

# Mathematical Formation and Analysis of COVID19 Pool Tests Strategies

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## Research Article

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# Mathematical Formation and Analysis of COVID19 Pool Tests Strategies

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**Abstract:** The excessive spread of the pandemic COVID19 around the globe has put mankind at risk. The medical infrastructure and resources are frazzled, even for world's top economies, due to the large COVID19 infection. To cope up with this situation, countries are exploring the pool test strategies. In this paper, a detailed analysis has been done to explore the efficient pooling strategies. Given a population and the known fact that the percentage of people infected by the virus, the minimum number of tests to identify COVID19 positive cases from the entire population is observed. The analysis reveals that the tests needed are very less when compared to the total population. This can be looked as an essential step towards efficient utilization of sparse available resources of COVID19 testing kits, especially for the countries having limited medical infrastructure.

**Keywords:** Coronavirus, COVID19, Pool Test, Binary Tree, Genetic Algorithm.

**Mathematics Subject Classification (2000)** MSC 05C05, MSC 092D30

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## 1. Introduction

In the history, the mankind has observed numerous infectious diseases, which not only affect the lives of many people but also instigate crisis interventions that are vanquished in extended period of time. Some such reported infectious diseases includes the outbreak of smallpox in 1520, cholera in 1817, plague in 1855, Spanish influenza in 1918, Asian Flu in 1957, Hong Kong Flu in 1968, HIV/AIDS in 1981, acute respiratory syndrome coronavirus (SARS-CoV) in 2002, H1N1 influenza and Swine Flu in 2009, the Middle East respiratory syndrome coronavirus (MERS-CoV) in 2012 and Ebola in 2014 [1].

Recently, a new outbreak of novel coronavirus has been identified in the Wuhan City of Hubei Province in South China, which is termed as SARS-CoV-2 or COVID-19. This virus expeditiously spreads within China and then across the globe. It has infected more than million and killed thousands of individual since December 2019. It is observed that the infected individual showed the common symptoms of fever, cough, sore throat and difficulty in breathing [2,3]. Over the time, it spreads to the lower respiratory tract of the

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patient and causes viral pneumonia and may lead to dyspnea, respiratory distress syndrome and multiple organ failure in severe cases [4]. The World Health Organization termed the outbreak of COVID-19 a pandemic on March 11, 2020 [5, 6]. The pandemic infected a major fraction of the world's population. As on June 1, 2020, there are 6,040,609 confirmed cases of COVID19, including 370,657 deaths, as reported by WHO, exceeding the equivalents of in family viruses' outbreak Middle East syndrome (MERS) and Severe Acute Respiratory Syndromes (SARS) [7].

The loss of lives and economic damage done by the pandemic is terrible with substantial uncertainty all over. A region wise death toll due to COVID 19 across the globe is depicted in Fig 1, which is fairly enormous and rapidly growing with time. Therefore, the exploration of both non-pharmaceutical and pharmaceutical knowledge towards pandemic is essential and need of the hour to make a real difference in everyone lives. The obstruction in avoiding wide-spread of COVID19 is majorly due to limited medical infrastructure and resources. In the world's low economies, the situation is even worse. This is due to the fact that limited testing has been carried out in lack of sufficient testing kits resulting in the non-identification of infected individuals, who are then spread the disease as carriers and will affect the COVID 19 control efforts.

Currently, there are two types of tests to confirm COVID19 infection [8]. The former is the test done to confirm the presence of the virus in an individual whereas in the later presence of antibodies is checked to establish the infection in an individual. However, these tests are very costly and time consuming. A

