Correlation between Spatial Fractal and Temporal Chaos During the Sliding Wear Process of AISI 5120 Steel

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Research Article

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Posted Date: September 8th, 2021

DOI: <https://doi.org/10.21203/rs.3.rs-859871/v1>

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Correlation between spatial fractal and temporal chaos during the sliding wear process of AISI 5120 steel

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Abstract

The purpose of this paper is to establish the relationship between surface morphology and friction coefficient in the wear process. Different wear stage tests of AISI 52100 ring sliding against AISI 5120 disc were designed and conducted on a rotating setup. The fractal and chaos theories were employed to study the nonlinear features of surface structure and friction signal from spatial and temporal scales. The results showed that 3D surface morphology has fractal nature. The fractal dimension D_s first increased and then stabilized at a maximum and finally decreases dramatically. The multifractal spectrum width $\Delta\alpha$ presented an contrary evolution trend. The friction coefficient signal has chaotic nature. The standard deviation of distance matrix STD obeyed the evolution rule of a bathtub curve. The correlation value between D_s and STD was -0.7727, and the correlation value between $\Delta\alpha$ and STD was 0.7130. The strong correlation between spatial and temporal scales is beneficial to on-line recognition and prediction of wear states in real time.

Keywords: Spatial fractal; Temporal chaos; Surface morphology; Friction coefficient.

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1. Introduction

Owing to unique properties of high strength and high wear resistance, AISI 5120 steel is conventionally used to produce critical mechanical components, such as cam, bearing, gear, gear shaft, and machine tool [1]. As friction pair material, the mating surface inevitably emerges material removal, weight loss, and surface damage under a harsh operating condition, eventually leading to failure of critical components [2-4]. This is an enormous threat to the service life span and performance of mechanical equipment. In order to overcome the early or sudden wear failure, it is vital to understand wear behaviors and the evolution mechanism. The surface roughness, weight loss, wear rate, wear width and depth, and surface morphology constantly vary with wear process. These information is easy to be acquired by using the conventional measurement or characterization methods. Hence there is a large number of related researches have emerged in the field of equipment and surface engineering.

Yang et al. [5] used the weight loss to investigate the influence of heat treatment temperature on the wear resistance of steel in sliding process. The results showed that the weight loss of austempered steel was less than that of tempered sample, and the weight loss increased with the wear time. Qin et al. [6] employed the technique of nanocrystal surface modification to strengthen AISI 52100 steel, it was found that the surface roughness and wear volume of treated surface were less than those of original surface. Zhou and Piao [7, 8] found that the wear scar width and mass loss of ultrasonic treated sample were obviously less than those of untreated sample. This phenomenon indicated that the treated sample had better wear resistance. Bejjani et al.

[9] utilized the parameter of 3D surface topography to qualitatively study the wear mechanism of tools when machining different materials. Liu et al [10] explored the tooth wear evolution to reveal the gear contact fatigue failure competition mechanism. The results showed that the surface roughness rapidly increased during the running-in wear stage.

The aforementioned researches mainly focus on the surface profile height or the amount of material removal. The characteristic and evolution of complex structures of surface morphology are rarely involved in the study. Actually, the structure of surface morphology is composed of the profiles at different scales, which has the nature of self-similarity and scale-invariance, and this feature is named as fractal. The fractal theory [11] is employed to reveal the complex nonlinear features of morphology, behavior, information, material and other objects from spatial scale. The surface morphology exhibits fractal behavior within a limited scale range, which can reflect the evolution of worn surface in the spatial scale. The characteristic parameters of fractal dimension and multifractal spectrum are put forward and used to study the nonlinear structural features of worn surfaces [12-15]. Xu et al. [16] investigated the wear evolution of sliding-rolling contact by using fractal dimension. The results showed that the fractal dimension of 3D surface first descended and fluctuated around a minimum, and finally ascended slowly in the wear process. Zuo et al. [17] studied the fractal behavior of worn surface in brass sliding against steel by using fractal dimension. The results showed that the value of fractal dimension was fraction in the range of 1-2. And the variation of fractal dimension reflected the morphology changes

of worn surface in different wear stages. Furthermore, Zuo et al.[18] employed the multifractal method to analyze the evolution of 3D surface morphology in the wear process of brass sliding against steel. The results showed that the spectrum width and spectrum difference obeyed the evolution rule of inverted bathtub curve. The worn surface had the multifractal nature with scale-invariance.

