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Research Article

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Posted Date: September 2nd, 2021

DOI: https://doi.org/10.21203/rs.3.rs-849511/v1

Version of Record: A version of this preprint was published at The International Journal of Advanced Manufacturing Technology on January 17th, 2022. See the published version at https://doi.org/10.1007/s00170-022-08718-y.

A Tool-path Planning Method Used in Computer Controlled Optical Surfacing Based on Improved Prim Algorithm

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Abstract: In view of the disadvantages of existing planning methods used in CCOS techniques, such as low efficiency and workpieces contain obvious mid-frequency error after polishing, a new tool-path planning method based on improved Prim algorithm was proposed, of which the core idea was consist by following steps: surface data reading, mesh generation, distribution of resident points determining and polishing path generating. After that, comparison of raster path and the path based on improved Prim algorithm was carried out by simulated experiments from aspects of path length and polishing texture. The results indicated that the path based on improved Prim algorithm could shorten path length as well as increase polishing efficiency, moreover, both the texture and mid-frequency errors can be improved by using the path presented. It was concluded that the presented planning method could improve polishing efficiency and machining quality. Then, comparison between raster path and the path based on improved Prim algorithm was carried out by simulated experiments, from two sides of path length and polishing texture. The results indicated that the path based on improved Prim algorithm could shorten path length as well as increase polishing efficiency, moreover, both the texture and mid-frequency errors would be improved by using the presented path. Finally, the validity of presented planning method was proved in machining experiments.

Keyword: CCOS, tool-path planning, improved Prim algorithm, path length, polishing texture.

1. Introduction

Computer controlled optical surfacing (CCOS) was developed in the 1970s as a new type of optical processing technology. It is widely used in aspheric optical lenses fabrication because of its high accuracy [1-4]. The researches of tool-path have important implication for the polishing craft of optical lenses. Reasonable tool-path can reduce processing time and increase polishing efficiency and obtain better surface shape. Two fundamental tool-paths are conventional now in CCOS techniques: raster path and spiral path. The former is in commonly used because it has advantages of simple operation and easy process control. However, there are too many unnecessary routes which increase the processing time and reduce efficiency. The spiral path is easy, stable and efficient, but it is only fit for the fabrication of spherical lens [5-6].

In order to improve the surface quality of workpiece in CCOS techniques, different tool-path optimizations were proposed by researchers. Deng et al. [7] developed a polishing tool-path based on an algorithm named adaptive programming model. It could improve the quality of aspheric surface however polishing efficiency had not been increased. Zhou et al. [8] studied spiral path and put forward a novel path based on uniform-area-increment spiral. The path could avoid the excessive machining in the central region and result in higher precision, but was only fit for the fabrication of spherical lens. Zhao et al. [9] adopted a polishing path generation for physical uniform coverage of the aspheric surface based on the Archimedes spiral in bonnet polishing. The revised path achieves lower roughness and surface tolerance than the traditional Archimedes path, which indicates that the revised path can achieve uniform physical coverage on the surface. Zhang et al. [10] proposed a new polishing path planning method for physically uniform overlap of polishing ribbons instead of traditional geometrically uniform coverage of polishing path on freeform surfaces. The path extraction algorithm (PEA) was given to predict the location of the adjacent polishing path. Proposed polishing path planning was implemented for a typical freeform surface and the polishing paths with physically uniform overlap of polishing ribbons between two adjacent paths were obtained. Han et al. [11] presented a tool paths generation strategy for polishing of freeform surface with physically uniform coverage. This strategy used surface expansion and re-parameterization techniques to avoid edge effect in polishing. The effectiveness and robustness of the developed polishing path generation technique were proved by case studies. Chen et al. [12] developed a trochoidal toolpath for the pad-polishing of freeform surfaces with global control of material removal distribution. The flexibility of a trochoidal toolpath has been applied in the pad-polishing of freeform surfaces using a tilted elastic disk, with global control of material removal distribution over part surfaces. The material removal variation along the guide-line of the trochoidal toolpath is addressed by using optimized trochoidal step and radius, after precisely calculating the effect of overlapped toolpath trajectories. Huang et al. [13] proposed a trajectory planning of optical polishing based on optimized implementation of dwell time. Simulation and experimental results show that the proposed dwell time algorithm and spline interpolation method can considerably improve the solution accuracy of dwell time and the convergence rate of the form error during polishing.