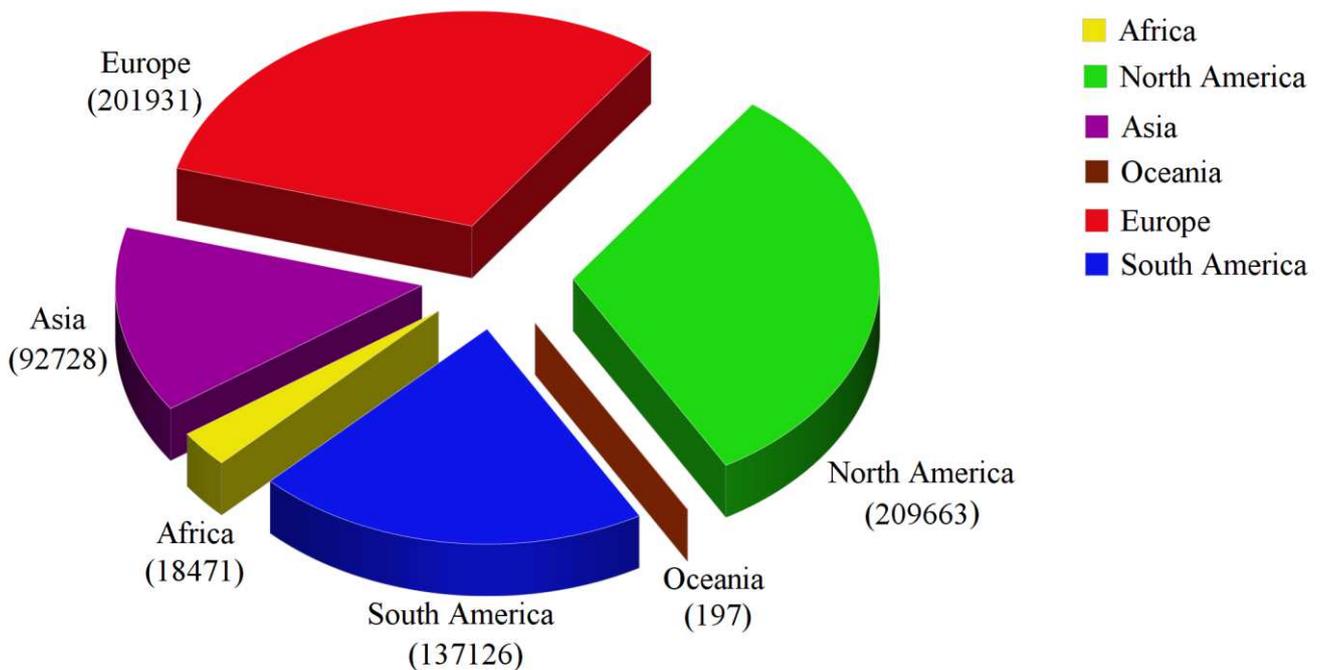


Fig. 1. Region wise total death toll due to COVID 19 as on July 31, 2020 [15].