Characterization on the fractal features of 3D surface morphology can be applied to monitor wear states. But the surface information is hardly obtained in real time without shutdown. Through relevant researches, it is found that the friction signals extracted from a friction system in real time, such as friction coefficient, friction vibration, friction noise, and friction temperature, contains abundant information about the wear behaviors. The friction signals have chaotic features, and their evolution rules corresponds to the different wear stages [19-22]. Therefore, the purpose of this paper is to establish the correlation between surface morphology and friction signal. This paper is organized as follows: In Section 2, taking the friction system of AISI 52100 steel sliding against AISI 5120 steel as research object, the wear experiments with different wear durations are performed in a ring-on-disc setup. In Section 3, the surface morphologies of specimens in different wear stages are acquired, and the friction coefficient signals are extracted from the friction system. In Section 4, the fractal dimension and multifractal spectrum of 3D surface morphology are analyzed. In Section 5, the chaotic feature and evolution of friction coefficient signal are investigated. And the relationship between fractal features and chaotic features are explored. Finally, the main conclusions are drawn in Section 6.

2. Experimental methodology

After quenching and grinding, the ring of AISI 52100 steel owned the property of 61 HRC hardness and the average surface roughness of 0.031 μm . After annealing and turning, the disc of AISI 5120 steel owned the property of 40 HRC hardness and the average surface roughness of 4.044 μm . The chemical compositions of AISI 52100 and AISI 5120 were noted in Table 1. The ring and disc were regarded as a friction pair, aiming to perform wear experiments. The mating surface was a zone with an external diameter of 34 mm and an internal diameter of 24 mm. As shown in Fig. 1, the ring specimen was installed onto a ring clamp by using a set screw and a dowel pin, and it did rotating movement with the speed of v during testing. The disc specimen was fixed on a disc clamp, and it kept still during testing. The friction torque signal generating from friction system was obtained by a torque sensor with sampling frequency of 300 Hz. Meanwhile, it was transmitted to a signal acquisition and analysis system.

Table 1 Chemical compositions of AISI 52100 and AISI 5120

Steel	Element	C	Si	Mn	Cr	S	P	Ni	Cu	Mo
AISI 52100	Content/ Wt%	0.18	0.17	0.50	0.70	≤ 0.035	≤ 0.035	≤ 0.30	≤ 0.30	≤ 0.15
		0.24	0.37	0.80	1.00					
AISI 5120	Content/ Wt%	0.95	0.15	0.25	1.40	≤ 0.025	≤ 0.025	≤ 0.30	≤ 0.25	≤ 0.10
		1.05	0.35	0.45	1.65

Fig. 1. Schematic diagram of experimental setup

The stage testing method [18, 23] was employed to study the evolution rule of surface morphology of AISI 5120 disc in the wear process. As displayed in Fig. 2, the first step was to implement a whole process of wear experiment, and the corresponding wear duration was t_0 . Then, the whole wear process was divided into n stages according to the variation of friction coefficient signal. The duration of each stage was $t_1, t_2, t_3, \dots, t_n$ in turn. In this paper, a whole process of wear experiment was conducted, and its wear duration was 1020 min. The corresponding variation of friction coefficient signal was displayed in Fig. 5(k). Its variation was divided into three stages of running-in, stable wear, and rapid wear. Thereby the test groups and the corresponding wear duration were determined. Namely, the test group $n=10$, the wear duration of each group was $t_1=30$ min, $t_2=60$ min, $t_3=120$ min, $t_4=220$ min, $t_5=320$ min, $t_6=560$ min, $t_7=800$ min, $t_8=860$ min, $t_9=900$ min, $t_{10}=1000$ min, respectively. It was emphasized that the ring and disc in each experiment were new specimens. Additionally, the normal load and the rotating speed of each experiment were 380 N and 600 rpm, respectively. The mating surface was under a mixed

lubrication state. All of the wear experiments were in the environment of normal temperature and atmospheric pressure.