The above several methods were mainly applied to global polishing of workpiece, but they were easy to cause low polishing efficiency in local polishing of workpiece. And there were reports revealed that tool-path had a significant impact on mid-frequency error [14-16]. Regular tool-path will produce polishing texture with regularity and higher mid-frequency error. In this paper, to improve polishing efficiency and polishing texture an optimized strategy including a new tool-path planning method based on improved Prim algorithm is proposed.

2. Tool-path planning based on improved Prim algorithm

The technology of CCOS polishing is to determine material removal amount by using computer to control pressure of polishing position, relative velocity and polishing time. Fig. 1 illustrates process flow chart of CCOS.

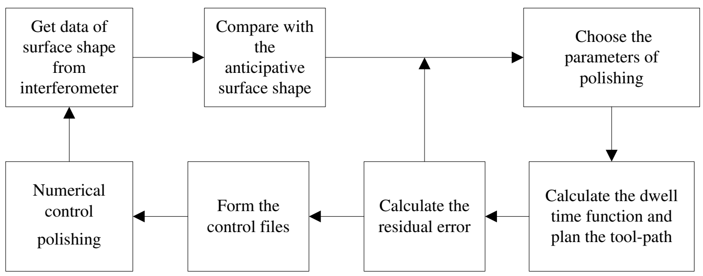


Fig. 1 Process flow chart of CCOS

2.1 Data processing of surface shape

Data processing of surface shape includes surface fitting, mesh generation, generation of eigenvalue and distribution of resident points determining. Reasonable distribution of resident points is the precondition of obtaining reasonable polishing tool-path. Before determining workpiece distribution of resident points, mesh generation was performed firstly. In different stage of polishing, while choosing different polishing tool size, the corresponding grid size is different. With increasing polishing tool size, the gird size increases. For instance, large grid can reduce polishing area and increase polishing efficiency in step of rough polishing. And polishing precision can be guaranteed through making grid size become smaller in step of fine polishing. When mesh generation was performed, the number of data points was set to be same in each grid. In order to express the surface height of each grid, a new parameter was defined, namely, eigenvalue of grid M. Then it is supposed that the number of data points was k in each grid and the values of each data point were m_1, m_2, m_3, ..., m_k respectively. According to the average of data in each grid, the eigenvalue of grid M could be expressed as

$$M = \frac{\sum_{i=1}^{k} m_i}{k} \tag{1}$$

While machining was carried out, resident points were determined by the eigenvalue of grid. And a critical value of processing e had been set at the beginning. The center points of grids were defined as resident points which satisfied condition $M \geq e$. Then distribution of resident points was generated. Therefore, according to this condition, the processing points which were unnecessary would be deleted. It can avoid redundant movement of tool to shorten the path length and reduce processing time thereby increasing polishing efficiency.

2.2 Principle of improved Prim algorithm

Prim algorithm is a kind of minimum spanning tree algorithm. The basic idea of Prim algorithm is that: firstly, all points in the undirected network which is connected by a number of points are divided into two sets: internal set which a point randomly was selected at the beginning and external set which is composed by the rest of the points. Secondly, a point in the internal set and external set are selected respectively to make the edge connecting by the two points be the shortest. And the point in the external set will be added to the internal set. Lastly, repeat until all points in the external set are added to the internal set. The minimum spanning tree is obtained [17,18]. However, the result of Prim algorithm is minimum spanning tree but not a path, thus, improved Prim algorithm is presented in order to find the better polishing path.

The process of improved Prim algorithm is that: the set of resident points R {P_1, P_2, P_3, ...P_n} is obtained according to distribution of resident points, as shown as Fig. 2. And a point P_i is selected from the set R to be the starting point and is deleted from the set R. The nearest point P_j to P_i is found out. If there are points of equal distance, the eigenvalues of the points whose equal distance is equal are compared to find the highest one. They are connected and P_j is defined as the new starting point. Then P_j is deleted from the set R, and the nearest point to P_j is found out in the rest points of the set R. Repeat until all the resident points are connected. The flow chart of improved Prim algorithm is shown in Fig. 3.

