

# Random Beamforming in Mobile mmWave Systems: Performance Evaluation and Parameters Optimization

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## Research

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## RESEARCH

# Random Beamforming in Mobile mmWave Systems: Performance Evaluation and Parameters Optimization

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## Abstract

Random Beamforming (RBF) is considered one of the most promising beamforming techniques especially for Massive Multiple Input Multiple Output (MIMO) mmWave systems. It has been proven to achieve optimal sum rate capacity for downlink MIMO systems for many applications. In this paper, the RBF is implemented in a mmWave MIMO systems with mobile users and the effect of this mobility is studied. Beside that, some parameters of the system have been optimized for different realistic scenarios. The mathematical derivation of the system model and the simulation using some practical values have been conducted. The resulted degradation in system throughput as a consequence of beamforming outage that results from users mobility is calculated. Many factors that affect the system throughput were considered in the derivation. The second part of the paper is going a further step in optimizing the network parameters for different operation scenarios. These parameters include the frame duration and beam width. Taking in consideration that the outage probability is a Convex optimization problem, the optimal values of these parameters were derived. for the walking and running users cases.

**Keywords:** mmWave; MIMO; random beamforming (RBF); Frame Duration; Beam Width; mobility

## 1 Introduction

Beamforming is one of the most important enabling technologies for the mmWave systems [1]. There are many types of beamforming that has been suggested recently and one of the most promising ones for massive MIMO mmWave systems is the Random Beamforming (RBF) [2–5]. Channel estimation on the other hand is considered one of the most active issues when dealing with mmWave systems [6]. Many channel models for mmWave systems (30-300 GHz frequency band) have been derived and studied in the literature for many years now and some of them include the Uniform Random Single-Path (UR-SP) that takes one path in consideration whether it is the line of sight (LoS) or the strongest non line of sight (NLoS) and Uniform Random Multiple-Path (UR-MP) that takes multipath channels in consideration [3, 4]. mmWave channels have been proven to be sparse with respect to the angle of arrival (AoA) domain where there is always one strong Line of Sight (LoS) and few weaker Non Line of Sight (NLoS) rays (or clusters of rays) between each transmitter and receiver. RBF is trying to utilize such sparsity in the downlink Channel State Information (CSI) to achieve a optimal capacity (linear sum rate)

with respect to the total number of antenna elements in the Base Station (BS) [3]. Using that fact, wireless systems with limited feedback from users (user equipments (UE)) to BS is achievable with the goal of faster systems, less overhead, and acceptable performance. The suggested RBF systems that use mmWave is assuming stationary (no mobility) users in each cell. So, the effect of mobility on the overall performance is investigated here in order to understand better the effect of user mobility on the achievable throughput when using random beamforming. See figure 1 for possible mobility scenario effect.

1.pdf

**Figure 1** Possible Performance degradation resulted from mobility when using the random beamforming

In the traditional random beamforming, BS's randomly create many beams that are supposed to point towards the UE's in each cell (opportunistically) as we assume that there are many users in each cell at any time. Once the UE receives the pilot signal (through any beam), it will send a feedback (the Signal to Noise Ratio (SNR) or the Signal to Interference plus Noise Ratio (SINR) in our case). Then, BS's collect all these received feedbacks and choose the UE with the SNR to send the data to it. The BS sends the intended data to that UE using the beam direction that was used in the beamforming initiation process. All of this have been proven to give great performance as long as the UE is fixed and immobile. But if the UE is moving during this process, the agreed on beam in each beamforming round can miss the user (as in fig 1 while the data is being sent from BS to the UE. And as we will see in the simulation results, the higher mobility the UE shows, the larger degradation in the system performance we will see.

The first part of our work is studying the mobility effect from different perspective [14]. Taking the same assumptions in [2] and [4] for system model and assuming we have many UE's in the system and each of them will have various mobility states (fixed, walking, running, biking, or riding a moving car). In each case, several factors were studied and their effect on the overall system operation was estimated. These factors include the beam width (of the beamforming), the distance between BS and the mobile UE (as a random variable), the mobility orientation (or the movement direction of each UE), and the maximum shift in UE position during the round trip time (RTT) of the messages during the beamforming and data transmission time. The effect of each of these factors was studied alone and then they were combined together in a closed form expression that describes the throughput degradation results from UE mobility [14]. The other possible effects of mobility like blockage (both from the user itself (self blockage) or from other obstacles in the environment), the doppler shift results from horizontal and vertical movement of UE during mobility, and user scheduling if there are many users in each beam coverage areas are good directions for further future work.

After studying the mobility effect and derive it both mathematically and through extensive simulation in [14], it was obvious that some systems parameters need to be optimized for different operation scenarios. The frame duration and the beam angle (or beam width, as they will be used interchangeably throughout the paper)

for RBF in mobile scenarios was studied as well in [17]. The goal of the work in [17] was to optimize the system performance for different users with different mobility states while using RBF. Taking in consideration the typical coverage distance for Base Stations in an Ultra Dense Networks [23] (i.e. no more than 200 meters), and the results of the work in [14], we derived the best frame duration and the beam width for each outage probability [17].

### 1.1 Related Works

Many studies have been conducted in this field as mobility is a major part of wireless communication systems nowadays. Studying the mobility effects on wireless communication systems that uses mmWave is a relatively new topic and has yet to be fully investigated. In [7], authors suggest that the normal walking speed of humans can be considered as stationary in the mmWave channel systems that uses IEEE 802.11ad default frame duration. But there is a possibility of 2.0 to 4.0 dB degradation in the average Block Error Rate (BLER) performance if we use narrow band communications [7]. Other research to study mobility effect on the overall system operation was conducted in [8]. In this work new mobility models were suggested to describe each different scenario taking in consideration the specific geometric properties of the used mmWave systems. The measurement experiments conducted in [8] show that the use of mmWave frequencies in small cells systems is feasible but the time variance of the propagation channel resulted from the mobility was very high [8].

In [13], an approximation model, based on the Quasi-Stationary (QS) assumptions was suggested to accelerate the computation in the Markovian framework. They also used that model to calculate the average system throughput for fixed and mobile users. The probability of handover for mobile users between different Base Stations were also considered in that work [13]. Although, directional transmission is considered very important to mmWave communications, the directional narrow beamwidth links can be easily degraded or disconnected because of the misalignment resulted from the mobility of the transmitter or the receiver [28]. In their work [28], and after performing many measurements, they found that that micro-scale mobility has bad effects on the link capacity of about 1% even though these links can suffer from large capacity degradation of over 50% in case of mobility for flying drones.

Although, many Orthogonal Multiple Access (OMA) based random beamforming have been studied in the literature [2–4], but the Non-Orthogonal Multiple Access (NOMA) has also been studied in [15]. In this work, the authors studied the performance of the coexistence of RBF and NOMA both for single beam and multiple beams used by the base station (BS) [15]. The special features of the mmWave frequencies allowed better scheduling and reduced outage probability in specific scenarios over the OMA- based systems. Also, it is worth mentioning that the concept of Random Beamforming (RBF) has been suggested for a while and did not start with the work in [2] as some form of RBF was suggested since 2003 [16].

Many researchers have been working on determining the optimal length (duration) of wireless frames for various applications and scenarios as in [18–20]. Minimizing the frame duration in large wireless networks (with multi-hops) have been found to be equivalent to the packet throughput maximization in [18]. In other words,

authors in [18] found that reducing the size of the frame duration to the minimum is the way to increase the packet throughput and optimizing the multicast trees of the network that is utilized by the streams of packets [18]. So, maximizing the transmission throughput of the network is done by minimizing the number of time slots in each frame [18]. Although they got a good results, their assumptions can not be generalized for all scenarios as it can depend on other system factors like the number of users, the available bandwidth, and the type of application the system used for.

Frame durations of the transmitter and the receiver are not the same in many systems and this is found to waste the system's frequency channel [19]. Such differences were studied in [19] where its effect on the wireless communications system achievable throughput was reduced. To solve such a problem, the receiver must set the frame length to be equal to that's of the sender and they can be selected for frame aggregation properly [19]. Even though the delay and the wasted time were reduced, and the system throughput was improved, but no closed form solution was derived for the optimum frame length in terms of the differences in slots durations.

Authors in [20] went a step further in their study and derived a closed form estimation of the optimal frame duration in Frame Slotted ALOHA protocol of the Radio Frequency Identification (RFID) systems. This is done by optimizing the efficiency of the Time-Aware Framed Slotted ALOHA (TAFSA) taking in consideration the variations of slot durations. Despite the fact that optimizing frame durations have been the interest of many researchers, but there is no (one solution fits all) for all scenarios so far. Frame duration optimization is also considered in this paper for the MIMO systems that uses Random Beamforming algorithm and mmWave frequencies. In the optimization process, the derived expected degradation (the outage probability) resulted from the UE mobility during the communication which was published in [14] is now used for parameter optimization of the system and for different scenarios.

In beamforming, we can set the beam width based on different factors. Setting the optimal beam width for different wireless mmWave systems that uses beamforming is the focus of many studies recently [21, 22]. In [21], a special type of wireless networks was studied that is called Small Cell Networks (SCNs) which uses UE's as relays and uses mmWave frequencies. These networks throughput was optimized in [21] using a coordinated meta-heuristically optimized beamwidth/alignment-delay approach. This study shows that the transmitters' and receivers' beamwidths need to be paired carefully to avoid having considerable Multi-User Interference (MUI) even in the Time Division Multiple Access (TDMA) systems [21]. In [22], a novel framework was suggested that combines the Matching Theory and Swarm Intelligence together in order to pair vehicles efficiently and optimize beamwidths of Tx and Rx. This study uses both of the the CSI and the Queue State Information (QSI) to establish communication links among vehicles in a Vehicle to Vehicle (V2V) network [22]. In this paper, optimizing the beamwidth is done by using many parameters of the dynamic mobile network for different situations in the systems that uses random beamforming and mmWave frequencies.

## 1.2 Contributions

Contributions of this paper can be summarized in the following points:

- Derive the mathematical expression for the performance degradation resulted from user mobility when using Random Beamforming (RBF) in mmWave systems that is part of small cells systems or what is called Ultra Dense Networks (UDN) that are part of the 5G and beyond systems. Part of this work was published in [14].
- Extensive simulation was done to prove the theoretical results for different practical scenarios and the parameters used in the simulation are listed in tables 1 and 2. Part of these simulation results were also included in [14].
- Different system parameters (Frame duration and Beam width) were optimized for different mobility scenarios and practical systems scenarios. Some of these results were published in our work in [17].

### 1.3 Organization

The organization of the rest of the paper is as follows. system assumptions, and the scenarios of work for stationary and mobile UE cases are explained in section 2. Section 3 shows the mobility effect and the outage probability derivation. Section 4 is showing the parameters optimization in case of mobility, whereas Section 5 is showing the numerical results. Finally, Section 6 concludes the paper and gives some ideas about the possible future work directions related to this work.

### 1.4 Notations

The following notations are used in this paper.  $a$  represents a column vector whereas  $A$  represents a matrix.  $A_{i,j}$  represents the  $i$ th row and  $j$ th column entry of the matrix  $A$ . For the matrix  $A$ ,  $A^H$ ,  $A^T$  indicate the conjugate transpose and the transpose respectively.  $x \sim CN(\mu, \Sigma)$  represents a random vector  $x$  that is complex Gaussian Distributed with the mean of  $\mu$  and the covariance matrix of  $\Sigma$ .  $\theta \sim Unif(a, b)$  is a representation of an angle  $\theta$  that is randomly distributed within  $[a, b)$ .  $E[x]$  denotes the statistical expectation of  $x$ . And finally,  $det(A)$ ,  $|x|$  and  $\|x\|$  are the determinant, the absolute and the norm values of  $x$ .

## 2 Methods

In this section, we introduce the assumptions about the downlink broadcast mmWave MIMO system with brief introduction to the original random beamforming algorithm.

### 2.1 Downlink Broadcasting MIMO System that uses mmWave

For the typical static MIMO system suggested in [2], the Gaussian broadcast channel is assumed which consists of  $n$  UEs spread within the coverage area of the Base Station (BS). Each UE is equipped with a number of antennas ( $N$ ) and the BS is equipped with ( $M$ ) antennas. Block fading channel model is assumed in our system where the channel propagation matrix is constant during the coherence time  $T$ . The number of users is assumed to be more than the number of BS antennas at all times ( $n \gg M$ ). Also, we assume that each UE has no more antennas than the BS antennas ( $N \leq M$ ) which is the normal case in practice. Our system can be described as in equation 1:

$$\mathbf{y}_i(t) = \sqrt{\rho_i} \mathbf{H}_i \mathbf{s}(t) + \mathbf{w}_i(t) \quad i = 1, \dots, n. \quad (1)$$

where  $\mathbf{s}(t)$  is the  $M$ -vector of transmitted signal by the BS,  $\mathbf{y}_i(t)$  is the  $N$ -vector of received signal by UE  $i$ ,  $\mathbf{H}_i$  is the  $N \times M$  channel matrix between the BS and UE,  $\mathbf{w}_i$  is the  $N$ -vector of additive white gaussian noise, and  $\rho_i$  is the SNR of the  $i$ -th user (averaged over the randomness of channel). One of the most important features of the mmWave signals is that they provide huge bandwidth, the ability to pack many antenna elements in small area (because of the small wavelength of such high frequencies) and the ability to be transmitted in narrow beams using the beamforming techniques which can be utilized in directional transmission with less interference and eavesdropping from other parties (other than the legitimate Tx and Rx) and that is why we use the mmWave with massive-MIMO in this context.