Fig. 2. Stage testing method

3. Experimental results

3.1 Surface morphology of specimens

After the experiments, ring and disc were cleaned with 97% alcohol in an ultrasonic cleaning machine and then dried at the vent. A shape measurement laser microscopic system (VK-X1000) was used to gather surface information. This measurement system had the property of 0.5 nm resolution in the height direction and 1 nm resolution in the width direction. The measured zone of ring surface was the worn area with a width of 2 mm and a length of 3 mm along the diameter direction. The measured zone of disc surface was the worn area with a width of 2 mm and a length of 4 mm along the diameter direction. The variations of surface morphology of ring and disc at different wear stages were displayed in Fig. 3 and Fig. 4, respectively.

As displayed in Fig. 3(a), the initial surface of an original ring emerged a smooth and uniform texture. As displayed in Figs. 3(b)-(e), the surface intermittently generated single and thin furrows from 30 min to 220 min, and the height of surface profile showed an increasing trend. With the extension of wear duration, the ring surface emerged several indentations, adhesive layers, deep furrows, debris, and

spalling zones, as shown in Figs. 3(f)-(h). From 860 min to the end, a large area of adhesive layer appeared on the ring surface, and the surface was completely destroyed, as shown in Figs. 3(i)-(l). It is inferred that the wear mechanism of a ring was adhesive wear accompanied with abrasive wear.

As displayed in Fig. 4(a), the initial surface of the turned disc exhibited a texture with uniformly distributed peaks and valleys. As displayed in Figs. 4(b)-(e), from 30 min to 220 min, the profile peaks of disc surface were sheared continuously, and the contact zone of mating surface increased continuously. As displayed in Figs. 4(f)-(h), from 320 min to 800 min, the more peaks were flattened, and the more valleys were filled. Namely, the mating surface remained flat with few furrows and indentations. As displayed in Figs. 4(i)-(l), from 860 min to the end, the surface morphology deteriorated gradually and continuously emerged the phenomenon of spalling, furrows, indentations, gullies, debris, deteriorated lubrication oil, and delamination. It is judged that the wear mechanism of a disc was also adhesive wear accompanied with abrasive wear.

Fig. 3. 3D morphology of ring surface: (a) Original surface; (b) Test 1 $t=30$ min; (c) Test 2 $t=60$ min; (d) Test 3 $t=120$ min; (e) Test 4 $t=220$ min; (f) Test 5 $t=320$ min; (g) Test 6 $t=560$ min; (h) Test 7 $t=800$ min; (i) Test 8 $t=860$ min; (j) Test 9 $t=900$ min; (k) Test 10 $t=1000$ min; (l) Test 11 $t=1020$ min.

Fig. 4. 3D morphology of disc surface: (a) Original surface; (b) Test 1 $t=30$ min; (c) Test 2 $t=60$ min; (d) Test 3 $t=120$ min; (e) Test 4 $t=220$ min; (f) Test 5 $t=320$ min; (g) Test 6 $t=560$ min; (h) Test 7 $t=800$ min; (i) Test 8 $t=860$ min; (j) Test 9 $t=900$ min; (k) Test 10 $t=1000$ min; (l) Test 11 $t=1020$ min.

3.2 Friction coefficient signals

During testing, the friction torque signal was extracted from the friction system in real time. Then the friction coefficient signal was obtained by formula calculation. For all of the experiments, the variations of friction coefficient signals were shown in Fig. 5. From 30 min to 120 min, the friction coefficient signal gradually increased with obvious fluctuation, as shown in Figs. 5(a)-(c). That because the contact zone of mating surface increased gradually. At the moment of 220 min, the value of friction coefficient signal emerged a decreasing trend, as shown in Fig. 5(d). Subsequently, the contact zone reached the maximum so that the friction coefficient signal stabilized at a minimum from 320 min to 800 min, as shown in Figs. 5(e)-(g). However, from 860 min to the end, the friction coefficient signal increased rapidly, and the mating surface was deteriorated. as shown in Figs. 5(h)-(k).