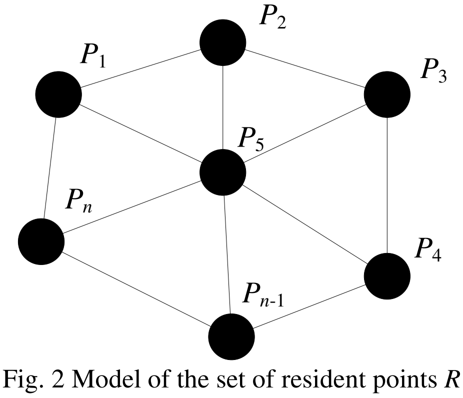


Fig. 2 Model of the set of resident points R

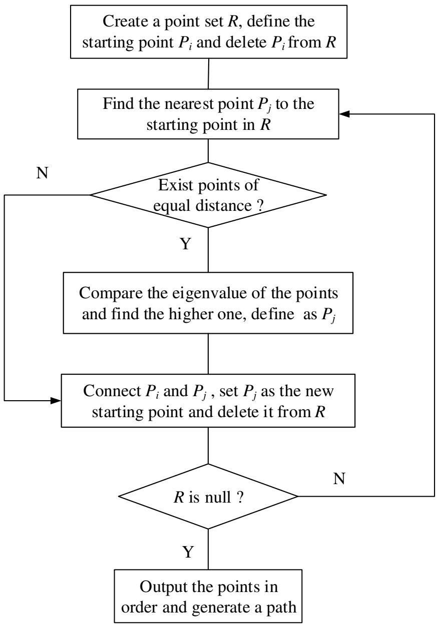


Fig. 3 The flow chart of improved Prim algorithm

The set of resident points R represented the projections of resident points in plane XOY, namely position of resident points. It could be expressed as:

$$P_i = (x_i, y_i), P_j = (x_j, y_j) \tag{2}$$

The distance of any two resident points D_{ij} was given by

$$D_{ij} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} \tag{3}$$

And the distance matrix of any two resident points D was expressed by

$$D = \begin{pmatrix} 0 & D_{12} & D_{13} & \cdots & D_{1i} & \cdots & D_{1n} \\ D_{21} & 0 & D_{23} & \cdots & D_{2i} & \cdots & D_{2n} \\ D_{31} & D_{32} & 0 & \cdots & D_{3i} & \cdots & D_{3n} \\ \vdots & \vdots & \vdots & \cdots & \vdots & \cdots & \vdots \\ D_{i1} & D_{i2} & D_{i3} & \cdots & 0 & \cdots & D_{in} \\ \vdots & \vdots & \vdots & \cdots & \vdots & \cdots & \vdots \\ D_{n1} & D_{n2} & D_{n3} & \cdots & D_{ni} & \cdots & 0 \end{pmatrix} \tag{4}$$

The distance matrix D was a matrix of size $n \times n$. According to flow chart of improved Prim algorithm, the point P_i was set to be starting point and the minimum D_{ij} which is greater than zero in i row and i column was found out. Then P_i and P_j were linked and i row and i column were deleted. The distance matrix of rest resident points D_1 was got. D_1 was expressed by

$$D_1 = \begin{pmatrix} 0 & D_{12} & D_{13} & \cdots & D_{1j} & \cdots & D_{1n} \\ D_{21} & 0 & D_{23} & \cdots & D_{2j} & \cdots & D_{2n} \\ D_{31} & D_{32} & 0 & \cdots & D_{3j} & \cdots & D_{3n} \\ \vdots & \vdots & \vdots & \cdots & \vdots & \cdots & \vdots \\ D_{j1} & D_{j2} & D_{j3} & \cdots & 0 & \cdots & D_{jn} \\ \vdots & \vdots & \vdots & \cdots & \vdots & \cdots & \vdots \\ D_{n1} & D_{n2} & D_{n3} & \cdots & D_{nj} & \cdots & 0 \end{pmatrix} \tag{5}$$

The distance matrix D_1 became a matrix of size $(n\text{-}1)\times(n\text{-}1)$. The point P_j was set to be starting point. Last, repeated until the distance matrix become null matrix. The algorithm was finished to output the points in order and generate polishing path.

