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Rui Yu (✉ yurui1983@foxmail.com)

Wuhan University <https://orcid.org/0000-0002-6633-2337>

Ziyin Song

Wuhan University

Tianyu Wu

Wuhan University

Wenquan Wu

Wuhan University

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Experimental realization of non-Abelian gauge field in circuit system

Ziyin Song*, Tianyu Wu*, Wenquan Wu*, Rui Yu†

¹*School of Physics and Technology, Wuhan University, Wuhan 430072, China.*

Synthetic gauge field, especially the non-Abelian gauge field, has emerged as a new way to explore exotic physics in a wide range of materials and platforms. Here we present the building blocks, consisting of capacitors and inductors, to implement the non-Abelian tunneling matrices and show that circuit system is an appropriate choice to realize the non-Abelian gauge field. To demonstrate the novel physics enabled by the non-Abelian gauge field, we provide a simple and modular scheme to design the Rashba-Dresselhaus spin-orbit interaction and topological Chern state in circuits. By measuring the spin texture and chiral edge states of the resonant frequency band structures, we confirm the spin-orbit effect and topological Chern state in circuits. Our schemes open a broad avenue to study non-Abelian gauge field and related physics in circuit platform.

Gauge field is a key concept in modern physics. It plays an essential role in high-energy physics, electromagnetism, condensed matter physics and many other disciplines. Gauge field is classified into Abelian and non-Abelian types according to whether their associated symmetry groups are commutative or not. The non-Abelian gauge field is characterized by non-commutative matrix type gauge potentials, which generate novel physics that is distinctly different from the Abelian case. The remarkable experimental progress of the past decade has opened a window into realizing the non-Abelian gauge field in a wide variety of physical systems, including cold atoms¹⁻⁵, photons⁶⁻¹¹, polaritons¹², and mechanical systems¹³. Many novel physical phenomena, such as the quantum anomalous Hall effect¹⁴, topological insulators^{5,11}, non-Abelian monopoles¹⁵ and real-space non-Abelian Aharonov-Bohm effect^{8,10} have been realized in the above systems. Despite these advances, the hunt for easily fabricated non-Abelian gauge field systems is still an essential research direction.

Circuit, consisting of lumped-parameter devices, is a system whose properties are determined by the device parameters and the connection topology of the electrical network. As the wires may connect devices via a bridge structure, this property allows ones to design electrical networks with braided structures in real space and obtains matrix type tunnelings between devices, which enables the circuit to be an ideal platform for the study of non-Abelian physics. Recently, many cir-

*These authors contribute equally to this work.

†yurui@whu.edu.cn

cuit systems have emerged, concerning the implementation of topological insulator and semimetal states^{16–22}, high-order topological states^{23,24}, and high-dimensional topological states^{25–27}. Most of these work exploit networks with capacitors and inductors to achieve topological properties protected by crystal symmetry or time-reversal symmetry. However, very few studies have been done on Chern insulator, which is an important type of topological state that does not require any crystal symmetry or time-reversal symmetry. Because the circuit network consisting of capacitors and inductors inherently possesses time-reversal symmetry, a circumspect design is required to break this symmetry and simultaneously open a topological energy gap to obtain the Chern state. Based on the Haldane model, two theoretical work has proposed the implementation of the topological Chern circuit using negative impedance converters and Hall resistors^{21,22}. Despite these two theoretical proposals, no synthesis and observation of the Chern circuit have been implemented experimentally yet.

In this paper, we present theoretical and experimental schemes for implementing non-Abelian gauge field in circuits. We first present the circuit modules that produce the Pauli-matrix-form tunneling matrices. Then, using these circuit modules, we experimentally realize two non-Abelian circuit systems. The first one implements the Rashba-Dresselhaus spin-orbit interaction (SOI) in a two-dimensional square lattice, which is time-reversal invariant. The second one applies active device, the integrator operational amplifier, to the first system in order to break the time-reversal symmetry, offering a non-Abelian route to realize the topological Chern state. These two examples demonstrate that circuit system could provide a flexible, modular, and highly controllable playground to investigate the non-Abelian gauge field and related remarkable physics.

