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Abstract

In a supply chain, inventory is the single largest source of costs for a company. This is due to the various physical and informational activities that accompany inventory management, primarily the holding and transportation of inventory. Companies are looking to streamline these activities and minimize the associated costs. One of the most coveted models for jointly solving these two problems is the Inventory Routing Problem (IRP), which will be the focus of this study. This paper deals with the case of a deterministic replenishment demand in a distribution network consisting of a supplier and a number of customers to be served by a single vehicle over a finite planning horizon. We study the impact of increasing supplier delivery times on the network costs. A mathematical model is developed and solved by an exact method. The results obtained will be analyzed and solutions will be suggested.

Keywords: Inventory Routing Problem · Lead-Time · Exact methods · Optimization · Supply Chain Network ·

1 Introduction

Logistics is a constantly evolving concept because of its direct impact on business growth. The ultimate goal of companies is to control the various costs of the supply chain, namely the costs of ordering, transportation, storage and return of stock. Indeed, logistics allows for better cost management by analyzing, planning, and anticipating any event that may occur during the supply process. One of the most well-known logistics problems is the Inventory Routing Problem (IRP). This problem has been the subject of much research over the past decade. It combines two important supply chain activities, namely inventory management and product transportation.

The first definition of PIT was proposed by Bell et al. (1983). These researchers viewed IRP as a process of periodically distributing a single product from a supplier to a set of customers with deterministic demand, using an infinite and homogeneous fleet of vehicles. The vehicles start and end their rounds at the supplier's site. Then, the IRP was defined by Federguen & Zipkin (1984) as an extension of the classical vehicle routing problem (VRP) while controlling the inventory level at different customers and suppliers. This problem consists of making simultaneous vehicle routing and inventory management decisions. Since its appearance, many studies have been done and simple and efficient mathematical formulations have been created to solve the IRP problem.

Among the IRP models existing in the literature, we find the model implemented by Archetti et al (2007) which consisted of an IRP where a supplier must replenish a set of customers over a time horizon. The objective of the model was to manage the inventory at the supplier and the different customers in the network, determine the best replenishment routes for all customers as well as the appropriate quantities to transfer from the supplier to the customers and the assignment of customers to specific delivery periods. The model considers a single product in a multi-period scenario and only one vehicle of a given capacity is available. The transportation and the inventory holding costs are proportional to the distance traveled and the inventory level at the customer and the supplier. The considered replenishment policies were the order up to level (OU) policy and the maximum level (ML) policy. Customer demands are deterministic and independent. Customer orders are managed by the supplier using the Vendor Management Inventory (VMI) approach.

Coelho et al. (2012) on the other hand considered a multi-period IRP. They formulated an IRP, with and without consistency requirements and developed a metaheuristic called Adaptive Large Neighborhood Search (ALNS) enhanced. These requirements are used not only to provide cost-effective solutions to their customers, but also to provide high-quality service to gain a competitive advantage. They considered six different consistency characteristics in IRP solutions (quantity consistency, vehicle fill rate, OU policy, driver consistency, partial driver consistency, visit spacing). They also analyzed the effect of different inventory policies, routing decisions, and delivery sizes. To evaluate the performance of their algorithm, they used a multitude of benchmarks among which are the instances of Archetti et al. (2007), starting with the use of a single vehicle and then multiple vehicles serving the same network structure.

In a very recent work, Amri-Sakhri et al. (2021) studied a joint supply chain problem dealing with transportation and inventory level control. Their problem was a combinatorial decision problem since the vehicle routing and inventory problems were solved jointly. Their IRP aimed to determine the best set of customers visited in each period, the delivery quantity for each customer, and the network routes to use in each replenishment period, so that total inventory and transportation costs were minimized. They used the classical genetic algorithm (GA) to solve their problem, and then used different genetic crossover operators to test their performance and impact on the result obtained and determine the most appropriate operator to incorporate in their GA.

In another work, Amri-Sakhri et al. (2022) studied the same network structure proposed in Amri-Sakhri et al. (2021) to solve a multi-period IRP problem. They used a new metaheuristic known as memetic algorithm (MA) which combines GA and Variable Neighbors Search methods. The proposed algorithm has been tested on different benchmarks including that of Archetti et al. (2007). Most of the MA results were able to provide optimal solutions for small instances and improve the quality of solutions for large instances.