2.3 Generating polishing path

Based on the above procedures, polishing path is generated through combining with data processing of surface shape and improved Prim algorithm. Fig. 4 shows the flow chart of generating polishing path. Firstly, data of surface shape was read and mesh generation was performed. According to Eq. (1), the eigenvalue of grid was got to determine distribution of resident points. Secondly, the processing order of resident points was obtained based on Eqs. (2), (3), (4) and (5). Finally, the polishing path was output. These points that were connected would be deleted from the set of resident points by controlling of the improved Prim algorithm. It guaranteed that the path was simply connected to be polishing path.

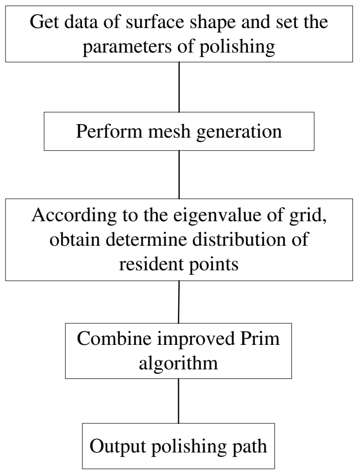


Fig. 4 The flow chart of generating polishing path

3. Path simulation and Comparison Study

In order to confirm the advantages of the improved Prim algorithm, comparison of raster path and the path based on improved Prim algorithm was carried out by simulated experiments from aspects of path length and polishing texture [19]. The chart of surface residual error from interferometer is shown in Fig. 5.

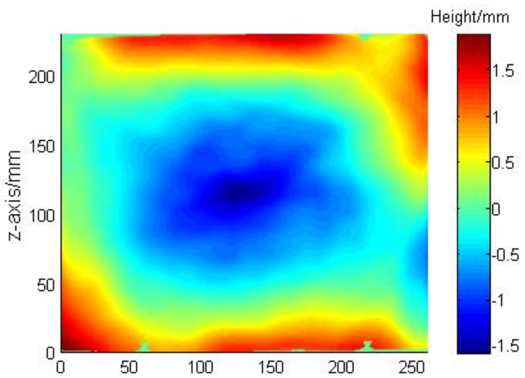


Fig. 5 The chart of surface residual error

3.1 Comparison of path length

Mesh generation was performed by setting the grid size to be $10mm\times10mm$. Raster path and the path based on improved Prim algorithm were obtained respectively through simulation, as shown as

Fig. 6 and Fig. 7. In Fig. 7, the condition of experiment was that the critical value of processing e was set to be zero in experiment, namely the eigenvalue of grid M was equal or greater than zero.

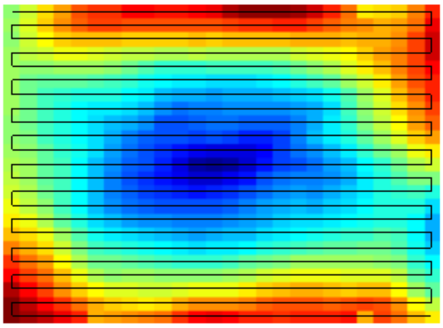


Fig. 6 Raster path

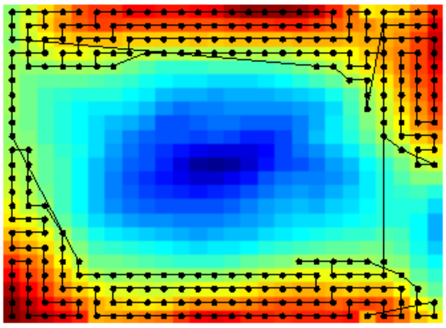


Fig. 7 The path based on improved Prim algorithm

It is easy to find from Fig. 6 and Fig. 7 that the length of raster path is 5970 mm and the length of the path based on improved Prim algorithm is 3513.5mm. The results show that the length of the path based on improved Prim algorithm is shorter than raster path by 41.15%. The path based on improved Prim algorithm can reduce the moving time of tool between resident points. Thus, processing time can be reduced to improve polishing efficiency.