Fig. 5. Friction coefficient signals: (a) Test 1 $t=30$ min; (b) Test 2 $t=60$ min; (c) Test 3 $t=120$ min; (d) Test 4 $t=220$ min; (e) Test 5 $t=320$ min; (f) Test 6 $t=560$ min; (g) Test 7 $t=800$ min; (h) Test 8 $t=860$ min; (i) Test 9 $t=900$ min; (j) Test 10 $t=1000$ min; (k) Test 11 $t=1020$ min.

4. Fractal and multifractal analysis of surface morphology

The surface roughnesses of ring and disc in each wear stage are measured three times. Because it is hardly to clearly show the too small error values in figure, only

the average values of surface roughnesses are displayed in Fig. 6. The surface roughness of the ring with an initial value of $0.031\ \mu\text{m}$ slightly raises to $0.081\ \mu\text{m}$ at 120 min. Then it gradually raises to $1.343\ \mu\text{m}$ from 120 min to 900 min, although there is a sudden drop at 320 min. From 900 min to the end, the surface roughness sharply raises to $2.559\ \mu\text{m}$. Overall, the surface roughness of ring shows an upward trend in the wear process. The surface roughness of the disc with an initial value of $4.044\ \mu\text{m}$ continuously declines to $2.191\ \mu\text{m}$ at 220 min. Then it fluctuates around $1.870\ \mu\text{m}$ from 320 min to 800 min. From 860 min to the end, the surface roughness rapidly raises to $4.027\ \mu\text{m}$. To sum up, the surface roughness of disc firstly declines, then remains stable, and finally raises in the wear process. This phenomenon can be explained by Fig. 3 and Fig. 4. During the wear process, the peaks of disc surface are continuously sheared, the materials are transmitted to the valleys. Hence, the disc surface becomes flat, and the surface roughness decreases obviously. Meanwhile, the ring surface appears a few debris and adhesion. With the production of debris and the deterioration of lubrication oil, the disc surface gradually emerges deep furrows, gullies, spalling and delamination. Correspondingly, the materials are transmitted to the ring surface.

Fig. 6. Surface roughness of ring and disc in different stages

The surface roughness of the disc follows the variation of “decrease-stabilization-increase”, which coincides with the three stages of running-in wear, stable wear and severe wear. Therefore, the nonlinear characteristics and evolution of disc surface morphology are discussed by using the fractal theory [11]. The fractal theory contains several quantitative characteristic parameters. For instance, the parameter of 2D fractal dimension is commonly used to characterize a 2D profile. The 3D surface consists of a number of 2D profiles, which contains more details and features. In order to accurately express the 3D morphology, Ausloos et al. [24] extends the 2D W-M curve (as shown in Eq. (1)) to a 3D surface, and they introduces the corresponding expression, as given in Eq. (2).

$$z(x) = L \left(\frac{G}{L} \right)^{d-1} \sum_{n=0}^{\infty} \gamma^{-(2-d)n} \cos(2\pi \gamma^n \frac{x}{L}) \quad (1)$$

Where, x is the coordinate of 2D profile in x-axis direction; $z(x)$ is 2D profile height; G is characteristic scale coefficient; γ is a constant greater than 1; $\gamma^n=1/L$; d is fractal dimension of 2D profile; n is frequency index; L is sampling length.

$$z(x, y) = \left(\frac{\ln \gamma}{M_s} \right) \sum_{m=1}^{M_s} A_m \sum_{n=-\infty}^{\infty} \left[1 - \cos(k_0 \gamma^n (x^2 + y^2)^{1/2}) \cos \left(\arctan(\frac{y}{x}) - \alpha_m \right) \right] \times \cos(\varphi_{m,n}) (k_0 \gamma^n)^{D_s-3} \quad (2)$$

