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Principles of design optimization for springs

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Abstract. The spring is the widespread resilient element which is used in the industrial machinery and automotive systems, as diesel fuel pumps, valvetrains, brakes, suspensions, seats, doors and control elements. For reducing impact events in some heavy trucks and railroad cars primarily, helical, or coil, springs are applied. In some vehicles torsion bars are used instead of the coil springs. The reduction of weight of the suspension springs causes the decrease of unsprung mass of the axle and has a positive influence on the comfort, traction and steering properties of the car. The development of modern passenger cars has highlighted a trend towards reduced package space for suspension components in order to maximize package space for occupants and loads. Such requirements lead to reduction in spring dimensions and wire cross-section. Springs can be found in high-precision testing devices, where springs play the role of energy harvesters. The efficient design procedures for spring elements are based on the modern simulation and optimization methods. The design formulas for linear helical springs with an inconstant wire diameter and with a variable mean diameter of spring are presented. Based on these formulas the optimization of spring for given spring rate and strength of the wire is performed.

DESIGN FORMULAS CYLINDRICAL SPRINGS

Cylindrical springs with circular wire

Helical springs are formed by wrapping wire or rod of uniform cross-section around a cylinder. We take as a reference frame cylindrical polar coordinate system (r, θ, z) . The axis z of the cylindrical polar coordinate system is aligned with the axis of the cylinder. A fixed distance between the successive coils of a spring is maintained, so that the axis of the wire forms a helix. When the distance between coils is small, the spring is called a closed-coiled spring. The governing equations for the closed-coiled helical spring were developed using semi-inverse Saint-Venant method by [Michell, 1899]. Unfortunately, the closed form of Saint-Venant solutions, which is well known in the theory of torsion of straight circular or elliptic rods, does not exist for curved rods in terms of elementary functions. Some approximate solutions for rectangular and circular cross-sections were delivered by [Wahl, 1929] and [Göhner, 1939, 1931, 1932]. The solutions for helical springs with circular cross-sections in terms of series of appropriate Legendre functions in toroidal coordinates were found by [Freiberger, 1949] and [Henrici, 1955]. The standard design formulas for helical springs [EN 13906, 2013, 2014], [Meissner et al, 2015] are based on these results.

Consider for the beginning the helical spring with a circular wire. The analysis of a cylindrical helical spring is based on the following conventional spring formulas for springs. The quantities defining a specific design of a spring are d , D , n , L_0 , where n is a number of active coils, d is the diameter of wire, D is the mean coil diameter, L_0 is a free length of the spring. The outer and inner diameters correspondingly are: $D_e = D + d$, $D_i = D - d$. Ignoring at first other certain complexities of spring technology, we can write simplified basic relations for an analysis of the compression spring. This simplified analysis considers spring ends as “plain”, so that only active coils are considered. The force acts ideally along the axis z of the cylindrical coordinate system.

The helical springs store elastic energy also by means of bending of wire. A torsion spring is a helical spring that works by torsion or twisting. The twist of helically coiled wire occurs about the axis of the coil by sideways forces or terminal moments applied to the ends of springs. The terminal moments twist the coil tighter or looser. The calculation formulas for the helical spring that is loaded by an axial force and terminal moment are provided below.

Consider the cylindrical helical spring with the mean diameter D . A coil spring can be wound in either a left hand or right hand direction. A left hand wound spring will spiral in the same direction as a left hand threaded screw. A right hand wound spring will spiral in the same direction as a right hand threaded screw. The lower spring end is assumed to be fixed. The right hand wound springs are considered hereafter.

Forces and moments in helical springs

Helical springs are formed by wrapping wire or rod of uniform cross-section around a cylinder (Leiseder, 1997). Let the angle α be the angle of inclination of the helix with any plane perpendicular to the axis of the coil (pitch

Figure 1. Helical spring, subjected to axial loading F and axial torque MT

angle, lead angle). We take as a reference frame cylindrical polar coordinate system (r, θ, z) . The axis z of the cylindrical polar coordinate system is aligned with the axis of the cylinder. A fixed distance between the successive coils of a spring is maintained, so that the axis of the wire forms a helix. When the distance between coils is small, the spring is called a close-coiled spring and in the consideration of stress in the wire, the torsion theory may be applied. Consider a close-coiled helical spring, which is subjected to axial loading by the axial force F and total axial torque M_θ . The torque $M_\theta = F_\theta R$ on the upper spring acts clockwise, if we look on the spring from above.

