Reduced Order Model-based Observer Design for Online Temperature Distribution Estimation in Lithium-ion Batteries

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Abstract Time/space separation-based modeling methods have been widely researched for estimating lithium-ion battery (LIB) thermal dynamics. However, these methods have been developed in an offline environment and may not perform well in real-time application since the battery systems in electric vehicles (EVs) are usually subject to external disturbances. Furthermore, the onboard measurements of temperature are often corrupted by significant error. To address these problems, we present a reduced model-based observer design for online temperature distribution estimation in LIBs. First, an extreme learning machine (ELM) based offline spatiotemporal model is constructed to approximate the thermal dynamics of LIB. Second, an adaptive reduced order observer is designed based on the offline model developed in the previous step. The offline model is then updated with the estimation results of the observer. As the performance of the estimator is highly related to the placement of sensors, a genetic algorithm (GA) based integrated optimization strategy is also developed to determine the optimal sensor location for online estimation. Finally, the whole temperature distribution is estimated in real time using the observer, the measured voltage, current and the limited available temperature data. Simulation results on a battery thermal system verify the effectiveness of this developed model.

Keywords *Spatiotemporal model · Lithium-ion batteries · State observer · Optimal sensor placement · Extreme learning machine*

1 Introduction

With oil prices rocketing and climate change concerns growing, energy and environmental problems due to oil consumption have become a major challenge of our time [1]. To solve this problem, new energy is being developed rapidly [2]. Lithium-ion batteries (LIBs) have been widely used as the power sources for electric vehicles (EVs) and hybrid electric vehicles (HEVs) because of their high energy density, high specific energy, and low environmental pollution [3, 4]. Since the life, efficiency, and safety of the battery all depend on its thermal behavior, an accurate battery thermal management system (BTMS) is indispensable to keep the battery operating within an optimal temperature range [5, 6].

The thermal process inside batteries belongs to a nonlinear distributed parameter system (DPS), which is described in nonlinear partial differential equations (PDEs) [7-10]. The input, output, even the state parameters of DPS can vary in both time and space directions [11, 12]. However, real time measurement of temperature distribution is very challenging because only a limited number of sensors can be placed in batteries. Therefore, a detailed mathematical model that can be used and updated online is required for online monitoring of the temperature distribution. Plenty of researchers have studied the accurate thermal model of LIBs, which can be classified into two parts: lumped thermal models and distributed thermal models.

The lumped thermal models assume that the temperature distribution in LIBs is uniform. That is to say, the temperature only varies in the time direction [13, 14]. Due to the simplicity of such models, they are widely used to characterize the thermal behavior of batteries. There are many studies on lumped models combined with the electrochemical model [15, 16] or equivalent-circuit electrical model [17-18], which have been successfully applied to battery cells or cylindrical batteries. These models are simple enough for temperature control and BTMS design. However, they do not consider the temperature difference in battery cell spaces [20]. To monitor and predict the temperature more accurately, distributed thermal models that can explain the temperature in time and space are needed. Various distributed models to characterize the thermal behavior of LIBs have been studied in recent years [21-24]. As with lumped models, most of the distributed thermal models were combined with an electrochemical model to

explain the heat generation. These models can provide very accurate information for battery design. However, due to the large amount of calculation, they are not suitable for online temperature monitoring and control related applications [25]. Recently, some studies have been done in the construction of reduced order model oriented for BTMS application. Cai and White [26] proposed a reduced electrochemical-thermal model for a LIB cell based on proper orthogonal decomposition (POD), which is also called the Kurhunen-Loève (KL) method. Muratori et al. [27] proposed a reduced order thermal model of prismatic and cylindrical LIB based on Laplace transformation. However, this method is only applicable to line models, and it cannot be used to model nonlinear battery thermodynamics effectively. Liu et al. [28] proposed a novel KL-based time/space separation model for a two-dimensional battery thermal process. An extreme learning machine (ELM) was applied to approximate the heat generation term and produced satisfactory model performance.

Though the above methods are suitable for online applications, they cannot perform in different working conditions, and they are very sensitive to external disturbances. Furthermore, the aging of electrochemical related parameters also leads to the time-varying thermal dynamics feature of LIB [4]. Therefore, the thermal model of LIB must be updated online to adapt to the external disturbances and variation of working conditions with few measured temperatures [29]. Some work has been reported on online thermal model development of batteries. However, most of these models do not take the temperature distribution into account.