3.2 Comparison of polishing texture

As discussed previously, regularity of the path directly affects polishing texture and has a significant impact on mid-frequency error. Mid-frequency error of workpiece with regular texture is obvious. Rather, the error of workpiece with disorderly and irregular texture is lower. In order to observe polishing texture clearly, simulations of two paths were carried out by intercepting a local part of workpiece whose size was $50mm \times 50mm$ (the range in x direction: 0~50mm, the range in y direction: 0~50mm). Fig. 8 reveals motion of polishing tool in CCOS. The polishing tool was in planetary motion. And the point A was chosen randomly in the polishing tool. The relative linear velocity v of the point A between polishing tool and workpiece was expressed as

$$\begin{aligned} v &= v_1 + v_2 \\ &= \sqrt{v_1^{\ 2} + v_2^{\ 2} - 2v_1v_2\cos(\pi - \beta - \theta)} \end{aligned} \tag{6}$$

where V_1—Revolution linear velocity of polishing tool,

V_2—Rotation linear velocity of polishing tool.

According to geometric relation of planetary motion and velocity formula of the point A, trajectory formula of the point A could be given by

$$r = \sqrt{r_1^{\ 2} + \rho_0^{\ 2} - 2r_1\rho_0\cos\beta} \tag{7}$$

where r_1—Distance between the point A and the center of polishing tool O_2,

ρ_0—Revolution radius.

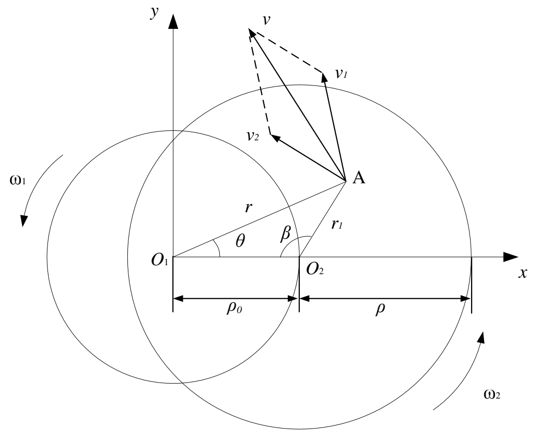
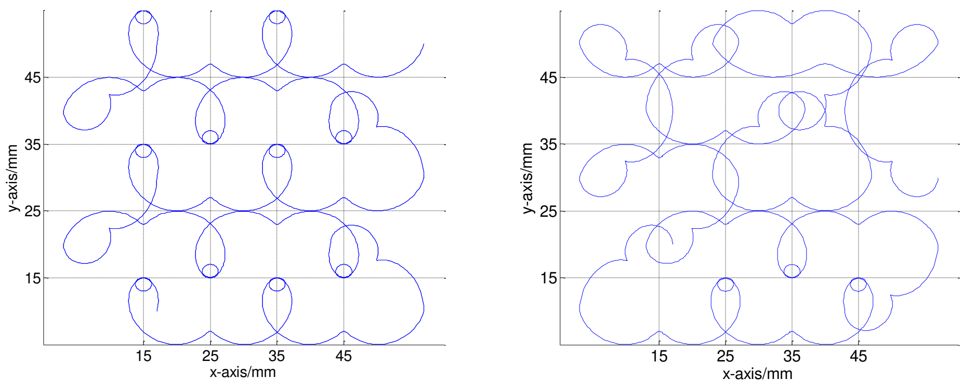