The circumferential force F_θ pushes towards the wire, applying pressure on the wire cross-section (Fig. 1A). The line of force coincides with the symmetry axis of the spring and consequently, the axis z of cylindrical coordinate system.

Consider the middle region of the spring, where the transition effects due to the end bars disappear and can be neglected. Due to the symmetry of the spring, all sections of the spring are deformed identically. In the Fig. 1B a portion of the spring has been isolated. In the absence of total axial torque, the cut section must have acting upon it a resultant force equal to F , and a couple, $M = FD/2$, for the free body is in equilibrium. The force and couple must lie in the same vertical plane. The couple M , acting in the vertical plane, has been shown as a vector, and is resolved into two components, $M \sin \alpha$ and $M \cos \alpha$, which lie in planes which are tangential and normal to the helix, respectively. The couple $M \sin \alpha$ tends to cause bending of the isolated portion of the spring wire, and $M \cos \alpha$ is acting so as to cause twisting. The magnitudes of the bending and twisting couples are equal to

$$M_B = M \sin \alpha - M_\theta \cos \alpha, \quad M_T = M \cos \alpha + M_\theta \sin \alpha$$

respectively.

For an extreme closed-coiled spring, when the angle α is very small, the couple M_B is negligible and M_T is approximately equal to M . A similar resolution of force F into components, F_N and F_S , which are causing normal and shearing components of stress, respectively, on the cut section, reads:

$$F_N = F \sin \alpha, \quad F_S = F \cos \alpha.$$

When the spring is close-coiled, the angle α is very small, and the force F_N is negligible. Then F_S will approximately equal to F .

For stress calculation the pitch can be neglected: $\alpha = 0$. One turn, or coil, of an undeformed helical spring becomes a torus, generated by rotating the cross-section about the z axis of the cylindrical coordinate system. The torus is assumed to be incomplete, i.e. the two ends of the turn are not joined. They carry equal in magnitude and opposite shear stress distributions with resultant F . The line of action of the resultant force F is coincident with the z axis.

Any segment of the coil is therefore in equilibrium under two opposite axial forces F with the same magnitude. Nonzero components of the shear stress in cylindrical coordinates are $\tau_{r\theta}, \tau_{\theta z}$. These components are independent of θ .

Stiffness and stored energy of cylindrical helical springs

Let the stroke of a helical spring in the absence of axial torque M_θ be the spring travel from released length L_{rel} to compressed length L_{comp} is $s = L_{rel} - L_{comp}$. Consider for definiteness a compression spring with a free length L_0 . Solid length L_c is the height at which the coils of the compressed spring close up. For the compression spring is valid $L_0 > L_{inst} > L_{fin} > L_c > 0$. The values F_{max} , F_{min} and F_c are considerably the spring loads at lengths L_{comp} , L_{rel} and L_c :

$$F_{min} = c(L_0 - L_{rel}), \quad F_{max} = c(L_0 - L_{comp}), \quad F_c = c(L_0 - L_c).$$

The extension spring is handled in the same way. The energy capacity of the linear spring could be expressed in terms either spring travel or spring force:

$$U_e = \frac{c}{2} [(L_{comp} - L_0)^2 - (L_{rel} - L_0)^2] = \frac{1}{2c} (F_{max}^2 - F_{min}^2) \quad (4)$$

and is equal to the work of applied forces on the total spring travel:

$$U_f = \frac{1}{2} (F_{\max} + F_{\min}) s.$$

The energy capacity of the linear spring loaded from its free state with the axial force F and the axial torque M_θ reads:

$$U_e = \frac{F^2}{2c} + \frac{FM_\theta}{c_{\theta F}} + \frac{M_\theta^2}{2c_\theta}.$$

The volume and the mass of the spring material of a cylindrical spring with constant, round cross-section is given by:

$$V = \frac{1}{4} \pi^2 d^2 D n, \quad m = \rho V. \quad (5)$$

where ρ is a density of spring material.