In this paper, the problem addressed is first the replenishment of a number of customers from a supplier. The supplier has a stock capable of covering the total demand of the different customers in the network. Deliveries are made by a single vehicle of limited capacity and over a periodic planning horizon. Customers cannot be out of stock. Next, we will modify our model by extending the supplier's delivery time. In the developed model, the supplier imposes specific lead times for replenishment. The objective of this work is to determine the impact of the variation of the replenishment lead time and to minimize the holding and transportation costs of the inventory under the new constraints. The main contributions of this paper can be summarized in the following points:

- Proposal of a new supply chain model considering customer demands, transportation and inventory holding costs, supplier lead time variation in a periodic planning horizon,
- During the replenishment period, customers are visited only once by the supplier's vehicle to deliver a quantity of stock capable of covering a horizon longer than a single period,
- Optimize the total cost of the network, i.e., the costs of holding and transporting inventory through the various nodes of the supply chain under new, more complex constraints,
- Solve the problem using an exact predefined method.

The rest of the paper is organized as follows. In the next section, we formally describe the studied problems and present the mathematical formulation of the developed IRP. Then, in the third section, we introduce the exact method used to solve our model and we present the experimental results of our computational tests, analyze them and propose some alternatives to avoid cost increase. The last section concludes our work and gives some perspectives for future research.

2 Problem definition and IRP model

In this paper, we study a supply chain composed of a supplier and a set of customers. Customers have a deterministic and independent demand for one product per period. The planning horizon under consideration covers three replenishment periods. Our study is in discrete time. The idea of this work is to study the effect of the variation of the lead-time (L) on a multi-period IRP.

In what follows, we begin by exposing the IRP model that uses an L equal to one delivery period. Then, we extend the formulation of the proposed model by increasing the L to two periods and study the effects on the network costs. Finally, we insert the LT technique into the non-replenishment period. Indeed, this problem has

not been sufficiently explored in previous research. But there are other important issues in the literature, closely related to our studied IRP, which will be mentioned later.

2.1 The proposed IRP model

Our problem can be defined as a Mixed Integer Programming (MIP) model in the sense that the supplier needs to define inventory levels alongside with routing decisions. The problem can be represented by a graph $G(N, E)$, where $N = N' \cup \{0\}$, N' is a set of customer nodes ($N' = \{1, \dots, n\}$) and 0 is the supplier node, and E is the corresponding set of arcs. The following notation is used to formulate the proposed model.

2.1.1 Indices

t : Period index ($t = 1, \dots, T$)

i, j : supplier or customer index, where $i, j = 1, \dots, N - 1$ represent customer

N : the set of nodes, in which 0 represents the supplier

2.1.2 Parameters

C_{ij} : cost of edge (i, j) traversed by the vehicle v at the period t

I_i^0 : the initial inventory level at the customers i

I_i^i : the initial inventory level at the supplier

U_i : maximum inventory level at the customers i

M_i : minimum inventory level at the customers i

d_i^t : customer i demand at the period t

Q : the capacity of the vehicle

r_0 : the quantity of products available at the supplier

r_i : the quantity of products consumed by customer i

p : the quantity of products transported from the supplier to the customers

h_i : the inventory holding costs at each node $i \in N$ per unit and per period

2.1.3 Decision variables

$X_{ij}^t = \begin{cases} 1 & \text{if the edge } (i, j) \in E \text{ is traversed by the supplier's vehicle at the period } t \\ 0 & \text{otherwise.} \end{cases}$

$Y_{iv}^t = \begin{cases} 1 & \text{if the customer } i \in N^0 \text{ is visited by the supplier's vehicle at the period } t \\ 0 & \text{otherwise.} \end{cases}$

I_i^t : the inventory level at node $i \in N$ at the period t

Z_i^t : the quantity carried by the supplier's vehicle after visiting the node i , in each period t

2.2 Formulation

2.2.1 First model formulation

In this part, we provide a mathematical formulation for the single vehicle IRP. The model is formulated as follows: The objective function aims to minimize the total operational costs that are the sum of the inventory costs at the supplier, the inventory costs at the customers and the costs of the routes over the time horizon:

$$\text{Min} \sum_{t=1}^T h_0 I_0^t + \sum_{t=1}^T \sum_{i=1}^{N'} h_i I_i^t + \sum_{t=1}^T \sum_{i=1}^N \sum_{j=1}^N c_{ij} X_{ij}^t \quad (1)$$

The constraint of the supplier inventory level at the period t :

$$I_i^t = I_i^{t-1} - r_0^t + p_0^t \quad \forall t \in T \quad (2)$$

Stock-out constraint at the depot: This constraint guarantees that for each time $t \in T$ the inventory level at the depot is enough to ship the total quantity delivered to the customers:

$$I_0^t \geq \sum_{i=1}^{N'} d_i^t \quad \forall t \in T \quad (3)$$

The constraint of the customer inventory level at the period t :

$$I_i^t = I_i^{t-1} - r_i^t + d_i^t \quad \forall i \in N'; \forall t \in T \quad (4)$$

This constraint enforces the inventory level to stay between lower bound and upper bound:

$$U_i \geq I_i^t \geq M_i \quad \forall i \in N'; \forall t \in T \quad (5)$$

This equality guarantee that the quantity delivered to customers from the supplier corresponds to the total demand.