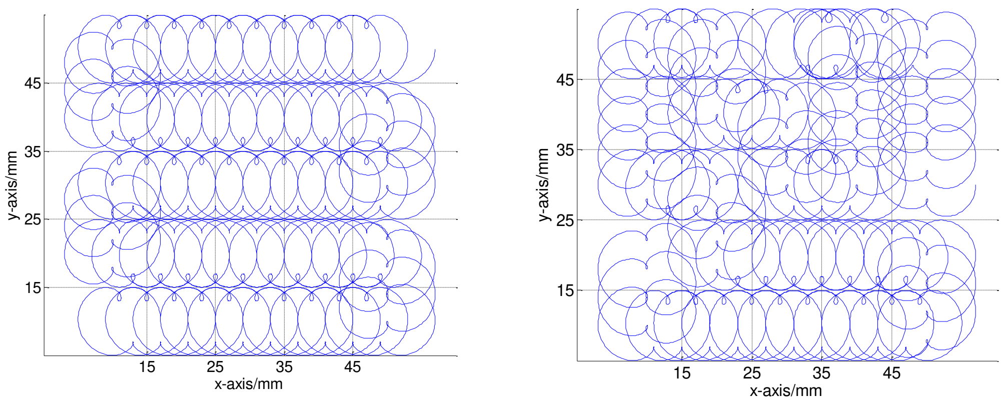
Fig. 8 Motion of polishing tool in CCOS

Polishing texture of the workpiece was reflected by drawing trajectory of the point A. The conditions of experiment were set to be: ρ=6.25mm (ρ is radius of polishing tool), ρ_0=5.0mm, r_1=2.0mm. It is potential that polishing texture will be different when the processing speed is different. The experiment was divided into two conditions: low-speed condition and high-speed condition. And the simulations were carried out for the two paths. Under conditions of the low-speed and high-speed, the trajectories of the point A are shown in Fig. 9 and Fig. 10, respectively. (low-speed: precession speed of polishing tool is 10 mm/min. High-speed: precession speed of polishing tool is 50 mm/min.)



(a) Raster path (b) The path based on improved Prim algorithm

Fig. 9 The trajectories of the point A in low-speed condition



(a) Raster path (b) The path based on improved Prim algorithm

Fig. 10 The trajectories of the point A in high-speed condition

It is discovered from Fig. 9(a) that under the condition of low-speed, the trajectory which the point A moved in accordance with the resident points order of raster path presents regular pattern of reciprocating motion. Similarly, it is shown from Fig. 10(a) that under the condition of high-speed, the point A moving in accordance with the resident point order of raster path, its trajectory presents regularity and definite symmetry. Therefore, the results indicate from Fig. 9(a) and Fig. 10(a) that the trajectories of the point A in raster path present regularity. And the polishing texture of workpiece processed presents regularity and symmetry that makes mid-frequency error after polishing be obvious. It is discovered from Fig. 9(b) that under the condition of low-speed, the trajectory the point A moved in accordance with the resident points order of the path based on improved Prim algorithm trajectory in the bottom area present regular pattern of reciprocating motion, in addition, the trajectory presents irregularity. Similarly, it is shown from Fig. 10(b) that under the condition of high-speed, the point A moving in accordance with the resident point order of the path based on improved Prim algorithm, its trajectory presents irregularity and asymmetry. Therefore, the results indicate from Fig. 9(b) and Fig. 10(b) that the trajectories of the point A in the path based on improved Prim algorithm present irregularity and definite complexity. And workpiece processed in this path gets disorderly and irregular polishing texture, thereby reducing mid-frequency error.

4. Experiment Validation

In order to verify the simulation results, the two polishing paths are successively used to process the same workpiece, and the experimental results are recorded and analyzed. The experiment was carried out with three times of processing. The first processing is used grating path, and the second and third processing are used the path based on the improved Prim algorithm. During the experiment, the path parameters are set unchanged to explore the influence of two paths on the polishing efficiency and the polishing texture of workpiece under the condition of constant parameters. The OP1000 three-axis CNC plane polishing machine and plane optical elements are used in the experiments (optical element material: K9 glass, size: 100mm×100mm×10mm), then the specific experimental steps are shown as follows:

1) The side of workpiece is marked for interferometric measurement, and the surface data of workpiece is read.

2) Reserved edge is set to conduct grid division.

3) The raster path is adopted to process the workpiece, and the workpiece is measured after processing. The processing time is recorded.

4) The corresponding characteristic values of grid according to the previously divided grid are obtained, and processing critical value of the path based on improved Prim algorithm is set. The path based on the improved Prim algorithm is generated according to the measured surface shape results, and is carry out in the second processing. The processing time is recorded.

5) The workpiece is measured after the second processing. A new path based on the improved Prim algorithm according to the surface shape results is generated, and is carry out in the third processing. The processing time is recorded.

The parameters and results of experiment are shown in Table 1(Prim Path represents the path based on the improved Prim algorithm). Figure 11~14 show the corresponding results of experiment in Table 1.

