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Optical Theodolite Based Rationality Check of Measurement Data from Pulse Radar

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ABSTRACT

Theodolite and radar are important measurement and control devices in aviation, aerospace and other fields. In order to make sure the tracking measurement and navigation control of aircraft and spacecraft safely, this paper creatively proposes a novel method to check the rationality of the three-dimensional measurement data (distance, azimuth and pitch angle) of the pulse radar by using the two-dimensional measurement data (azimuth and pitch angle) of the optical theodolite. Specifically, three different rationality checking functions are constructed for the three different cases of unreliable ranging channel, unreliable angle measuring channel and unreliable ranging and angle measuring channels, and are used for the rationality checking of radar data in the above three cases respectively. Simulation results show that the correct rate is more than 95% as long as the data measured by optical theodolite are reliable.

1. Introduction

The data collected by measuring equipment may contain outliers or abnormal data, which is an important factor affecting the safety of automatic operation process of dynamic system. In other words, to find and discriminate outliers and patchy outliers in real time is the premise and foundation to ensure the safe operation of the automation system.

For different engineering background and automation requirements, there are a lot of researches on outlier detection and diagnosis methods^[1-3]. For example, Ma Dazhong, et al. (2019) proposed a cyber-physical abnormality diagnosis method using data feature fusion for pipeline network, Zou Lei, et al. (2021) proposed a new representation to model the measurement outlier by an impulsive signal whose minimum interval length and minimum norm are larger than certain thresholds that are adjustable according to engineering practice, Liu Fang, et al. (2019) proposed an efficient Top- n local outlier detection algorithm for fast detecting top- n local outliers in large-scale datasets.

In the field of tracking and control of aircraft, the outliers and systematic deviations in the measurement data of tracking equipment are also difficult problems that affect the automation of measurement and control process. As everyone knows, the theodolite and radar are important tracking and measuring devices in the process of motion measurement and control. The tracking

measurement data of theodolite and radar are important basis for calculating the running trajectory, and plays an important role in the safety control of the motion bodies and the error analysis of the guidance system^[1]. Once the tracking and measuring data of theodolite or radar have large deviation or outliers, these wrong data will lead to navigation failure, or even make the missile or carrier rocket unable to reach the destination normally, or even lead to serious accident disaster. Therefore, in some important engineering fields such as aeronautics and astronautics, it is necessary to check online the rationality of the measurement data from the practical process of using theodolite and radar to track automatically the space flying target and locate and navigate autonomously.

In the field of engineering, we extend the outlier screening to the rationality check of data. The so-called rationality check of data, also known as data rationality test, refers to the use of appropriate data analysis methods or mechanism analysis methods to determine whether the sampling data is normal data, so as to find the sampling data that obviously deviate from the real situation. In other words, the purpose of rationality test is to identify and eliminate (or to repair) the data that deviates from the real situation and the unreasonable data. The unreasonable data can be various, such as outliers, non-outlier data obtained by equipment following the wrong target, and data with obvious large deviation.

How to accurately ensure the quality and correctness of measurement data, and how to effectively identify and correct unreasonable data, are the key issues in the field of process automation and engineering data processing.

The actual scene studied in this paper is an aerospace TT & C network composed of multiple sets of optical theodolites and radars. In the process of tracking a flying target, the theodolite and radar adopt the chain type alternate relay tracking with local time overlapping. The problem we need to solve is to check the rationality of the sampling data by using the redundant data brought by the overlapping relay tracking of the TT & C network equipment. Usually, the optical theodolite can only collect the data of azimuth and elevation angle of the flying target relative to the rotation center of the theodolite, which is abbreviated as (A_G, E_G) ; the radar can collect the data of the radial distance, azimuth and elevation angle of the flying target relative to the rotation center of the radar, which is abbreviated as (R_L, A_L, E_L) .

To check the rationality of optical theodolite measurement data with radar measurement data, we only need to translate and rotate the coordinate system of two different stations, compare the theodolite measurement data with the converted radar measurement data, and then analyze and judge the comparison residual. The practical method for this problem is mature^[4]. However, if only one optical theodolite has overlapping tracking arc with radar, it is very difficult to check and judge the rationality of radar data with theodolite measurement data.