To solve the previously mentioned problems, we propose an online estimation scheme with real time updates of the identified offline spatiotemporal model for accurate battery management. The aim of this research is to design a reduced model-based, observer-based online temperature distribution scheme for LIBs. First, a KL-ELM-based offline spatiotemporal model is identified for the battery thermal process. Second, an adaptive reduced order observer is designed based on the offline model. Then, the offline model is updated with the estimation results of the observer. As the performance of the estimator is highly related to the placement of sensors, a genetic algorithm (GA) based integrated optimization strategy is also developed to decide the optimal sensor location for online estimation. Finally, the whole temperature distribution is estimated in real time

Fig. 1 Structural sketch of the lithium-ion batteries

using the observer, input current, the measured voltage, and the limited available temperature data. The proposed method is simple and easy to implement.

The contributions of this work can be summarized as follows:

- 1) A reduced order, model-based observer design scheme is developed for online temperature estimation of the battery.
- 2) A GA-based integrated optimization method is proposed for optimal sensor placement.
- 3) Simulations on a battery verify the effectiveness of the proposed method.

2 Problem description and methodology

2.1 Problem description

The thermal dynamics of the LIB under investigation is a typical nonlinear DPS. Because the heat flux in the z direction is negligible compared to those in the x and y directions of Fig 1, a two-dimensional thermal model is studied here. According to heat transfer law, the fundamental heat transfer equation of a LIB can be described as follows [9, 22, 27]:

$$\rho c \frac{\partial T}{\partial t} = \frac{\partial}{\partial x} \left(k_x \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left(k_y \frac{\partial T}{\partial y} \right) + Q \quad (1)$$

The boundary conditions and the initial condition are described as follows:

$$-k_x \frac{\partial T}{\partial x} \Big|_{x=x,y} = h(T - T_{air}), T_0 = T(x, y, 0) \quad (2)$$

Here, $T(x, y, 0)$ is temperature distribution of the LIB; $\rho(kg/m^3)$ and $c(J/kg^\circ C)$ are density and specific heat of the LIB, respectively; $k_x(W/m^\circ C)$ and $k_y(W/m^\circ C)$ are thermal conductivities along the x and y direction,

respectively; the heat generation term Q is an unknown nonlinear function of input current I and measured voltage V of the LIB; $h(W/m^2\text{°C})$ is the convective heat transfer coefficient on the surfaces of the LIB, and T_{air} is the environment's temperature.

For this type of thermal process, it is difficult to accurately model the temperature distribution due to the following issues:

- 1) Time-varying process: due to the unknown environmental disturbance and battery aging, the thermal process is time-varying, which is hard to model.
- 2) Limited number of sensors: a reduced number of sensors is available for online model adaptation.

Fig. 2 Reduced order model-based observer design scheme

2.2 Methodology

To solve the problems mentioned, the reduced order, model-based observer is designed for estimating the online temperature distribution in LIBs. As shown in Fig. 2, this method contains the following main ideas:

1) Offline spatiotemporal model identification: First, the KL method is applied to calculate the optimal spatial basis functions $\{\varphi_i(x, y)\}_{i=1}^{\infty}$ for time/space separation. Second, with the obtained basis functions, mathematical expression of the reduced order model is derived by Galerkin's method, and the unknown structure and parameters are then determined using the ELM algorithm. Finally, the offline spatiotemporal model is reproduced using time/space reconstruction.

2) Online observer design: First, an adaptive reduced order observer is designed based on the obtained offline spatiotemporal model, and the stability is assured by Lyapunov stability theory. Second, the GA-based integrated optimization method is developed for optimal sensor placement. Finally, the real-time temperature distribution is predicted using the observer, input current, measured voltage, and the limited available temperature data.

