

# Structural Stability, Elasticity, Thermodynamics and Electronic Structures of L12-Type Ni<sub>3</sub>X (X=Al, Ti, V, Nb) Phases Under External Pressure Condition

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## Research Article

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# Structural stability, elasticity, thermodynamics and electronic structures of $L1_2$ -type $Ni_3X$ ( $X=Al, Ti, V, Nb$ ) phases under external pressure condition

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**Abstract:** In this paper, the impacts of external pressure on structural stability, elasticity, thermodynamics and relevant electronic structures of  $L1_2$ -type  $Ni_3X$  ( $X=Al, Ti, V, Nb$ ) phases were investigated using the first-principles methods. The lattice parameters( $a, b, c$ ) and volume( $V$ ) of the  $Ni_3X$  phases decrease with increasing pressure. however, the elastic constants( $C_{ij}$ ), bulk modulus( $B$ ), shear modulus( $G$ ), and Young's modulus ( $E$ ) increase. The calculated elastic constants indicate that the mechanical stability and ductility of  $Ni_3X$  phases enhance with increasing pressure. The mechanical anisotropy of  $Ni_3X$  phases are enhanced by the raised pressure. Electronic analysis shows that increase pressure makes Ni-d-orbital and X( $X=Al, Ti, V, Nb$ ) -d-orbital hybridization stronger and electron transfer increases. The sequence in regard to electron aggregation strength is  $Ni_3Ti > Ni_3Nb > Ni_3V > Ni_3Al$ . It is more directly reflected in the charge density difference maps. This is consistent with the analysis results of the enthalpy of formation( $\Delta H$ ) and Debye temperature ( $\Theta_D$ ).

**Keywords:**  $Ni_3X$ ; elastic constants; thermodynamic; electronic structures; First-principles

## 1. Introduction

Ni-based single-crystal superalloys are one the most important high- temperature materials. Which are widely used for modern aircraft engines owing to high

temperature strength<sup>[1]</sup>, thermal stability<sup>[2-4]</sup>, creep resistance<sup>[5,6]</sup> and oxidation resistance<sup>[7-9]</sup> at high temperatures. The excellent mechanical properties of nickel-based single-crystal superalloys at high temperatures are depend mainly on the effect of the  $\gamma'$ -Ni<sub>3</sub>Al strengthening phase in the alloy<sup>[10]</sup>. The large number of investigations on the  $\gamma'$ -Ni<sub>3</sub>Al strengthening phase were found that the strengthening effect cannot meet the requirements for the performance of nickel-based single crystal alloys under high temperature conditions any more<sup>[11-13]</sup>. Recent studies have proven the mechanical characteristics of nickel-based single crystal alloys can be significantly reinforced by adding Ti, V, Nb, Ta<sup>[11-13]</sup>, and promotes the formation of the  $L1_2$ -type  $\gamma'$ -Ni<sub>3</sub>X (X=Ti, V, Nb) phases.

Over the past decades, there are currently few reports on Ni<sub>3</sub>Ti (Hexagonal), Ni<sub>3</sub>V (Tetragonal), Ni<sub>3</sub>Nb (Tetragonal), Ni<sub>3</sub>Ta (Tetragonal). Li *et al.*<sup>[14]</sup> investigated the mechanical properties of  $\gamma''$ -Ni<sub>3</sub>Ta under variable pressure by means of theoretical simulations, and explored the internal mechanisms affecting its stability and mechanical strength. K. Santhy *et al.*<sup>[15]</sup> identifies the phase of stability of Ni<sub>3</sub>(Ti, V) through first principles calculations. Tomasz Czeppe *et al.*<sup>[16]</sup> research effects of Ti and high cooling rate on the phase equilibrium and properties of Ni<sub>3</sub>(Al, V) alloys. Gong *et al.*<sup>[17]</sup> analyzed the effect of alloying elements on the occupation preference, structural stability and mechanical performance of  $\gamma''$ -Ni<sub>3</sub>Nb based on density functional theory.

In recent research, the calculation of phases based on density functional theory mainly focused on under fixed conditions. For example, Hou *et al.*<sup>[18]</sup> calculated the structural, elastic, thermodynamic and electronic behaviour of Ni<sub>3</sub>Al, Ni<sub>3</sub>Ga and Ni<sub>3</sub>Ge under assumed pressures. The elastic properties, thermodynamic stability, lattice anisotropy and Debye characteristic temperature of Ni<sub>3</sub>X were found to increase with increasing external pressure. Mao *et al.*<sup>[19]</sup> explored the mechanical stability, microscopic electronic structure and thermodynamic properties of Mg<sub>2</sub>Sr by applying different pressures. Liu *et al.*<sup>[20]</sup> investigated the impact of pressure on the microscopic crystal structure, electron distribution properties and bonding ion configuration of the MgCu<sub>2</sub> Laves phase. The results of Zhao *et al.*<sup>[21]</sup> showed that

the variation of pressure could improve the mechanical stability, microscopic electronic structure and thermodynamic properties of Ni<sub>3</sub>V intermetallic compound and improve the mechanical strength of the alloy. Chen *et al.* [22] performed first-principles calculations on DO<sub>22</sub>-type Al<sub>3</sub>V and Al<sub>3</sub>Nb intermetallic compounds and found that pressure has a profound effect on the structure, mechanical properties and electronic properties of Al<sub>3</sub>V and Al<sub>3</sub>Nb. These studies are reported to provide new ideas and directions for the study of material properties, and they provide new theoretical approaches for the design, development and stability of new materials.

Based on the above research reports, first-principles calculations was used to investigate the structural stability( $\Delta H$ ), elastic constants( $C_{ij}$ ), bulk modulus ( $B$ ), Yong's modulus ( $E$ ), shear modulus ( $G$ ), Debye temperature ( $\Theta_D$ ), and electronic structures of L1<sub>2</sub>-type Ni<sub>3</sub>X (X=Al, Ti, V, Nb) phases under external pressure condition. The aim of this work is to provide theoretical basis for the exploration of material structure, organization and properties of L1<sub>2</sub>-type  $\gamma'$ -Ni<sub>3</sub>X, and also provides important theoretical references for subsequent experimental studies and alloy design.

## 2. Model and Computational details

The L1<sub>2</sub>-type  $\gamma'$ -Ni<sub>3</sub>X (X = Al, Ti, V, Nb) phase has a face-centered cubic structure [23] (FCC) with space group  $Pm-3m$  (No. 221). All calculations are conducted using the plane-wave-pseudopotential approach based on the DFT [24]. The generalized gradient approximation of the function PBE [25] is used to represent the electron exchange-related potential. The geometry of the primal unit of Ni<sub>3</sub>X was optimized by the Broyden-Fletcher-Goldfarb-Shanno (BFGS) scheme [26]. The cutoff energy of the atomic wave function was set to 440 eV and the K-point was given as 8×8×8. The valence electron configurations of Ni-3d<sup>8</sup>4s<sup>2</sup>, Ti-3d<sup>2</sup>4s<sup>2</sup>, V-3d<sup>3</sup>4s<sup>2</sup>, Nb-4d<sup>4</sup>5s<sup>1</sup>, and Ta-4f<sup>14</sup>5d<sup>3</sup>6s<sup>2</sup> were regarded. The convergence tolerance was set to an energy of 2.0 × 10<sup>-5</sup> eV/atom. The SCF convergence threshold was established at 2.0 × 10<sup>-6</sup> eV/atom, with forces on an individual atom lower than 0.03 eV/Å, stress deviations smaller than 0.05 Pa, and tolerance biases shorter than 0.001 Å.