We start with the Yang-Mills Hamiltonian with non-Abelian gauge field²⁸

$$H_{YM} = \frac{1}{2m}[(p_x + A_x)^2 + (p_y + A_y)^2], \quad (1)$$

where m is the effective mass of carrier and the gauge field component A_x and A_y are Hermitian matrices. Assuming that the vector potentials take the form of $A_x = (\alpha' + \beta')\sigma_x$ and $A_y = (\alpha' - \beta')\sigma_y$, which do not commute with each other. Substituting $A_{x(y)}$ into Hamiltonian (1), we get the standard Rashba-Dresselhaus spin-orbit interaction (SOI)^{29,30}

$$H = \frac{p^2}{2m} + \alpha(p_x\sigma_x + p_y\sigma_y) + \beta(p_x\sigma_x - p_y\sigma_y) + const., \quad (2)$$

where $\sigma_{x(y)}$ are Pauli matrices acting on the pseudo spin, $p^2 = p_x^2 + p_y^2$. $\alpha = \alpha'/2m$ and $\beta = \beta'/2m$ are the Rashba and Dresselhaus SOI constants, respectively. Above derivations explicitly show the mapping between non-Abelian gauge field and SOI. By applying a pseudo-spin rotation $\sigma_x \rightarrow -\sigma_y$ and $\sigma_y \rightarrow \sigma_x$, eq. (2) is equivalent to the Rashba SOI $H_R = -\alpha(p_x\sigma_y - p_y\sigma_x)$ and the Dresselhaus

SOI $H_D = -\beta(p_x\sigma_y + p_y\sigma_x)$ that have been used in much literature. In this paper, we investigate the implementation of the non-Abelian gauge field of the Rashba-Dresselhaus SOI type in the circuit.

We now outline the experimental scheme, based on capacitors and inductors, which generates effective non-Abelian gauge field in circuit lattice. The scheme is a generalization of the ones proposed in Refs.^{16,17} to realize quantum spin Hall states. As schematized in fig. (1 a), m and n cells consist of inductors L which are connected head-to-tail and form a loop. While the C cell consisting of capacitors connects the nodes in m and n cells. According to Kirchhoff's current law, the admittance equation of the circuit in fig. (1 a) is given as (see Supplementary Information)

$$\begin{pmatrix} i_m \\ i_n \end{pmatrix} = \begin{pmatrix} \frac{1}{j\omega L} \mathbf{L} - j\omega C \mathbf{C}_{mm} & j\omega C \mathbf{C}_{mn} \\ j\omega C \mathbf{C}_{mn}^T & \frac{1}{j\omega L} \mathbf{L} - j\omega C \mathbf{C}_{nn} \end{pmatrix} \begin{pmatrix} v_m \\ v_n \end{pmatrix} \quad (3)$$

where ω is the frequency of the AC signal, $j = \sqrt{-1}$, m and n are cell indexes, $i_{m(n)} = (i_{m(n)1}, i_{m(n)2}, \dots, i_{m(n)N})$, $v_{m(n)} = (v_{m(n)1}, v_{m(n)2}, \dots, v_{m(n)N})$. $i_{m(n)\tau}$ is the sum of the currents flowing into node $m(n)\tau$, $v_{m(n)\tau}$ is the voltage at node $m(n)\tau$, $\tau = 1, 2, \dots, N$, and N is the number of nodes in the inductor cell. Matrix $\mathbf{L}$ in eq. (3) characterizes the admittance matrix of inductors in m and n cells, which is given as $\mathbf{L} = (P + P^{-1} - 2I_N)$, where I_N is the N -by- N unit matrix, P is the permutation matrix for permuting 1 to 2, 2 to 3, ..., $N-1$ to N , and N to 1. Matrix $\mathbf{C}_{mn}$ describes the connection configuration of capacitors in C cells.