The spring stiffness, or spring rate, is the force required to produce a unit deflection. For close-coiled helical springs the force-deflection characteristic is approximately linear and can be calculated from the geometry and shear modulus G of the spring material $c = G d^4 / 8 D^3 n$. One must differ the basic and corrected shear stress in the spring with an axial load F . The formula for basic stress τ , or uncorrected stress, is obtained by dividing the torsion moment acting on the wire $M_T = F D / 2$ by the section modulus in torsion [Kobele, 2021] giving

$$\tau = M / W_T = 8 D F / \pi d^3.$$

The corrected stress τ_c is calculated by multiplying the basic stress τ by the correction factor $k = k(w)$, such that $\tau_c = k \tau$. The ratio of mean coil diameter to wire diameter $w = D / d$ is known as the spring index. A low index indicates a tightly wound spring (a relatively large wire size wound around a relatively small diameter mandrel giving a high rate). The correction factor accounts for stress concentration due to curvature of the spring as well as direct shear.

The Henrici correction factor reads:

$$k = 1 + \frac{5}{4w} + \frac{7}{8w^2} + \frac{155}{256w^3} + \dots$$

The correction factor due to Bergsträsser is:

$$k = \frac{w + \frac{1}{2}}{w - \frac{3}{4}}.$$

Wahl factor:

$$k = \frac{4w-1}{4w-4} + \frac{615}{1000w}$$

is also frequently used for stress correction.

Fatigue life and damage accumulation criteria

If the spring is to operate a definite, prescribed number of times through a deflection S , it must be designed so that the material does not fail in fatigue. A fatigue criterion for compression spring design is usually assumed to be [Spring Design Manual, 1996]:

$$\left(\frac{\tau_m + \tau_a}{\tau_w} + \frac{\tau_a}{\tau_e} \right) S_f \leq 1, \quad (6)$$

where

$\tau_m = (\tau_{\max} + \tau_{\min}) / 2$ is a mean stress in operation,

$\tau_a = (\tau_{\max} - \tau_{\min}) / 2$ is the alternating stress or stress amplitude,

τ_w is a working stress (strength),

τ_e is a endurance limit for completely reversed stress, and

S_f is a factor for safety.

The safety factor S_f for simplicity is assumed to be 1.

Both τ_w and τ_e usually vary with wire diameter in a manner approximated by $c_1 + c_2 / d^{c_3}$, where c_1, c_2, c_3 are experimentally acquired constants of the material and differ, of course, for τ_w and τ_e . Accordingly, both τ_w and τ_e have maximum values for a certain small wire diameter.

The fatigue life of springs is also frequently based on the damage evaluation from the Smith-Topper rules [Smith, Watson, Topper, 1970] or to [Landgraf, 1973]. According to Smith-Topper rule, the governing parameter for damage characterization is a product of total strain range and maximum stress. For discussion regarding applicability of Smith-Topper rule for automotive applications see [Fuchs et al, 1977]. During spring deformation the wire undergoes torsion, where the pure shear stresses predominate. Applying this approach to shear deformation, the Smith-Watson-Topper parameter transforms to:

$$p_{SWT.\tau} = \sqrt{G \gamma_a \tau_{\max}}. \quad (7)$$

Here $\gamma_a = \tau_a / G$ is the shear strain amplitude. The damage parameter is plotted versus number of reversals, so that damage per range between two reversals is a function of damage parameter. The accepted damage for a selected material during fatigue life of the spring is characterized by the condition:

$$p_{SWT.\tau} \equiv \sqrt{\tau_a \tau_{\max}} \leq p_{SWT.0}. \quad (8)$$

The experimentally acquired constant $p_{SWT.0}$ depends on material properties and accepted damage level for application under consideration ([Kobelev, 2018, Chapter 8]).

The fatigue behavior of the springs depends highly upon the surface treatment, mainly the shot peened layer on the surface. The highly inhomogeneous stresses in the shot peening layer are responsible for the crack arrest due to the compression stresses. The simulation methods must adequately describe the stress origin and depth variation of shot peening stresses. Cold formed springs also preserve another kind of residual stress due to the coiling. The influence of residual stresses on damage accumulation must be accounted for in fatigue calculations. The fatigue life of springs will be discussed in details in Chapter 8. The mechanical properties of spring materials were comprehensively discussed in [Yamada, 2007].

3 COMPRESSION AND TORQUE OF CYLINDRICAL HELICAL SPRINGS

Spring rates of non-cylindrical helical springs

The study of the optimization problem requires some generalization of common design formulas that account the variation of the mean diameter of the spring body as well the wire diameter along its length.