$$p = - \sum_{i=1}^{N'} d_i^t \quad \forall t \in T \quad (6)$$

The following constraints ensure the OU policy requirements imposing that, if a customer is visited, the quantity delivered is such that the maximum inventory level is reached:

$$d_i^t \geq U_i Y_i^t - I_i^{t-1} \quad \forall i \in N'; \forall t \in T \quad (7)$$

$$d_i^t \leq U_i Y_i^t \quad \forall i \in N'; \forall t \in T \quad (8)$$

Capacity constraint at the customers, this constraint guarantees that for each customer $i \in N$ the maximum quantity U_i is never exceeded:

$$d_i^t \leq U_i - I_i^{t-1} \quad \forall i \in N'; \forall t \in T \quad (9)$$

The vehicle capacity constraint:

$$\sum_{i=1}^{N'} d_i^t \leq Q \quad \forall t \in T \quad (10)$$

The next constraint ensures that a single vehicle is available:

$$\sum_{i=0}^N Y_{i0}^t \leq 1 \quad \forall t \in T \quad (11)$$

The flow conservation constraint:

$$\sum_{i=0}^N Y_{ij}^t = \sum_{i=0}^N Y_{ji}^t \quad \forall j \in N'; i \neq j; \forall t \in T \quad (12)$$

Sub-tour elimination constraints for each vehicle route at each period t :

$$Z_i^t - Z_j^t + QX_{ij}^t \leq Q - d_j^t \quad \forall i, j \in N'; i \neq j; \forall t \in T \quad (13)$$

$$d_i^t \leq Z_i^t \leq Q \quad \forall i \in N'; \forall t \in T \quad (14)$$

Non-negativity and integrality constraints:

$$X_{ijv}^t \in \{0, 1\} \quad \forall i, j \in N'; i \neq j; \forall t \in T \quad (15)$$

$$X_{0j}^t \in \{0, 1, 2\} \quad \forall j \in N'; \forall t \in T \quad (16)$$

$$Y_i^t \in \{0, 1\} \quad \forall i \in N'; \forall t \in T \quad (17)$$

$$d_i^t = 0 \quad \forall i \in N'; \forall t \in T \quad (18)$$

$$Z_i^t \geq 0 \quad \forall i \in N'; \forall t \in T \quad (19)$$

2.2.2 Second model formulation

We conserve the constraints of the first model for the delivery periods $t = 1$ and $t = 3$, only the quantity transferred constraint (6) will be replaced by the constraints (20) to (22). This transformation is the consequence of the L modification at the supplier. In this model, no replenishment is done when $t = 2$. The constraints (20) to (22) will ensure the stability of the model and avoiding stock out in the second period $t = 2$.

$$p \leq - \sum_{t=1}^2 \sum_{i=1}^{N'} d_i^t \quad (20)$$

$$p = 0 \quad (21)$$

$$p \leq - \sum_{i=1}^{N'} d_i^3 \quad (22)$$

3 Computational Experiments

3.1 Implementation and Benchmarks

We implement our problem using CPLEX 12.5.1.0. and run on a PC with a 2.40 GHz Intel Core i7-5500U CPU @ processor and 8 GB of RAM. To evaluate the performance of the selected solver, we used instances of the benchmark defined for the single vehicle IRP created by Archetti et al (2007). The characteristics of these instances are described, briefly, as follows:

- Time horizon $T = \{3\}$;
- Number of customers: 10 sets of small instances, each set consisting of 5 instances of 5, 10, 15, ..., 50 customers;
- Product quantity r_i consumed by customer i and randomly generated as an integer number in the interval $[10, 100]$;
- Product quantity r_0 made available at the supplier during the period t : $\sum_{i=1}^{N'} r_i$;
- Maximum inventory level U_i at customer i , it is given by $r_i g_i$ with $g_i \in \{2, 3\}$ and represents the number of time units needed to consume the quantity U_i ;
- Starting inventory level I_0^0 at the supplier: $\sum_{i=1}^{N'} U_i$;

- Starting inventory level I_i^0 at the customer i : $U_i - r_i^t$;
- Inventory cost at customer $i \in N'$ is h_i : randomly generated in the intervals $[0.01, 0.05]$;
- Inventory cost at the supplier $h_0 = 0.03$;
- The number of vehicles is equal to $v = 2$. The capacity of the first vehicle is given by: $\frac{3}{2} * \sum_{i=1}^{N'} r_i$ and the capacity of the second vehicle is equal to $\sum_{i=1}^{N'} r_i$;
- Transportation cost $C_{ij} : [\sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}]$, where the points (x_i, y_i) and (x_j, y_j) are obtained by randomly generating each coordinate as an integer number in the interval $[0, 500]$;
- Replenishment policy: The order up to level (OU) policy, in which every visit to a customer brings its inventory to the maximum level (ML), more details on the OU policy in Bertazzi et al. (2002).