The current conservation condition requires the sum of currents entering each node in the circuit is zero, i.e. $i_{m(n)\tau} = 0$. Therefore, eq. (3) can be formulated in the form of a tight-binding Hamiltonian (see Supplementary Information)

$$\begin{pmatrix} -\mathbf{C}_{mm} & U \mathbf{C}_{mn} U^\dagger \\ U \mathbf{C}_{mn}^T U^\dagger & -\mathbf{C}_{nn} \end{pmatrix} \begin{pmatrix} \tilde{v}_m \\ \tilde{v}_n \end{pmatrix} = \frac{\mathbf{\Lambda}}{\omega^2 L C} \otimes I_2 \begin{pmatrix} \tilde{v}_m \\ \tilde{v}_n \end{pmatrix}, \quad (4)$$

where matrix U is chosen to diagonalize the matrices P and $\mathbf{L}$, $\mathbf{\Lambda} = U \mathbf{L} U^\dagger$, $\tilde{v}_{m,n} = U v_{m,n}$, and the eigenvalue ω characterizes the resonant frequencies of the circuit system. It is interesting to note that, by designing the connection configurations in C cells, the tunneling matrices in terms of Pauli matrices can be obtained. Below, we discuss the case in which the inductor cell contains three nodes. The case with more nodes is discussed in the supplementary information. For three nodes case, following the derivation given above, we get (see Supplementary Information) $P = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$, $U = \frac{1}{2\sqrt{3}} \begin{pmatrix} 2 & 2 & 2 \\ -1-j\sqrt{3} & -1+j\sqrt{3} & 2 \\ -1+j\sqrt{3} & -1-j\sqrt{3} & 2 \end{pmatrix}$, and $\mathbf{\Lambda} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -3 \end{pmatrix}$. If $\mathbf{C}_{mn} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$, i.e., the capacitors are connected as depicted in the upper portion of fig. (1 c), we obtain $U \mathbf{C}_{mn} U^\dagger = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$. These

Figure 1: **The building blocks for generating non-Abelian gauge potential.** (a) Schematic diagram of the inductor cells m, n and the capacitor connection cell C . In cell m and n , inductors are connected head-to-tail and form a loop structure. Capacitors in cell C connect the nodes in cells m and n . (b to e) The designed capacitor cells that give the $\pm\sigma_{0,1,2,3}$ types of tunneling matrices.

results show that for the last two two-fold degenerate states in Λ , they have a matrix operator σ_1 mixing their internal degrees of freedom while tunneling from cell m to cell n . Employing the same method, the modules C which enable $\pm\sigma_{0,1,2,3}$ types of tunneling are designed as depicted in fig. (1 b to e). These modules are non-commutative with each other and constitute the building blocks for our implementation of the non-Abelian gauge field.

With these building blocks in hand, we now present the scheme for implementing the circuit system with Rashba-Dresselhaus SOI type non-Abelian gauge field. As shown in fig. (2), we choose the $\sigma_x(\sigma_y)$ modules as the nearest neighbor connections in the $x(y)$ direction and the σ_0 modules as the next nearest neighbor connections in the x and y directions. An advantage of the circuit system is demonstrated here, namely the ability of connecting devices at any distance by wires, which is difficult to be achieved in rigid materials. For the two-dimensional circuit network given in fig. (2 a), using similar method applied to derive eq. (3) and eq. (4), we obtain Hamiltonian

$$(h_1(\mathbf{k}) \oplus H_{RD}(\mathbf{k}))(\tilde{v}_1, \tilde{v}_2, \tilde{v}_3)^T = \frac{-3}{\omega^2 LC}(0 \oplus I_2)(\tilde{v}_1, \tilde{v}_2, \tilde{v}_3)^T, \quad (5)$$

where $\oplus$ stands for matrix direct sum, $h_1(\mathbf{k}) = 2(\cos k_1 + 3 \cos k_2 + \cos 2k_1 + \cos 2k_2) - 13$, $H_{RD}(\mathbf{k}) = d_0(\mathbf{k})\sigma_0 + d_1(\mathbf{k})\sigma_1 + d_2(\mathbf{k})\sigma_2$, $d_0(\mathbf{k}) = 2 \cos 2k_1 + 2 \cos 2k_2 - 13$, $d_1(\mathbf{k}) = 2 \cos k_1$,