Where, $z(x, y)$ is profile height of 3D surface corresponding to (x, y) ; $((\ln \gamma)/M_s)^{1/2}$ is normalization factor; the scale parameter $\gamma=1.5$; M_s is the number of overlaps of surfaces; A_m is surface amplitude; k_0 is horizontal variation; $\alpha_m \in [0, \pi]$; $\varphi_{m,n} \in [0, 2\pi]$; D_s is fractal dimension of 3D surface, and $2 < D_s < 3$.

On the basis of Eq. (3) [25, 26], the 3D root mean square method of fractal dimension of 3D surface morphology is proposed by Zuo et al. [27], is given as Eq.

(4)-Eq. (7).

$$S(\Delta x, \Delta y) \sim \left(\sqrt{\Delta x^2 + \Delta y^2} \right)^{2(3-D_s)} \quad (3)$$

$$V(\Delta x, \Delta y) = Var(\Delta x, \Delta y)^{1/2} \sim \left(\sqrt{\Delta x^2 + \Delta y^2} \right)^{(3-D_s)} \quad (4)$$

When $\Delta x = \Delta y = r$, take logarithm on both sides of Eq. (3), following as

$$\log V(r) = (3-D_s) \left(\log \sqrt{2} + \log r \right) + C \quad (5)$$

Where, C is a constant; r is a scale; $V(r)$ is a measure. The slope of $\log V(r) - (\log \sqrt{2} + \log r)$ equals to the slope of $\log V(r) - \log r$. Thereby, Eq. (4) can be simplified as

$$\log V(r) = (3-D_s) \log r \quad (6)$$

Fractal dimension D_s of 3D surface is given as

$$D_s = \lim_{r \rightarrow 0} \left(3 - \frac{\log V(r)}{\log r} \right) \quad (7)$$

The fractal dimension D_s of 3D surface is a quantitative parameter to describe the self-similarity and scale-invariance of the surface structure. According to Eq. (7), the fractal dimension D_s of disc surface in Fig. 4 is calculated and displayed in Fig. 7. The fractal dimension D_s of each surface is a fraction in the range of 2-3. To be specific, the D_s of original surface is 2.853 before testing. From 30 min to 220 min, the D_s continuously increases from 2.874 to 2.903. From 320 min to 800 min, the D_s remains stable around 2.908. From 860 min to the end, the D_s dramatically decreases to 2.832. In the whole wear process, the D_s obeys the evolution rule of an inverted bathtub curve. The D_s is a fraction, indicating that the 3D morphology of worn surface has fractal nature. Typically, the greater the D_s , the finer the surface structure, the stronger the self-similarity. Comparing Fig. 6 with Fig. 7, it is found that the variation of disc

surface roughness is consist with the evolution of fractal dimension D_s , and their constancy marks the stability of the friction system.

Fig. 7. Fractal dimension D_s of disc surface in different stages

The multifractal is a set of fractals in different dimensions, and its purpose is to describe the characteristics of a complex object at different levels. The multifractal spectrum $f(\alpha)$ and the multifractal spectrum width $\Delta\alpha$ are utilized to analyze the local singularity of worn surface. The $f(\alpha)$ indicates the probability distribution of the singular index α , and it is a fractal dimension of a subset with the same α , so $f(\alpha)$ is defined as multifractal spectrum. Based on the method of box counting dimension, the measures of $f(\alpha)$ and $\Delta\alpha$ are as follows:

$$\mu(q, \varepsilon) = k_1 k_2 \sum_{\alpha} \varepsilon^{-f(\alpha) + q\alpha} \quad (8)$$

Where, $\mu(q, \varepsilon)$ is a partition function; q is a weight factor; ε is the length of box; k_1 and k_2 are constants.