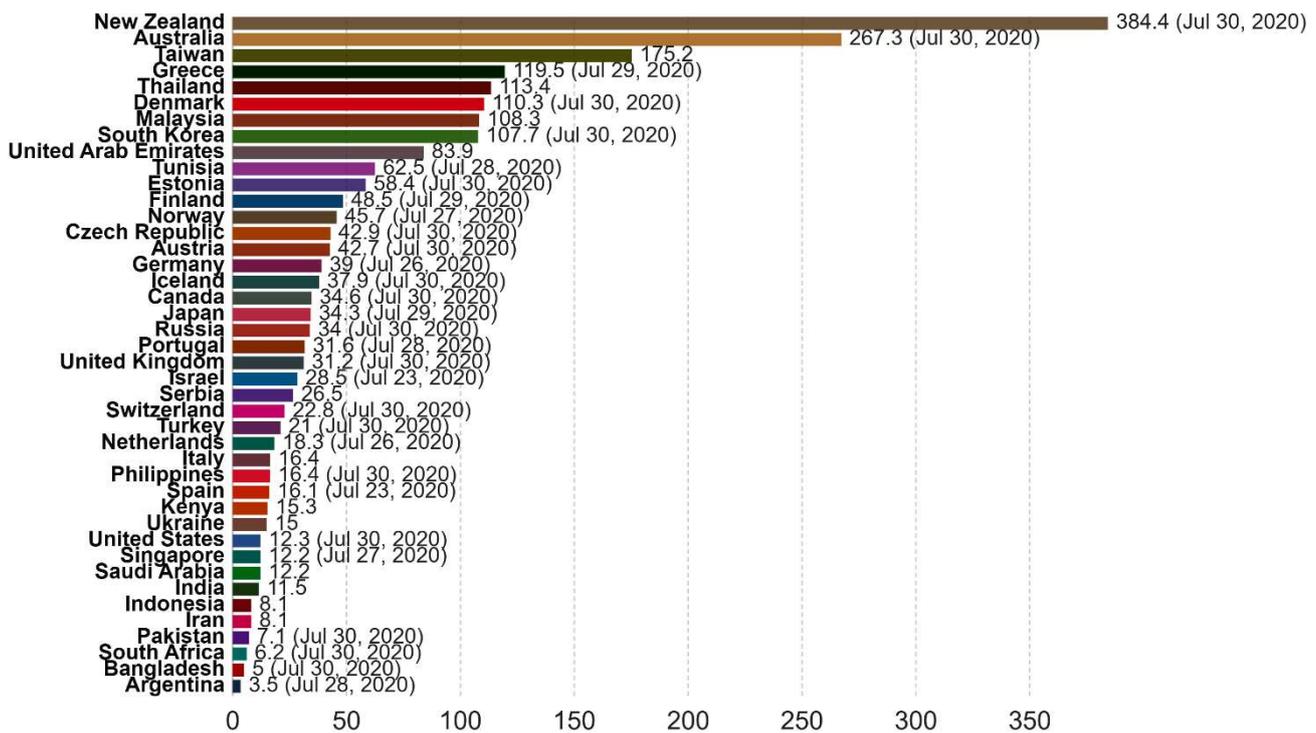


Fig. 2. Total average number of COVID19 test per confirmed case as on July 31, 2020 [15].

comprehensive information on the total number of COVID-19 tests per confirmed case is depicted in Fig. 2. It is clear from the figure that total number of tests conducted is having remarkable disparity revealing the limited availability of resources with some of the countries. Another way of looking at the extent of testing is with respect to the scale of outbreak in different countries. The same is depicted in Fig. 3, where the total test conducted on a daily basis against the total new confirmed cases per million people on a daily basis is illustrated. From the figure it is clear that few countries are doing ten or hundred times fewer tests than countries having the similar number of new confirmed cases. This essentially exhibiting the limited resource availability with some of the countries. To further elaborate on this the COVID19 tests per thousand people versus GDP per capita is depicted in Fig. 4. From figure, it is revealed that the total tests administered are very low with the countries with low GDP, which further validates the fact that these countries are having limited medical resources, which in turn effecting the COVID19 testing of the complete population.

To cope up with this situation, one can pool the specimens followed by the COVID 19 test to confirm the infection. Several countries (including South Korea, Israeli and India) adopted the pool test strategy to increase their testing capacity for COVID19 [9, 10, 11, 12, 13,14]. However, the main challenge here is to find the optimal pool size without which pool test sometimes more expensive than the individual testing. To the best of the author’s knowledge, optimal pool size is an untouched problem till date.