3. Offline spatiotemporal model identification

According to the theorem of separation of variables, the spatiotemporal variable can be approximately separated into the following time/space decoupled form:

$$T_n(x, y, t) = \sum_{i=1}^n \varphi_i(x, y) a_i(t) \quad (3)$$

where $\varphi_i(x, y)$ is the spatial basis function; $a_i(t)$ is the corresponding low order temporal model, and n is the model order. Since $\varphi_i(x, y)$ can be estimated using KL decomposition [26], the next stage is to estimate the expression of the temporal model $a_i(t)$. Substituting (3) into the physics-based model (1), the equation residual can be described as follows:

$$R = \frac{\partial T_n}{\partial t} - (k_0 \frac{\partial^2 T_n}{\partial x^2} + k_1 \frac{\partial^2 T_n}{\partial y^2} + k_2 F(T_n) + k_2 Q_n(x, y, t)) \quad (4)$$

With Galerkin's method [30], a discrete form of (4) can be written in the following form:

$$a_1(k+1) = \sum_{j=1}^n k_{ij} a_j(k) + \bar{k}_i q_i(k) \quad (5)$$

where k_{ij} and $\bar{k}_i$ are constants. Then (5) can be described in a component form as:

$$a(k) = K_1 a(k-1) + K_2 q(k-1) \quad (6)$$

$$T(x, y, k) = C a(k) \quad (7)$$

where: $a(k) = [a_1(k), \dots, a_n(k)]^T$, $K_1 = \{k_{ij}\}_{n \times n}$,

$K_2 = \text{diag}(\bar{k}_1, \dots, \bar{k}_n)$, $q(k) = [q_1(k), \dots, q_n(k)]^T$, $C = [\varphi_1(x, y), \dots, \varphi_n(x, y)]$.

The unknown nonlinear function $q(k)$ in (6) can be approximated using a single, hidden layer feedforward neural (SLFN) network. Then (6) can be expressed as follows:

$$a(k) = K_1 a(k-1) + K_2 \sum_{p=1}^N \beta_p G(\omega_p \cdot z(k-1) + \eta_p) \quad (8)$$

where N is the number of hidden neurons of the SLFN; β_p is the output weights connecting the corresponding hidden neurons and the output neuron of the

networks; ω_p is the input weights connecting the corresponding hidden neurons and the input neurons; η_p is the threshold of corresponding hidden neurons, and $G(\cdot)$ is the activation function of the hidden layer, $z(k) = [I(k), V(k)]^T$.

The value of corresponding temporal coefficients $a(k)$ corresponding to the spatiotemporal measurements $\{T(x, y, k)\}$ can be obtained from the following equation [27]:

$$a_i(k) = \langle \varphi_i(x, y), T(x, y, k) \rangle \quad (9)$$

Then the model parameters in equation (6) can be identified with a set of data $\{z(k), a(k)\}_{k=1}^{n_t}$.

Model (8) can be expressed in linear regression form as:

$$a(k) = H(k-1)\theta \quad (10)$$

where: $H(k-1) = [a(k-1), G(\omega_1 \cdot z(k-1) + \eta_1), \dots, G(\omega_N \cdot z(k-1) + \eta_N)]$, $\theta = [K_1, K_2\beta_1, \dots, K_2\beta_N]^T$ is the parameters' vector to be estimated.

Here, ELM algorithms [31] will be introduced to identify the unknown parameters that exist in (10). With ELM, ω_p and η_p are randomly generated independent of the training data and each other. Once they are estimated, the value is fixed in the following learning procedure.

Equation (8) can be written in matrix form as:

$$a = H\theta \quad (11)$$

where $a = [a(2), a(3), \dots, a(n_t)]^T$, and $H = [H(1), H(2), \dots, H(n_t-1)]^T$. Then the parameters θ can be calculated analytically as:

$$\hat{\theta} = H^\dagger a \quad (12)$$

where $H^\dagger$ is the Moore-Penrose generalized inverse of matrix H .

While the reduced order model (8) is trained very well, the offline spatiotemporal model of the battery thermal process can be reconstructed using time/space synthesis form as in equation (3).

4. Online observer design

For the model developed in the previous section to be updated in real time to adapt to the variation of working environments and external disturbances, we propose an adaptive observer based online temperature estimation scheme for

online battery temperature distribution estimation with the help of a few sensors. As shown in Fig. 2 (b), based on the offline spatiotemporal model obtained in Section III, the online estimation scheme includes two steps. The first step is the reduced observer design, and the second step is the sensor location design.