Table 1. The parameters and results of experiment

Serial Number	Processing Path	Grid Size (mm)	Feed Speed (mm/min)	Processing Critical Value (um)	Processing Time (min)	Surface Shape PV (um)
0	Initial surface shape	/	/	/	/	2.07
1	Raster path	10	50	/	14.4	1.68
3	Prim Path	10	50	0.01	12.5	1.28
4	Prim Path	10	50	0.01	12	1.07

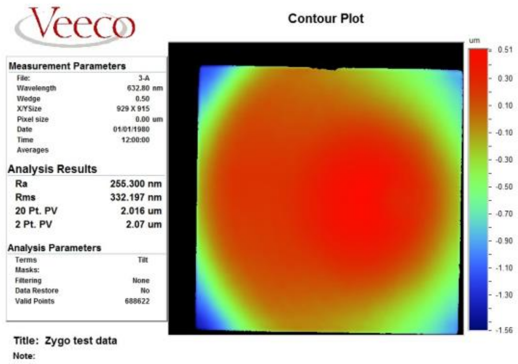
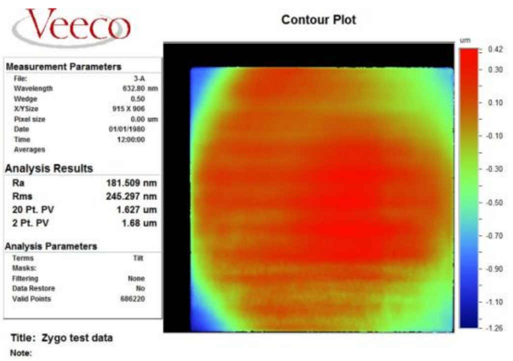
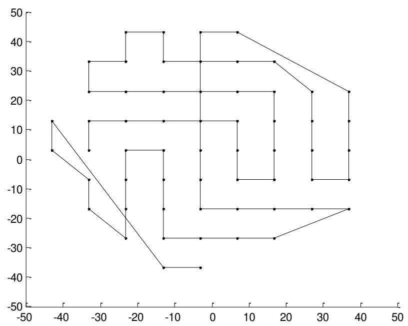


Fig. 10 Initial surface shape of workpiece (PV=2.07um)

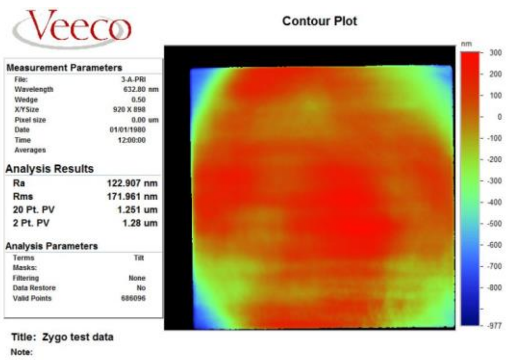


(a) Surface shape of workpiece (PV=1.68um)

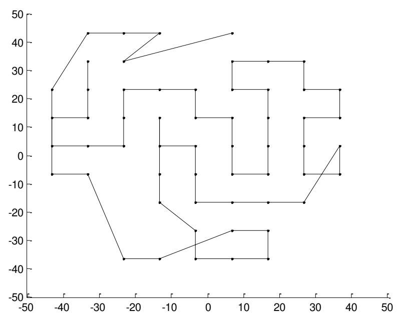


(b) Path based on improved Prim algorithm (after first processing)

Fig. 12 The first processing experiment with raster path



(a) Surface shape of workpiece (PV=1.28um)



(b) Path based on improved Prim algorithm (after second processing)

Fig. 13 The second processing experiment with path based on improved Prim algorithm

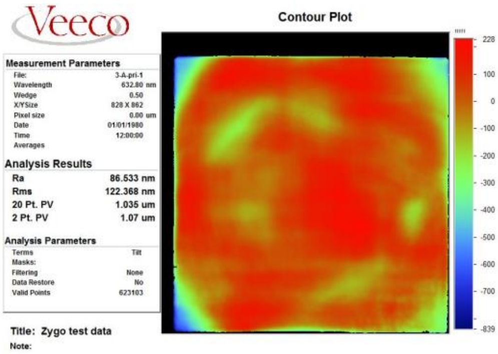


Fig. 14 The third processing experiment with path based on improved Prim algorithm: surface shape of workpiece (PV=1.28um)