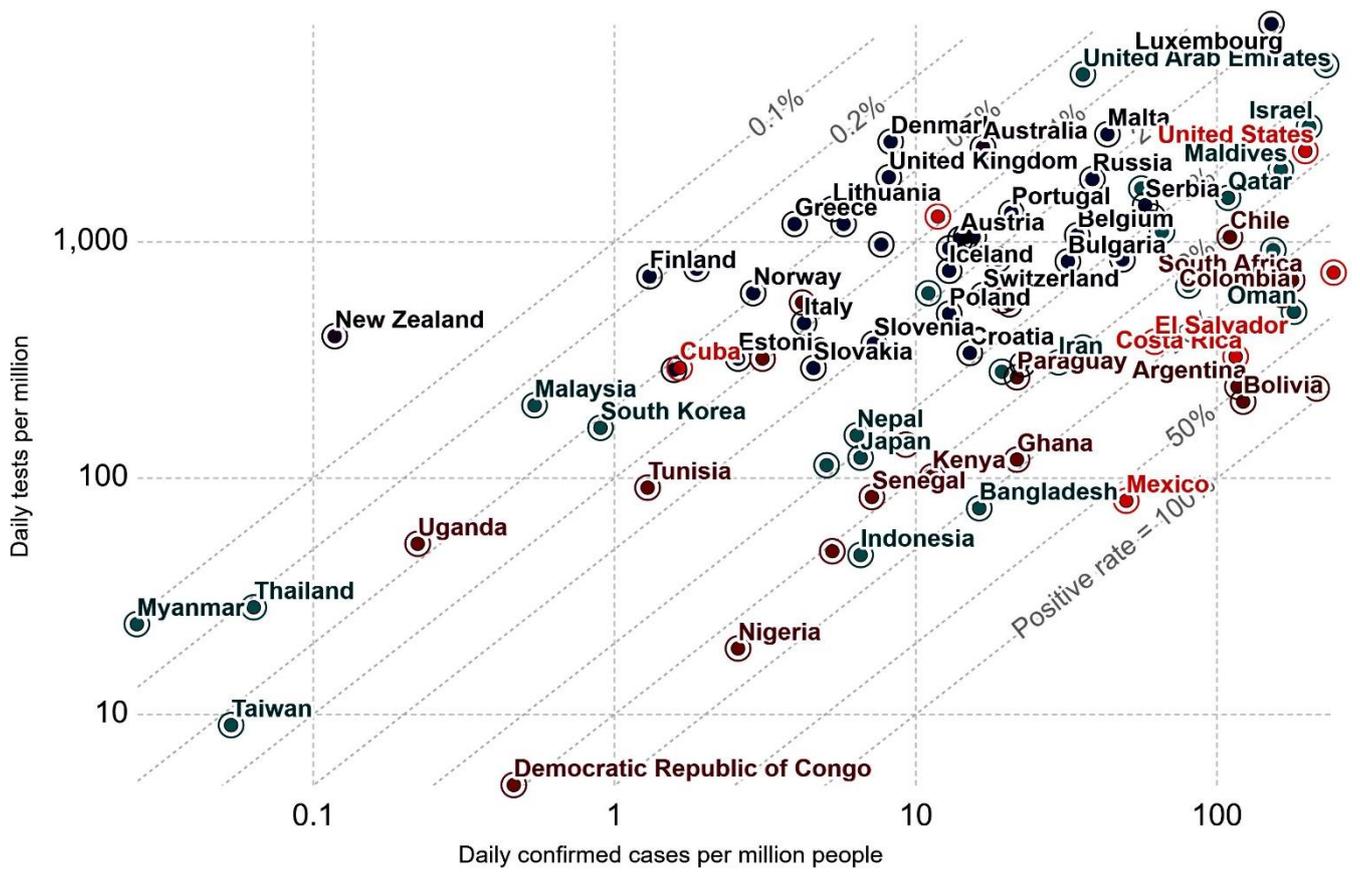


Fig. 3. Daily tests against daily new confirmed cases per million as on July 31, 2020 [15].