Other assumptions include:

(a) The total power transmitted by the BS is expressed as  $E [\|\mathbf{s}\|^2] = M$ . In other words, the average Tx power per antenna is a unit.

(b) For each user in the coverage area, the averaged SNR is expressed as  $\rho$ .

## 2.2 System Throughput

It is well known that in any MIMO system, if the receiver has the full information about the communication channel (i.e. CSI), then the capacity of the MIMO system can scale as  $\min(M, N) \log \rho$  [10]. But in case of not having full CSI information (which is the case in most practical situations and especially with mmWave channel that is sparse in nature), then the system capacity (or the achievable data rate) of any point-to-point (P2P) in a MIMO system will scale as  $\min(M, N)(1 - \min(M, N)/T) \log \rho$ . Here,  $T$  is representing the the coherence time of the channel [11]. In the special case of ( $N=1$ ), the system capacity (throughput) is given by (2) [2]:

$$R = E \left\{ \max_{P_1, \dots, P_n, \sum P_i = M_\rho} \log \det \left( 1 + \sum_{i=1}^n \mathbf{H}_i^H P_i \mathbf{H}_i \right) \right\}. \quad (2)$$

where  $M_\rho$  represents the total power (averaged). It can be seen that the capacity scales as  $(M \log \log n)$ .

## 2.3 Random Beamforming

As mentioned earlier, beamforming is now considered the way to go when designing mmWave MIMO systems. So, BSs need to transmit data in an aligned beams to the UE's all the time during the transmission. Traditional approach in doing so is by estimating the positions of receiver devices (or UEs) by using the Global Positioning System (GPS) for example. There are many other localization techniques but they all share the same overhead as the GPS. After estimating the location of the UE, BS aims the beams of data to the desired UE position. However, the location information usually comes with a substantial overhead and the GPS information is usually having high error margin with respect to such highly sensitive

applications. One of many alternative approaches, named the random beamforming [3], states that when the BS has messages to broadcast, it should form random beams in random directions. Taking our assumption in consideration, there should be many UEs in each cell and each of the BS beams will point to one UE at least. The BS then sends the pilot signal, receives acknowledgement from the UE, and start transmitting the data in that direction. By using this approach, we avoid the need for the GPS like localization techniques and get high accuracy beamforming. To sum it up, [3] mentions that “*RBF (random beamforming) achieves linear sum rate scaling w.r.t. the number of transmit antennas and, furthermore, yields optimal sum rate performance when the number of transmit antennas is large, if the number of users increases linearly w.r.t. the number of transmit antennas.*” Based on that, RBF is promising better performance and high throughput for mmWave communication systems that use MIMO in a dense environment. Each round of the Random beamforming is described in algorithm 1.

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**Algorithm 1** Random Beamforming for MIMO Systems using mmWave

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- 1: BS constructs  $S$  random orthonormal beam vectors  $\{u_1, \dots, u_S\}$ .
  - 2: BS chooses a normalized direction  $\theta$  randomly and transmits the beam  $x$  to in that direction.
  - 3: **for** UEs receiving  $\text{SNR} \geq$  a predefined threshold **do**
  - 4:     UE sends the SINR to the BS
  - 5: BS sends data stream to the UE having the maximum received signal power using the beamforming vector  $x$ .
- 

Given the above assumptions and the algorithm, the system achievable capacity can be expressed (approximately) as:

$$\begin{aligned}
 R &\approx E \left\{ \sum_{m=1}^M \log(1 + \max_{1 \leq i \leq n} \text{SINR}_{i,m}) \right\}. \\
 &= ME \left\{ \log(1 + \max_{1 \leq i \leq n} \text{SINR}_{i,m}) \right\}. \tag{3}
 \end{aligned}$$

Where  $\text{SINR}_{i,m}$  is the maximum Signal to Interference plus Noise Ratio and  $m$  is the index of that SINR and  $M$  is the number of transmit antennas [2].

### 3 Outage Probability Caused by UE Mobility

In this section, we analyze the expected performance loss of the random beamforming algorithm that resulted from the mobility of UEs during the communication process.

#### 3.1 Mobility Assumptions

The following assumptions are considered when analysing the performance degradation of the system explained in previous sections.

- A simplified channel model is assumed to be UR-SP, where we consider only the strongest signal path between the BS and each UE. Without loss of generality, the single path here between the BS and the UE can be the LOS or the strongest NLOS in case there is blockage in front of the LOS path.
- Number of antenna elements in each UE is less than that of the BS.
- The number of UEs in each cell denoted by  $K$  is large.

- As mentioned above, the CSI is acquired by sending a pilot signal before the beginning of the beamforming process from the BS to all the UEs' and we assume that all the CSI for all users is available once the beamforming starts.
- The received signal by each UE  $k$  is described as follows:

$$y_k = \mathbf{h}_k^H \mathbf{x} + n_k, \quad k = 1, \dots, K. \quad (4)$$

where  $\mathbf{h}_k$  is the vector of each user  $k$  channel,  $\mathbf{x}$  is the Tx signal vector, and  $n_k$  is the noise.

- Channel model is represented by:

$$\mathbf{h}_k = \alpha_k \sqrt{M} \mathbf{a}(\theta_k). \quad (5)$$

where  $\alpha_k$  is user  $k$  channel gain factor,  $M$  is the BS No. of antennas,  $\theta_k$  is the main path angle for the user  $k$ ,  $\mathbf{a}(\cdot)$  is the vector of array steering. During the beamforming process, each user is sending back the averaged power received calculated as:

$$|y_k|^2 = |\mathbf{h}_k^H \mathbf{x}|^2 + \frac{1}{N_s}. \quad (6)$$

with  $N_s$  represents the number of training samples (if there are more than one) during the beamforming training time.

To sum it up, our analysis of the mobility effect on the system capacity is based on these factors:

- The beam width, some realistic values are assumed.
- The direction of movement of each UE, treated as a random gaussian variable.
- The mobility speed and its relationship to the channel variations and alignment accuracy.
- The distance between the BS and each UE that is part of the beamforming process and its corresponding Round Trip Time (RTT).

Figure 2 shows an example of the UEs distribution within any beam's area:

[scale=0.3]2.png

**Figure 2** Beam Area with Side Boarders.

### 3.2 Degradation Analysis

Unlike the basic scenario, UE's can move while the RBF is going on. Such movement causes the derived optimal sum rate to degrade depending on many factors. When we have mobile UE, the agreed on beam between the BS and the UE may miss the right direction because the UE will be in a different position. So, here we analyse the factors that can decide the final resulted degradation because of such mobility.

### 3.2.1 Beamforming Round Trip Time

This time is the period of time taken for the UE to send its feedback (SINR) to the BS and start receiving the data from the BS if selected as the current receiver. This time is mainly decided by propagation time of the electromagnetic (EM) waves, the time taken to process the received feedback in the BS which includes decoding the received packets that have the SINR from many UE's, choosing the intended UE to transfer the data to, and preparing data for transmission, and the data frame duration time. The physical limit in our analysis is the propagation time. To simplify the analysis, we assume the processing time in the BS is negligible when compared with the propagation time.

So, the round trip time can be calculated as in equation 7:

$$T_{RTT} = \frac{2 * d}{c} + FD. \quad (7)$$

where  $d$  represents the distance between the BS and the UE when performing the random beamforming,  $FD$  is the used frame duration, and  $c$  is the speed of light. Note that the TRR is double the propagation time as it is the two way time duration.

### 3.2.2 The Edge Probability

Which is the probability of having the UE (or UEs) on the edge of the beam area (the shaded region of the figure 2). The simplified shape of the physical area of each beam created by the BS has two narrow borders that represent the area where if the UE is in it, then there is a chance that it might leave the beam coverage area as a result of mobility during the RTT. We calculate such probability using this equation:

$$Pr(BeamEdge) = \frac{\text{Border Area}}{\text{Total Beam Area}}. \quad (8)$$

where

$$\text{Border Area} = 2D_{\max}L_{BS}. \quad (9)$$

and

$$D_{\max} = v_{\max}T_{RTT}. \quad (10)$$

$D_{\max}$  represents the maximum distance that the UE can travel during the RTT and it is different depending on the UE location and its speed,  $L_{BS}$  is the transmission range or the coverage distance of the BS in each cell,  $v_{\max}$  is the maximum speed of mobility of the UE in any scenario. Also we have:

$$\text{Total Area} = \frac{1}{2}B_B B_H. \quad (11)$$

where  $B_B$  is the length of the modeled beam base that is calculated using the approximated trigonometric calculations as:

$$\text{BeamBase} = \frac{2B_H}{\tan \theta_B}. \quad (12)$$

and  $B_H$  is the length (or height) of the beam area and it is the BS maximum coverage length or the ( $L_{BS}$ ),  $\theta_B$  is the Beam angle (i.e. beamwidth), and  $\theta_{BB}$  is the beam base angle which is approximated by:

$$\theta_{BB} = \frac{\pi}{2} - \frac{\theta_B}{2}. \quad (13)$$

Fig. 3 is showing the Physical approximate implementations of the numerical parameters explained above:

[scale=0.4]3.png

**Figure 3** Beam Approximate Shape with the Parameters used in calculating the Edge Probability

### 3.2.3 The Probability of Moving Away

It is the probability that the UE is within or close to any of the two edges of the beam and that it will move out of the beam area which can be represented as a special type of distribution called Triangular distribution (fig. 5) [9]. This kind of distribution is maximized when the UE location is on the outside edge of the beam area and minimized when it is as far away from it as possible:

$$f(x | a, b, k) = \begin{cases} \frac{2(x-a)}{(k-a)(b-a)} & a \leq x \leq b \\ \frac{2(k-x)}{(k-a)(k-b)} & b \leq x \leq k \\ 0 & (x < a), (x > k) \end{cases} \quad (14)$$

where:  $a$  here expresses the largest distance the UE can be away from the beam edge and still possible to leave the beam area but with low probability,  $b = k$  is the variable representing the location of the UE when it is exactly on the outer border of the beam area (the peak value in the triangular distribution) where the possibility of the UE leaving the beam area is maximized, The maximum possible distance between  $a$  and  $b$  is the previously explained  $D_{\max}$ , and finally the variable  $x$  which represent the random location of the user within the beam borders (between the two extremes  $a$  and  $b$ ). Fig. 4 is explaining the relationship between the UE location and the probability of leaving the beam coverage area:

Figure 5 represents the special case of the Triangular Distribution and its parameters that is used in (14):

[scale=0.4]4.png

**Figure 4** Relationship Between UE location and Beam leaving probability: (a) UE far enough from the Beam Border so that probability is low. (b) UE is closer to the beam border which increases that probability. (c) UE is very close to the beam border, which means that the beam leaving probability is higher.

[scale=0.3]5.eps

**Figure 5** The Triangular Distribution used to Express the Beam Leaving Probability distribution

### 3.2.4 All Parameters Together in One Formula

To put all individual parameters discussed earlier in one formula, we first mention our assumptions for clarification:

- Random Beamforming for a MISO broadcast system with one BS and many UEs.
- ULA with M antennas in the BS.
- The BS sector is covering 180 degrees with adjustable number of beams.
- Number of beams range from 10 to 120 (corresponds to  $18^\circ$ - $1.5^\circ$  as a beam angle (width)).
- Based on the number of beams, different triangles of beam coverage are created each time.
- We calculate the beam coverage total area and beam borders where there is a possibility that the user will leave the beam coverage area before the data arrives at it (as a result of mobility) which leads to throughput degradation.

Taking all these assumptions in consideration, we get the theorem 1:

**Theorem 1** *Knowing the capacity of a MIMO downlink broadcast system with static UEs and BS (as in [2]), then the system capacity will suffer the following degradation when there are mobile users in the system:*

$$Th_D = \sum_{n=1}^N (Pr_n(OnBeamEdge) * Pr_n(AwayFromBeam)). \quad (15)$$

$$= \sum_{n=1}^N \left( \frac{2 * D_{max} * L_{BS}}{0.5 * \frac{2 * L_{BS}}{\tan(\frac{\pi - \theta}{2})}} \right) * f(E(x) | a, b, k). \quad (16)$$

$$= \sum_{n=1}^N 2 * Speed * RTT * \tan\left(\frac{180 - \theta}{2}\right) * f(E(x) | a, b, k). \quad (17)$$

where  $Th_D$  is the expected throughput degradation of the studied system,  $Pr_n(OnBeamEdge)$  is the probability that the UE  $n$  can leave beam area during the random beamforming round trip time (RTT),  $Pr_n(AwayFromBeamCenter)$  is the probability that the

direction of mobility of the UE is towards the edges of the beam area,  $N$  is the number of users in the BS coverage area (beam area),  $Speed$  is the Expected speed of the UE (as listed in table 1),  $RTT$  is the round trip time of the message between the BS and UE,  $\theta$  is the beam angle (beamwidth),  $F(x | a, b, k)$  is the triangular distribution from equation 14, and  $x$  is the random variable representing the UE position within the beam borders and its expectation is expressed as in (18):

$$E(x) = \int_{-\infty}^{\infty} x * f(x)dx. \quad (18)$$

*Proof* The throughput degradation probability derived here is the sum (summation) of all the outage probabilities for all mobile users being served by the BS at each RBF round and it is the intersection (multiplication) of the probability that the UE is on the beam edge and the probability that it moves away from the beam center and that is why it was calculated as in 17.  $\square$

## 4 System Parameters Optimization

After studying the effect of mobility on the system performance, we investigated the feasible system parameters (i.e. the frame duration and the beam width) and the best values for each of them for different system scenarios and practical applications.