Figure 2: The Rashba-Dresselhaus spin-orbit interaction circuit. (a) Schematic and circuit diagram of the two dimensional Rashba-Dresselhaus SOI circuit. The green (yellow) blocks are the nearest neighbor connection modules $+\sigma_{1(2)}$ in the $x(y)$ direction. The purple blocks are the next-nearest neighbor connection module $+\sigma_0$ in x and y directions. Their configuration details are given in fig. (1). The cyan blocks are the inductor cells, where each node connects to ground by a capacitor with the same capacitance as the capacitors used in the $\sigma_{0,1}$ modules. (b) Resonant frequency band structures of the circuit. The Rashba-Dresselhaus type splitting is present at $(\pm\pi/2, \pm\pi/2)$ points. (c) Printed circuit board layout of the Rashba-Dresselhaus SOI circuit. The red dashed box indicates a unit cell. The green, orange and purple dashed box correspond to the σ_1 , σ_2 , and σ_0 modules, respectively. (d) The experimentally measured frequency dispersions and spin texture compared with the theoretical results. The red (blue) blocks are the experimentally measured spin in the positive (negative) x -direction. The red (blue) dashed-lines are results calculated from model Hamiltonian $H_{RD}(\mathbf{k})$. The k -line passes through the $(\pi/2, \pi/2)$ point along the k_1 direction.

and $d_2(\mathbf{k}) = 2\sqrt{3}\cos k_2$. We are interested in $H_{RD}(\mathbf{k})$, which presents Rashba-Dresselhaus SOI type frequency band structures near $(\pm\pi/2, \pm\pi/2)$ points as shown in fig. (2 b). Around these four points, the low-energy properties are accurately described by a Rashba-Dresselhaus SOI Hamiltonian. For example, expanding $H_{RD}(\mathbf{k})$ near $(\pi/2, \pi/2)$ point and keeping to the first order of $\mathbf{k}$, we obtain $H(\mathbf{k}) = -17\sigma_0 + \alpha(k_1\sigma_1 + k_2\sigma_2) + \beta(k_1\sigma_1 - k_2\sigma_2)$, where $\alpha = -\sqrt{3} - 1$ and $\beta = \sqrt{3} - 1$ are the Rashba and Dresselhaus SOI constants, respectively. In fig. (2 a), we connect the nodes in the inductor cells to ground with capacitors, which do not affect the hopping matrices but shift the frequency of $h_1(\mathbf{k})$ and $H_{RD}(\mathbf{k})$. The printed circuit board fabricated according to the schematic diagram is depicted in fig. (2 c). The experimentally measured resonant frequency band structures and the spin texture (red and blue color blocks) compared to the results derived from model Hamiltonian $H_{RD}(\mathbf{k})$ (dashed-lines) are shown in fig. (2 d). The shape of the bands, as well as the spin texture, fits well with the theoretical results. However, the tolerances and the internal resistance of the devices result in a certain broadening and about $0.1 \times 10^5 \text{Hz}$ shift of the band structures. Up to now, we have synthesized the Rashba-Dresselhaus SOI in the circuit lattice. This scheme can be easily generalized to 1D or 3D lattice for the study of other types of SOI and related novel physics.