## 3. Results and discussion

### 3.1. Structural properties

The stability of a material is closely related to the enthalpy of formation of the crystals that make up the material. The enthalpy of formation is the total energy of the crystal minus the energy of the reactants. When the value of the enthalpy of formation is less than zero, it indicates that the crystal structure is thermodynamically stable; otherwise, the crystal structure is unstable. In this research, the enthalpy of formation ( $\Delta H$ ) of L1<sub>2</sub>-type  $\gamma'$ -Ni<sub>3</sub>X (X=Al, Ti, V, Nb) can be determined by the following expression.

$$\Delta_r H_m^\theta = \Delta_f H_m^\theta(\text{product}) + \Delta_f H_m^\theta(\text{reactant}) \quad (1)$$

$$\Delta H = \frac{5eN_A}{2} [E_{\text{Total}}^{\text{Ni}_3\text{X}} - (3E_{\text{Total}}^{\text{Ni}} + E_{\text{Total}}^{\text{X}})] \quad (2)$$

where  $E_{\text{Total}}^{\text{Ni}_3\text{X}}$  represents the total energy of L1<sub>2</sub>-type  $\gamma'$ -Ni<sub>3</sub>X;  $N_A$  stands for Avogadro's constant;  $e$  is for the basic charge;  $E_{\text{Total}}^{\text{Ni}}$  and  $E_{\text{Total}}^{\text{X}}$  are the energy per atom in the Ni and X crystals, respectively.

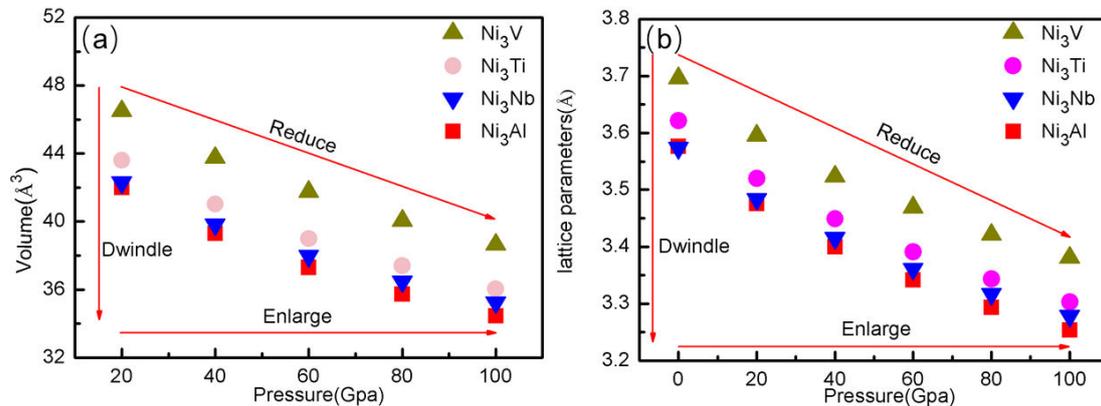
The calculated values of the lattice parameters  $a$ ,  $b$ ,  $c$ , the formation enthalpy ( $\Delta H$ ), volume at 0 GPa, and experimental and theoretical data studied by other researchers are listed in Table 1. This indicates that the calculated lattice constants are less than 1% of the theoretical allowable error from the experimental<sup>[27]</sup> and theoretical data<sup>[15, 28-30]</sup>. The  $\Delta H$  of L1<sub>2</sub>-type  $\gamma'$ -Ni<sub>3</sub>X (X=Al, Ti, V, Nb) phases at 0 GPa are -48.15, -53.05, -20.49, -21.82 kJ/mol, respectively. It indicates that L1<sub>2</sub>-type  $\gamma'$ -Ni<sub>3</sub>X phase is thermodynamically stable, the stability of the  $\gamma'$ -Ni<sub>3</sub>X has following sequence: Ti > Al > Nb > V, and the  $\Delta H$  of Ni<sub>3</sub>Ti, Ni<sub>3</sub>V are relatively close to -45.14 KJ/mol and -16.27 KJ/mol with previous research findings<sup>[15]</sup>, indicating that the feasibility of the calculation method and the simulative results are accurate.

**Table 1** Lattice parameters and formation enthalpy ( $\Delta H$ ) of L1<sub>2</sub>-type  $\gamma'$ -Ni<sub>3</sub>X (X=Al, Ti, V, Nb) phase at 0 GPa.

Phase	Species	Lattice constant(Å)	V(Å <sup>3</sup> )	$\Delta H$ (KJ/mol)
Ni <sub>3</sub> Al	This study	a=b=c=3.577	45.753	-48.15
	Experiment [27]	a=b=c=3.572		
	Other calculation [28]	a=b=c=3.574		
Ni <sub>3</sub> Ti	This study	a=b=c=3.621	47.494	-53.05

	Other calculation [29]	(Hexagonal)		-38.29
	Other calculation [15]	a=b=c=3.610		-45.14
Ni <sub>3</sub> V	This study	a=b=c=3.746	52.570	-20.49
	Other calculation [30]	(Tetragonal)		-26.70
	Other calculation [15]	a=b=c=3.569		-16.27
Ni <sub>3</sub> Nb	This study	a=b=c=3.696	50.498	-21.82
	Other calculation [31]	(Tetragonal)		-39.50

Fig. 1 shows the curves of lattice constants and cell volume (V) of L1<sub>2</sub>-type  $\gamma'$ -Ni<sub>3</sub>X as a function of pressure. It can be seen that a (a=b=c) and V decrease with the increase of pressure, indicating that the increase of pressure leads to the compression of cell volume and the atomic distances become smaller. However, as the pressure increases, the decrease in the interatomic distance slows down and the volume change becomes slow. This is due to the fact that the reduction of interatomic distances under high pressure disrupts the balance of interatomic interaction forces and leads to strong repulsion between atoms, which makes it difficult for the crystal to continue to be compressed under high pressure.



**Fig.1** Variations of the lattice parameters and unit cell volume of L1<sub>2</sub>-type  $\gamma'$ -Ni<sub>3</sub>X (X=Ti, V, Nb, Ta) with 0 to 100GPa.