### 4.1 Frame-Duration Optimization

Based on the results from the last section, we construct the following frame duration optimization problem as a convex optimization:

$$\begin{aligned} & \text{maximize} && T_{RTT} \\ & \text{subject to} && Th_D \leq \Psi. \\ & \text{And} && d \leq 200 \end{aligned} \quad (19)$$

where  $\Psi$  is the outage accepted margin or threshold,  $T_{RTT}$  is the round trip time from the equation 7 above,  $d$  is the distance between the BS and UE, and  $Th_D$  is outage Expectation calculated in the equation 17 above.

### 4.2 Beam-Width Optimization

Next, we optimize the Beam width for any possible outage probability when random beamforming is used with mobile users. This also has been proven to be a convex optimization problem [14]. This optimization is expressed as:

$$\begin{aligned} & \text{Minimize} && \theta \\ & \text{subject to} && Th_D \leq \Psi. \\ & \text{And} && d \leq 200 \end{aligned} \quad (20)$$

where  $\theta$  (beamwidth) is one of the system parameters that need to be minimized in order to reduce the number of competing UEs in each beam and each time slot of

the scheduled data transmission for the MIMO system. This minimization is aimed also in increasing the number of random beams that each BS can create and serve which means more UEs served in a shorter time.

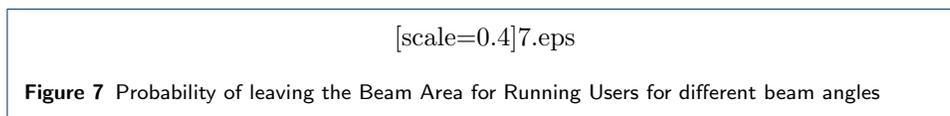
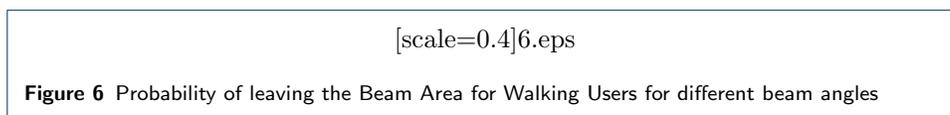
### 5 Numerical Results and Discussion

Mathematical modeling explained above with the parameters listed in table 1 are used in obtaining the numerical results to prove the feasibility of the suggested framework. All simulation parameters are similar to those in [2–4] for fair comparison. Up to the authors’ best knowledge, no similar works were done on the RBF with the same assumptions, so the performance evaluation was done on several operation scenarios to show the contribution.

**Table 1** Mathematical Derivation Parameters

Parameters	Specifications
Beam Angles (widths)	1.5, 2, 3, 4, 6, 9, and 18 Degrees
Speed Range for Walking	3-5 Km/h
Speed Range for Running	16-24 Km/h
Speed Range of Biking	15-40 Km/h
Speed Range of Car riding user	65-112 Km/h
Cell Radius (Beam Coverage Distance)	200 meters
Mobility Model	Random Way Point (RWP).
Number of Nodes (UEs)	1000.
Outage Threshold	1, 2, 5, 10, 15, 25, and 50%.

The Expected Throughput Degradation (%) resulted from mobility of UE is shown in the Figures 6 to 9. It has been observed that the probability of misalignment between the BS and any UE in its coverage area decreases as the distance between them decreases. This makes sense as this decreases the RTT and the possible movement during this RTT. Also, it can be seen clearly that larger beamwidth gives lower probability of misalignment and degradation in system capacity with mobile UEs. As this means more freedom to the users to move within the beam before the misalignment happens.



The relationship between the average throughput degradation (outage probability) and the mobility speed range is explained in the figures 10 to 13. It can be seen that there is a linear relationship between these two parameters as we see that faster moving UEs causes larger misalignment that results in larger capacity degradation:

In order to confirm the validity of our above mentioned mathematical modeling and their simulation results, a well known mobility model that is called the Random Waypoint (RWP) [12] was used for different scenarios to estimate the real degradation in throughput for different beamwidths and UE speeds. The results of such analysis is summarized in table 2:

[scale=0.4]8.eps

**Figure 8** Probability of leaving the Beam Area for Biking Users for different beam angles

[scale=0.4]9.eps

**Figure 9** Probability of leaving the Beam Area for Users Riding a Car for different beam angles

[scale=0.4]10.eps

**Figure 10** Expected Throughput Degradation for different Walking Speeds.

[scale=0.4]11.eps

**Figure 11** Expected Throughput Degradation for different Running Speeds.

[scale=0.4]12.eps

**Figure 12** Expected Throughput Degradation for different Biking Speeds.

[scale=0.4]13.eps

**Figure 13** Expected Throughput Degradation for different Car Speeds.

**Table 2** Simulation Parameters

Parameters	Specifications
Mobility Modeled as	Random Way Point (RWP).
Beam Angles in degrees	1.5, 2, 3, 4, 6, 9, and 18 Degrees.
UE Range of Speed	0.2 - 31 m/s.
Pause Intervals Range	[0 1] seconds.
Movement Intervals Durations	[2 6] seconds.
Direction Interval	[-180 180] Degrees.
# of UE Nodes	1000.
Beam Coverage Distance	200 meters

from the Simulation of the mobility (using the random waypoint mobility model) of UE with different speeds, locations, directions, and different distances between the BS and UE, we got the results shown in Fig. 14 for different Beam widths. As it can be seen clearly, these results are compatible with our mathematical analysis results where the probability of UE leaving the beam coverage area is about 4.5% in the worst case scenario:

[scale=0.5]14.eps

**Figure 14** Throughput Degradation Resulted from Simulated Mobility for Random Speeds, Directions, and Distances between BS and UE

As for the frame duration and beam width optimization, the optimization was performed using the convex optimization software called (cvx) from Stanford University described in [24]. We use Convex optimization as it was shown in [14] that the outage probability is a convex with respect to different parameters (i.e. frame duration and beamwidth) for the Mobile RBF. The default solver of the cvx is the SDPT3 and it was used in this optimization scenario. The computational complexity of the SDPT3 by default is  $O(n^3)$  where  $n$  is the number of variables, but as our problem is sparse, then the real complexity is much better than that. Also, this optimization is only required after collecting enough data from the network to set the parameters and can only be called and executed whenever the type of the applications or services of the network change dramatically. The results for frame duration optimization are shown in the figures 15 for UE with walking speed and 16 for UE with running speed users:

[scale=0.19]15.eps

**Figure 15** Optimal Frame Duration for Different Outage Probability Percentages with Walking Speed Range UEs

[scale=0.19]16.eps

**Figure 16** Optimal Frame Duration for Different Outage Probability Percentages with Running Speed Range UEs

Figures 15 and 16 shows clearly that a high degradation in the system capacity is expected when the mobile UEs are moving with high speeds or when the transmitted frame durations are more than (10 msec). This means that for such system to work, there is some kind of compromise that need to be forced as we can get longer frames with a larger probability of misalignment and retransmission or short frames with better delivery probability but might not fit well for bandwidth hungry applications. This can be application dependent and decided prior to network operation. It is worth noting that the default frame duration agreed on in the 5G standard is as short as (1 msec) for single frames and can be as small as half a millisecond for subframe length [13].

[scale=0.19]17.eps

**Figure 17** Optimal Beam Angle for Different Outage Probabilities with Walking UEs

[scale=0.19]18.eps

**Figure 18** Optimal Beam Angle for Different Outage Probabilities with Running UEs

The other parameter optimized in our work is the beamwidth (or beam angle) for each beam created during the random beamforming process and the results for this optimization for both walking and running speeds are as in figures 17 and 18:

The same tradeoffs between the speed and degradation are clear as in the case of optimal frame duration above. Shorter frames give better performance in the mobile scenarios and require many narrow beams. On the other hand, few wider beams are necessary for less degradation in the mobile UE situation. Finally, we can say that the optimum values for both the frame length and the beamwidth in order for the Mobile RBF to work well is when the FD is ( $\leq 10$ msec) and the beamwidth is ( $\leq 60^\circ$ ). The results in figures 6 to 18 show clearly the relationship among the system setup and parameters and give clear insights about how to set the values of these parameters in realistic environments. This can be generalized beyond only the mobile scenario of the RBF and it can set the stage to decide when to use the beamsteering and when to accept the tolerable outage probability depending on the use case type.

The range of values that we got for our parameters fits with the values in the well known standards like the 802.11ad standard for the Wireless Local Area Networks (WLAN) [14] where the highest FD is in the range of (10-100 msec) and as small as parts of the millisecond [13]. On the other hand, the resulted beamwidth for most of the situations above is also within the acceptable range where it can be as narrow as few degrees [14] or as wide as 100 degree [25]. We did not list the results for faster moving UEs (bike and car riding) because it is obvious that supporting such speeds required more sophisticated beam alignment or tracking techniques as the ones suggested in [26] and [27].

## 6 Conclusions and Future Work

In this paper, the mobility effect on the performance of Random Beamforming (RBF) was studied. It seems that any beam angle (width) more than 3 degrees is giving acceptable throughput even with mobility. Random Beamforming got a linear sum rate as the number of the users increased linearly with the number of antennas in the BS (for static users). So, with the same assumption here, the throughput degradation is proportional to (1/distance) which means that the farther the UE, the less probability it will get out of the Beam coverage area because of mobility. Throughput degradation is proportional to (1/beamwidth) which means that the wider the beam, the less probability of users leave the beam as a result of mobility. Even with mobility, BS can still deliver at least one message for the UE before it leaves the Beam coverage area with a very high probability. Different system parameters (that include communication frame duration and beamforming

beamwidth have also been optimized in this paper while taking in consideration the users mobility and other operation conditions.

Some future directions for this work can include dealing with the more realistic channel models for the mmWaves like the Uniform Random Multiple Paths (UR-MP) or the Saleh-Valenzuela (SV) channel model. Dealing with heterogeneous networks where many base stations are required to cooperate and coexist to serve shared users is also an interesting direction for future improvement of this work. Utilizing deep learning techniques for better understanding of the mobility behaviour and suggest adaptive beamforming that take such behaviour modeling in consideration is another way to go from here.

#### Abbreviations

RBF: Random Beamforming; MIMO: Multiple Input Multiple Output; UR-SP: Uniform Random- Single Path; LoS: Line of Sight; NLoS: Non-line of Sight; UR-MP: Uniform Random- Multiple Path; AoA: Angle of Arrival; CSI: Channel State Information; BS: Base Station; UE: User Equipment; SNR: Signal to Noise Ratio; SINR: Signal to Interference plus Noise Ratio; RTT: Round Trip Time; BLER: Block Error Rate; QS: Quasi-Stationary; OMA: Orthogonal Multiple Access; NOMA: Non-Orthogonal Multiple Access; RFID: Radio Frequency Identification; TAFSA: Time-Aware Framed Slotted ALOHA; SCN: Small Cell Networks; MUI: Multi-User Interference; TDMA: Time Division Multiple Access; QSI: Queue State Information; V2V: Vehicle to Vehicle; UDN: Ultra Dense Networks; P2P: Point to Point; GPS: Global Positioning System; EM: Electromagnetic; RWP: Random Waypoint; WLAN: Wireless Local Area Network; SV: Saleh-Valenzuela

#### ETHICS APPROVAL AND CONSENT TO PARTICIPATE

Not applicable

#### CONSENT FOR PUBLICATION

Not applicable

#### AVAILABILITY OF DATA AND MATERIAL

All the simulation tools used are open source and available online.

#### Competing interests

The authors declare that they have no competing interests.

#### Funding

Not applicable

#### Author's contributions

Both authors have the same contributions.