In the existing methods of using optical theodolites to detect the rationality of radar data, in order to solve the above problem, it is usually assumed that at least two theodolites have overlapping tracking arcs with radar at the same time. In this case, a direct method is to fuse the azimuth and elevation data measured by two optical theodolites, and use K-formula or L-formula to calculate the target position coordinates^[5-7], so as to realize the rationality check of radar measurement data. Obviously, this method is essentially to use four measurement data of two theodolites to check three measurement data of one radar, and does not solve the problem of using only two measurement elements of one optical theodolite to realize the correctness and rationality check of the three measurement elements from the pulse radar.

In practical engineering, although both theodolite and radar can be used to track targets, their accuracy is different. The accuracy of theodolite is higher than that of radar^[8]. In addition, it is difficult to avoid outliers and time-varying deviations in the measurement data of the pulse radar, whether using reflection tracking or response tracking. Once there is only one theodolite and radar overlapping observation arc in the TT&C network, whether the rationality of radar measurement data can be checked with the measurement data of a single theodolite becomes a technical bottleneck with practical needs^[9]. The purpose of data rationality check is to determine whether the data is normal and reasonable, and to find abnormal data in sampling data set. The function of the pulse radar measurement data rationality check is to ensure that the radar measurement data used for trajectory calculation is accurate and reliable, and to avoid the bad influence of the abnormal measurement data^[8].

This paper attempts to establish a new kind of comparative residual analysis method to check the rationality of pulse radar measurement data which just uses the angle data from a single theodolite. In order to achieve the above goal, this paper designs the detection algorithm in following three cases: in section 2 and section 3, the practical methods to check the rationality of radar ranging data and radar angle data are proposed respectively; section 4 designs the joint check method of radar angle measurement and ranging data rationality. Simulation and result analysis are given in section 5 to verify the effectiveness of the above methods and algorithms.

2. Rationality Check of Angle Data When Radar Ranging Data is Normal

For the pulse radar measurement data, the distance (R_L) and angle (A_L, E_L) are from two different measurement channels. The common situation is that the distance R_L and angle (A_L, E_L) are not necessarily both outliers or abnormal in engineering tracking processes. Therefore, this paper considers the rationality test of angles in case that the radar ranging data R_L is normal.

2.1 Angles Conversion Algorithm Based on Distance R_G

In order to clearly describe the relationship between different measurement data in space, we need to introduce several classical coordinate system concepts, including geocentric Cartesian coordinate system and station Cartesian coordinate system.

The so-called station coordinate system refers to a three-dimensional Cartesian coordinate system with the center of rotation of the station equipment as the origin, the Y-axis pointing to the zenith, and the X-axis pointing to the north of the earth. Denote the optical theodolite station coordinate system as $O_G-X_GY_GZ_G$, and the pulse radar station coordinate system as $O_L-X_LY_LZ_L$.

Assuming that an optical theodolite and a pulse radar track an aircraft synchronously (see Figure 1). The azimuth and pitch angle (A_G, E_G) of the target M relative to $O_G-X_GY_GZ_G$ and the radial distance and angle (R_L, A_L, E_L) of the target M relative to $O_L-X_LY_LZ_L$ are obtained respectively. In this section, we intend to establish a method for checking the rationality of the radar's angle measurement data when the distance data measured by the radar is normal.

Figure 1. Schematic diagram of simultaneous tracking of theodolite and radar

Because the optical theodolite can't get the ranging data, it can't directly check and calibrate the measurement data of pulse radar through data conversion. In order to overcome the above difficulties, we try to establish a set of coordinate transformation algorithms in this section, as shown below:

Our intuitive idea is to try to estimate the angle data of the target M tracked by the optical theodolite in $O_L-X_L Y_L Z_L$, that is, the reasonable angle measurement data that the radar sampling should get. Figure 1 intuitively describes the way to calculate angle of the target M in $O_L-X_L Y_L Z_L$ based on the theodolite angle measurement data.