Then, the quality index $\tau(q)$ is defined as:

$$\tau(q) = \lim_{\varepsilon \rightarrow 0} \left[\frac{\log \mu(q, \varepsilon)}{\log \varepsilon} \right] = \lim_{\varepsilon \rightarrow 0} \left[\frac{\log \varepsilon^{\min(-f(\alpha) + q\alpha)} + \log(k_1 k_2)}{\log \varepsilon} \right] = \min(-f(\alpha) + q\alpha) \quad (9)$$

So, $f(\alpha)$ is given as:

$$f(\alpha) = \min(-\tau(q) + q\alpha) \quad (10)$$

And α and $\Delta\alpha$ are given as:

$$\alpha = \frac{d\tau(q)}{d(q)}; \quad \Delta\alpha = \alpha_{\max} - \alpha_{\min} \quad (11)$$

Based on the 3D morphology of disc surface in Fig. 4, the multifractal spectrums of their 2D profiles are measured and displayed in Fig. 8. The multifractal spectrum in each wear stage appears a shape of invert hook, and the left endpoint is always higher than the right endpoint. It is obvious that the range of $f(\alpha)$ has hardly changed, yet the range of α firstly narrows gradually and then expands continuously. To quantify the range change of α , the $\Delta\alpha$ in each wear stage is computed and displayed in Fig. 9. The $\Delta\alpha$ of original disc surface is a maximum of 1.502, then it declines to 0.949 at 120 min. From 220 min to 860 min, the $\Delta\alpha$ remains around 0.093. Finally, the $\Delta\alpha$ rapidly ascends to 1.397. The $\Delta\alpha$ reflects the fluctuation degree of worn surface profile. The original and damaged surface have the most fluctuation, and the running-in surface has the least fluctuation. The $\Delta\alpha$ obeys the evolution rule of a bathtub curve, not only corresponding to the three processes of running-in, stable wear and severe wear, but also corresponding to the evolution of fractal dimension D_s in the whole wear process.

Fig. 8. Multifractal spectrum of disc surface: (a) Original specimen; (b) Test 1 $t=30$ min; (c) Test 2 $t=60$ min; (d) Test 3 $t=120$ min; (e) Test 4 $t=220$ min; (f) Test 5 $t=320$ min; (g) Test 6 $t=560$ min; (h) Test 7 $t=800$ min; (i) Test 8 $t=860$ min; (j) Test 9 $t=900$ min; (k) Test 10 $t=1000$ min; (l) Test 11 $t=1020$ min.

Fig. 9. Multifractal spectrum width $\Delta\alpha$ in different stages

5. Chaotic analysis of friction coefficient signal

The chaos theory is commonly used to analyze the nonlinear feature of discrete signal, aiming to reveal the inner attributes of a complex system from temporal scale [28]. The chaotic system has the specific property of boundness, ergodicity, inner randomness, sensitivity to initial values, long-term unpredictability, and fractal. The convergent phase trajectory composed of phase points is a symbol of chaotic system, and it is called as chaotic attractor. To analyze the chaotic characteristic of friction coefficient signal, based on the theory of Taken's reconstruction [29], the univariate time series of friction coefficient signal is reconstructed to a 3D space, following as:

$$\mathbf{X} = [\mathbf{X}_1, \mathbf{X}_2, \Lambda, \mathbf{X}_N]; \quad \mathbf{X}_i = [x_i, x_{i+\tau}, x_{i+2\tau}, \Lambda, x_{i+(m-1)\tau}] \quad (12)$$

Where, $\{x_1, x_2, x_3, \dots, x_n\}$ is a set of time series, n is the number of discrete point x_i ;

$i=1, 2, \dots, N$; N is the number of reconstructed vector X_i , $N=n-(m-1)\tau$; m is the embedding dimension; τ is the time delay.