#### Acknowledgements

not applicable

#### Author details

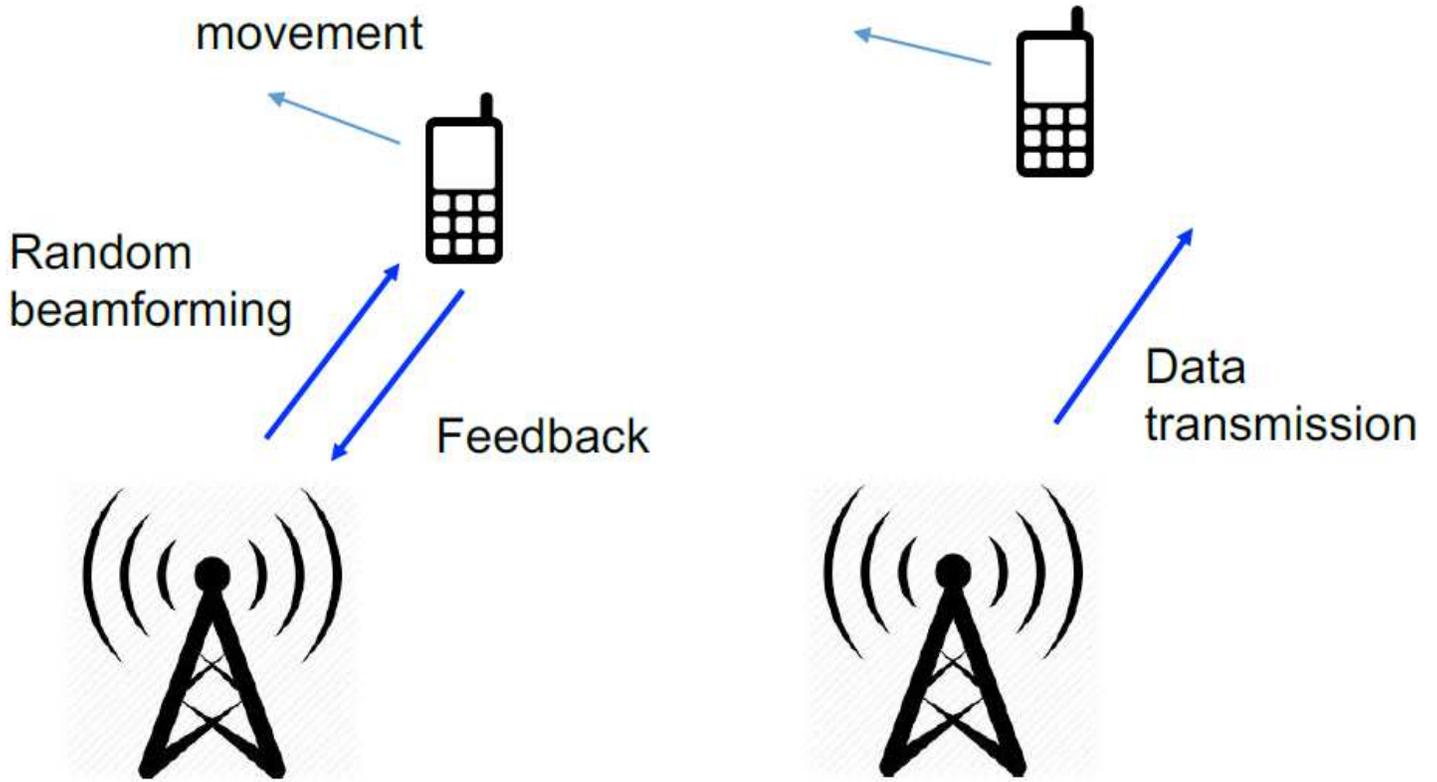
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## Figures



(a) Establishing the connection

(b) Misalignment due to mobility

Figure 1

Possible Performance degradation resulted from mobility when using the random beamforming

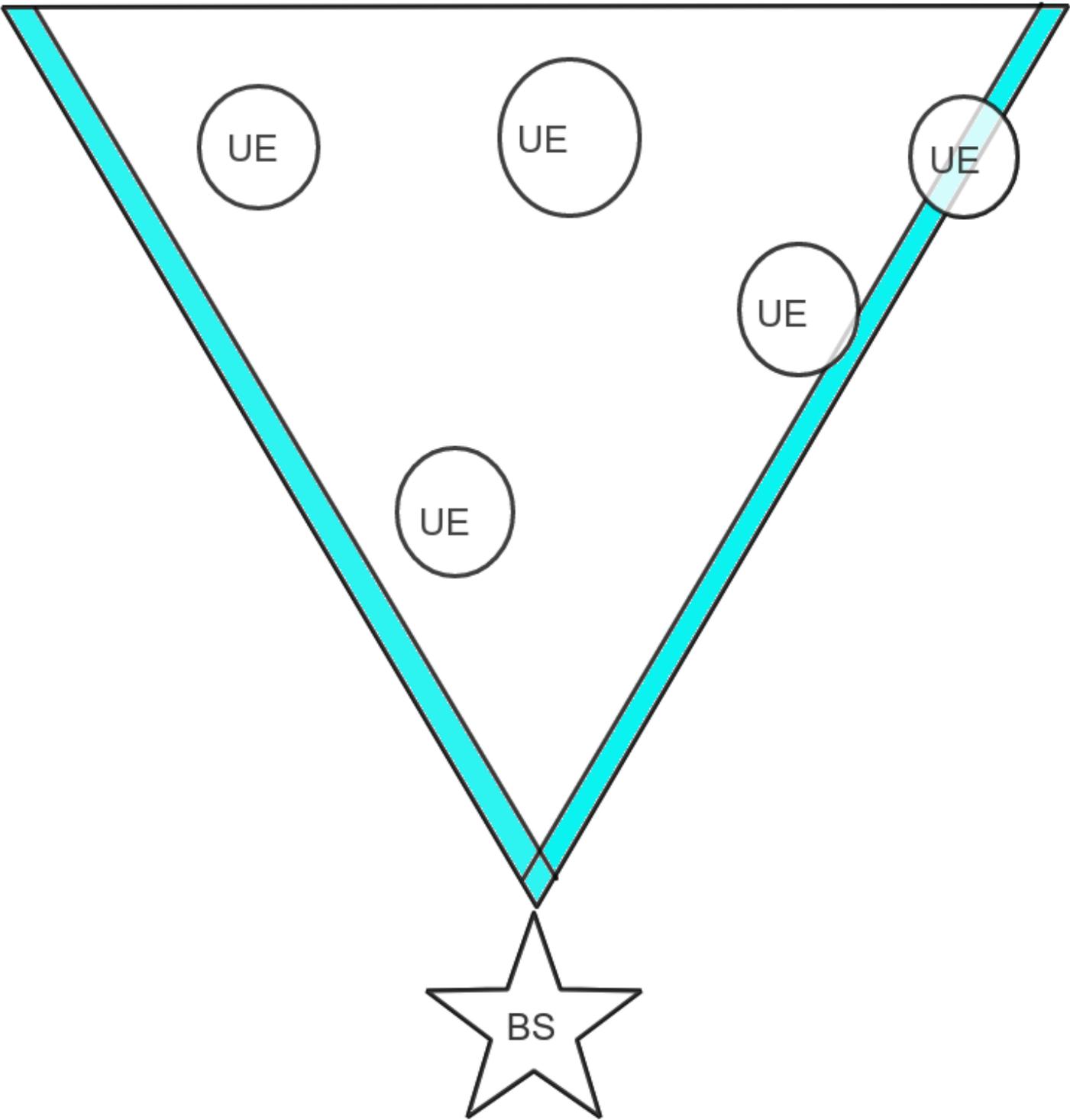
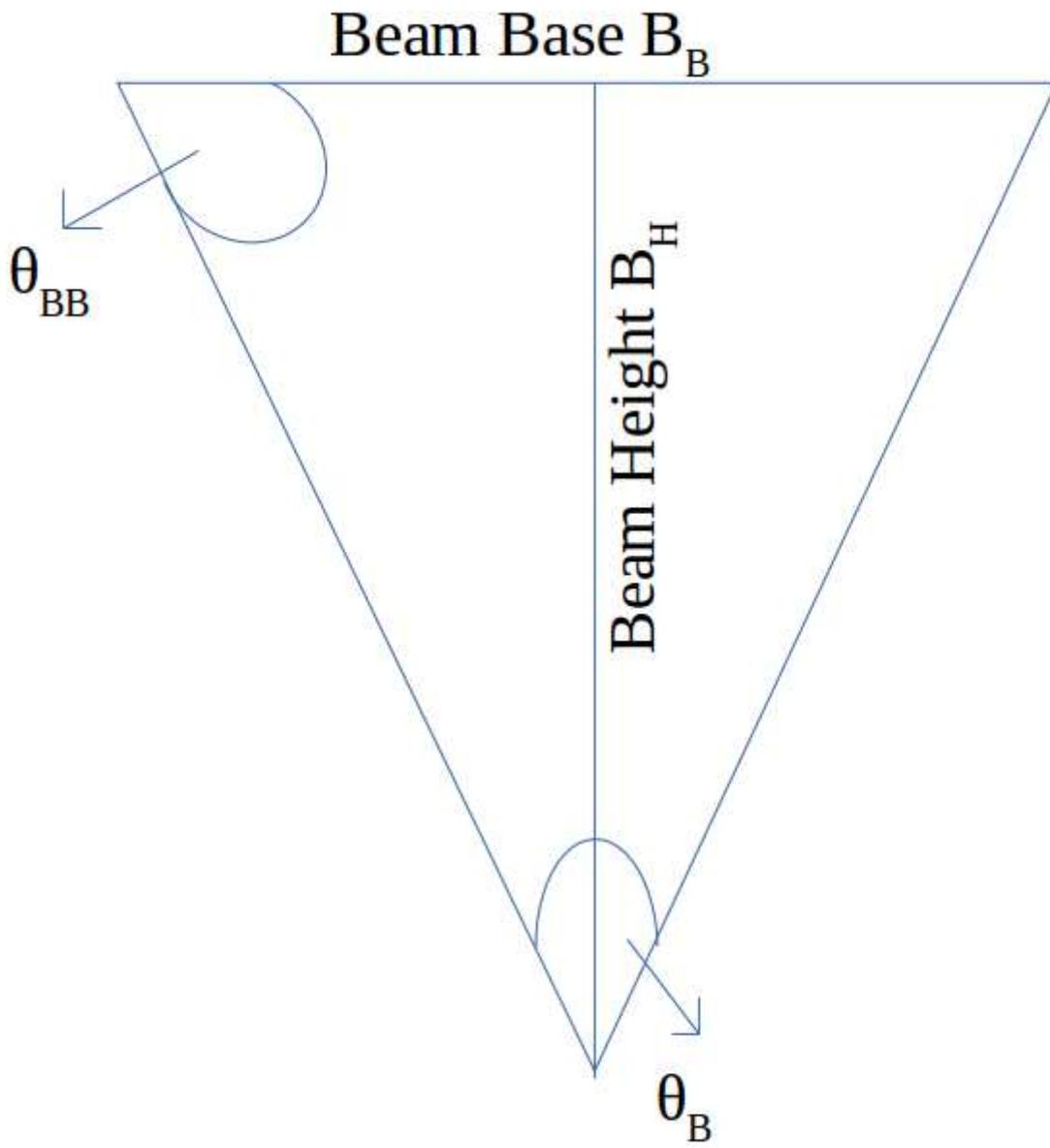


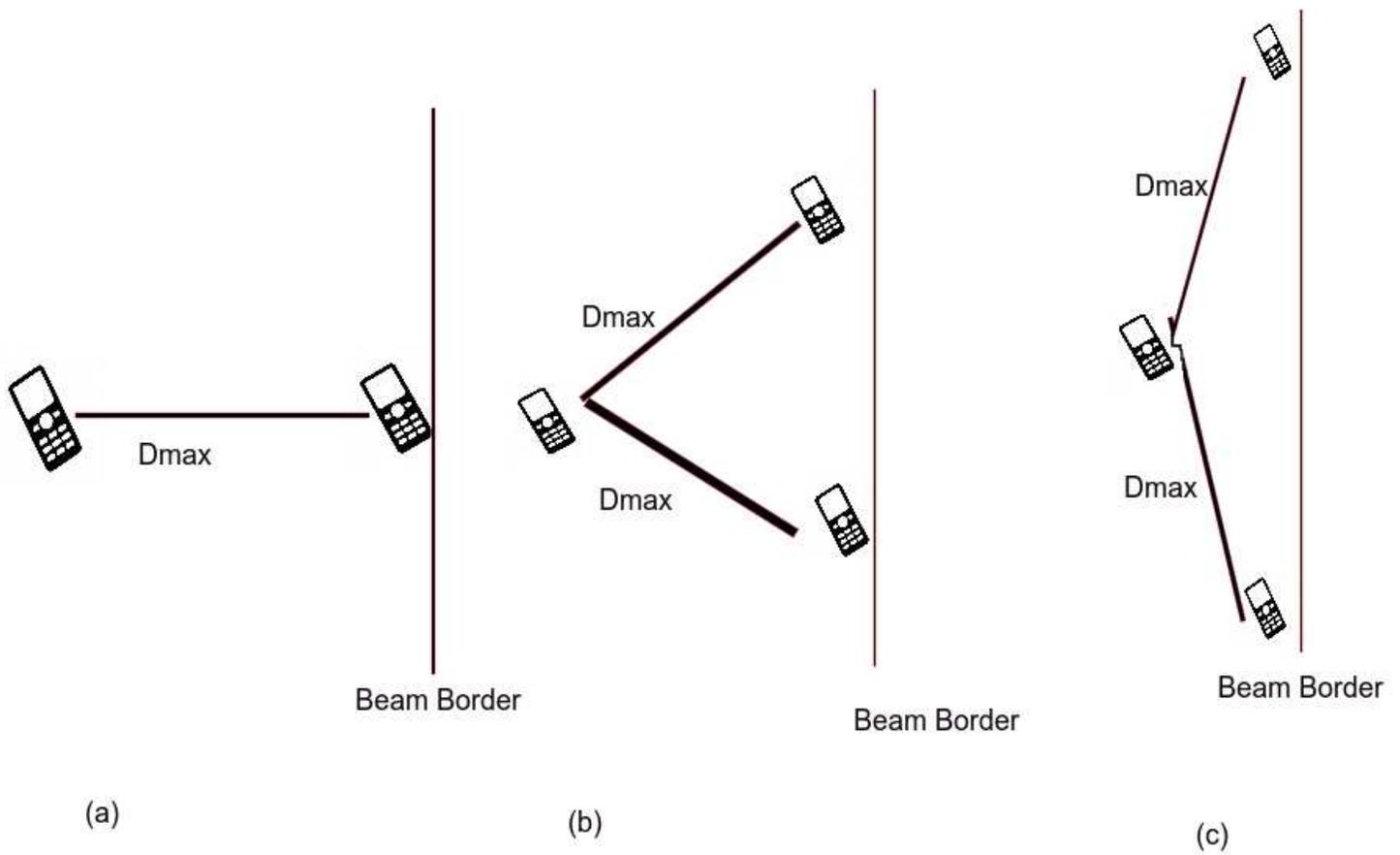
Figure 2

Beam Area with Side Borders.