We now turn to discuss the implementation of topological circuit with a non-zero Chern number based on the Rashba-Dresselhaus SOI circuit. The model Hamiltonian $H_{RD}(\mathbf{k})$ is time-reversal invariant, i.e. $\hat{T}H_{RD}(\mathbf{k})\hat{T}^{-1} = H_{RD}(-\mathbf{k})$, where $\hat{T} = \sigma_1 K$ and K is the complex conjugation operator. The frequency band structures of Hamiltonian $H_{RD}(\mathbf{k})$ cross at $\pm(\pi/2, \pi/2)$ points and have no gaps in the Brillouin zone. Our strategy to realize the Chern state is to introduce a proper term which is able to break the time-reversal symmetry and open a topological gap. We choose a $d_3(\mathbf{k})\sigma_3$ term with $d_3(\mathbf{k})$ being an odd function of $\mathbf{k}$ to break the time-reversal symmetry. Together with the requirement of opening a topological gap, we design a circuit illustrated in fig. (3 a, b). The main device we use here is the integrator operational amplifier, which is connected in series with a capacitor and then in parallel with an inductor. The circuit depicted in fig. (3 a, b) leads to a similar Hamiltonian as given in eq. (5), with $h_1(\mathbf{k}) = 9j/(2CR\omega) + 8\cos k_1 + 12\cos k_2 - 23$ and a new term $d_3(\mathbf{k})\sigma_3$ in the two-by-two block Hamiltonian (see Supplementary Information) $H_C(\mathbf{k}) = d_0(\mathbf{k})\sigma_0 + d_1(\mathbf{k})\sigma_1 + d_2(\mathbf{k})\sigma_2 + d_3(\mathbf{k})\sigma_3$, where $d_0(\mathbf{k}) = -23$, $d_1(\mathbf{k}) = 2\cos k_1$, $d_2(\mathbf{k}) = 2\sqrt{3}\cos k_2$, $d_3(\mathbf{k}) = -3\sqrt{3}/(2CR\omega) + 2\sqrt{3}\sin k_1 + 2\sqrt{3}\sin k_2$. The resonant frequency of $h_1(\mathbf{k})$ is calculated as $\omega_1 = 9j/(2CR(23 - 8\cos k_1 - 12\cos k_2))$. The real part of ω_1 is zero and the imaginary part is positive. Therefore the corresponding voltage $ve^{j\omega_1 t}$ and current $ie^{j\omega_1 t}$ decay exponentially with time. In Hamiltonian $H_C(\mathbf{k})$, $d_3(\mathbf{k})$ is a function of ω . The eigenvalues ω can be obtained by solving $\text{Det}[\sum_{i=0}^3 d_i(\mathbf{k})\sigma_i + 3/\omega^2 LC\sigma_0] = 0$. In order to determine the topological nature of Hamiltonian $H_C(\mathbf{k})$, we calculate the Chern number by the Wilson loop method. The

Figure 3: The topological Chern circuit. (a) Schematic and circuit diagram of the two dimensional Chern circuit. The cyan blocks $L \parallel \int$ are the inductor modules, which are detailed in (b). The green (yellow) blocks are the nearest neighbor connection module in the $x(y)$ direction and the gray blocks are the nearest neighbor connection module in x and y directions. Their configuration details are given in fig. (1). (b) Schematic of a cyan cell in a. The integrator operational amplifier is marked by a green dotted box. The integrator is serially connected with a red capacitor (the capacitance is three times the capacitance of the black capacitor in the integrator) and then connected in parallel with the inductor L . Each node has a grounded resistor, which has twice the resistance compared to the resistors in the integrator. (c) Printed circuit board layout of one unit cell of the topological Chern circuit. The devices in the blue dashed box correspond to $L \parallel \int$ module given in fig. (b). The green (yellow) dashed box correspond to the $\sigma_1 - i\sqrt{3}\sigma_3$ ($\sigma_2 - i\sqrt{3}\sigma_3$) module in fig. (a). (d) The calculated Wilson loop of the model Hamiltonian $H_C(\mathbf{k})$, which indicates the topological Chern number $C = 1$. (e) Comparison of the experimentally measured edge states (black) with the theoretical results (blue curves) of a 30×3 unit cells circuit. The strip shape circuit has period boundary conditions in the x direction and open boundary conditions in the y direction. The figure on the left (right) shows the edge states excited by a voltage source connected at the $y = 1$ ($y = 3$) boundary.

calculated Wilson loop shown in fig. (3 d) indicates the Chern number of the system is 1.