For the friction coefficient signals in different wear stages, 10,000 data points are selected at 30 min, 60 min, 120 min, 220 min, 320 min, 560 min, 800 min, 860 min, 900 min, 1000 min, and the end, respectively. Subsequently, each set of data is reconstructed based on Eq.(12). In order to present the phase trajectory in 3D space, the high-dimensional matrix X is projected to a 3D space based on the method of principal component analysis [30]. Thereby the phase trajectory of each stage is obtained and shown in Fig. 10. Aiming at clearly expressing the variation of phase trajectory, the coordinate intervals of X, Y, and Z in Figs. 10(a)-(c), (i) and (k) are (0.2-0.6), (-0.2-0.2), (-0.2-0.2), respectively. The coordinate intervals of X, Y, and Z in Figs. 10(d)-(h) are (0.15-0.3), (-0.04-0.04), (-0.02-0.02), respectively. As shown in Fig. 10(a), the phase trajectory constantly overlaps but never crosses in a larger space at 30 min. From 120 min to 220 min, the phase trajectory shows a trend of continuous convergence, as shown in Figs. 10(b)-(d). From 320 min to 800 min, the phase trajectory converges to a minimal space and remains relatively stable, as shown in Figs. 10(e)-(g). However, the phase trajectory begins to spread and escape from a specific space from 860 min to the end, as shown in Figs. 10(i)-(k).

The characteristic parameter of standard deviation of distance matrix STD is employed to quantify the convergence degree of the phase trajectory. The smaller the STD , the larger the convergence degree, indicating the chaotic attractor has good stability. The calculation formula is given as:

$$\mathbf{d} = \begin{bmatrix} d_{1,1} & d_{1,2} & \Lambda & d_{1,N} \\ d_{2,1} & d_{2,2} & \Lambda & d_{2,N} \\ \mathbf{M} & \mathbf{M} & \mathbf{O} & \mathbf{M} \\ d_{N,1} & d_{N,2} & \Lambda & d_{N,N} \end{bmatrix}; \quad d_{i,j} = |\mathbf{X}_i - \mathbf{X}_j| \quad (12)$$

$$STD = \sqrt{\frac{1}{N^2} \sum_{i,j=1}^N (d_{i,j} - \mu)^2} \quad (13)$$

Where, $\mathbf{d}$ is a distance matrix; $d_{i,j}$ is an euclidean distance between two phase points of $\mathbf{X}_i$ and $\mathbf{X}_j$; μ is the mean value of $\mathbf{d}$.

The *STDs* of friction coefficient signals in different wear stages are calculated and displayed in Fig. 11. From 30 min to 220 min, the *STD* gradually decreases from 0.0533 to 0.0026. Then, the *STD* remains relatively stable at the minimum from 320 min to 800 min. Ultimately, the *STD* dramatically increases to 0.0419 from 860 min to the end. Correspondingly, the chaotic attractor evolves from an unstable state to a stable state and then to an unstable state, which also obeys the variation rule of an inverted bathtub curve.

Fig. 10. Phase trajectory of friction coefficient signal: (a) Test 1 $t=30$ min; (b) Test 2 $t=60$ min; (c) Test 3 $t=120$ min; (d) Test 4 $t=220$ min; (e) Test 5 $t=320$ min; (f) Test 6 $t=560$ min; (g) Test 7 $t=800$ min; (h) Test 8 $t=860$ min; (i) Test 9 $t=900$ min; (j) Test 10 $t=1000$ min; (k) Test 11 $t=1020$ min.

Fig. 11 Standard deviation of distance matrix STD in different stages

Observing the variation curves of fractal dimension D_s and multifractal spectrum width $\Delta\alpha$ of 3D surface morphology and the STD of phase trajectory of friction coefficient, there are certain correlations among them. To measure the overall error and the correlation degree of any two among D_s , $\Delta\alpha$ and STD , covariance $cov(x, y)$ and correlation coefficient $r(x, y)$ are calculated and noted in Table 2. The negative values of $cov(D_s, \Delta\alpha)$ and $cov(D_s, STD)$ indicate that the evolution trend of fractal dimension D_s is contrary to the multifractal spectrum width $\Delta\alpha$ and the standard deviation of distance matrix STD . Oppositely, the positive value of $cov(\Delta\alpha, STD)$

indicates that the evolution trend of both $\Delta\alpha$ and STD is consistent. Furthermore, the larger $r(D_s, \Delta\alpha)$ illustrates that the morphology parameters can better characterize the surface features from spatial scale. The 0.7727 of $r(D_s, STD)$ and 0.7130 of $r(\Delta\alpha, STD)$ illustrate that there is a great correlation between D_s and STD , so is between $\Delta\alpha$ and STD . That is to say, the fractal features of the 3D surface morphology can be primarily reflected by the chaotic features of the friction coefficient signal.