### 3.2. Elastic properties

The elastic constant is a physical quantity used to characterize the elastic properties of a material, and it has an extremely important influence on the overall performance of the material. This physical quantity makes it possible to correlate the mechanical properties of a material with its stiffness, mechanical stability and stress-strain relationship. Fig. 2 shows the trend of the elastic constants ( $C_{ij}$ ) of Ni<sub>3</sub>X (X=Al, Ti, V, Nb) at 0 to 100GPa. It can be seen that the stability of Ni<sub>3</sub>X phase under

external pressure satisfies the following constraints on the mechanical stability of cubic crystals <sup>[32-33]</sup>:

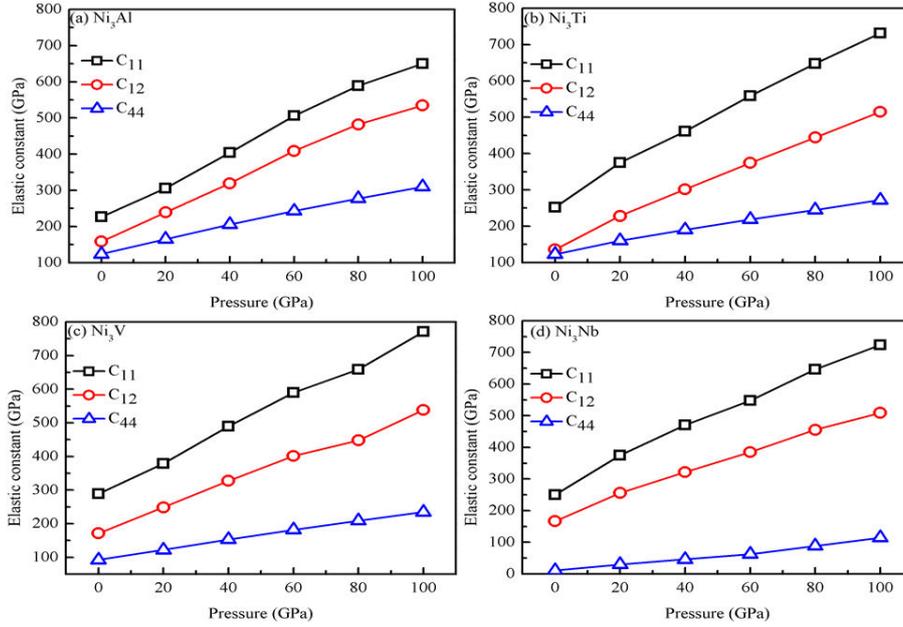
$$C_{11} - C_{12} > 0, C_{11} > 0, C_{44} > 0, C_{11} + 2C_{12} > 0 \quad (3)$$

which indicates they are mechanically stable under compression. Furthermore, it is observed that  $C_{11}$ ,  $C_{12}$  and  $C_{44}$  increase with pressure increasing,  $C_{11}$  possesses relatively large values, among which  $C_{11}$  is more sensitive to the change of pressure than  $C_{12}$ ,  $C_{44}$ . Since  $C_{11}$  represents the elasticity in length, whereas  $C_{12}$  and  $C_{44}$  are on behave of the elasticity in shape, explain the off diagonal and elastic shear characteristic of cubic crystals <sup>[34]</sup>, respectively. Under zero pressure, the  $C_{44}$  value of  $Ni_3Nb$  is the smallest and close to zero, it shows that  $Ni_3Nb$  have almost no shape elasticity at 0 GPa. The larger change in  $C_{11}$  is due to the strain on the length, which produces a certain volume change without changing the shape of the crystal.

**Table 2** The elastic constants ( $C_{ij}$ ), bulk modulus (B), Yong's modulus (E), shear modulus (G), Poisson's ratio ( $\nu$ ) of  $Ni_3X$  (X=Al, Ti, V, Nb) at 0 to 100GPa.

Phase	P(GPa)	$C_{11}$	$C_{12}$	$C_{44}$	B	G	E	G/B	$\nu$
<b>Ni<sub>3</sub>Al</b>									
	0	226.9	158.9	123.9	181.6	74.1	195.7	0.41	0.320
	20	306.0	239.3	165.0	261.6	88.2	237.9	0.34	0.348
	40	404.0	319.3	205.9	347.5	110.7	300.2	0.32	0.356
	60	506.5	409.0	243.0	441.5	129.5	353.9	0.29	0.367
	80	589.4	482.0	277.5	517.8	146.0	400.4	0.28	0.371
	100	650.1	534.7	310.3	573.1	161.0	441.6	0.28	0.372
<b>Ni<sub>3</sub>Ti</b>									
	0	252.1	135.8	123.0	174.5	91.1	232.8	0.52	0.278
	20	374.8	228.2	160.3	227.1	117.1	299.8	0.51	0.280
	40	461.0	301.8	190.0	354.9	134.0	357.1	0.38	0.332
	60	558.8	374.4	218.7	435.8	154.7	415.0	0.35	0.341
	80	647.9	444.6	245.1	512.4	172.2	464.6	0.34	0.349
	100	731.7	515.0	272.2	587.2	188.1	509.9	0.32	0.355
<b>Ni<sub>3</sub>V</b>									
	0	288.7	171.2	92.3	210.3	77.0	205.9	0.37	0.337
	20	378.5	248.4	121.9	291.8	94.7	256.4	0.32	0.354
	40	489.1	327.5	153.1	381.4	118.5	322.1	0.31	0.359
	60	589.4	401.4	181.8	464.1	139.5	380.4	0.30	0.363
	80	658.2	447.7	208.7	517.9	158.6	431.7	0.31	0.361
	100	770.9	538.4	234.4	615.9	176.9	484.3	0.29	0.369

Ni <sub>3</sub> Nb									
0	250.3	166.6	10.6	194.5	19.1	55.5	0.1	0.452	
20	375.4	256.1	29.4	295.8	39.2	112.6	0.13	0.437	
40	470.5	321.7	45.8	371.3	55.6	158.9	0.15	0.429	
60	548.2	385.0	62.2	439.4	69.3	197.5	0.16	0.425	
80	646.0	455.4	88.6	519.0	91.2	258.5	0.18	0.417	
100	723.5	509.0	114.7	580.5	111.6	314.6	0.19	0.410	



**Fig.2** The elastic constants ( $C_{ij}$ ) of Ni<sub>3</sub>X (X=Al, Ti, V, Nb) at 0 to 100GPa.

The bulk modulus (B), shear modulus (G), Yong's modulus (E) and Poisson's ratio ( $\nu$ ) of Ni<sub>3</sub>X (X=Al, Ti, V, Nb) at 0 to 100GPa can be calculated straightforwardly by the Voigte-Reusse-Hill method<sup>[35-37]</sup>, using these elastic constants from Fig.2. The calculation equations are as follows.