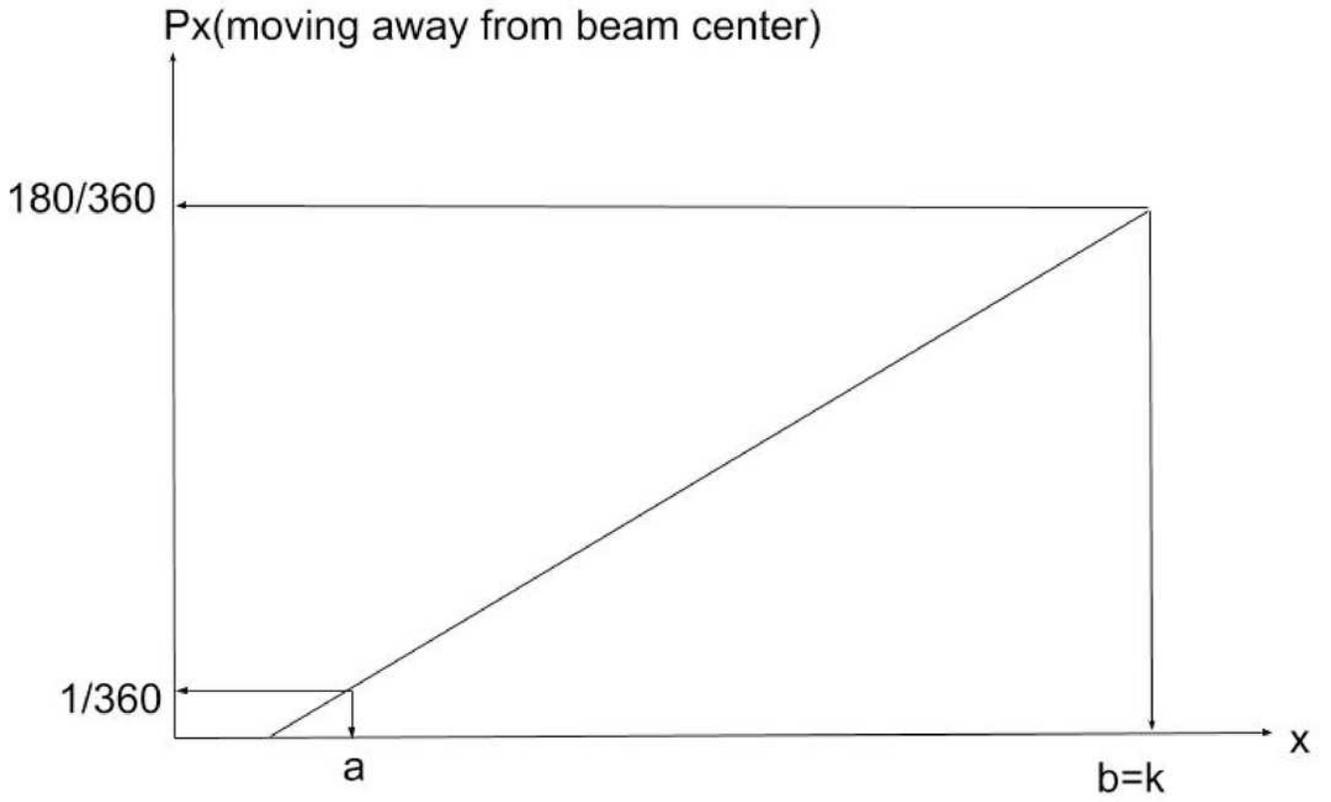
**Figure 3**

Beam Approximate Shape with the Parameters used in calculating the Edge Probability



**Figure 4**

Relationship Between UE location and Beam leaving probability: (a) UE far enough from the Beam Border so that probability is low. (b) UE is closer to the beam border which increases that probability. (c) UE is very close to the beam border, which means that the beam leaving probability is higher.



**Figure 5**

The Triangular Distribution used to Express the Beam Leaving Probability distribution

