917
30	100	5.0	58.00	1.63	2527
40	50	4.5	41.50	1.44	1570
40	65	4.8	53.95	1.59	2033
40	80	5.1	66.40	1.72	2676
40	100	5.5	83.00	1.87	3949
50	50	4.7	59.50	1.65	1536
50	65	5.1	77.35	1.82	2232
50	80	5.5	95.20	1.97	3327
50	100	6.0	100.00	2.00	7005

5 Conclusion

(1) From the modeling results, the maximum abutment stress in the web pillar along the pillar strike direction appears somewhere ahead of the web pillar with the maximum mining depth, and the abutment stress distribution along the dip direction presents approximately a bowl shape. By combining the relationship between the abutment stress and the ultimate strength of the web pillar with the failure characteristics of the yield zone, the instability mechanism of the web pillar is revealed, that is, when the abutment stress of the web pillar is larger than its ultimate strength, shear failure occurs. Based on the characteristics of the abutment stress distribution and the yield zone distribution, the retaining widths of the web pillars are designed and evaluated.

(2) According to the characteristics of the abutment stress distributions of web pillars, a mechanical bearing model of the web pillar is established. Based on the cusp catastrophe theory, a cusp catastrophe model of pillar-overburden is constructed, and the sufficient and necessary conditions for the web pillar instability are derived.

(3) By means of the mechanical model of chamber and surrounding rocks, the formula for calculating the yield zone width at both sides of the web pillar is obtained and can be used in combination with the instability criterion of web pillars to validate the rationality of the designed pillar widths and provide guidance on safety mining and recovery efficiency in open-pit highwall mining.

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References:

- [1] Wang R, Yan S, Bai J B, Chang Z G, Song Y B. Theoretical analysis & study of the destabilization on damaged width of rib pillar in open-pit highwall mining[J]. Rock and Soil Mechanics, 2019,40(08):1-13.
- [2] Song Z L, Fan J F, Qi W H, et al. On the establishment of the evaluation system for green mining in large open-pit coal mines[J]. Journal of Safety and Environment, 2016,41(S2):350-358.
- [3] Bunting D. Chamber pillars in deep anthracite mines[J]. Trans AIME, 1911,42:236-245.
- [4] Greenwald H P, Howarth H C, Hartman I. Experiments on strength of small pillars of coal in the Pittsburgh bed[M]. U. S. Bureau of Mines, Washington, Technical Paper 605, 1939.
- [5] Holland C T, Gaddy F L. Some aspects of permanent support of overburden on coal beds[J]. Proceedings of West Virginia Coal Mining Institute, 1957:43-65.
- [6] Holland C T. The strength of coal in mine pillars, In: Proceedings of the 6th symposium on rock mechanics[J]. University of Missouri, Rolla, MO, 1964:450-456.
- [7] Salamon M D G, Munro A H. A study of the strength of coal pillars[J]. Journal of the South African Institute of Mining and Metallurgy, 1967,68:55-67.
- [8] Bieniawski Z T. The effect of specimen size on compressive strength of coal[J]. International Journal of Rock Mechanics and Mining Sciences & Geomechanics Abstracts 1968,5(4):325-326.
- [9] Sheorey P R. Pillar strength considering in situ stress[J]. Workshop on coal pillar mechanics and design, 1992:122-127.
- [10] Galvin J M, Hebblewhite B K, Salmon M D G. University of New South Wales coal pillar strength determinations for Australian and South African mining conditions[J]. Proceedings of 2nd workshop on coal pillar mechanics and design, 1999:63-72.
- [11] John L. Porathur, S. Karekal, et al. Web pillar design approach for Highwall Mining extraction[J]. International Journal of Rock Mechanics and Mining Sciences, 2013,64:73-83.
- [12] Lukáš Ď, Richard Š. Numerical Analysis of the Stability of Lignite Pillars[J]. Procedia Engineering, 2017,191:310-316.
- [13] Alexandros V D, Andreas B. Evaluating performance of lignite pillars with 2D approximation techniques and 3D numerical analyses[J]. International Journal of Mining Science and Technology, 2017,27:929-936.
- [14] Hikaru S, Yanlong C, Akihiro H, et al. Application of Highwall Mining System to Recover Residual Coal in End-walls[J]. Procedia Earth and Planetary Science 2013,6:311-318.
- [15] Zhang L Y, Deng K Z, Zhu C G, Xing Z Q. Analysis of stability of coal pillars with multi-coal seam strip mining[J]. Transactions of Nonferrous Metals Society of China 2011,21(S3):s549-s555.
- [16] Han X. An unmanned intelligent mining machine: China, 10123925.5[P].2014-06-18.
- [17] Xiong G J. Limit equilibrium theory of axisymmetric based on rigorous conditions and its application calculation of earth pressure[D].Shanghai Jiao Tong University, 2019.
- [18] Guo W B, Deng K Z, Zou Y F.Study on failure and instability of strip coal pillar by catastrophic theory[J]. Journal of China University of Mining & Technology, 2005,34(01):80-84.
- [19] Guo W B, Deng K Z, Zou Y F.Cusp catastrophic model of instability of strip coal pillar along strike[J]. Chinese Journal of Rock Mechanics and Engineering, 2004,23(12):1996-2000.
- [20] Wilson A H, Sun J L. Study on a determining size of the coal[J]. Mine Surveying, 1973,9(01):30-42.
- [21] Gu S C, Fan Q, Liu W. Study on calculating the width of plastic zone in two sides of rectangular coal drift[J]. Mining R D, 2015,35(01):60-63.
- [22] Gu S C, Fan Q, Chen X. A calculation method of plastic zone width for rectangular coal roadway[J]. Coal Safety, 2014,45(09):24-27+31.