Firstly, we use the geodetic data (longitude λ , latitude μ , and geodetic elevation ξ , etc.) of the station to establish the transformation relationship Ω_{Ge} and origin coordinates (x_{Ge}, y_{Ge}, z_{Ge}) of the station in the geocentric rectangular coordinate system. The related methods have been introduced in detail in the literatures^[10-12]:

$$\Omega_{Ge} = \begin{pmatrix} -\sin \lambda & \cos \lambda & 0 \\ \cos \lambda & \sin \lambda & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 0 & 1 \\ -\sin \mu & \cos \mu & 0 \\ \cos \mu & \sin \mu & 0 \end{pmatrix} \quad (1)$$

and

$$\begin{pmatrix} x_{Ge} \\ y_{Ge} \\ z_{Ge} \end{pmatrix} = \begin{pmatrix} \left(\frac{a_g}{\sqrt{1-e_g^2 \sin^2 \mu}} + \xi \right) \cos \mu \cos \lambda \\ \left(\frac{a_g}{\sqrt{1-e_g^2 \sin^2 \mu}} + \xi \right) \cos \mu \sin \lambda \\ \left(\frac{a_g - a_g e_g^2}{\sqrt{1-e_g^2 \sin^2 \mu}} + \xi \right) \cos \mu \cos \lambda \end{pmatrix} \quad (2)$$

where, a_g is the long half axis of the reference ellipsoid, dimension m; e_g is the first eccentricity of the reference ellipsoid.

Secondly, the transformation relationship between the coordinate system $O_G-X_GY_GZ_G$ and $O_L-X_LY_LZ_L$, as well as the the coordinate calculation of rotation center of the optical theodolite station and the pulse radar are established:

(a). The rotation matrix Ω_L and the coordinates $(x'_{oL}, y'_{oL}, z'_{oL})$ of the pulse radar station origin (O_L) in $O_G-X_GY_GZ_G$ can be determined by formulae (1) and (2)

(b).The rotation matrix Ω_G and the coordinates $(x'_{oG}, y'_{oG}, z'_{oG})$ of the optical theodolite origin (O_G) in $O_L-X_LY_LZ_L$ can be determined by formulae (1) and (2).

Using the theodolite measurement data (A_G, E_G) and the unknown variable quantity R_G to be determined, the position coordinates of the target M in $O_L-X_LY_LZ_L$ can be established as follows.

$$\begin{pmatrix} x_L \\ y_L \\ z_L \end{pmatrix} = \begin{pmatrix} X'_{ol} \\ Y'_{ol} \\ Z'_{ol} \end{pmatrix} + R_G \Omega_G \begin{pmatrix} \cos A_G \cos E_G \\ \sin E_G \\ \sin A_G \cos E_G \end{pmatrix} \quad (3)$$

It is not difficult to derive from the above conversion relationship that, according to the measurement data of the optical theodolite, the radial distance R_R , azimuth angle A_R , and pitch angle E_R of the target M in $O_L-X_LY_LZ_L$ can be expressed as the formula (4) and (5).

$$\begin{cases} A_R = \text{arctanh}\left(\frac{\tilde{z}_L}{\tilde{x}_L}\right) + \begin{cases} 0 & \tilde{x}_L \geq 0, \tilde{z}_L \geq 0 \\ \pi & \tilde{x}_L < 0 \\ 2\pi & \tilde{x}_L \geq 0, \tilde{z}_L < 0 \end{cases} \\ E_R = \arcsin(\tilde{y}_L / R_G) \end{cases} \quad (4)$$

where, $\tilde{x}_L = x_L - X'_{oG}$, $\tilde{y}_L = y_L - Y'_{oG}$, $\tilde{z}_L = z_L - Z'_{oG}$, and the distance R_G is unknown.

Since the radial distance between the optical theodolite and the target M is unknown, therefore, in order to calibrate and detect the measurement data from the pulse radar, the estimated value of R_G must be given in advance.

2.2 Estimation of RG Based on Normal Radar Ranging

It can be seen from Figure 1 that the measured angle (A_G, E_G) of the optical theodolite can only tell us that the target M should be located in a certain position on the ray, and that the target M should be located on a sphere whose distance from the rotation center of the range pulse radar (O_L) is equal to R_L . In other words, the target M should satisfy the following formula:

$$\begin{cases} \frac{X - X_{OG}}{\cos A_G} = \frac{Y - Y_{OG}}{\tan E_G} = \frac{Z - Z_{OG}}{\sin A_G} = t \\ (X - X_{OL})^2 + (Y - Y_{OL})^2 + (Z - Z_{OL})^2 = R_T^2 \end{cases} \quad (5)$$

where the (X_{OG}, Y_{OG}, Z_{OG}) is the origin coordinate of optical theodolite, the (X_{OL}, Y_{OL}, Z_{OL}) is the rotating center coordinate of the pulse radar in O_G - X_G Y_G Z_G , R_T is the true distance between the target M and the rotation center of the theodolite.