With the feedback of the error between the estimated and online measured temperature, the observer can be designed based on the identified offline spatiotemporal model, as:

$$\hat{a}(k+1) = f(\hat{a}(k), I(k), V(k)) + L(T_m - \hat{T}_m) \quad (13)$$

$$\hat{T}_m(k) = C_m \hat{a}(k) \quad (14)$$

where T_m is the real time temperature data; m is the number of online sensors and $m \ll M$; $M = n_x \times n_y$ is the number of offline sensors that collect temperature distribution data for offline spatiotemporal model identification; $\hat{T}_m$ is the reconstructed temperature field with the estimated $\hat{a}(k)$. L is the gain matrix of the observer determines the rate at which $\hat{a}(k) \rightarrow a(k)$. C_m is of the following form:

$$C_m = \begin{bmatrix} \varphi_1(X_1) & \cdots & \varphi_n(X_1) \\ \vdots & \ddots & \vdots \\ \varphi_1(X_m) & \cdots & \varphi_n(X_m) \end{bmatrix} \quad (15)$$

The performance of the observer is highly related to both the gain matrix L and the placement of the sensors m . In this paper, the choice of the sensor placement m is integrated with the design of the observer for online temperature distribution estimation. The goal is to find the appropriate choice of m and value of L simultaneously so that the following objective function $J(m, L, \hat{T})$ can be minimized:

$$\text{Min} J(m, L, \hat{T}) = \frac{1}{2} \|\hat{T}(X, k) - T_m(X, k)\|_Q \quad (16)$$

Subject to:

$$\begin{cases} \hat{a}(k+1) = f(\hat{a}(k), I(k), V(k)) + L(T_m - \hat{T}_m) \\ \hat{T}_m(k) = C_m \hat{a}(k) \end{cases}$$

where Q is a positive definite weighting matrix.

The integration cost function is a nonlinear constrained optimization problem. It is very hard and complex to optimize the sensor placement parameter m and the

observer gain matrix L together using analytical optimization methods. To solve this problem, a hierarchical optimization framework [32] is adopted here to decompose the integration optimization problem into two nested steps: embedded observer design (inner loop) and master sensor placement optimization (outer loop). In the inner loop, an adaptive observer based on the offline spatiotemporal model is designed. In the outer design loop, a GA-based algorithm is used to optimize the sensor placement problem. A detailed description can be found in the following subsection.

4.1 Adaptive observer design

Given the designed sensor number and location, the observer is designed to guarantee stability and convergence. With the ELM-based low-order model developed in Section III, the observer can be designed as:

$$\hat{a}(k) = K_1 \hat{a}(k-1) + g(I(k-1), E(k-1)) + L(T_m - \hat{T}_m) \quad (17)$$

$$\hat{T}_m(x, y, k) = C_m \hat{a}(k) \quad (18)$$

where: $g(k) = [g_1(k), g_2(k), \dots, g_n(k)]^T$

The gain L is chosen for $K_1 - LC_m$ in order to be stable. The proof is present as follows.

With the estimation error $e_k = a(k) - \hat{a}(k)$ and the measurement error $e_T = T_m(k) - C_m \hat{a}(k) = C_m e_k$ the following can be obtained:

$$\begin{aligned} e_k &= a(k) - \hat{a}(k) \\ &= a(k) - \left(K_1 \hat{a}(k-1) + g(I(k-1), E(k-1)) + L(T_m - \hat{T}_m) \right) \\ &= (K_1 - LC_m)e_{k-1} = \omega e_{k-1} \end{aligned} \quad (19)$$

Consider the following Lyapunov function:

$$V_k = e_k^T e_k \quad (20)$$

Then:

$$\begin{aligned} \Delta V_k &= V_k - V_{k-1} = e_k^T e_k - e_{k-1}^T e_{k-1} \\ &= e_{k-1}^T \omega^T \omega e_{k-1} - e_{k-1}^T e_{k-1} \\ &= e_{k-1}^T (\omega^T \omega - 1) e_{k-1} \end{aligned} \quad (21)$$

$$\begin{aligned}
&= (\|\omega\|^2 - 1)e_{k-1}^T e_{k-1} \\
&= (\|\omega\|^2 - 1)(\|e_{k-1}\|^2)
\end{aligned}$$

According to equation (21), $\Delta V_k < 0$, when $\|\omega\| < 1$; therefore, the output error e_T will converge to zero.