The formulas for stiffness and spring rate for a general non-cylindrical helical spring with an arbitrary variable cross-section are derived below following Castigliano's method [Teodorescu, 2013]. . The elastic energy stored in the spring:

$$\begin{aligned}
 2U_e &= \int_l \left[\frac{M_T^2}{G I_T} + \frac{M_B^2}{E I} \right] dl, \\
 \frac{M_T^2}{G I_T} + \frac{M_B^2}{E I} &= M^2 \left(\frac{\cos^2 \alpha}{G I_T} + \frac{\sin^2 \alpha}{E I} \right) \\
 &+ 2MM_\theta \cos \alpha \sin \alpha \left(\frac{1}{G I_T} - \frac{1}{E I} \right) + M_\theta^2 \left(\frac{\sin^2 \alpha}{G I_T} + \frac{\cos^2 \alpha}{E I} \right)
 \end{aligned} \tag{9}$$

The angle α is inclination of the helix with any plane perpendicular to the axis of the coil . For the element of length the following expression is valid:

$$dl = \frac{D(\theta)d\theta}{2 \cos \alpha}. \tag{10}$$

The total length and the mass of the spring wire correspondingly are:

$$l = \int_0^{2\pi} \frac{D(\theta)d\theta}{2 \cos \alpha}, \quad m = \rho \int_0^{2\pi} \frac{D(\theta)A(\theta)d\theta}{2 \cos \alpha}. \tag{11}$$

The substitution for the moment $M(\theta) = FD(\theta)/2$ leads to the formula for the stored energy [Kobelev, 2018]. The expression for elastic energy delivers the compression spring rate c , the compression-twist springs rate $c_{\theta F}$ and the twist springs rate c_θ of an arbitrary non-cylindrical helical spring:

$$\begin{aligned}
 \frac{1}{c} &= \frac{\partial^2 U_e}{\partial F^2} = \int_l \frac{D^2(\theta)}{4} \left(\frac{\cos^2 \alpha}{G I_T} + \frac{\sin^2 \alpha}{E I} \right) dl, \\
 \frac{1}{c_{\theta F}} &= \frac{\partial^2 U_e}{\partial F \partial M_\theta} = \int_l \frac{D(\theta)}{2} \cos \alpha \sin \alpha \left(\frac{1}{G I_T} - \frac{1}{E I} \right) dl, \\
 \frac{1}{c_\theta} &= \frac{\partial^2 U_e}{\partial M_\theta^2} = \int_l \left(\frac{\sin^2 \alpha}{G I_T} + \frac{\cos^2 \alpha}{E I} \right) dl.
 \end{aligned} \tag{12}$$

Assuming for small pitch $\cos \alpha \cong 1, \sin \alpha \cong 0$ we get the following representation for compression (or extension) spring rate:

$$\frac{1}{c} = \int_0^{2\pi n} \frac{1}{G I_T} \left(\frac{D(\theta)}{2} \right)^3 d\theta \quad (13)$$

4 HELICAL SPRINGS OF MINIMAL MASS

Optimization problem

The designer of the springs deals with the problems that require minimum weight or volume of the spring material because of space limitations or material cost limitations. The formulas are established that express minimum volume and weight in terms of the given requirements. The designer obtains the boundaries for weight and volume and what parameters must be changed for further weight reduction. We allow the mean diameter of the spring body together with the wire diameter to be the functions of the polar angle along the spring wire, so that:

$$D = D(\theta), \quad d = d(\theta), \quad \theta = 0..2\pi n.$$

For the analytical treatment, we constrain ourselves to the following optimization problem: Minimize the mass of the spring $m \rightarrow \min_{D,d}$ assuming the spring rate is equal to a given positive constant c^* :

$$c(D, d) = c^* \quad (14)$$

and the forces at installed height F_1 and full stroke F_2 are prescribed, the fatigue conditions (6), (8) fulfilled, and the ideal stress at full stroke $\tau = M_T / W_T \leq \tau_w$ are limited.

Optimization of helical springs for maximal stress

Consider at first the practically important case of the non-cylindrical springs with variable circular cross-section: the stress at solid height must be less than τ_w to protect the spring from inadvertent damage. This restriction, applied on the basic shear stress at solid height:

$$\tau \equiv \frac{8F_c D}{\pi d^3} \leq \tau_w, \quad F_c = c(L_c - L_0).$$

This inequality could be expressed in terms of wire diameter $d(\theta) \geq d_1(\theta)$. In this inequality the optimal diameter of wire

$$d_1(\theta) \equiv \sqrt[3]{\frac{8F_c D(\theta)}{\pi \tau_w}} \quad (15)$$

is the solution of algebraic equation $\tau = \tau_w$ with respect to $d(\theta)$.