$$B = \frac{1}{3}(C_{11} + 2C_{12}) \quad (4)$$

$$G_V = \frac{1}{5}(C_{11} - C_{12} + 3C_{44}) \quad (5)$$

$$G_R = \frac{5(C_{11} - C_{12})C_{44}}{3(C_{11} - C_{12}) + 4C_{44}} \quad (6)$$

$$G = \frac{1}{2}(G_V + G_R) \quad (7)$$

$$E = \frac{9GB}{3B + G} \quad (8)$$

$$\nu = \frac{3B - E}{6B} \quad (9)$$

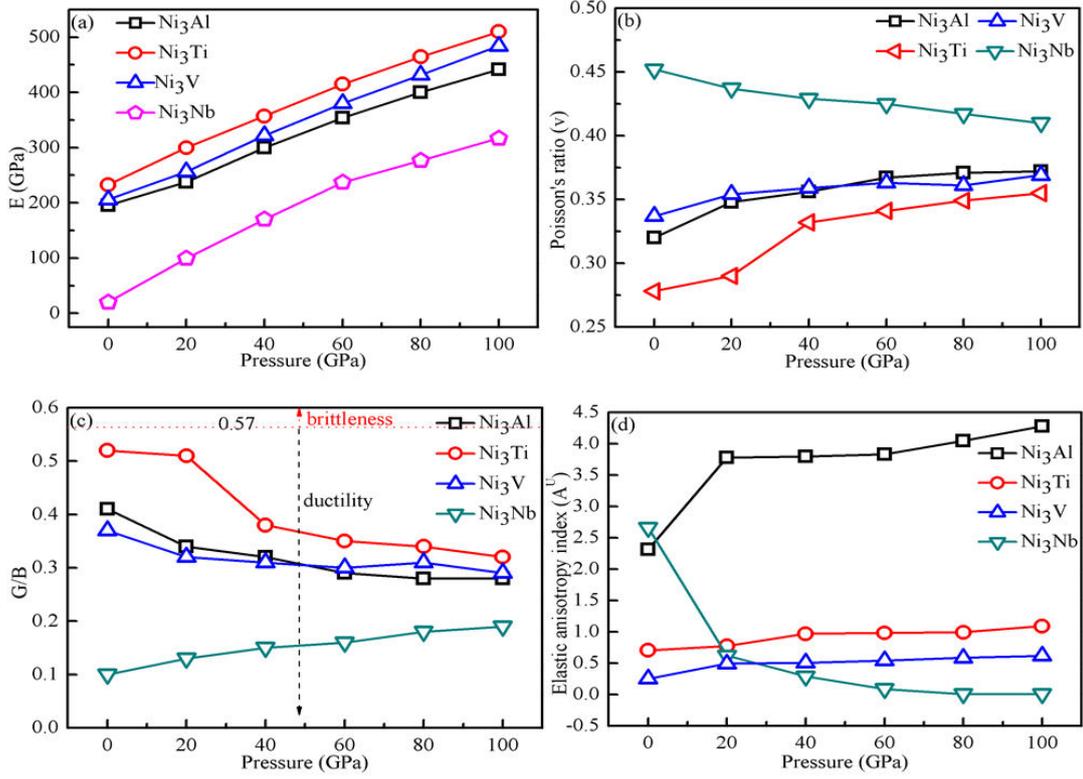
According to the methods above, the  $B$ ,  $G$ ,  $E$  and  $\nu$  of  $\text{Ni}_3\text{X}$  ( $\text{X}=\text{Al}$ ,  $\text{Ti}$ ,  $\text{V}$ ,  $\text{Nb}$ ) at 0 to 100GPa can be obtained, as shown in Table 2 and Fig 3. It is generally accepted that  $G$  is usually used to measure the resistance of a metallic material to its own deformation, while  $B$  is used to characterize the resistance of the material to volume change. The figure clearly shows that the values of bulk modulus for different intermetallic compounds at the same pressure are much larger than the shear modulus, implying that  $\text{Ni}_3\text{X}$  tends to resist volume change more than shape change under high pressure.  $E$  is a physical quantity that describes the resistance of a solid material to deformation. It is used as a measure of the stiffness of an isotropic elastomer, the larger the  $E$ , the less deformable and more rigid the material. Thus,  $\text{Ni}_3\text{Al}$ ,  $\text{Ni}_3\text{Ti}$  and  $\text{Ni}_3\text{V}$  are much stiffer than  $\text{Ni}_3\text{Nb}$ . The hardness decreases in the following order:  $\text{Ni}_3\text{Ti} > \text{Ni}_3\text{V} > \text{Ni}_3\text{Al} > \text{Ni}_3\text{Nb}$ . It is clear that the values of  $B$ ,  $G$  and  $E$  in Fig.3(a), (c) show an increasing trend with increasing pressure, which indicates that pressure can increase the elastic modulus of the material and thus improve the hardness of the metal material itself. Fig. 3(b), (c), (d) show  $G/B$ ,  $\nu$ , and  $A^U$ . Pugh<sup>[38,39]</sup> proposed a rough method for judging the toughness and tough-brittle transition behavior of materials using the ratio of  $G/B$  after an in-depth study. The brittleness of a material is associated with high  $G/B$  values, while low  $G/B$  values lead to ductility of the material<sup>[38]</sup>. The critical value of  $G/B$  to distinguish between ductility and brittleness of a material is about 0.57<sup>[39]</sup>. Figure 3(c) shows that the calculated  $G/B$  values for different pressures are below 0.57, indicating that  $\text{Ni}_3\text{Ti}$ ,  $\text{Ni}_3\text{V}$ ,  $\text{Ni}_3\text{Al}$  and  $\text{Ni}_3\text{Nb}$  all show ductility. Meanwhile, the increase in pressure leads to smaller  $G/B$  values for  $\text{Ni}_3\text{Al}$ ,  $\text{Ni}_3\text{Ti}$  and  $\text{Ni}_3\text{V}$ , indicating that the ductility of  $\text{Ni}_3\text{X}$  is improved at high pressure. Conversely, higher pressures resulted in worse ductility of  $\text{Ni}_3\text{Nb}$ . Poisson's ratio furnishes pertinent information on bonding capabilities<sup>[40]</sup>, and for metallic materials,  $\nu$  is usually between 0.25 and 0.5. the calculated values of  $\text{Ni}_3\text{Al}$ ,  $\text{Ni}_3\text{Ti}$ ,

Ni<sub>3</sub>V and Ni<sub>3</sub>Nb at different pressures are between 0.25 and 0.5, and, the Poisson's ratio values of Ni<sub>3</sub>Al, Ni<sub>3</sub>Ti and Ni<sub>3</sub>V increase correspondingly with increasing pressure, which translates into better ductility of the material. On the contrary, Ni<sub>3</sub>Nb has poor ductility.

Elastic anisotropy is an important property of materials, which reveals the difference in mechanical properties in different directions. The anisotropy of the crystal can be characterized by the universal anisotropy index. It is defined as <sup>[41,42]</sup>:

$$A^U = 5\frac{G_V}{G_R} + \frac{B_V}{B_R} - 6 \quad (10)$$

When the value of universal anisotropy index is not equal to 1, it indicates that the material is anisotropic, otherwise the crystal is isotropic. Fig.3(d) shows that the value of  $A^U$  is not equal to 1, it is obvious that Ni<sub>3</sub>X phases are anisotropic materials. For the Ni<sub>3</sub>X (X = Al, Ti, V) phase, the value of  $A^U$  increases with increasing external pressure, which indicates that anisotropy can be enhanced. On the contrary, the anisotropy of Ni<sub>3</sub>Nb decreases. Moreover, the anisotropy decreases in the following order at 0 GPa. Ni<sub>3</sub>Nb > Ni<sub>3</sub>Al > Ni<sub>3</sub>Ti > Ni<sub>3</sub>V. In other words, Ni<sub>3</sub>Nb exhibits stronger anisotropy at 0 GPa than the other phases. This is closely related to its smallness  $C_{44}$  at zero pressure.