3.2 Computational Results

In this study, the maximum level of the reconstituted customer stock can cover for some customers only two consumption periods and three periods for the others, which can be explained by the parameter $g_i \in \{2, 3\}$, (subsection 5.1). This information has limited the scope of our model, which can only be tested in the case of $L > 3$. In their work, Archetti et al (2007) solved a multi-period IRP in the case of $L = 1$. Here, we will extend L to two periods, for the same instances. Table 1 gives the average results of the multi-period IRP with $L = 1$ and $L = 2$, tested on the small instances of Archetti et al (2007) using the same resolution algorithm of Branch & Cut (B&C), each instance row corresponding to the average results of five instances with the same number of customers. The first column is linked to the set of instances. The second column gives the average results provided in Archetti et al (2007) for a single vehicle IRP with $L = 1$. Columns 3 to 6 show the results of our experiments on the model IRP with $L = 2$. Column 3 gives the average results of the developed problem. Column 4 gives the average computing time “CPU” (in seconds) required to obtain these results. Column 5 indicates the average percent gap between the cost of the incumbent solution (upper bound) and the cost of the best LP relaxation, when the optimal solution is not obtained within the limits of the system memory. Finally, column 6 shows the average variation rate in the total cost of the supply network, ranging from an IRP with $L = 1$ to an IRP with $L = 2$.

Table 1 Average solution values for the single vehicle IRP with $L = 1$ and $L = 2$.

Model	Archetti et al. [17]	Developed IRP ($L=2$)			%Evaluation
Instance	z_1	z_2	CPU (s)	%GAP	z_1 to z_2
Small-5	1418,76	1791,83	0,46	0,00	26,30
Small-10	2228,67	2566,23	10,17	0,00	15,15
Small-15	2493,47	3157,80	580,40	0,00	26,64
Small-20	3053,02	3944,57	1993,59*	4,89	29,20
Small-25	3451,15	4320,23	4158,36*	8,10	25,18
Small-30	3643,22	4453,52	1764,39*	3,93	22,24
Small-35	3846,87	5022,88	2005,02*	5,97	30,57
Small-40	4125,70	5245,79	1967,78*	5,49	27,15
Small-45	4270,61	5599,97	1439,82*	6,91	31,13
Small-50	4810,87	6267,08	1458,13*	11,00	30,27

*: *aborted prematurely with out of memory status*

Looking at the results in the table above, we see that the variation of L affects all parameters of the replenishment chain and leads to a considerable increase in the total cost of the network. Customers try to replenish their inventory to cover their demand during periods of no replenishment, which has a direct effect on the increase in inventory movements and cost of ownership. The number of customers with priority demand will change. As a result, the transport map will change, which should include more customers in the first and third delivery periods. This variation has a visible influence on the objective function which has increased well, and we find that in all cases when we increase the L of delivery, the total cost of the distribution network also increases, with an average of 26.38%. The performance of the CPLEX solver decreases when the number of clients increases, and the results obtained correspond more and more to the upper bounds of the solutions and not to the optimal solutions.

Indeed, CPLEX has its limits which depend on the growth of the instances and the number of variables to be treated. In some cases, the upper bound will be considered due to the computational requirements of the CPLEX processor which are excessively long for problems with more than 20 clients. However, CPLEX determines the optimal solution values in 16 instances. In 14 instances, the solver obtains results with an optimality deviation of

less than 6%. For other instances (up to 50 clients), the maximum percentage of error between the optimal solutions and the obtained solutions (upper bound) does not exceed 11%. Despite these discrepancies, the results obtained prove the advantage of using the B&C method applied to IRP with $L = 2$, as a viable alternative for obtaining excellent quality results for small instances.

4 Conclusion

In this paper, we studied a multi-period IRP, in which we aimed to satisfy customer demands and find the optimal amount of product to ship in each delivery period. We also tried to find the best delivery routes by an optimization process for each period under several constraints. In this context, we set up a mathematical model to allow us to examine the impact of the increase of the suppliers' delivery time on the supply chain network. We found that moving from periodic replenishment to longer replenishment lead time increases the overall logistics costs of the network.

To solve the problem of the increase in total network cost due to the variation in delivery time, several techniques have been proposed in the literature that can be combined with our IRP to give new variants capable of improving our results. Among these solutions, we find the Collaborative Vehicle Routing Problems (CVRP), where collaboration occurs when two or more actors in the supply chain share their resources and/or some information to serve the same final customers, Bahinipati and Deshmukh (2012). The most well-known VRPCs are the Vehicle Routing Problem with Satellite Facilities (VRPSF), Jezic et al. (2015) and the Vehicle Routing Problem with Lateral Transshipment (VRPLT), Amri-Sakhri et al. (2021).

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