Fig. 3 GA-based optimal sensor location decision

4.2 GA-based optimal sensor location decision

As shown in Fig. 3, the optimal sensor locations are decided by solving the cost function, where the observer gains have been decided in the previous part. This optimization problem is difficult to solve by analytical methods since it may be non-convex and not differential with respect to the design variable. Here, a genetic algorithm (GA) based method is proposed to solve the optimization problem.

GA is derived from the mechanisms of natural selection and nature genetics. By searching for the fitness through an iterative procedure, the optimal (or sub-optimal) solution can be found without calculating the derivatives of the function to be optimized. The GA manipulation is based on a population of individuals, and each individual represents a possible solution of the optimization problem. In the GA-based optimal sensor placement design, it is assumed the number of online measurements m is fixed. The same data used for offline model identification can be used for sensor location design. Each population consists of M elements where m elements are given 1 and others are given 0. The procedures for implementing the GA algorithm are summarized as below:

Step 1: Population initialization: generate a random population of n individuals.

Step 2: Fitness calculation: evaluate the fitness of each individual by the cost function $J(m, L, \hat{T})$

Step 3: New population generation: create a new population by repeating the following process:

1) Selection: select the parent individuals for crossover according to their fitness value: the individual with the best fitness is preserved and used to replace the individual with the worst individual; other individuals are selected using roulette wheel selection.

2) Crossover: two randomly selected individuals from the parents will be crossed over with a certain crossover probability to generate the next generation.

3) Mutation: to maintain the number of sensors, nd randomly chosen elements are mutated, with nd denoting the difference between the number of sensors in the new generation (indicated as 1 in individuals) and the number of sensors m predesigned for online estimation.

Step 4: Repeat steps 2 and 3 until a stopping criterion is met. Then, the best solution in the current population is returned.

5. Simulations verification

5.1 Physical model

To evaluate the effectiveness of the proposed online temperature estimation strategy, simulations and comparisons are conducted in this section. The two-dimensional thermal model proposed by Kim et al. [21] is simulated for model verification. In their model, the heat generation term is described as follows:

$$Q = aJ \left[E_{oc} - (V_p - V_n) - T \frac{dE_{oc}}{dT} \right] + a_p r_p i_p^2 + a_n r_n i_n^2 \quad (22)$$

where a is the specific area of battery; J is the current density; V_p and V_n are the potentials of the positive and negative electrodes; a_p and a_n are the specific area of battery positive and negative electrodes; and i_p and i_n are linear current density (Am⁻¹) in the positive and negative electrodes, respectively. The last two terms in (22) are the resistive heating due to current movement in the positive and negative metal, respectively. The detailed description can be expressed as follows:

Fig. 4 Input signals for model identification: **(a)** input current, **(b)** measured voltage

$$\nabla^2 V_p(x, t) = -r_p i_p \quad (23)$$

$$\nabla^2 V_n(x, t) = r_n i_n \quad (24)$$

$$J = Y(V_p - V_n - U) \quad (25)$$

where the resistance r (r_p or r_n) is calculated as:

$$r = \frac{1}{h_c S_c + h_e S_e} \quad (26)$$

Y and U were written as functions of battery depth of discharge (DOD) following Gu's approach as:

$$U = a_0 + a_1(DOD) + a_2(DOD)^2 + a_3(DOD)^3 \quad (27)$$

$$Y = a_4 + a_5(DOD) + a_6(DOD)^2 \quad (28)$$

$a_0 \sim a_6$ are constants. The value of other parameters can be seen in Kim et al. [21]. The physical model was simulated using the finite element method conducted in COMSOL software. The data collected from COMSOL can be employed for model estimation and verification of the developed online modeling strategy.

5.2 Model estimation and verification

With the known physical model of a battery, appropriate input signals must be designed to fully excite the state of charge and thermal dynamics in the entire working range, and then informative output temperature samples can be collected for model identification. In this paper, the input current was selected using random input signals as shown in Fig. 4(a). The corresponding measured voltage is shown in Fig. 4(b), which is also used as the input signals of the proposed

Fig. 5 Temperature distribution at different times for model identification: (a) 60s, (b) 180s, (c) 240s, (d) 300s

Fig. 6 Predicted errors of the ELM based spatiotemporal model using training data at different times: (a) 60s, (b) 180s, (c) 240s, (d) 300s

model. In total, 300 data were collected from COMSOL with sampling interval $\Delta t = 1s$. A total 15×20 thermocouples were evenly placed along the x and y

Fig. 7 Optimized sensor location for online estimation

Fig. 8 Temperature distribution at different times for model

directions, which were used for temperature data collection. The temperature distribution at 60s, 180s, 240s, and 300s are presented in Fig. 5.