**Fig.3** The function curve of pressure and Poisson's ratio, Young's modulus, bulk modulus, shear modulus and anisotropy index.

### 3.3 Anisotropy of acoustic velocities and Debye temperature

#### 3.3.1 Anisotropy of acoustic velocities

The propagation of sound velocity is anisotropic in solid, which is contingent on the symmetry of the crystal structure and the direction of dissemination. The relationship is shown by the following formula<sup>[43]</sup>.

$$\left| C_{ijkl} n_i n_j - \rho v^2 \delta_{ik} \right| = 0 \quad (11)$$

$$v(k) = \frac{dw}{dk} \quad (12)$$

In this work, only the pure propagation modes of the cubic structure crystal  $Ni_3X$  in [100], [110], and [111] directions are considered; the sound velocities in equations (11) and (12) can be solved by the elastic constants and phonon frequencies. (13)-(16) denote the sound velocities of crystals in different crystal directions, respectively.

Cubic crystal<sup>[44,45]</sup>:

For [100] direction:

$$[1 \ 0 \ 0] = [ \ 0 \ 0 ] = [ \ ]^c \quad (\text{polarization}) \quad (13)$$

$$[100]v_l = \sqrt{C_{11}/\rho}; [010]v_{l1} = [001]v_{l2} = \sqrt{C_{44}/\rho}; \quad (14)$$

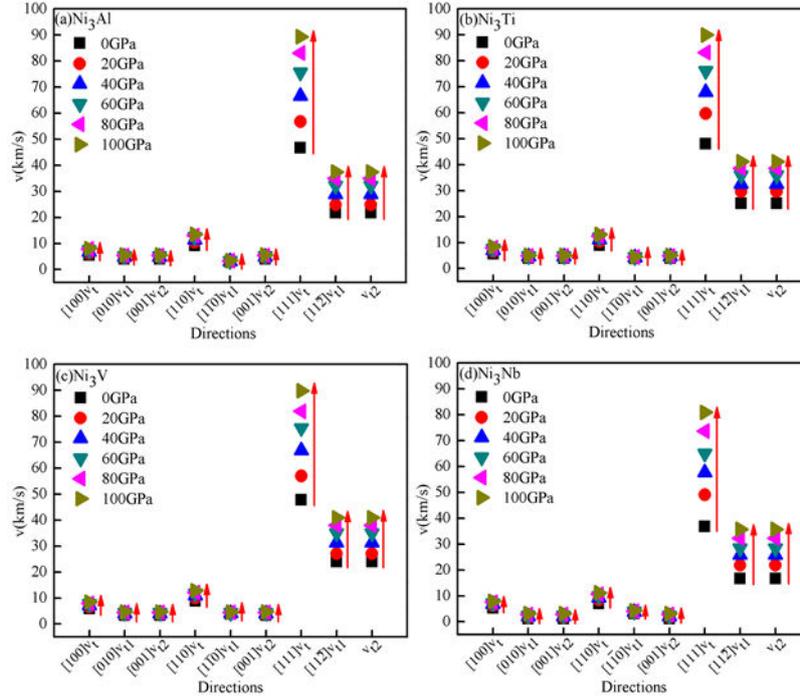
For [110] direction:

$$\begin{aligned} [110]v_l &= \sqrt{(C_{11} + C_{12} + 2C_{44})/\rho}; [1\bar{1}0]v_{l1} = \sqrt{(C_{11} - C_{12})/\rho}; \\ [001]v_{l2} &= \sqrt{C_{44}/\rho} \end{aligned} \quad (15)$$

For [111] direction:

$$\begin{aligned} [111]v_l &= \sqrt{(C_{11} + 2C_{12} + 4C_{44})/3\rho}; \\ [11\bar{2}]v_{l1} = v_{l2} &= \sqrt{(C_{11} - C_{12} + C_{44})/3\rho} \end{aligned} \quad (16)$$

Where,  $C_{ijkl}$  represents the elastic constant of  $\text{Ni}_3\text{X}$  under the assumed pressure;  $n_i$  and  $n_l$  denote the polarization direction and the propagation direction of sound velocity, respectively;  $w$  is the vibration frequency of the sound wave per unit period,  $v$  is the propagation velocity of sound ( $v_l$ : the propagation velocity of sound in the longitudinal direction,  $v_{l1}$ : is the sound velocity of the first transverse wave, and  $v_{l2}$ : the sound velocity of the second transverse wave). the calculated results of the sound velocity of  $\text{Ni}_3\text{X}$  along each direction and at varying pressures is illustrated in Fig. 4. The elastic anisotropy of the crystal can be reflected by the propagation rate of the sound velocity in different directions within the crystal. For example,  $C_{11}$  determines the longitudinal sound velocity along the [100] direction,  $C_{22}$  determines the longitudinal sound velocity along the [010] direction,  $C_{33}$  determines the longitudinal sound velocity along the [001] direction, while  $C_{44}$ ,  $C_{55}$  and  $C_{66}$  determine the transverse sound velocity of  $\text{Ni}_3\text{X}$  along the [100], [010] and [001] directions, respectively.



**Fig.4** The sound velocity in different directions increases of the L12-type  $\gamma'$ -Ni3X (X=Ti, V, Nb) phase with different of pressure

In Fig.4, the outcome shows that the sound velocity in different directions increases of the L12-type  $\gamma'$ -Ni3X (X=Ti, V, Nb) phase with the increase of pressure. Moreover, the speed increase rates in the [001], [110] and [111] directions are different, which indicates the existence of anisotropy of sound velocity. Among the three longitudinally propagating sound velocities in Fig.4, the longitudinal wave velocity along the [111] direction ( $[111]_{v1}$ ) is the fastest, which may be due to the difference of  $C_{11}$  and  $C_{12}$ , resulting in the difference in sound velocity propagation. This also explains the anisotropic variation of the Ni3X phase.