Non-zero Chern numbers are associated with the chiral edge states at the boundary of the system, which are usually measured experimentally to verify the topological properties of the system. To measure the edge states, we fabricate a printed circuit board (fig. (3 c)) containing 30×3 unit cells with periodic boundary conditions in the x direction and open boundary conditions in the y direction. Excitation source connects at node 1 in the $(x = 1, y = 1)$ cell with frequencies sweeping from 160 kHz to 260 kHz. Measuring the amplitude and the phase of the voltages at all nodes as a function of frequency, we obtain the resonant frequency bands as shown in fig. (3 e). Analogous to the Chern insulator in solid materials, the Chern state in circuit has a bulk gap for the resonance frequency bands but exhibits chiral propagating edge modes that traverse the gap. The chiral edge modes are topologically protected by the Chern number and are immune to the defects and device tolerances. The blue dashed curves in fig. (3 e) are the frequency bands of a strip shape circuit calculated by using the model Hamiltonian $H_C(\mathbf{k})$. On the two edges of the strip, the edge states have opposite chiralities near 200 kHz. Since the edge states are highly localized in the y direction, we can only detect the edge states located on the $y = 1$ ($y = 3$) boundary with the excitation source applied at the same boundary, while the edge state on the $y = 3$ ($y = 1$) boundary can not be excited. These results are consistent with the property of the topological edge states. The issues of the curves becoming broadened can be improved by choosing high-precision devices and using inductors with lower internal resistance.

As evidenced by above two examples, our scheme suggests a circuit approach to study physics related to non-Abelian gauge field. A wide variety of non-Abelian physics, such as the non-Abelian Aharonov-Bohm effect, the non-Abelian Aharonov-Casher effect, the Hofstadter's moth, and the novel motion of particles in gauge field, etc., may be studied in the circuit system. We present a mapping between the Kirchhoff equation for the circuit and the tight-binding Hamiltonian. The resonant frequency of the circuit corresponds to the eigenvalue of the Hamiltonian. The node voltage corresponds to the eigenstates. Resonant frequencies and node voltages are easily measured in electrical experiments, which provides a convenient way to verify the physical properties of the system. The on and off states of circuit branches are easy to be controlled by switches, which makes it possible to change the circuit structure and obtain controllable gauge field and topological states. The property that topological states are insensitive to impurities and defects has potential applications in circuit. Based on electro-mechanical-acoustic analogy, our scheme may be generalized to implement non-Abelian physics in mechanical and acoustic systems. In combination with superconducting devices, our proposal has the potential to investigate topological states in superconducting system.

References

1. Osterloh, K., Baig, M., Santos, L., Zoller, P. & Lewenstein, M. Cold Atoms in Non-Abelian Gauge Potentials: From the Hofstadter "Moth" to Lattice Gauge Theory. *Phys. Rev. Lett.* **95**, 010403 (2005).
2. Lin, Y.-J., Jimenez-Garcia, K. & Spielman, I. B. Spin-orbit-coupled Bose-Einstein condensates. *Nature* **471**, 83–86 (2011).
3. Huang, L. *et al.* Experimental realization of two-dimensional synthetic spin-orbit coupling in ultracold Fermi gases. *Nature Physics* **12**, 540–544 (2016).
4. Wu, Z. *et al.* Realization of two-dimensional spin-orbit coupling for Bose-Einstein condensates. *Science* **354**, 83–88 (2016).
5. Cooper, N. R., Dalibard, J. & Spielman, I. B. Topological bands for ultracold atoms. *Rev. Mod. Phys.* **91**, 015005 (2019).
6. Terças, H., Flayac, H., Solnyshkov, D. D. & Malpuech, G. Non-Abelian Gauge Fields in Photonic Cavities and Photonic Superfluids. *Phys. Rev. Lett.* **112**, 066402 (2014).
7. Iadecola, T., Schuster, T. & Chamon, C. Non-Abelian Braiding of Light. *Phys. Rev. Lett.* **117**, 073901 (2016).
8. Chen, Y. *et al.* Non-Abelian gauge field optics. *Nature Communications* **10**, 3125 (2019).
9. Rechcińska, K. *et al.* Engineering spin-orbit synthetic Hamiltonians in liquid-crystal optical cavities. *Science* **366**, 727–730 (2019).
10. Yang, Y. *et al.* Synthesis and observation of non-Abelian gauge fields in real space. *Science* **365**, 1021–1025 (2019).
11. Ozawa, T. *et al.* Topological photonics. *Rev. Mod. Phys.* **91**, 015006 (2019).
12. Lim, H.-T., Togan, E., Kroner, M., Miguel-Sanchez, J. & Imamoglu, A. Electrically tunable artificial gauge potential for polaritons. *Nature Communications* **8**, 14540 (2017).
13. Fruchart, M., Zhou, Y. & Vitelli, V. Dualities and non-Abelian mechanics. *Nature* **577**, 636–640 (2020).
14. Jotzu, G. *et al.* Experimental realization of the topological Haldane model with ultracold fermions. *Nature* **515**, 237–240 (2014).