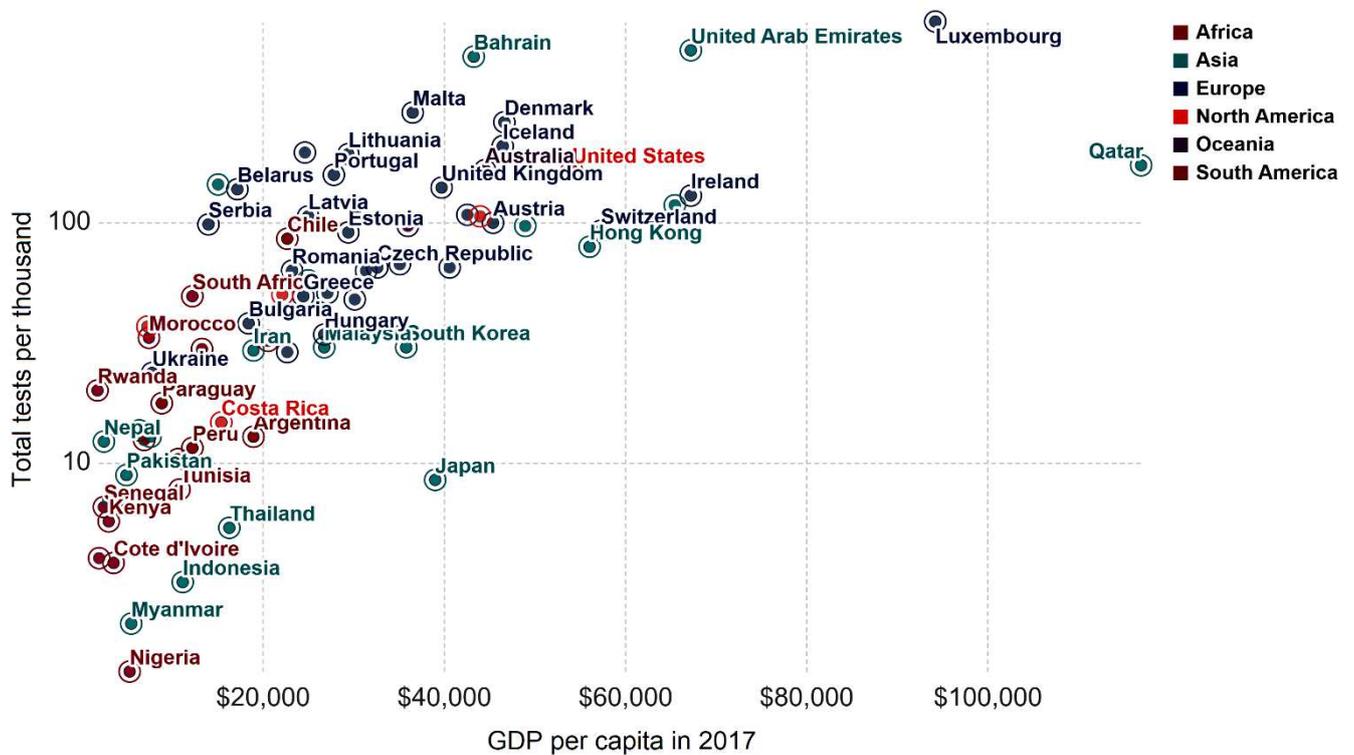


Fig. 4. Total COVID19 tests per thousand people vs. GDP per capita as on July 31, 2020 [15].

Therefore, in this paper, the formal definition of COVID19 Pool Test problem is discussed and an efficient solution is explored. The core idea is to provide an efficient way to obtain the optimal pool size so that the testing of complete population can be done in minimal number of test. In other words, given a population and the known fact that the percentage of people infected by the virus, the minimum number of tests to identify COVID19 positive cases from the entire population is observed. The two different pooling strategies, namely, (1) Single Pool and (2) General Pool Testing are proposed. The solution to these testing strategies are also proposed which are based on the binary tree and genetic algorithm (GA). The analysis reveals that the tests needed are very less when compared to the total population. This can be looked as an essential step towards efficient utilization of sparse available resources of COVID19 testing kits, especially for the countries having limited medical infrastructure. From the analysis, it has been found that the proposed solution is efficiently finding the optimal pool size and further revealing that the total test administered is fairly less when compared to the administering test of every individual in the population. This can be seen as a positive sign in this critical phase and provides an exigent way to utilize available medical resources systematically.

The rest of the paper is organized as following: Section 2 presents the proposed coronavirus pool test problems followed by their solutions in Sections 3-4. Section 5 essentially provides the experimental results and discussions. Finally, Section 6 presents the concluding remarks.

## 2. COVID19 Pool Test Problem Statement

In this section, the formal definitions of proposed corona pool test strategies are discussed. These strategies are termed as: (1) single pool test problem, and (2) general pool test problem. A basic structure of the COVID19 Pool Test Problem can be seen in Fig. 5 and the details can be summarized as follows:

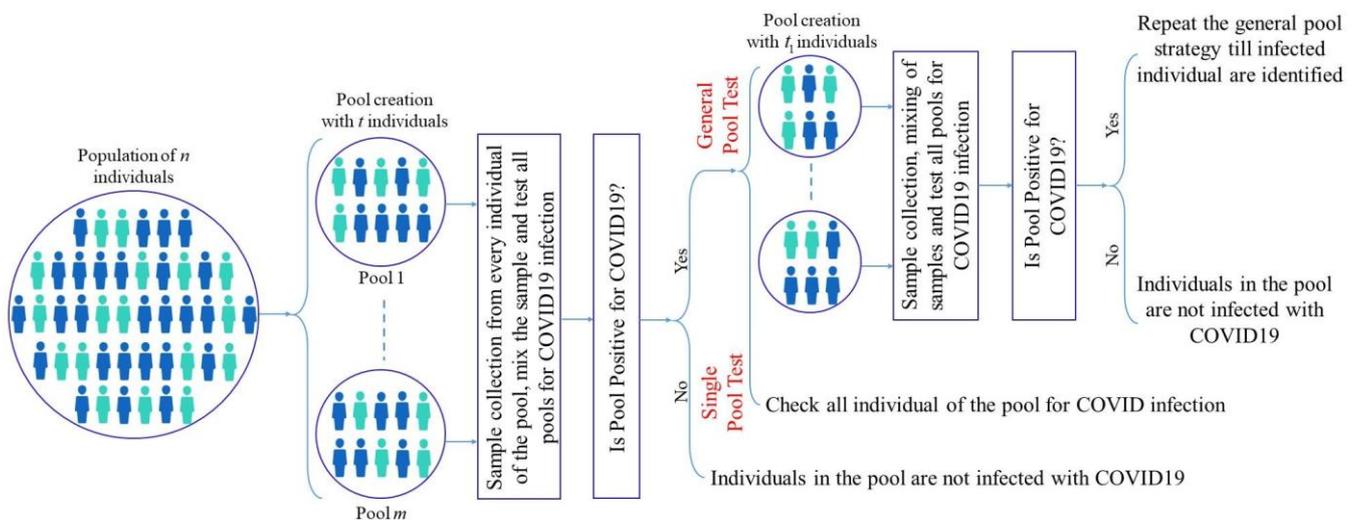


Fig. 5. Basic structure of COVID19 Pool Test problem.

### *2.1. Single COVID19 pool test problem*

In Single COVID19 pool test, successive grouping technique is adopted till the infected individuals are isolated. The process begins with portioning the population of size  $n$  into groups/pools of individuals of some size (say  $t < n$ ) followed by collecting multiple copies of samples of every individual in the pool. Then, a mixture of a copy of the samples of all the individuals from the pool is taken and the COVID19 test is conducted. Such a test is called a pool test. If the test is positive, then all the individuals of the pool are administered individual COVID19 test to identify the infected individuals in the population. This is the coarser formulation of the problem wherein the pools of the population are created only once.

### *2.2. General COVID19 pool test problem*

In the general COVID19 pool test problem, the single COVID19 pool test is initially considered and the outcome of the same is noted. If it tests positive then instead of administering the test on every person in the pool, the pool is further divided into random pools of some size (say  $t_1 < t_0 = t$ ) and again a mixture of a copy of the samples of all the individuals in the corresponding pool is taken and tested. Same step is repeated for each and every pool formed. If the pool test of the pool results into positive then at least one of the persons in that pool is infected by COVID19 and if it results into negative then none in the pool are infected by the COVID19. In the next level, take all the individuals belonging to the pool whose COVID19 test was positive and divide them into random groups of some size (say  $t_2 < t_1$ ) again and continue this iterative process till all the COVID19 infected individuals are identified. For the simplicity, the process started with the assumption that the pool tests of the entire population was tested positive, i.e., at least one individual in the entire population is infected by COVID19 because if the population is tested negative in the very beginning then there is no point of conducting any kind of test for COVID19 infection.

## **3. Mathematical formation of Single COVID19 Pool Test and its solution**

In this section, all of the efforts are concentrated on the single pool test described in Section 2.1. The problem that now arises is what should be the group size  $t$  so as to administer minimum number of tests, i.e., a pool should comprise of how many maximum numbers of individuals such that the tests that are performed overall are least among all possible pool sizes. To solve the problem, an assumption is considered which states that the percentage of COVID19 positive individual is known. This value might not be exact but can always be predicted using the available data of the population. Thus, the single pool test problem can be formulated mathematically as:

Statement: Let  $n$  be the size (total number of individuals) of the population,  $P$  be the percentage of COVID19 infected patients. Then, what should the pool size  $t$  be so as to perform a minimum number of tests to identify all the infected individuals in the worst case. The worst case is when there is at most one infected person lies in every pool.

The solution of this problem can be obtained as follows: Given that the maximum number of COVID19 positive patients is  $p = \left\lceil \frac{nP}{100} \right\rceil$ . If the pool size is  $t$  then total tests administered in first level is  $\left\lceil \frac{n}{t} \right\rceil$ . Out of the total tests administered in the first level, at most  $p$  pool will test positive. Thus, the total number of individuals on whom the test is to be administered again are at most  $pt$  resulting into total tests  $f(t)$ , where

$$f(t) = \left\lceil \frac{n}{t} \right\rceil + pt \quad (1)$$

also it is observed that  $1 < t \leq \frac{n}{p} - 1$  when  $p | n$  and  $1 < t \leq \left\lceil \frac{n}{p} \right\rceil$  when  $p \nmid n$ . Thus, the problem reduces to the following minimization problem:

$$\min f(t) \quad (2)$$

$$\text{s.t. } 2 \leq t \leq \frac{n}{p} - 1 \text{ when } p | n \text{ and } 2 \leq t \leq \left\lceil \frac{n}{p} \right\rceil \text{ when } p \nmid n \quad (3)$$