According to the results of experiment in Table 1, under the same conditions, the processing time of path based on improved Prim algorithm is shorter than that of raster path. The path based on improved Prim algorithm can improve the polishing efficiency. Fig. 11~14 reflect the polished texture of two polishing paths. Fig. 11 is surface shape of workpiece before processing. The raster path is used for processing and the workpiece surface shape is measured. As can be seen from Fig. 12(a), surface shape of workpiece presents obvious stripe marks and regular texture. Fig. 13(a) is surface shape of workpiece processed by path based on improved Prim algorithm (after first processing). The stripe marks of surface shape are not obvious. Figure 13(b) is path based on improved Prim algorithm (after second processing) generated according to surface shape of workpiece in Figure 13(a). The path based on improved Prim algorithm (after second processing) is adopted to process the workpiece, and stripe marks of surface shape are gradually disappeared, as shown in Figure 14. It can be seen from when the path based on improved Prim algorithm is used for processing, the surface shape PV presents a downward trend and the surface accuracy of workpiece is improved. Then the results of three processing experiments are processed by a Fourier transform to get the spectrum diagram of workpiece, as shown in Figure 15. The intermediate frequency error of workpiece after processing experiment of raster path is mainly contain periodic components which is 9.01 mm, and the intermediate frequency error of workpiece after two processing experiments of Path based on improved Prim algorithm is mainly contain periodic components which is 8.197 mm. It can reduce the intermediate frequency error of workpiece effectively through processing experiments of path based on improved Prim algorithm. Therefore, the experimental results are consistent with the simulation results, which verifies the feasibility of proposed method.

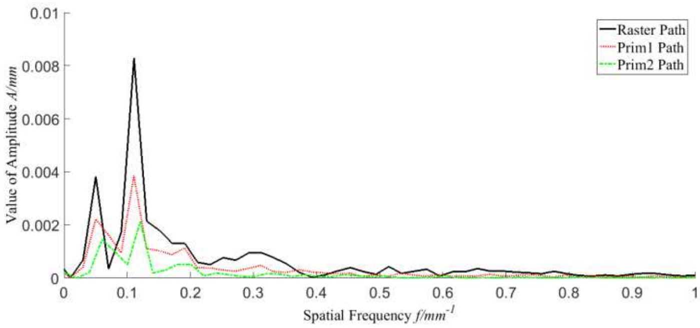


Fig. 15 The spectrum diagram of workpiece after three processing experiments

5. Conclusion

(1) After analyzing the influence of polishing craft for optical lenses that polishing path exerted, an optimized strategy including a new tool-path planning method based on improved Prim algorithm is proposed. Firstly, mesh generation is performed and distribution of resident points is determined according to the eigenvalue of grid. Then improved Prim algorithm is applied to the polishing path planning. It can obtain simply connected path to be polishing path.

(2) Comparison of raster path and the path based on improved Prim algorithm was carried out by simulated experiments from aspects of path length and polishing texture. The results indicate that: from the comparison of Fig. 6 and Fig. 7, the length of the path based on improved Prim algorithm is shorter than raster path by 41.15%. The path based on improved Prim algorithm can reduce processing time and increase the polishing efficiency effectively. And through Fig. 9 and Fig.10 the path based on improved Prim algorithm has been found to be different with raster path in the aspect of polishing texture. The polishing texture of the former present irregularity and disorderliness thereby mid-frequency error can be reduced.

(3) The processing experiments are carried out. The experimental results show that when the path based on improved Prim algorithm is used for processing, the surface shape PV presents a downward trend and the surface accuracy of workpiece is improved. Moreover, it can reduce the intermediate frequency error of workpiece effectively through processing experiments of path based on improved Prim algorithm. Thus, the experimental results are in good agreement with the simulation results, which proves the feasibility of proposed method.

Acknowledgments This work was supported by Laboratory of Robotics and Intelligent Systems (CASQuanzhou) and Scientific and Technological Project of Quanzhou (No.2020C071)

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