15. Sugawa, S., Salces-Carcoba, F., Perry, A. R., Yue, Y. & Spielman, I. B. Second Chern number of a quantum-simulated non-Abelian Yang monopole. *Science* **360**, 1429–1434 (2018).
16. Ningyuan, J., Owens, C., Sommer, A., Schuster, D. & Simon, J. Time- and Site-Resolved Dynamics in a Topological Circuit. *Phys. Rev. X* **5**, 021031 (2015).
17. Albert, V. V., Glazman, L. I. & Jiang, L. Topological Properties of Linear Circuit Lattices. *Phys. Rev. Lett.* **114**, 173902 (2015).
18. Lee, C. H. *et al.* Topoelectrical Circuits. *Communications Physics* **1**, 39 (2018).
19. Luo, K., Yu, R. & Weng, H. Topological Nodal States in Circuit Lattice. *Research* **2018**, 6793752 (2018).
20. Lu, Y. *et al.* Probing the Berry curvature and Fermi arcs of a Weyl circuit. *Phys. Rev. B* **99**, 020302 (2019).
21. Hofmann, T., Helbig, T., Lee, C. H., Greiter, M. & Thomale, R. Chiral Voltage Propagation and Calibration in a Topoelectrical Chern Circuit. *Phys. Rev. Lett.* **122**, 247702 (2019).
22. Haenel, R., Branch, T. & Franz, M. Chern insulators for electromagnetic waves in electrical circuit networks. *Phys. Rev. B* **99**, 235110 (2019).
23. Imhof, S. *et al.* Topoelectrical-circuit realization of topological corner modes. *Nature Physics* **14**, 925–929 (2018).
24. Ezawa, M. Higher-order topological electric circuits and topological corner resonance on the breathing kagome and pyrochlore lattices. *Phys. Rev. B* **98**, 201402 (2018).
25. Yu, R., Zhao, Y. X. & Schnyder, A. P. 4D spinless topological insulator in a periodic electric circuit. *National Science Review* **7**, 1288–1295 (2020).
26. Wang, Y., Price, H. M., Zhang, B. & Chong, Y. D. Circuit implementation of a four-dimensional topological insulator. *Nature Communications* **11**, 2356 (2020).
27. Zhang, W. *et al.* Topoelectrical-circuit realization of a four-dimensional hexadecapole insulator. *Phys. Rev. B* **102**, 100102 (2020).
28. Yang, C. N. & Mills, R. L. Conservation of Isotopic Spin and Isotopic Gauge Invariance. *Phys. Rev.* **96**, 191–195 (1954).

223 29. Bychkov, Y. A. & Rashba, É. I. Properties of a 2D electron gas with lifted spectral degeneracy.
224 *Soviet Journal of Experimental and Theoretical Physics Letters* **39**, 78 (1984).

225 30. Dresselhaus, G. Spin-Orbit Coupling Effects in Zinc Blende Structures. *Phys. Rev.* **100**, 580–
226 586 (1955).

227 **Methods**

228 The circuit was implemented on FR4 printed circuit board. Components consist of inductors L (1.8
229 μH with a $\pm 5\%$ tolerance and 43 $m\Omega$ series resistance), capacitors C (47 nF with a $\pm 5\%$ toler-
230 ance), operational amplifier AD8057ARTZ-REEL7, and resistors R (15 Ω with a $\pm 1\%$ tolerance),
231 R_2 (30 Ω with a $\pm 1\%$ tolerance) , R_3 (100 $M\Omega$ with a $\pm 1\%$ tolerance). The voltage, including the
232 amplitude and phase, is probed by Rohde&Schwarz vector network analyzer ZNL6 5kHz-6GHz.