Generally, the origin coordinate of the theodolite can be considered $X_{OG} = Y_{OG} = Z_{OG} = 0$ in O_G - X_G Y_G Z_G , so, we get a quadratic equation

$$(1 + \tan^2 E_G)t^2 - 2(X_{OL} \cos A_G + Y_{OL} \tan E_G + Z_{OL} \sin A_G)t + X_{OL}^2 + Y_{OL}^2 + Z_{OL}^2 - R_T^2 = 0 \quad (6)$$

There are three cases: the ray and the sphere may not intersect, do not intersect at two points or not tangent to a point (see Figure 2). Combining with Figure 2, we give three estimation algorithms of R_G in different situations, i.e. non intersection, intersection and tangent.

Figure 2. Diagram of relative relationship between ray and sphere

A. Estimation of R_G in the Case of Tangency

Solving quadratic equation (5), when t has only one solution, the ray is tangent to the ball. In this case, the tangent point can be considered as the location of target M. Substituting t into the ray trajectory equation (5), we get the formula (7).

$$X = t \cos A_G \quad Y = t \tan E_G \quad Z = t \sin A_G \quad (7)$$

and the estimator of R_G is the formula (8).

$$R_G = \sqrt{X^2 + Y^2 + Z^2} \quad (8)$$

B. Estimation of R_G in the Case of Intersection

To solve the quadratic equation in the case that t has two solutions t_1 and t_2 , the ray intersects with the sphere at two points, the midpoint of the two intersection points is the location of target M. Substituting t_1 and t_2 into the ray trajectory equation in formula (5), we get the formula:

$$\begin{cases} X_1 = t_1 \cos A_G & Y_1 = t_1 \tan E_G & Z_1 = t_1 \sin A_G \\ X_2 = t_2 \cos A_G & Y_2 = t_2 \tan E_G & Z_2 = t_2 \sin A_G \end{cases} \quad (9)$$

Then the estimator of R_G can be gotten as following:

$$R_G = \sqrt{\left(\frac{X_1+X_2}{2}\right)^2 + \left(\frac{Y_1+Y_2}{2}\right)^2 + \left(\frac{Z_1+Z_2}{2}\right)^2} \quad (10)$$

C. Estimation of R_G in Disjoint Case

To solve the quadratic equation (6) in the case that t has no solution, the ray does not intersect the sphere. In this case, it can be concluded that the point on the sphere at the location which is the position of target M at the shortest distance between the ray and the sphere. The calculation program is as follows:

Assuming that the coordinate (X_{OL}, Y_{OL}, Z_{OL}) of the vertical point from to ray is (X_C, Y_C, Z_C) , we substitute it into the ray trajectory equation (5) and get the formula:

$$X_C = t \cos A_G \quad Y_C = t \tan E_G \quad Z_C = t \sin A_G \quad (11)$$

Because the quantity product of the vertical direction vector $(X_{OL} - X_C, Y_{OL} - Y_C, Z_{OL} - Z_C)$ and the ray direction vector $(\cos A_G, \tan E_G, \sin A_G)$ is zero, it can get the following formula (12).

$$\cos A_G(X_{OL} - X_C) + \tan E_G(Y_{OL} - Y_C) + \sin A_G(Z_{OL} - Z_C) = 0 \quad (12)$$

To substitute the formula (11) into the formula (12), we get the following formula:

$$t = \frac{X_{OL} \cos A_G + Y_{OL} \tan E_G + Z_{OL} \sin A_G}{1 + \tan^2 E_G} \quad (13)$$