Rewrite the formula for spring rate (12), taking into account that for all possible cross-sections the stress conditions require that $d(\theta) \geq d_1(\theta)$:

$$c(D, d) = \pi G \left(4 \int_0^{2\pi} \frac{D^3(\theta)}{d^4(\theta)} d\theta \right)^{-1} \geq \pi G \left(4 \int_0^{2\pi} \frac{D^3(\theta)}{d_1^4(\theta)} d\theta \right)^{-1}.$$

Substitution of the expression (15) for the optimal diameter of wire into the last expression reduces the stiffness requirement (9) to the following inequality:

$$c^* = c(D, d) \geq 4\pi G \left(\frac{F_c}{\pi \tau_w} \right)^{4/3} \left(\int_0^{2\pi} D^{5/3}(\theta) d\theta \right)^{-1}. \quad (16)$$

Otherwise, the expression (13) for the mass of spring after the substitution (15) results in the second inequality:

$$m \geq \frac{1}{2} \pi \rho \left(\frac{F_c}{\pi \tau_w} \right)^{2/3} \left(\int_0^{2\pi} D^{5/3}(\theta) d\theta \right). \quad (17)$$

Well known, that the inequalities of the same sign can be multiplied. The multiplication of the inequalities (16) and (17) results in a final lower boundary for spring mass:

$$m \geq \frac{2 \rho G F_c^2}{\tau_w^2 c^*} \quad (18)$$

This important inequality establishes the exact lower boundary for the mass of spring of arbitrary variable shape and variable circular cross-sections, designed to fulfill the stress condition at solid length:

$$m \geq m_1 = \frac{2 \rho G F_c^2}{\tau_w^2 c^*}. \quad (19)$$

The elastic potential energy per unit volume (elastic energy density) is equal to $\tau_w^2 / 2G$. The elastic potential energy per unit mass (specific elastic energy density) is $\tilde{U}_e = \tau_w^2 / 2\rho G$. This value is the material constant in state of shear. The inequality (19) indicates the mass in terms of stored elastic energy in the spring:

$$m \geq m_1 = \frac{F_c^2}{\tilde{U}_e c^*}.$$

Design for fatigue life

The spring is to operate a definite number of cycles through a deflection s measured as additional compression from L_0 . The application of a similar optimization procedure, as applied above, for the fatigue condition (6) leads to optimal wire diameter

$$d_2(\theta) = \sqrt[3]{\frac{8D(\theta)}{\pi} \left(\frac{F_{\max}}{\tau_w} + \frac{F_{\max} - F_{\min}}{2\tau_e} \right)} \quad (20)$$

and mass lower boundary:

$$m \geq m_2 = \frac{2\rho G}{c^*} \left[\frac{F_{\max}}{\tau_w} + \frac{F_{\max} - F_{\min}}{2\tau_e} \right]^2 \quad (21)$$

Hence, these expressions determine the optimal spring, acceptable from the viewpoint of fatigue life criterion (6).

Instead, when the accumulated damage according to Smith-Topper rule (8) is the measure for fatigue life, then the optimal wire diameter is:

$$d_3(\theta) = \sqrt[3]{\frac{8D(\theta)}{\pi p_{SWT.0}} \sqrt{\frac{F_{\max}(F_{\max} - F_{\min})}{2}}} \quad (22)$$

Accordingly, the mass of spring, designed to comply with the Smith-Topper rule, satisfies the condition:

$$m \geq m_3 = \frac{2\rho G}{c^*} \frac{F_{\max}(F_{\max} - F_{\min})}{2p_{SWT.0}^2} \quad (23)$$

CONCLUSIONS

It was proved, that the optimal wire shape, determined from the certain equal stress condition, guarantees the lowest possible mass of the spring. This mass depends only on the ultimate allowable stress for the spring material, the load at full stroke and the spring stiffness. This is an important milestone for comparison of different spring designs and spring materials. As the density and shear module are almost the same for all spring steels, the spring quality parameter can serve as the benchmarking property for spring design.

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