Table 2 Covariance of correlation coefficient among D_s , $\Delta\alpha$, and STD

$cov(D_s, \Delta\alpha)$	$cov(D_s, STD)$	$cov(\Delta\alpha, STD)$	$r(D_s, \Delta\alpha)$	$r(D_s, STD)$	$r(\Delta\alpha, STD)$
-0.0035	-0.0003	0.0015	-0.8826	-0.7727	0.7130

6. Conclusions

Different wear stage tests of AISI 52100 sliding against AISI5120 are implemented on a rotating ring-on-disc setup. The 3D surface morphology and friction coefficient signal are extracted from the friction system. Based on fractal theory, the fractal and multifractal features of surface morphology are analyzed. Based on chaos theory, the chaotic features of friction coefficient signal are investigated. Significantly, the relationship between surface morphology and friction coefficient is explored. The main conclusions are as follows:

- 1) In the whole wear process, the surface roughness of AISI 52100 ring increases continuously. The surface roughness of AISI 5120 disc decreases first and then keeps stable and finally increases dramatically. To sum up, the ring surface gradually tends to the failure state. The disc surface firstly tends to an optimum state and then to a failure state, ensuring a long stable period of the mating surface. The main wear mechanism of ring sliding against disc is adhesive wear

accompanied with abrasive wear.

- 2) For the 3D surface morphology of disc, its fractal dimension D_s obeys the evolution rule of an inverted bathtub curve of “increase-stability-decrease”. Conversely, the multifractal spectrum width $\Delta\alpha$ obeys the evolution rule of a bathtub curve of “decrease-stability-increase”. A larger D_s or a smaller $\Delta\alpha$ indicates that the surface morphology has fractal nature with finer morphology structure, stronger self-similarity and more intense complexity.
- 3) The phase trajectory of reconstructed friction coefficient signal constantly overlaps but never intersects. It firstly converges to a specific minimal space and then keeps relatively stable, finally diverges and escapes from the space, as the wear process goes on. The standard deviation of distance matrix STD obeys the evolution rule of a bathtub curve of “decrease-stability-increase”. The convergent phase trajectory is named as a chaotic attractor. The STD stabilizing around a minimum indicates that the friction coefficient signal has chaotic nature and maintains at a stable state.
- 4) There is a negative correlation between D_s and $\Delta\alpha$ of surface morphology. These two fractal parameters can jointly reveal the nonlinear characteristics of 3D surface morphology from the spatial scale. There is a negative correlation between D_s and STD , and there is a positive correlation between $\Delta\alpha$ and STD . These findings establish the correlation between fractal in spatial scale and chaos in temporal scale. And they further establish the correlation between 3D surface morphology and friction coefficient signal in the same friction system. The evolution of fractal feature of surface morphology can be used to recognize wear states. The evolution

of chaotic feature of friction coefficient signal can be employed to on-line and real-time recognize the wear states. These researches have a great application prospect in monitoring and predicting on the condition of mechanical equipment operation or tool wear.

Acknowledgments

This project is supported by the National Natural Science Foundation of China (Grant Nos. 52105215, 52175194, 51675483, 52075047), Fundamental Research Funds for the Provincial Universities of Zhejiang (RF-A2019008).

Declarations

Conflict of interest The authors declare that they have no conflict of interest.

Data Availability Statements The datasets generated during and/or analyzed during the current study are available from the corresponding author on reasonable request.

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