### 3.3.2 Debye temperature

The Debye temperature ( $\Theta_D$ ) is an important physical parameter that reflects the degree of dynamic distortion of the solid lattice and the strength of the interatomic bond. Many physical quantities of matter are connected with it, as elasticity, hardness, specific heat and melting point, so it is essential to study the Debye temperature.  $\Theta_D$  can be calculated from the data of elastic constants in Table 2 according to the following equation [46-48]:

$$\Theta_D = \frac{h}{k_B} \left[ \frac{3n}{4\pi} \left( \frac{N_A \rho}{M} \right) \right]^{1/3} v_m \quad (16)$$

$$v_m = \left[ \frac{1}{3} \left( \frac{2}{v_l^3} + \frac{1}{v_s^3} \right) \right]^{(-1/3)} \quad (17)$$

$$v_l = \sqrt{\frac{3B+4G}{3\rho}} \quad (18)$$

$$v_s = \sqrt{\frac{G}{\rho}} \quad (19)$$

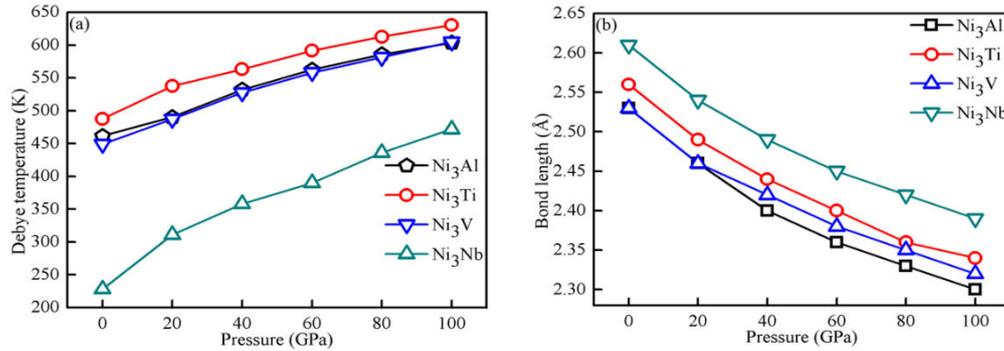
The  $h$  in equation (16) represents Planck's constant,  $K_B$  stands for the Boltzmann's constant,  $n_i$  is the number of atoms in a single  $Ni_3X$  cell,  $N_A$  is Avogadro's number,  $\rho$  is the density of a single  $Ni_3X$  cell,  $M$  is the molecular weight of  $Ni_3X$ , respectively.  $v_m$ ,  $v_l$  and  $v_s$  stand for the average, longitudinal and shear sound velocities, respectively. Table.3 displays the dependence of  $v_m$ ,  $v_l$  and  $v_s$  with pressure interval from 0 to 100GPa. The computed values of Debye temperature ( $\Theta_D$ ) of  $Ni_3X$  at variable pressures are demonstrated in Fig. 5. The Debye temperature of  $Ni_3Al$  and  $Ni_3V$  at 0 GPa are 461.45 K, 448.83K respectively, which is close to the existing experimental values derived from the measurement of elastic constants at chamber temperature<sup>[21]</sup>. Which are in line with the findings of this paper. It can be found that  $Ni_3Ti$  has the highest  $\Theta_D$  among the five phases. Debye temperature rises with incremental pressure for  $Ni_3X$  phases, and the growth rate gradually decreases.

In addition, as mentioned above, the Debye temperature ( $\Theta_D$ ) can be utilized to depict the intensity of covalent bonds in solids. the larger the value of  $\Theta_D$ , the stronger the covalent bonds. Therefore, the covalent bonds in  $Ni_3X$  become stronger as the pressure increases.

**Table 3** Sound velocity, Debye temperature ( $\Theta_D$ ) of  $Ni_3X$  (X=Al, Ti, V, Nb) phases

Phase	Pressure	$v_m$ (m/s)	$v_l$ (m/s)	$v_s$ (m/s)	$\Theta_D$ (k)
Ni3Al	0	4207.93	6152.01	3162.58	461.45
	20	4470.68	6880.14	3317.77	490.27
	40	4849.46	7584.10	3586.32	531.81

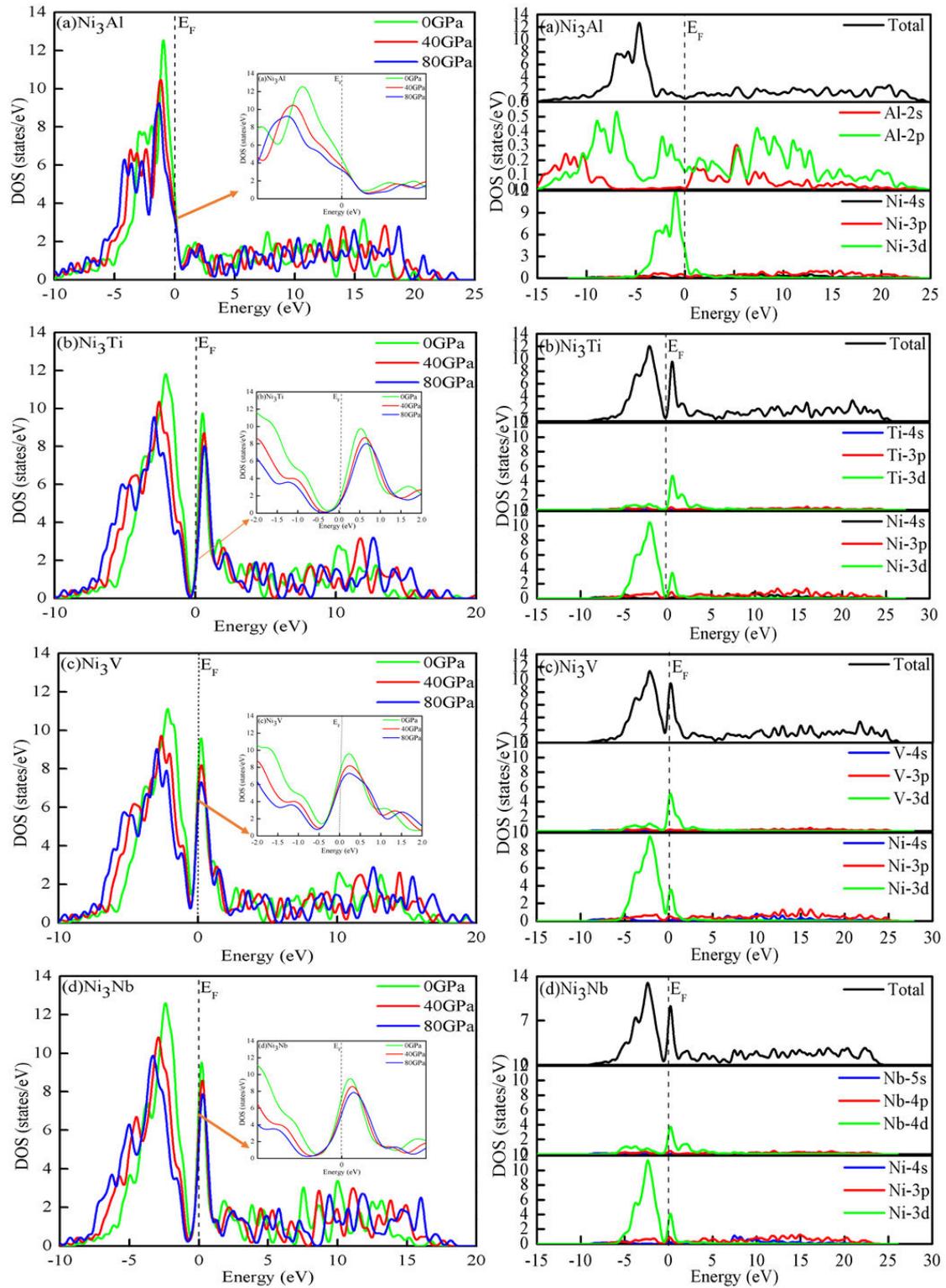
	60	5129.87	8220.99	3775.00	562.56
	80	5340.12	8661.35	3920.81	585.61
	100	5504.08	8937.02	4040.39	603.59
Ni <sub>3</sub> Ti	0	4446.32	6132.45	3401.70	487.60
	20	4900.93	7102.18	3692.49	537.45
	40	5135.15	7660.46	3839.26	563.13
	60	5393.67	8182.37	4016.07	591.48
	80	5587.69	8605.96	4046.10	612.76
	100	5749.31	8977.2	4253.29	630.48
Ni <sub>3</sub> V	0	4092.86	3053.58	6156.80	448.83
	Other calculation [21]	3835.20	3432.78	6442.70	507.12
	20	4442.07	3288.78	6909.67	487.13
	Other calculation [21]	4196.65	3648.77	7248.24	570.23
	40	4809.94	3551.61	7578.06	527.47
	60	5088.3	3749.84	8094.33	558.00
	80	5300.28	3910.35	8385.85	581.24
	100	5516.44	4054.6	8896.58	604.95
Ni <sub>3</sub> Nb	0	2079.96	1466.94	4981.47	228.09
	20	2835.67	2015.41	6005.79	310.97
	40	3263.52	2328.63	6588.92	357.89
	60	3557.53	2542.98	7043.09	390.13
	80	3976.12	2853.68	7563.29	436.03
	100	4302.19	3099.52	7922.38	471.85