The distance d between the point (X_{OL}, Y_{OL}, Z_{OL}) and the ray is equal to the distance between the point and the vertical point (X_C, Y_C, Z_C) as follows:

$$d = \sqrt{(X_{OL} - X_C)^2 + (Y_{OL} - Y_C)^2 + (Z_{OL} - Z_C)^2} \quad (14)$$

In the formula (14), the point (X_C, Y_C, Z_C) can be substituted and eliminated by the formula (11) and the formula (13). Therefore, the estimator of R_G can be obtained.

$$R_G = \sqrt{(d - R_T)^2 + X_C^2 + Y_C^2 + Z_C^2} \quad (15)$$

2.3 Residuals Generation and Threshold Design

Comparative analysis is one of the common methods to check the rationality of measurement data, and it has a wide range of applications in the field of aircraft measurement and control^[10]. In the

process of radar measurement data processing, the cross comparison technology of different source data is usually used to judge the quality of measurement data and identify abnormal data.

In this section, based on the angle measurement data and range estimation value of a single optical theodolite, a method for rationality check of ranging data of pulse radar is established. The specific method is as follows:

1) In order to optimally estimate the radial distance (R_G) between the theodolite and the target by the calculation methods in section 2.2, we substitute the theodolite measurement data (R_G, A_G, E_G) into formula (4), and use the optical theodolite measurement data to calculate the radial distance (R_R), azimuth angle (A_R), and pitch angle (E_R) of the target M in $O_L-X_L Y_L Z_L$.

2) For moving target M, whether the measured data are the (R_L, A_L, E_L) from the compared radar or the (R_R, A_R, E_R) calculated back from the theodolite measurement data, the alignment residual sequence at time t_i is as follows:

$$\begin{cases} \Delta A(t_i) = A_L(t_i) - A_R(t_i) \\ \Delta E(t_i) = E_L(t_i) - E_R(t_i) \end{cases} \quad (16)$$

where the vector ($\Delta A(t_i), \Delta E(t_i)$) is the difference between two kinds of data from two stations.

3) The detection threshold is determined based on the comparison residuals. The threshold constant includes the azimuth threshold constant $c_{\Delta A}$ and the pitch angle threshold constant $c_{\Delta E}$. The method for obtaining the threshold constant is as follows^[15]:

$$\begin{cases} c_{\Delta A} = 1.483 \text{ med } \{ |\Delta A(t_1)|, \dots, |\Delta A(t_N)| \} \\ c_{\Delta E} = 1.483 \text{ med } \{ |\Delta E(t_1)|, \dots, |\Delta E(t_N)| \} \end{cases} \quad (17)$$

2.4 Radar Data Checking Based on Comparison Residuals

The basic way to check the rationality of the angle data measured by the radar is to construct a threshold function as shown in the following formula:

$$\phi(x, c) = \begin{cases} 1, & |x| > c \\ 0, & |x| \leq c \end{cases} \quad (18)$$

and judge according to the following rules:

- a. If $\phi(\Delta A(t_i), c_{\Delta A})\phi(\Delta E(t_i), c_{\Delta E}) = 1$, then data ($A_L(t_i), E_L(t_i)$) is reasonable;
- b. If $\phi(\Delta A(t_i), c_{\Delta A}) = 1, \phi(\Delta E(t_i), c_{\Delta E}) = 0$, then data $A_L(t_i)$ is reasonable and $E_L(t_i)$ is not reasonable; vice versa.
- c. If $\phi(\Delta A(t_i), c_{\Delta A}) = 0, \phi(\Delta E(t_i), c_{\Delta E}) = 1$, then data $E_L(t_i)$ is reasonable and $A_L(t_i)$ is not reasonable; vice versa.

If the measurement data at a single time is unreasonable, the single unreasonable data is likely to be an outlier; if multiple consecutive measurement data are unreasonable, these multiple unreasonable data may be patchy outliers, or they may have occurred systematic deviation.

3. Ranging Checking based on Normal Angles

In this section, we consider the estimation algorithm for the distance R_G between the target and the theodolite when the angle data (A_L, E_L) measured by the radar is not reasonable.