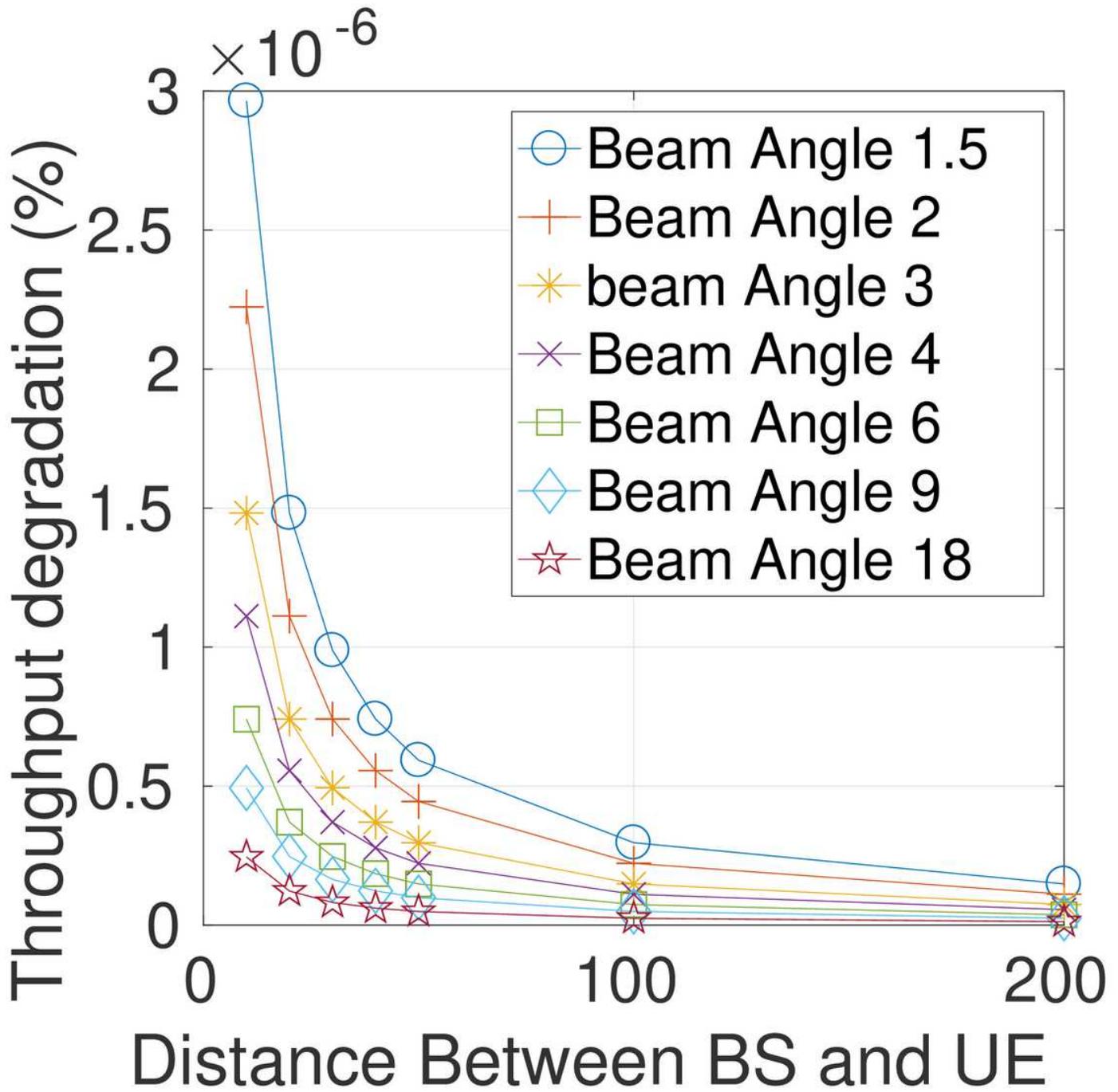


Figure 6

Probability of leaving the Beam Area for Walking Users for different beam angles

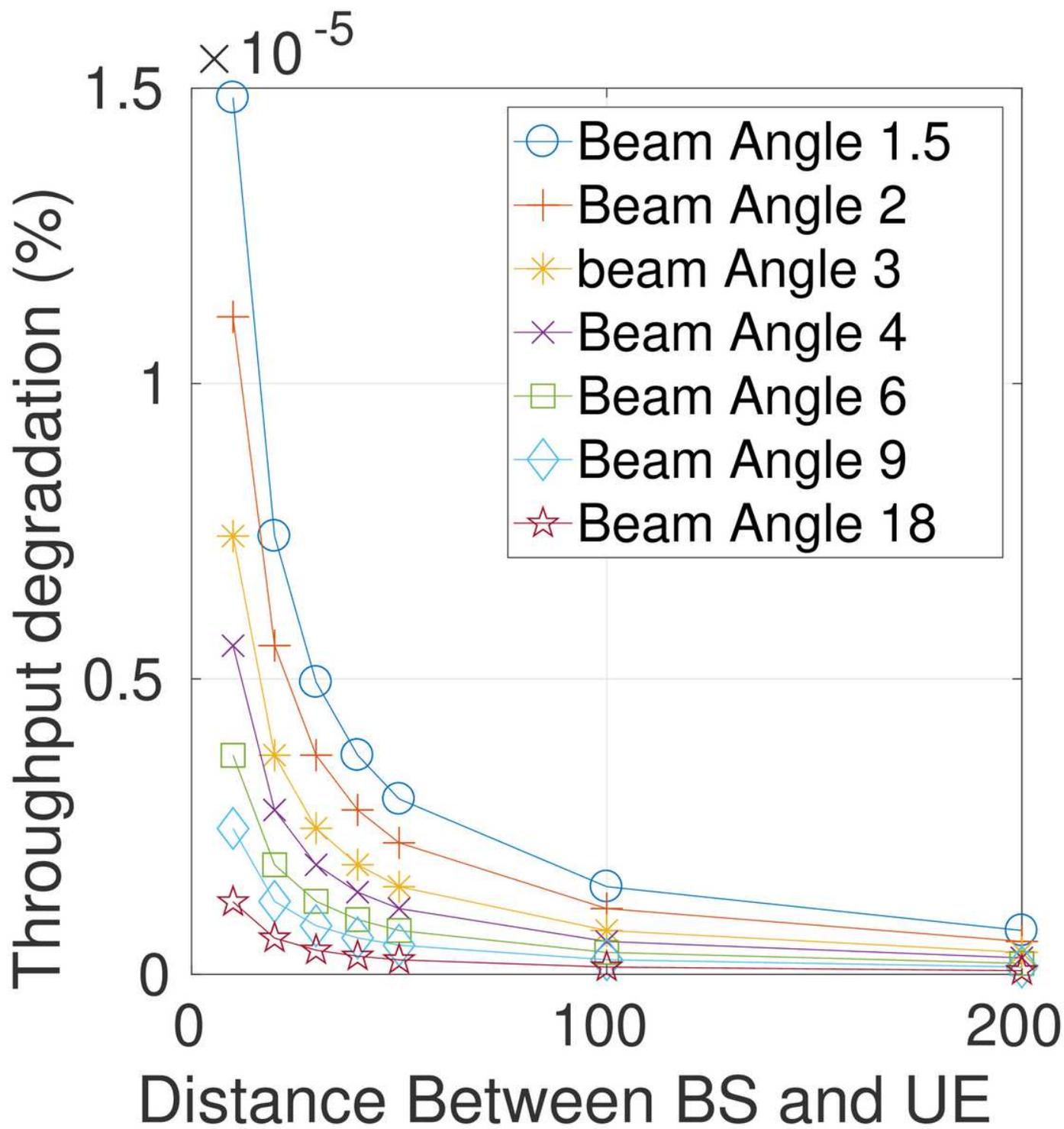


Figure 7

Probability of leaving the Beam Area for Running Users for different beam angles

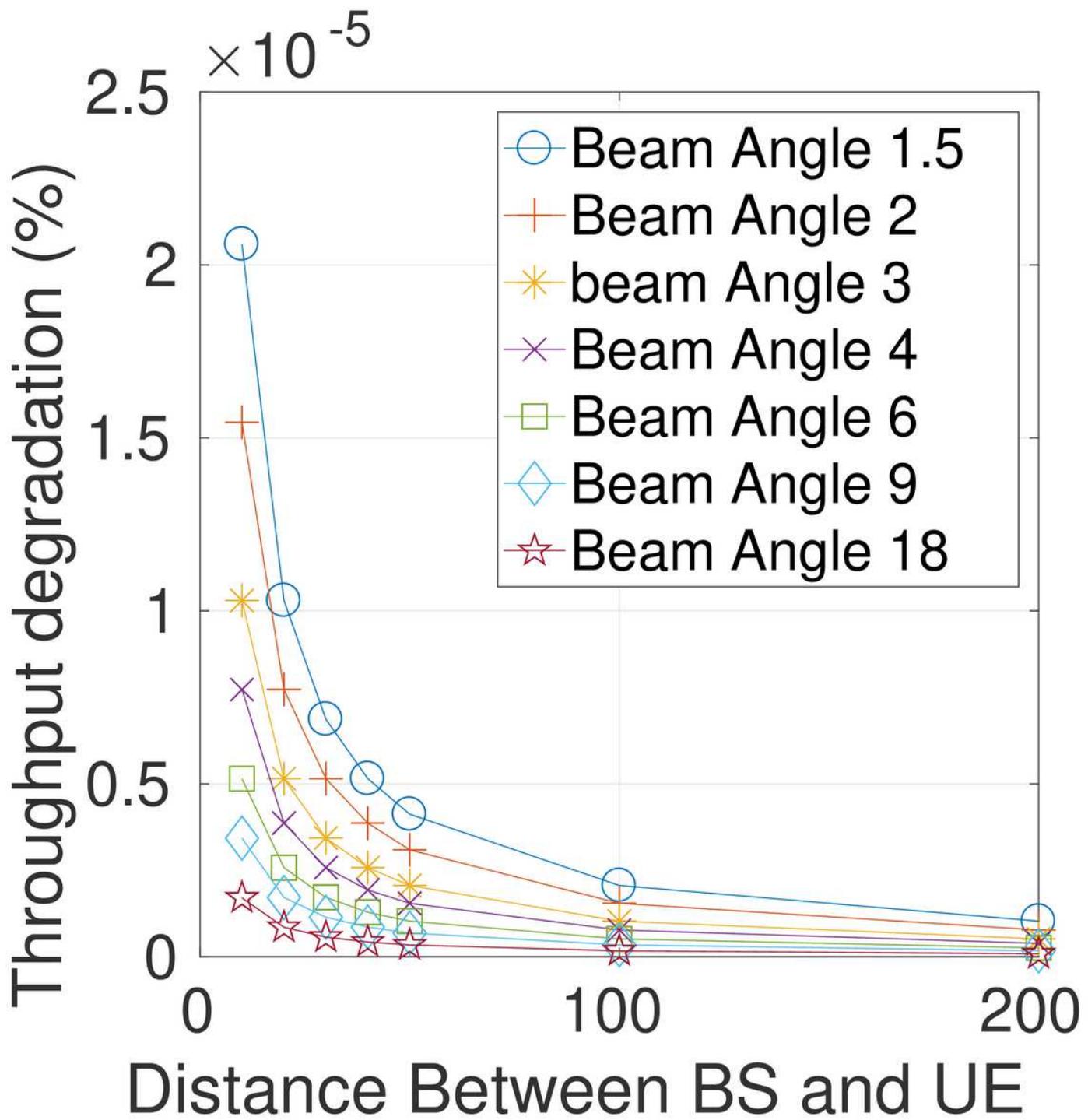


Figure 8

Probability of leaving the Beam Area for Biking Users for different beam angles

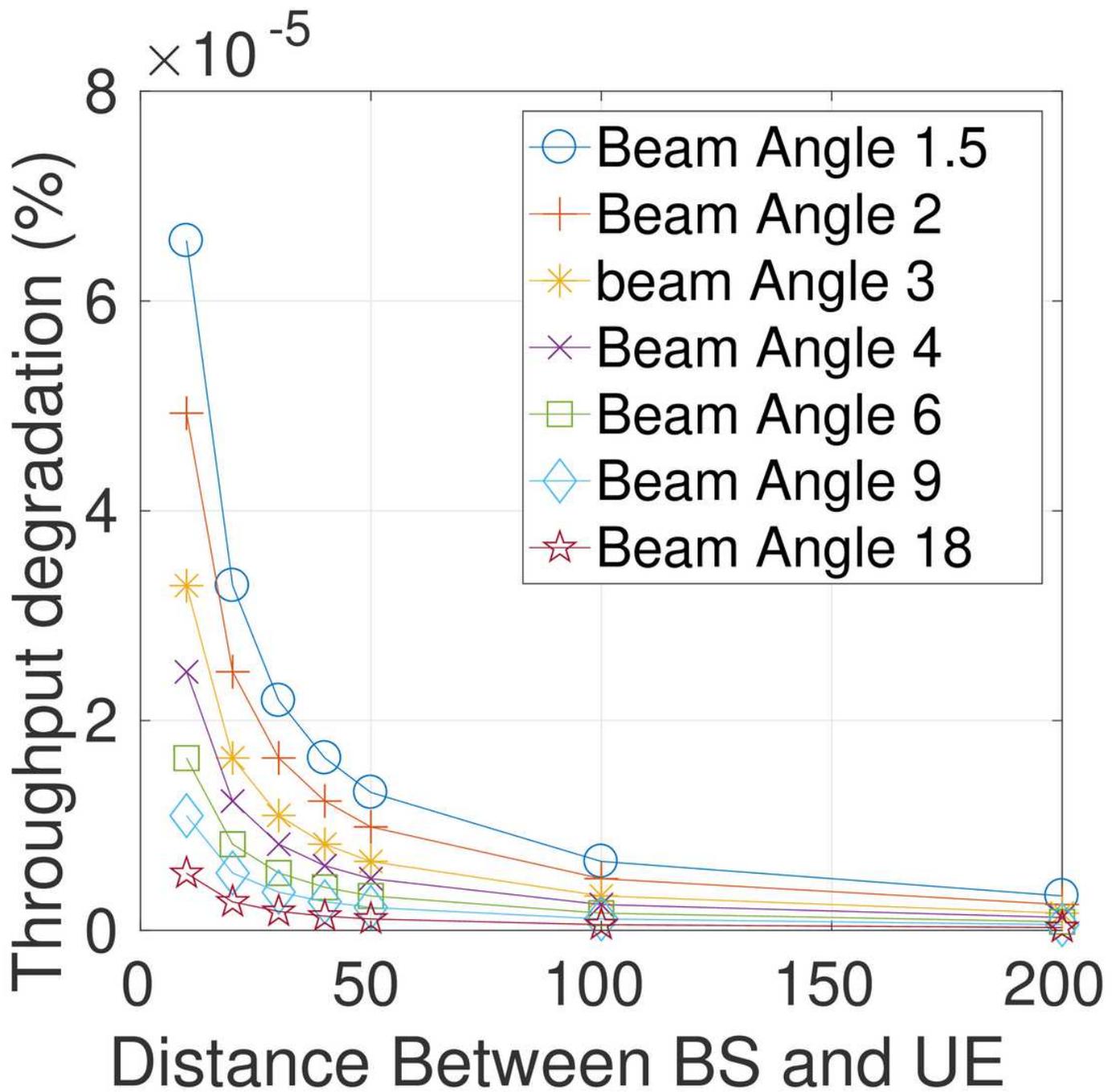


Figure 9

Probability of leaving the Beam Area for Users Riding a Car for different beam angles

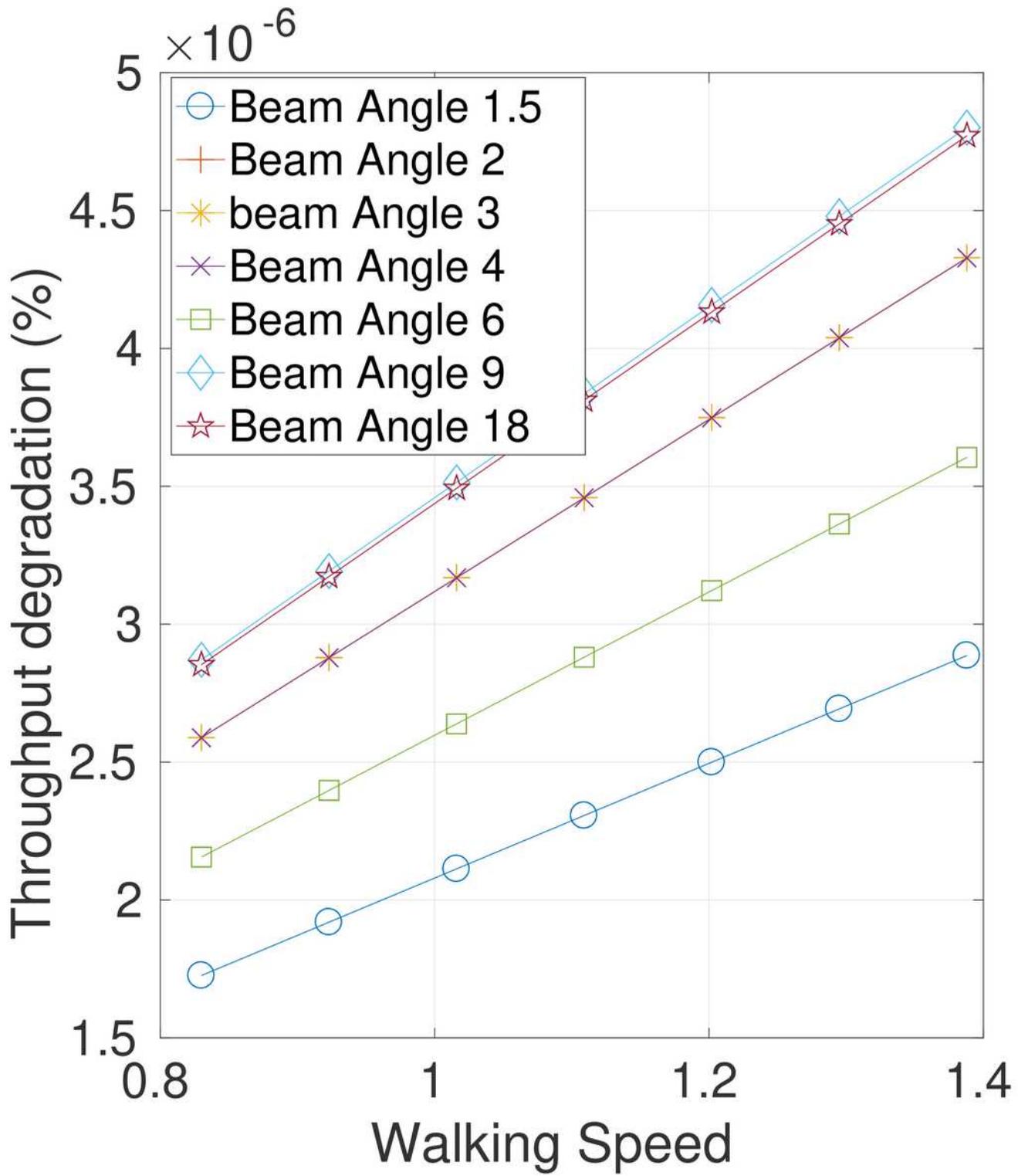


Figure 10

Expected Throughput Degradation for different Walking Speeds.