**Fig.5** Sound velocity, Debye temperature ( $\Theta_D$ ) of Ni<sub>3</sub>X (X=Al, Ti, V, Nb) phases

### 3.4 Electronic structure

To in addition recognize the bonding traits and to look into the impact of stress on the digital structure, the whole density of states and partial density of states of Ni<sub>3</sub>X are calculated in this paper, as proven in Fig. 6. Here it is solely the TDOS of Ni<sub>3</sub>X at 0, 40 and 80 GPa that it has plotted to describe the hassle extra clearly.



**Fig.6** Density of states of  $Ni_3X$  ( $X=Al, Ti, V, Nb$ ) at 0,40,80GPa.

As seen in Fig. 6, many energy states occupy the Fermi level, which means that the  $Ni_3X$  phase demonstrates metallic properties. In addition, the peak height of TDOS can be significantly reduced by increasing the pressure, and the distribution range of TDOS can be expanded. For  $Ni_3Al$ ,  $Ni_3Ti$ ,  $Ni_3V$  and  $Ni_3Nb$ , the dominant

bond-forming peaks in the vicinity of the Fermi level are dominated by the Ni-3d and Al-3p states, the Ni-3d and Ti-3d states, and the Ni-3d and V-3d states. In particular, the hybridization between Ni and X atoms is apparent, which forms a covalent bonding feature.

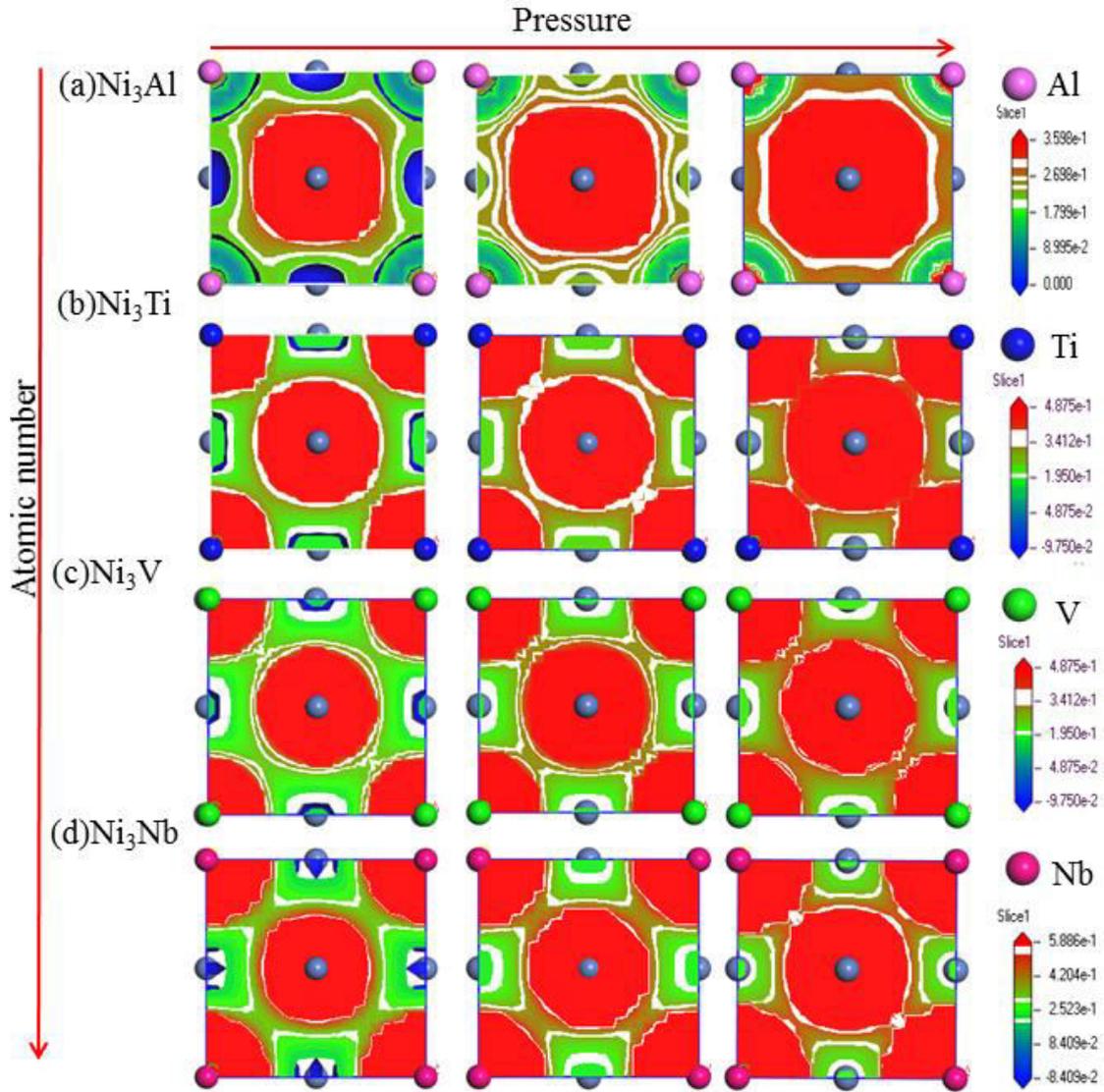
To reap perception into the chemical bonding homes of Ni<sub>3</sub>X beneath pressure, Mulliken charges <sup>[49]</sup> as properly as bond lengths have been analyzed to quantitatively examine the impact of stress on structural, elastic, thermodynamic, and digital properties. The ionic configurations are listed in Table 4. At 0 GPa, the charges transferred from X to Ni atoms for Ni<sub>3</sub>Al, Ni<sub>3</sub>Ti, Ni<sub>3</sub>V, and Ni<sub>3</sub>Nb are 0.25, 0.40, 0.30, and 0.28 e, respectively. In addition, the charges transferred from X to Ni atoms expand with growing pressure. But the bond length size decreases with the ascend of pressure. This is the physical root of the impact of pressure on the structure, elastic constants and Debye temperature, etc. This offers an appropriate rationalization for the variant of lattice constant and volume with pressure.