233 **Data availability**

234 The data that support the findings of this study are available from the corresponding author upon
235 reasonable request.

236 **Code availability**

237 The codes that support the findings of this study are available from the corresponding author upon
238 reasonable request.

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242 **Author contributions** R.Y. designed the research, performed the research, and wrote the paper. Z.Y.S.,
243 T.Y.W. and W.Q.W performed the experiment and data analysis under the supervision of R.Y. All authors
244 contributed to discussions and manuscript revision.

245 **Competing Interests** The authors declare that they have no competing interests.

246 **Correspondence and requests for materials** should be addressed to R. Y.

Figures

Figure 1

The building blocks for generating non-Abelian gauge potential. (a) Schematic diagram of the inductor cells m , n and the capacitor connection cell C . In cell m and n , inductors are connected head-to-tail and form a loop structure. Capacitors in cell C connect the nodes in cells m and n . (b to e) The designed capacitor cells that give the $\pm\sigma_0, 1, 2, 3$ types of tunneling matrices.

Figure 2

The Rashba-Dresselhaus spin-orbit interaction circuit. (a) Schematic and circuit diagram of the two dimensional Rashba-Dresselhaus SOI circuit. The green (yellow) blocks are the nearest neighbor connection modules $+\sigma_1(2)$ in the x(y) direction. The purple blocks are the next-nearest neighbor connection module $+\sigma_0$ in x and y directions. Their configuration details are given in fig. (1). The cyan blocks are the inductor cells, where each node connects to ground by a capacitor with the same capacitance as the capacitors used in the $\sigma_0,1$ modules. (b) Reso-nant frequency band structures of the

circuit. The Rashba-Dresselhaus type splitting is present at $(\pm\pi/2, \pm\pi/2)$ points. (c) Printed circuit board layout of the Rashba-Dresselhaus SOI circuit. The red dashed box indicates a unit cell. The green, orange and purple dashed box correspond to the σ_1 , σ_2 , and σ_0 modules, respectively. (d) The experimentally measured frequency dispersions and spin texture compared with the theoretical results. The red (blue) blocks are the experimentally measured spin in the positive (negative) x-direction. The red (blue) dashed-lines are results calculated from model Hamiltonian $HRD(k)$. The k-line passes through the $(\pi/2, \pi/2)$ point along the k_1 direction.

Figure 3

The topological Chern circuit. a) Schematic and circuit diagram of the two dimensional Chern circuit. The cyan blocks $L \parallel F$ are the inductor modules, which are detailed in (b). The green (yellow) blocks are the nearest neighbor connection module in the x(y) direction and the gray blocks are the nearest neighbor

connection module in x and y directions. Their configuration details are given in fig. (1). (b) Schematic of a cyan cell in a. The integrator operational amplifier is marked by a green dotted box. The integrator is serially connected with a red capacitor (the capacitance is three times the capacitance of the black capacitor in the integrator) and then connected in parallel with the inductor L. Each node has a grounded resistor, which has twice the resistance compared to the resistors in the integrator. (c) Printed circuit board layout of one unit cell of the topological Chern circuit. The devices in the blue dashed box correspond to L II F module given in fig. (b). The green (yellow) dashed box correspond to the $\sigma_1 - i\sqrt{3}\sigma_3$ ($\sigma_2 - i\sqrt{3}\sigma_3$) module in fig. (a). (d) The calculated Wilson loop of the model Hamiltonian $HC(k)$, which indicates the topological Chern number $C = 1$. (e) Comparison of the experimentally measured edge states (black) with the theoretical results (blue curves) of a 30×3 unit cells circuit. The strip shape circuit has period boundary conditions in the x direction and open boundary conditions in the y direction. The figure on the left (right) shows the edge states excited by a voltage source connected at the $y = 1$ ($y = 3$) boundary.

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