3.1 Estimation of R_G When (A_L, E_L) is Reasonable

Denoting the angle data measured by the pulse radar as (A_L, E_L) in $O_L-X_L Y_L Z_L$, the rotation matrix between the coordinate system $O_G-X_G Y_G Z_G$ and $O_L-X_L Y_L Z_L$ is Ω_L , the direction cosine of motion target M in $O_G-X_G Y_G Z_G$ is as follows.

$$\begin{pmatrix} l_1 \\ m_1 \\ n_1 \end{pmatrix} = \Omega_L \begin{pmatrix} \cos E_L \cos A_L \\ \sin E_L \\ \cos E_L \sin A_L \end{pmatrix} \quad (19)$$

Using the formula (19), the azimuth and elevation angles of the target M in the theodolite station coordinate system $O_G-X_G Y_G Z_G$ can be calculated as follows

$$\begin{cases} A_T = \arctan\left(\frac{n_1}{l_1}\right) + \begin{cases} 0 & l_1 \geq 0, n_1 \geq 0 \\ \pi & l_1 < 0 \\ 2\pi & l_1 \geq 0, n_1 < 0 \end{cases} \\ E_T = \arcsin(m_1) \end{cases} \quad (20)$$

Based on the angle measurement data (A_T, E_T) calculated according to formula (20) and the measurement data (A_G, E_G) measured by the optical theodolite, the optimal coordinate estimation of the target M in $O_G-X_G Y_G Z_G$ can be determined by the following formula^[10,14]:

$$\hat{X} = \frac{C_1}{2C}, \hat{Y} = \frac{C_2}{2C}, \hat{Z} = \frac{C_3}{2C} \quad (21)$$

where,

$$C_1 = \begin{vmatrix} b_1 & -\sum_{i=1}^2 l_i m_i & -\sum_{i=1}^2 l_i n_i \\ b_2 & \sum_{i=1}^2 (1 - m_i^2) & -\sum_{i=1}^2 n_i m_i \\ b_3 & -\sum_{i=1}^2 n_i m_i & \sum_{i=1}^2 (1 - n_i^2) \end{vmatrix}, C_2 = \begin{vmatrix} \sum_{i=1}^2 (1 - l_i^2) & b_1 & -\sum_{i=1}^2 l_i n_i \\ -\sum_{i=1}^2 l_i m_i & b_2 & -\sum_{i=1}^2 n_i m_i \\ -\sum_{i=1}^2 l_i n_i & b_3 & \sum_{i=1}^2 (1 - n_i^2) \end{vmatrix}, C_3 = \begin{vmatrix} \sum_{i=1}^2 (1 - l_i^2) & -\sum_{i=1}^2 l_i m_i & b_1 \\ -\sum_{i=1}^2 l_i m_i & \sum_{i=1}^2 (1 - m_i^2) & b_2 \\ -\sum_{i=1}^2 l_i n_i & -\sum_{i=1}^2 n_i m_i & b_3 \end{vmatrix},$$

$$C = \begin{vmatrix} \sum_{i=1}^2 (1-l_i^2) & -\sum_{i=1}^2 l_i m_i & -\sum_{i=1}^N l_i n_i \\ -\sum_{i=1}^2 l_i m_i & \sum_{i=1}^2 (1-m_i^2) & -\sum_{i=1}^N n_i m_i \\ -\sum_{i=1}^2 l_i n_i & -\sum_{i=1}^2 n_i m_i & \sum_{i=1}^N (1-n_i^2) \end{vmatrix}$$

and

$$\begin{cases} b_1 = \sum_{i=1}^2 [(1-l_i^2)X_{0i} - l_i(n_i Z_{0i} + m_i Y_{0i})] \\ b_2 = \sum_{i=1}^2 [(1-m_i^2)Y_{0i} - m_i(n_i Z_{0i} + l_i X_{0i})] \\ b_3 = \sum_{i=1}^2 [(1-n_i^2)Z_{0i} - n_i(l_i X_{0i} + m_i Y_{0i})] \end{cases}$$

where the (X_{0i}, Y_{0i}, Z_{0i}) is the location coordinate of the radar ($i=1$) and theodolite ($i=2$) in $O_G-X_G Y_G Z_G$, and

$$\begin{pmatrix} l_2 \\ m_2 \\ n_2 \end{pmatrix} = \Omega_G \begin{pmatrix} \cos E_G \cos A_G \\ \sin E_G \\ \cos E_G \sin A_G \end{pmatrix} \quad (22)$$

Therefore, the estimator of R_G between the target M and the theodolite is the formula (23).

$$R_G = \sqrt{\hat{X}^2 + \hat{Y}^2 + \hat{Z}^2} \quad (23)$$

3.2 Rationality Check of Range data Measured by Radar

By combining the expressions (3) and (4) in Section 2.1 with the optical theodolite measurement data (A_G, E_G) and the above estimated R_G stated in formulae (23), we can get the radial distance R_R , azimuth angle A_R , and pitch angle E_R of the target M in $O_L-X_L Y_L Z_L$.