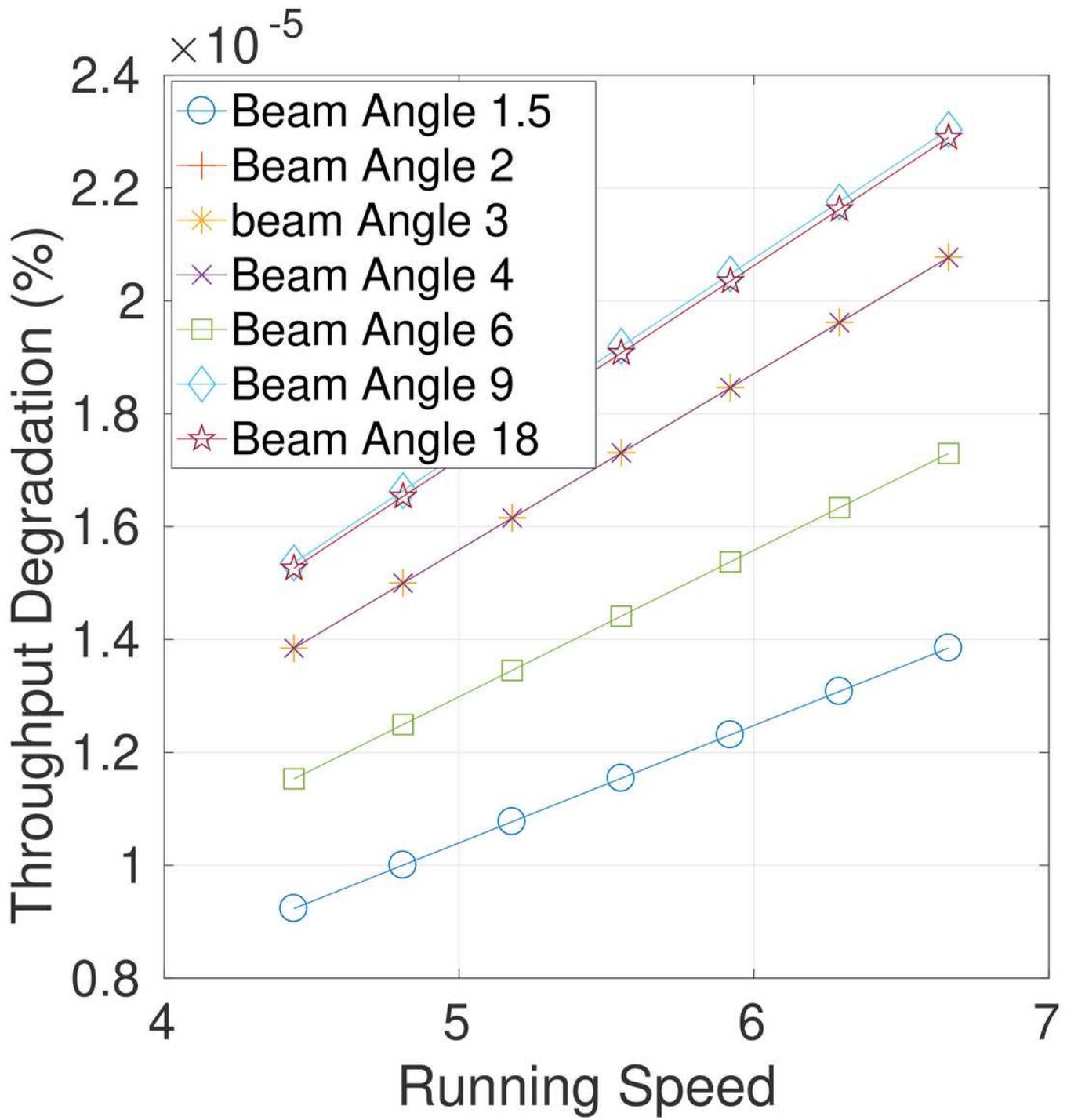


Figure 11

Expected Throughput Degradation for different Running Speeds.

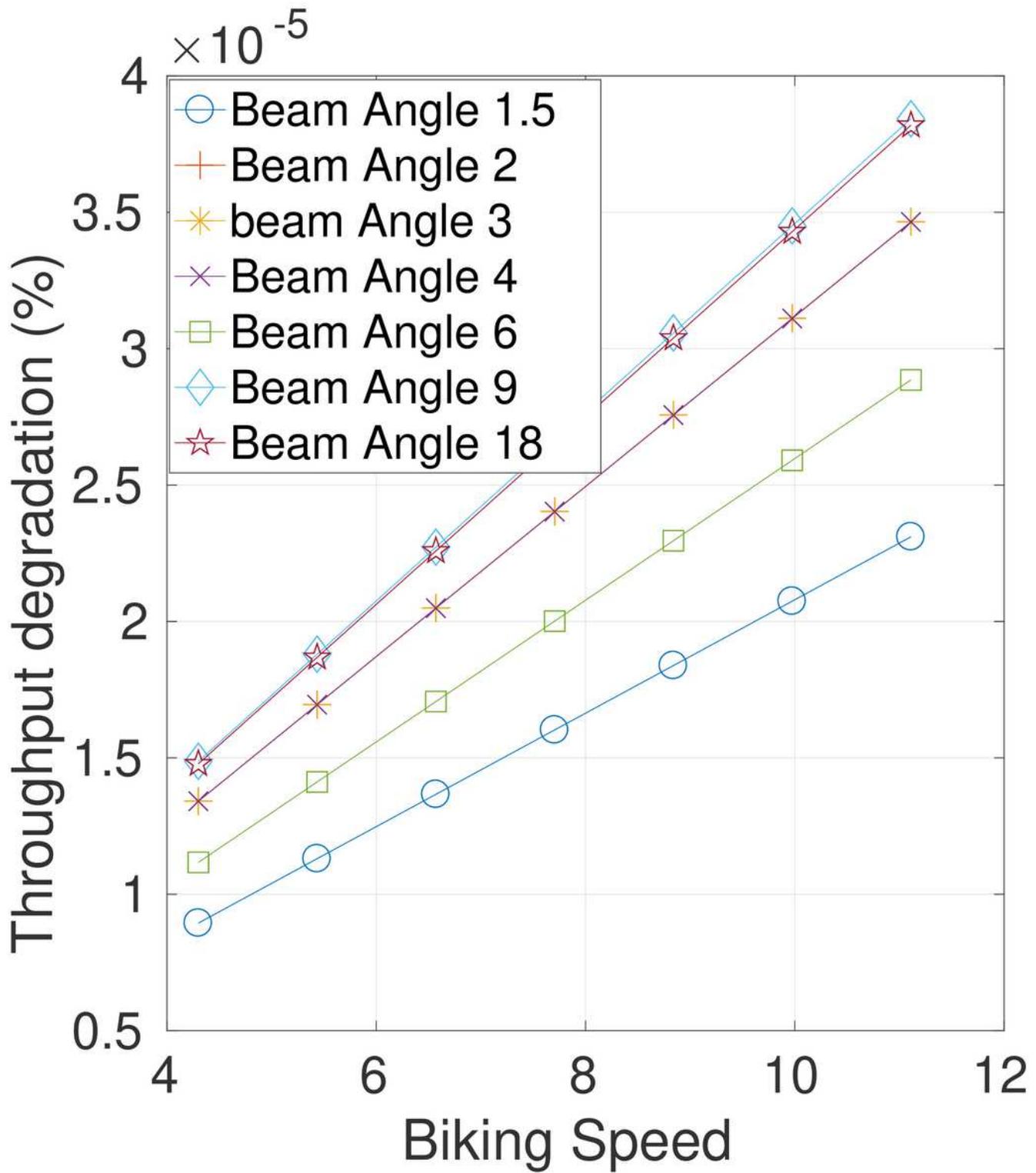


Figure 12

Expected Throughput Degradation for different Biking Speeds.

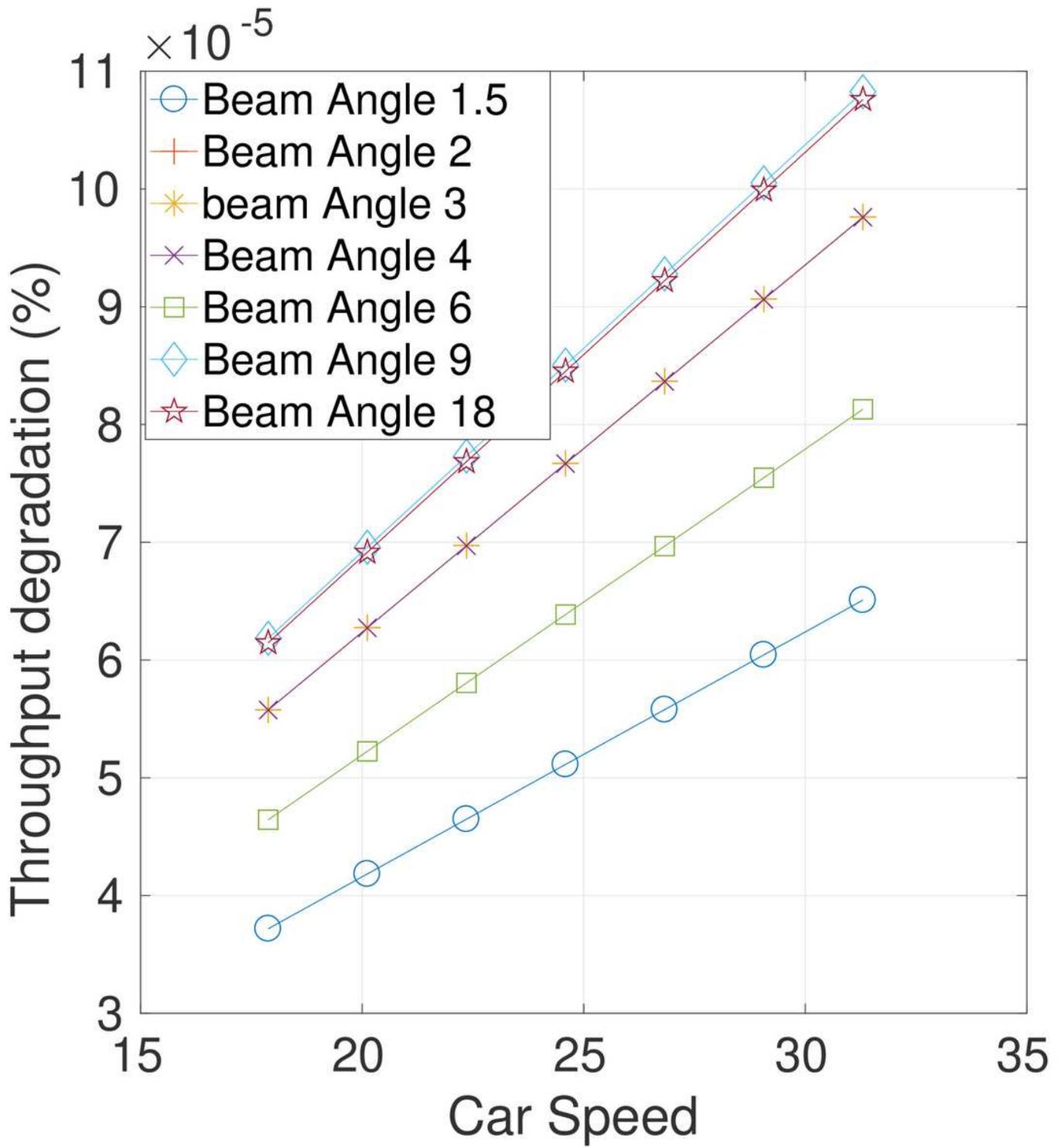


Figure 13

Expected Throughput Degradation for different Car Speeds.

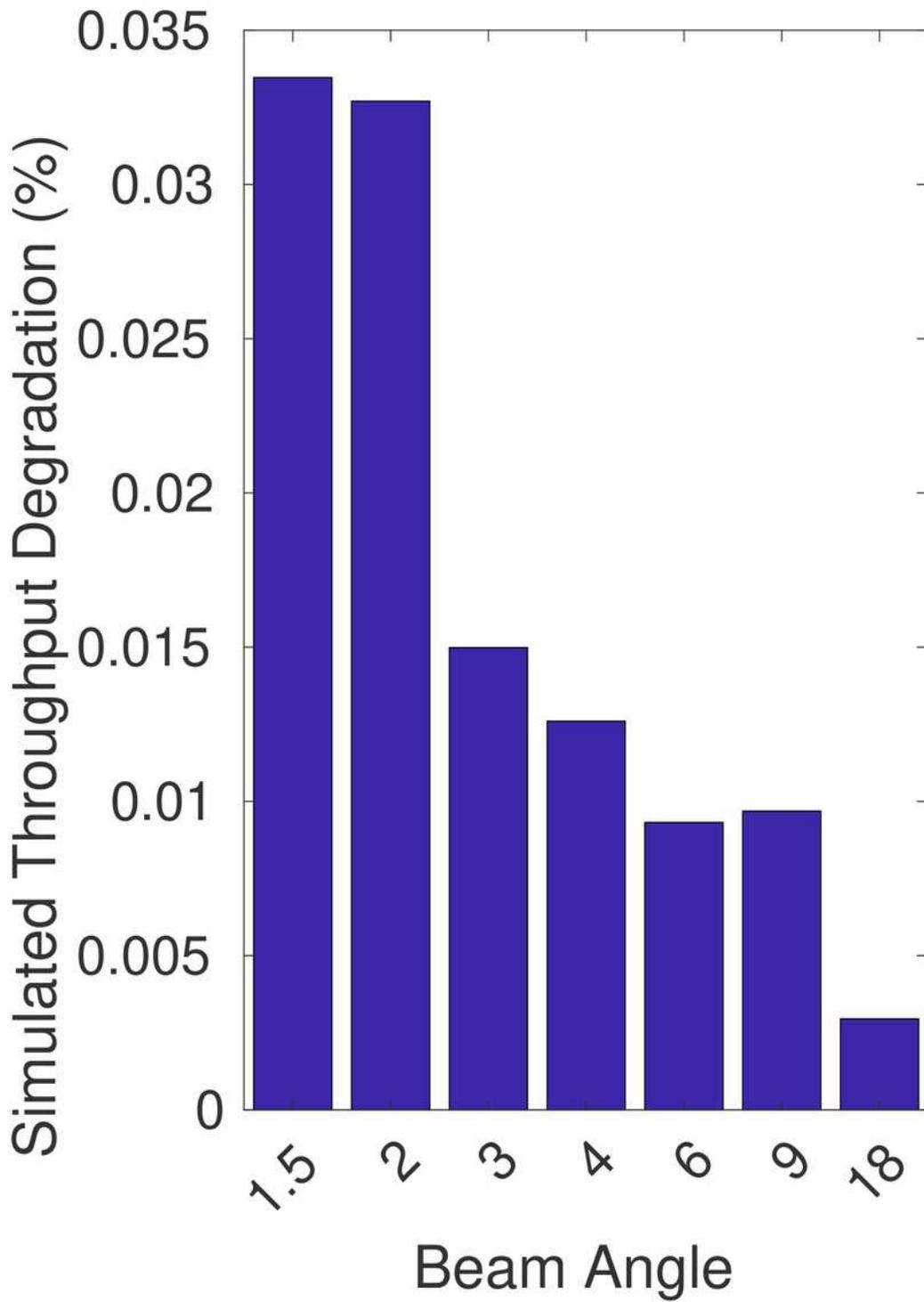


Figure 14

Throughput Degradation Resulted from Simulated Mobility for Random Speeds, Directions, and Distances between BS and UE