With the collected temperature data, the KL method is first applied to calculate the dominant spatial basis functions for time/space separation, where the order of basis functions is selected as 5. Then the low-order temporal model described in (8) is determined. Finally, the spatiotemporal dynamic of LIB can be reconstructed based on the obtained spatial basis functions and determined low-order temporal model. The predicted errors of the ELM-based spatiotemporal model corresponding to the actual temperature data in Fig. 5 are shown in Fig. 6.

Fig. 9 Prediction error of proposed model with sensor location optimized on the testing data at different times: (a) 60s, (b) 180s, (c) 240s, (d) 300s

Fig. 10 Prediction error of proposed model with evenly placed sensors on the testing data at different times: (a) 60s, (b) 180s, (c) 240s, (d) 300s

Obviously, the derived ELM-based spatiotemporal model can approximate the thermal behavior of the original system very well.

When the ELM-based spatiotemporal model is trained properly, a state observer as described in section IV can be designed. Four sensors were used for online temperature data collection. The GA-based optimal sensor placement strategy described in section IV. B is used to design the sensor location for online estimation. The optimization result is shown in Fig. 7, where the crossed points denote the obtained sensor location for online estimation through optimization. The circle points denote the sensors for model validation that will be used later. To test the performance of the observer, another input current $I(t) = -25 + 10\sin(t - 0.2)$ is given to the thermal process. Under this designed input current, 300 temperature measurements are collected for model verification. Temperature data at different times are shown in Fig. 8.

For comparison, an observer with the same structure while the four sensors are averagely distributed is also established, and the prediction errors of the ELM-based observer with optimized sensor placement and evenly placed sensors on the testing data at the corresponding time in Figure 6-16 is shown in Fig. 9 and Fig. 10 respectively.

For clearer comparison, the thermal dynamic at two different locations (as shown in Fig. 7) of the measured value and the estimation results of the observer are shown in Fig. 11 and Fig. 12, respectively. For performance comparison, the relative results of the observer with the four averaged averagely distributed sensors and results from the offline model with no sensors are also shown. In Fig. 11 and Fig. 12, the solid line represents the estimated result of the observer with the optimized sensor location; the dash dot line represents the estimation result of the observer with averagely distributed sensors, and the dashed line represents the estimation result of the observer with averagely distributed sensors.

The temporal normalized absolute error (TNAE) results, which are defined as follows, is also shown in Fig. 13.

$$TNAE = \frac{1}{n_t} \sum_{t=1}^{n_t} |e(x, y, t_i)| \quad (29)$$

where $e(x, y, t_i)$ is the predicted error.

From Fig. 7 to Fig. 13, it is apparent that the observer can perform well under the simulated working conditions. The performance of the observer with

optimized sensor location is better than both the performance of the observer with averagely distributed sensors and the performance of the offline model.

Fig. 11 Thermal dynamic at point 1 for performance comparison: (a) temperature evolution, (b) predicted errors

Fig. 12 Thermal dynamic at point 2 for performance comparison: (a) temperature evolution, (b) predicted errors

Fig. 13 Performance comparison in terms of TNAE

6. Conclusion

In this paper, a systematic online estimation scheme for temperature distribution in LIBs was presented. First, an ELM-based spatiotemporal model was developed to approximate the thermal behavior. Second, an adaptive observer was designed for the determined ELM-based spatiotemporal model. Finally, the spatiotemporal

temperature distribution of the battery was estimated online using the designed observer, input current, the measured voltage, and limited temperature values. As the performance of the estimator is highly related to the placement of sensors, a GA-based integrated optimization method was also proposed to decide the optimal sensor location for online estimation. The effects of the sensor location and the observer gain matrix were both considered in designing the sensor location. Numerical simulations and comparisons demonstrated the effectiveness of the proposed method.

Declarations

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Conflict of interest

The authors declare that they have no conflict of interest.

Availability of data and material

All data generated during the study are available from the corresponding author by request.

Code availability

Custom code

Compliance with ethical standards

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