The rationality check process of the measurement data obtained in the radar tracking process is as follows:

a. We compare the calculated result (R_R, A_R, E_R) with the actual measured data (R_L, A_L, E_L) of pulse radar to form a series of three-dimensional residuals

$$\begin{cases} \Delta R(t_i) = R_L(t_i) - R_R(t_i) \\ \Delta A(t_i) = A_L(t_i) - A_R(t_i) \\ \Delta E(t_i) = E_L(t_i) - E_R(t_i) \end{cases} \quad (t_i = t_0 + ih, i = 1, 2, \dots, N) \quad (24)$$

b. The threshold constant $c_{\Delta R}$ is determined with the residual series:

$$c_{\Delta R} = 1.483 \text{ med } \{|\Delta R(t_1)|, \dots, |\Delta R(t_N)|\} \quad (25)$$

where the N is the number of samples

c. If the angle measurement data obtained by the pulse radar tracking target M is normal, the rule to check rationality of the ranging data measured by the pulse radar is as follows: $R_L(t_i)$ is judged to be reasonable if $|\Delta R(t_i)| \leq c_{\Delta R}$; otherwise, $R_L(t_i)$ is judged as abnormal data.

4. Data Checking if Reliability of Radar is Unknown

In practical engineering, it is common that we do not know in advance which sampling data is normal in the range measurement channel and angle measurement channel of the pulse radar. So, in this section, the data rationality check methods are discussed when there are no prior knowledges about the data quality of pulse radar measurement channels.

Step 1. Assuming that the angle measurement data from the optical theodolite are reliable and the quality of radar measurement data are unknown, the range measurement data R_L and the angle measurement data (A_L, E_L) sampled from the pulse radar are inversely calculated as the data (R_F, A_F, E_F) of target M in $O_G-X_GY_GZ_G$.

In detail, assuming that the rotating center of the pulse radar is O_L , the coordinate of which is $(X'_{OL}, Y'_{OL}, Z'_{OL})$ in $O_G-X_GY_GZ_G$, the rotation matrix between these two station coordinate systems is the matrix Ω_F , it can be derived that the position coordinate of target M in $O_G-X_GY_GZ_G$ is (X_G, Y_G, Z_G)

$$\begin{pmatrix} x_F \\ y_F \\ z_F \end{pmatrix} = \begin{pmatrix} X'_{OL} \\ Y'_{OL} \\ Z'_{OL} \end{pmatrix} + R_L \Omega_F \begin{pmatrix} \cos A_L \cos E_L \\ \sin E_L \\ \sin A_L \cos E_L \end{pmatrix} \quad (26)$$

Using the formula (4), the azimuth angle A_F and the pitch angle E_F in $O_G-X_GY_GZ_G$ can be obtained as follows.

$$\begin{cases} A_F = \arctan\left(\frac{\tilde{z}_F}{\tilde{x}_F}\right) + \begin{cases} 0 & \tilde{x}_F \geq 0, \tilde{z}_F \geq 0 \\ \pi & \tilde{x}_F < 0 \\ 2\pi & \tilde{x}_F \geq 0, \tilde{z}_F < 0 \end{cases} \\ E_F = \arcsin\left(\frac{y_F}{\sqrt{\tilde{x}_F^2 + \tilde{y}_F^2 + \tilde{z}_F^2}}\right) \end{cases} \quad (27)$$

Where $\tilde{x}_F = x_F - X'_{OL}$, $\tilde{y}_F = y_F - Y'_{OL}$, $\tilde{z}_F = z_F - Z'_{OL}$.