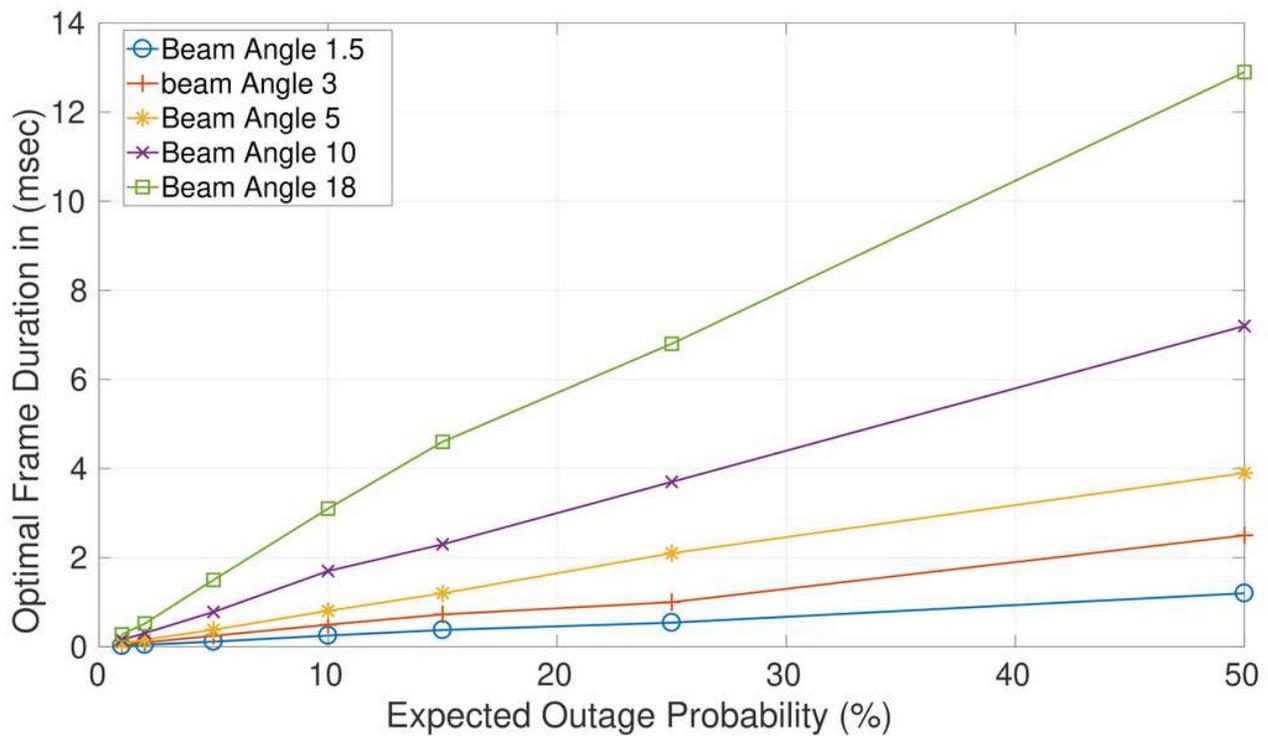


Figure 15

Optimal Frame Duration for Different Outage Probability Percentages with Walking Speed Range UEs

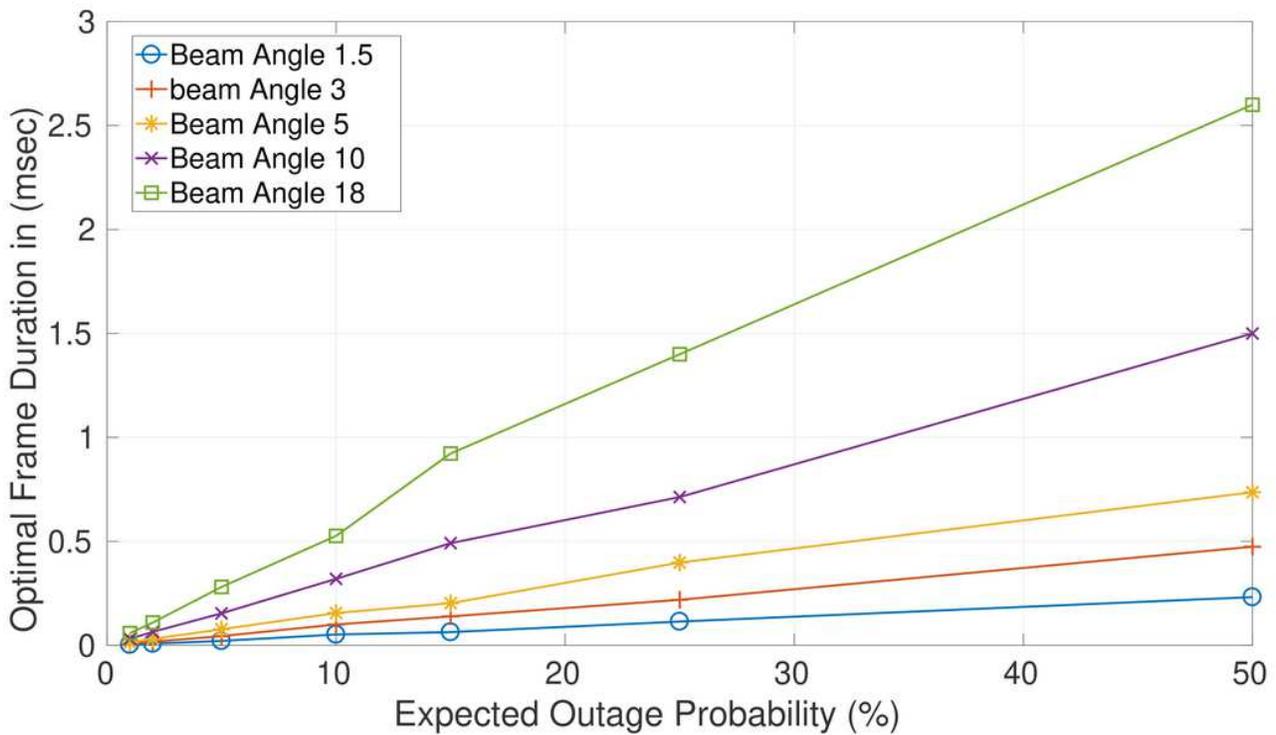
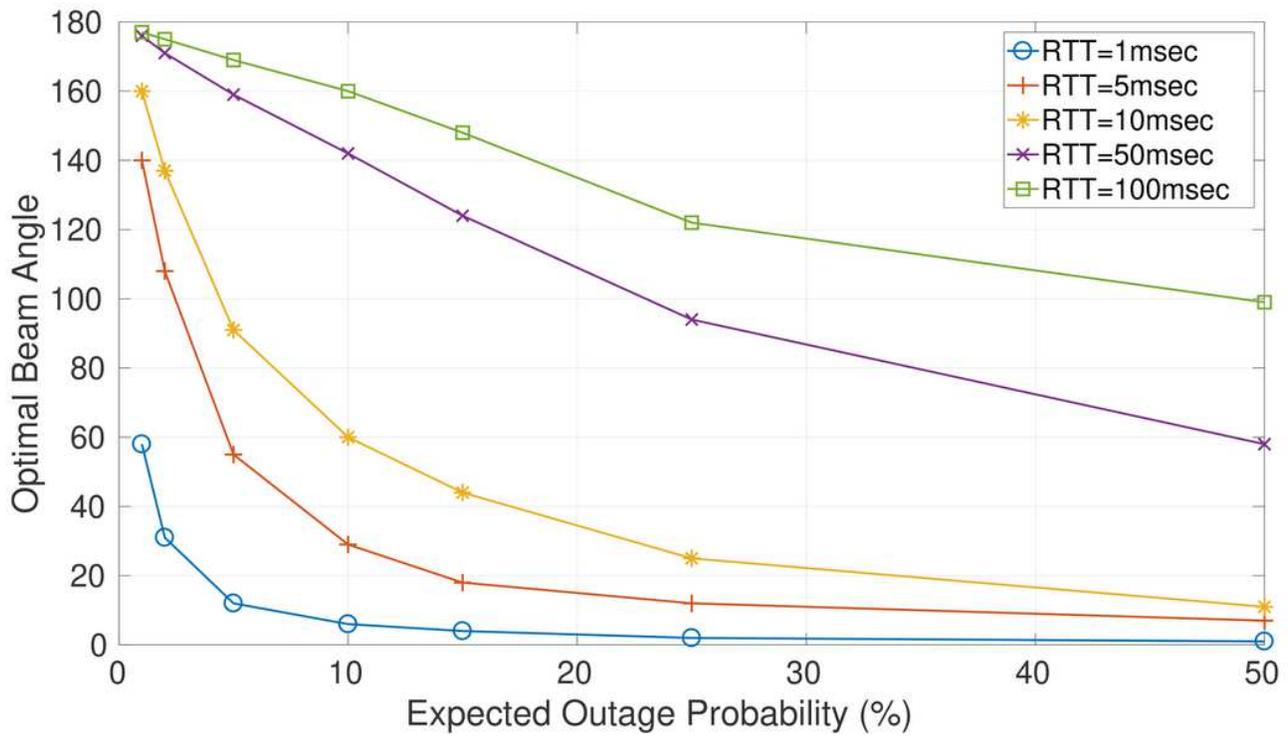


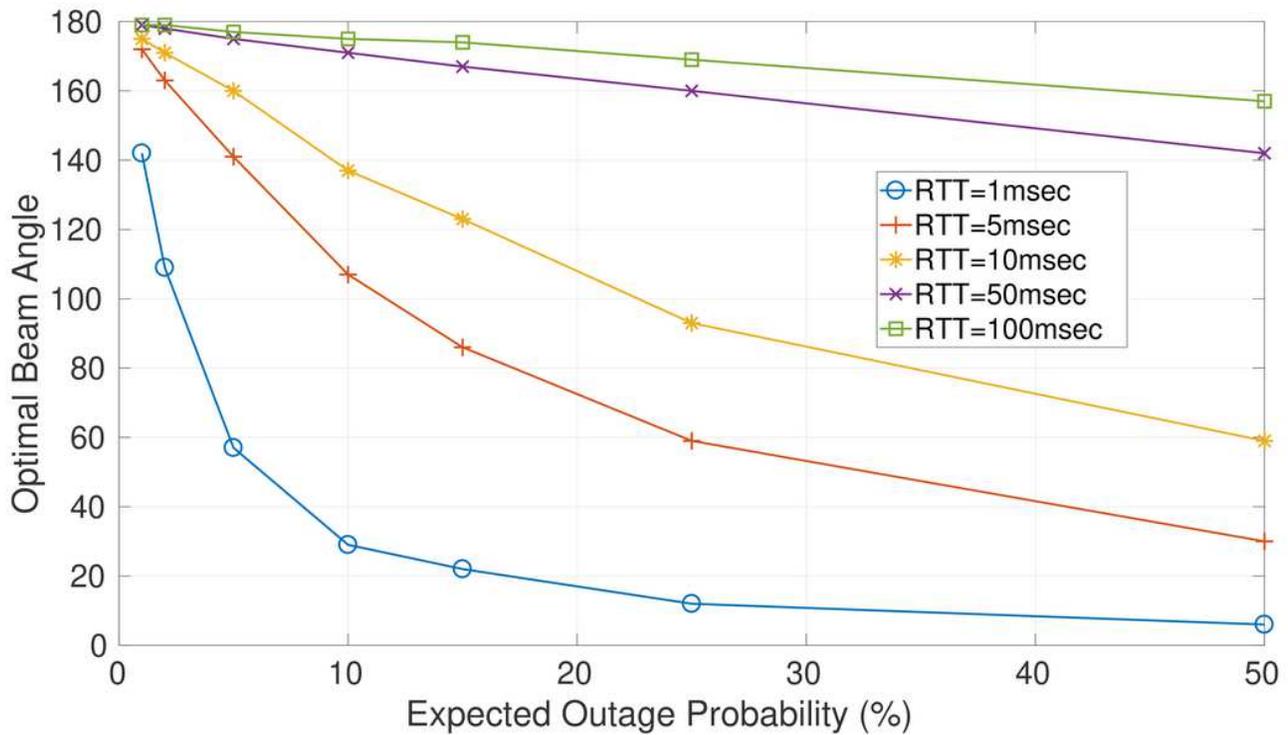
Figure 16

Optimal Frame Duration for Different Outage Probability Percentages with Running Speed Range UEs



**Figure 17**

Optimal Beam Angle for Dfferent Outage Probabilities with Walking UEs



**Figure 18**

Optimal Beam Angle for Different Outage Probabilities with Running UEs