**Table 4** Mulliken Charge, Bond length of Ni<sub>3</sub>X (X=Al, Ti, V, Nb) phases at 0 to 100GPa

Pressure (GPa)	Ni <sub>3</sub> Al		Ni <sub>3</sub> Ti		Ni <sub>3</sub> V		Ni <sub>3</sub> Nb	
	Bond length (Å)	Mulliken Charge(e)	Bond length (Å)	Mulliken Charge(e)	Bond length (Å)	Mulliken Charge(e)	Bond length (Å)	Mulliken Charge(e)
0	2.53	Ni <sup>-0.08</sup> Al <sup>+0.25</sup>	2.56	Ni <sup>-0.13</sup> Ti <sup>+0.4</sup>	2.53	Ni <sup>-0.1</sup> V <sup>+0.3</sup>	2.61	Ni <sup>-0.09</sup> Nb <sup>+0.28</sup>
20	2.46	Ni <sup>-0.07</sup> Al <sup>+0.22</sup>	2.49	Ni <sup>-0.2</sup> Ti <sup>+0.59</sup>	2.46	Ni <sup>-0.15</sup> V <sup>+0.44</sup>	2.54	Ni <sup>-0.19</sup> Nb <sup>+0.58</sup>
40	2.4	Ni <sup>-0.06</sup> Al <sup>+0.19</sup>	2.44	Ni <sup>-0.26</sup> Ti <sup>+0.78</sup>	2.42	Ni <sup>-0.19</sup> V <sup>+0.58</sup>	2.49	Ni <sup>-0.29</sup> Nb <sup>+0.86</sup>
60	2.36	Ni <sup>-0.05</sup> Al <sup>+0.16</sup>	2.4	Ni <sup>-0.32</sup> Ti <sup>+0.96</sup>	2.38	Ni <sup>-0.24</sup> V <sup>+0.72</sup>	2.45	Ni <sup>-0.37</sup> Nb <sup>+1.11</sup>
80	2.33	Ni <sup>-0.04</sup> Al <sup>+0.13</sup>	2.36	Ni <sup>-0.38</sup> Ti <sup>+1.14</sup>	2.35	Ni <sup>-0.29</sup> V <sup>+0.86</sup>	2.42	Ni <sup>-0.45</sup> Nb <sup>+1.35</sup>
100	2.3	Ni <sup>-0.03</sup> Al <sup>+0.10</sup>	2.34	Ni <sup>-0.44</sup> Ti <sup>+1.31</sup>	2.32	Ni <sup>-0.33</sup> V <sup>+0.99</sup>	2.39	Ni <sup>-0.53</sup> Nb <sup>+1.58</sup>

To further analyze the difference interaction between Al, Ti, V, Nb atoms and Ni atoms, this article studies the charge density (Fig 7 is the charge density diagram of Ni<sub>3</sub>X phases.) distribution of Ni<sub>3</sub>X on the [010] crystal plane. From the point of view of electron transfer between atoms, Ni atoms lose more electrons in Ni<sub>3</sub>X (X=Ti, V, Nb), and the lost electrons are transferred to the space between Ni atoms and Ti, V, and Nb (the red part in Fig 7). With the pressure increases, the electron transfer tendency increases. The order of electron aggregation strength between doping atoms is Ti>Nb>V>Al, therefore, the interaction between Ti atoms and Ni atoms is the

strongest, and the interaction between Ta atoms and Ni atoms is the weakest. This is consistent with the analysis results of the enthalpy of formation, the Debye temperature and the density of states.



**Fig.7** The charge density of  $Ni_3X$  ( $X=Al, Ti, V, Nb$ ) at 0,40,80GPa.

#### 4 Conclusion

In this paper, the structural stability( $\Delta H$ ), elastic constants( $C_{ij}$ ), bulk modulus ( $B$ ), Yong's modulus ( $E$ ), shear modulus ( $G$ ), Debye temperature ( $\Theta_D$ ), and electronic structures of L12-type  $Ni_3X$  ( $X=Al, Ti, V, Nb$ ) phases under external pressure condition were systematically studied by first principles calculations. The conclusion is as follows:

- (1) The relationship between elastic constant, elastic modulus,  $G/B$  value and

Poisson's ratio under pressure shows that  $\text{Ni}_3\text{X}$  has good shear stability and ductility. And the  $C_{ij}$ ,  $B$ ,  $G$ ,  $E$  values increase with the increase of pressure.

(2) All  $\text{Ni}_3\text{X}$  phases have the sound velocity anisotropy, and the increase of external pressure leads to the enhancement of anisotropy. The [111] direction has strong longitudinal anisotropy, which is closely related to the large  $C_{ij}$  of  $\text{Ni}_3\text{X}$  phases.

(3) The sound velocity and Debye temperature increase of the  $\text{Ni}_3\text{X}$  phases with the increase of the applied pressure. The magnitude of the Debye temperature under the applied pressure is:  $\text{Ni}_3\text{Ti} > \text{Ni}_3\text{Al} > \text{Ni}_3\text{V} > \text{Ni}_3\text{Nb}$ .

(4) Electronic analysis shows that increase pressure results in improving the bond strength and crystal stability of  $\text{Ni}_3\text{X}$  phases. The degree of charge hybridization and Ni-X bond strength increases with increasing pressure. In those phases, the charge accumulation of  $\text{Ni}_3\text{Ti}$  is the largest and the crystal structure is the most stable.

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### **Declarations**

**Conflicts of interest:** The authors declare that they have no conflict of interest.

### **Availability of data and material**

The raw/processed data required to reproduce these findings can be shared upon request.

### **Code availability**

The code for the simulations can be provided upon request.

**Authors' contribution:** **Y H Wu:** Investigation, Methodology, Formal analysis, Validation, Writing - original draft, Writing - review & editing. **J S Chen:** Writing - review & editing, Validation. **J Y Ji:** Formal analysis, Methodology, Validation. **Y Z Zhang:** Conceptualization, Methodology. **Q Z Wang:** Writing - review & editing. **K Xiong:** Conceptualization, Methodology.

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