Step 2. The angle measurement data (A_F, E_F) obtained by inverse calculation can be used to compare with the original angle measurement data (A_G, E_G) from the optical theodolite:

$$\begin{cases} \Delta A_G(t_i) = A_G(t_i) - A_F(t_i) \\ \Delta E_G(t_i) = E_G(t_i) - E_F(t_i) \end{cases} \quad (t_i = t_0 + ih, i = 1, 2, \dots, N) \quad (28)$$

Step 3. Using the formula (17) and the residual series (28) to determine the detection threshold constants $c_{\Delta A_G}$ and $c_{\Delta E_G}$;

Step 4.Checking: if $\phi(\Delta A_G(t_i), c_{\Delta A_G}) = 1, \phi(\Delta E_G(t_i), c_{\Delta E_G}) = 0$, then the measurement data (R_G, A_G, E_G) of target M in $O_G-X_GY_GZ_G$ is reasonable at time t_i ; otherwise, the measurement data (R_G, A_G, E_G) or part of the data (R_G, A_G, E_G) are abnormal.

5. Simulation and Result Analysis

In order to verify the validity of the rationality test algorithm of pulse radar measurement data based on a single optical theodolite, in this section, we take the nominal trajectory data of a flight test and the measurement data of a single radar as the simulation source data, and select the sampling interval of 0.1s, a total of 1859 points. The azimuth and pitch angle of the target relative to the optical theodolite coordinate system are calculated and plotted in Figure 3. The radar measurement data curve is shown in Figure 4.

Figure 3. Angles measured by the optical theodolite

Figure 4. Sampling data measured by the pulse radar

a. Reasonability checking of the angles (A_L, E_L) in the case that the ranging data R_L is normal. The normal radar ranging data and abnormal angle data of the pulse radar are computed and drawn

in Figure 5, which are based on the data of azimuth angle(A_L) and pitch angle(E_L) as shown in Figure 4. In simulation, the time series of measurement data shown in Figure 4 are randomly added with 2% gross errors from the 10th, that is, there are 37 random outliers in 1859 data.

Figure 5. Angle measurement data of pulse radar with outliers.

The data rationality check based on comparative residuals proposed in section 2 is used to discriminate the measurement data from the pulse radar. Simulation results are shown in Figure 6, and the data with the red mark in Figure 6 are the identified outliers.

Figure 6. Rationality check results of radar angle measurement data.

b. Ranging rationality check in the case that the angle measurement data from the pulse radar is reasonable. In order to simulate the normal angle data and abnormal ranging data from the pulse radar, we add outliers to the ranging distance R_L in Figure 4 and get the ranging distance data with outliers as shown in Figure 7. The data rationality check algorithm based on comparative residuals is used to check the measured data from pulse radar. Simulation results are shown in Figure 8, where the data with the blue mark are the identified outliers.

Figure 7. RL data with outliers and R rationality check results.

c. Rationality check of radar measurement data in the case that the quality of measurement data is unknown. In order to simulate the unknown data quality of pulse radar, outliers are added to the distance R_L , azimuth A_L and elevation E_L shown in Figure 4, and we get the measurement data curve including outliers, as shows in Figure 8.

Figure 8. Measurement data of pulse radar with outliers.

The rationality check method of data based on comparison residual proposed in section 3 is used to discriminate the measured data of pulse radar. The simulation results are shown in Figure 9, and the data at the red marked points in the figure are identified outliers.

Figure 10. Rationality check results of radar measurement data.

Based on the above simulation results, it can be seen that the proposed algorithm can effectively check the rationality of radar measurement data and identify outliers. The statistical check effect is that the outlier recognition accuracy rate is better than 95%.

6. Conclusions

The rationality check of sampling data is very important to ensure the reliability of data automatically processing and the correctness of results. In this paper, a series of rationality check methods for sampling data series from the pulse radar are ingeniously established and implemented in three different situations: the first case is that the ranging measurement data from the pulse radar is normal, but whether the angle measurement data from the pulse radar is normal is unknown; the second is that the angle measurement data is normal, but it is not clear whether the ranging measurement from the pulse radar is normal; the third situation is that we don't know whether the overall situation of radar tracking measurement data is normal.

In this paper, a method and algorithm for calibrating or verifying the quality of tracking measurement data of pulse radar by using a theodolite is established. Simulation results show that the above methods and algorithms are effective: for the three typical situations stated above, the accuracy rate is more than 95%.

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