Let us define

$$f(t) = \left\lceil \frac{n}{t} \right\rceil + pt \quad (4)$$

Since  $\left\lceil \frac{n}{t} \right\rceil$  is an increasing function of  $\frac{n}{t}$  thus the same value of  $t$  minimizes Eqns. (2) and (3). By second

derivative test we get  $t = \sqrt{\frac{n}{p}}$  as the point of minima of equations  $g(t)$  and thus of  $f(t)$  as well. Observe

that while doing the above, only those values of  $p$  for which  $t = \sqrt{\frac{n}{p}}$  lies within the range of  $2 \leq t \leq \frac{n}{p} - 1$

. In case  $P$  exceeds this value, test should be administered to each person individually. In theory,  $t$  can be

any real number but it is desirable to have a natural number and so both  $t = \left\lceil \sqrt{\frac{n}{p}} \right\rceil$  and  $t = \left\lceil \sqrt{\frac{n}{p}} \right\rceil - 1$  are

checked for the minimum value. Also, the minimum value is less than equal to  $n$  when the condition on  $p$  is satisfied. Let us validate the proposed solution in view of an example:

Consider a city wherein the population is  $n=128$  and percentage of corona cases are  $P=4.68\%$ , which leads to total infected individuals  $p = \left\lceil \frac{128 \times 4.68}{100} \right\rceil = 5.99 \approx 6$  and total number of tests administered in first level as  $t = \sqrt{\frac{128}{6}} = 4.62 \approx 5$ . This has led to minimum number of tests (using Eqn. 1) required  $f(5) = \left\lceil \frac{128}{5} \right\rceil + 6 \times 5 = 26 + 30 = 56$ . Further, by virtue of minima, if any other value of  $t$  is considered, the total number of tests to be done will be more than 56. For instance, if  $t=6$  is considered then total tests come out to be 58. Therefore, it is concluded that there is the requirement of at most 56 tests for the entire population of the city.

#### 4. Mathematical formation of General COVID19 Pool Test and its solution

In this section, all of the efforts are concentrated on the general pool test as described in Section 2.2. The problem remains the same as described in the previous section except that the group size is determined once only whereas the construction of pools will be done at multiple levels. Thus, in general pool test problem, the pool size at every level needs to be determined so that the total number of tests are minimized. Let us start the discussion, under the assumption that the percentage of COVID19 positive individuals is known. Thus, the general pool test problem can be formulated mathematically as:

Statement: Let  $n$  be the size (total number of individuals) of the population,  $P$  be the percentage of COVID19 infected individuals in the population. Then, what should the pool sizes  $t_i$  be so as to perform minimum number of tests to identify all the infected individuals in the worst case. The worst case is when there is at most one infected person lies in a pool in every level ( $i$ ).

The solution of this problem can be obtained as follows: Given that the maximum number of COVID19 positive patients  $p = \left\lceil \frac{nP}{100} \right\rceil$ . Let  $t_0$  be the size of first pool. Thus, number of pools formed are  $\left\lceil \frac{n}{t_0} \right\rceil$  each of size  $t_0$  except at most one of size less than  $t_0$ . Out of these  $\left\lceil \frac{n}{t_0} \right\rceil$  pools at most  $p$  pools will have a

positive result in their pool test. If  $p \geq \left\lceil \frac{n}{t_0} \right\rceil$  then all the individuals need to be tested again by grouping with some other size in the worst case. Let us choose  $t_0$  such that  $p < \left\lceil \frac{n}{t_0} \right\rceil$  and doing so at most  $pt_0$  out of  $n$  individuals need to be tested again in the second level. Let  $t_1$  be the pool size for this level then number of pools formed are at most  $\left\lceil \frac{pt_0}{t_1} \right\rceil$  each of size  $t_1$  except at most one of size less than  $t_1$ . Observe that  $t_1 < t_0$  because otherwise  $p > \frac{pt_0}{t_1} \Rightarrow p \geq \left\lceil \frac{pt_0}{t_1} \right\rceil$  since  $p \in \mathbb{N}$ , which contradicts the fact that the number of COVID19 positive individuals  $p$  need to be less than number of pools formed  $\left\lceil \frac{pt_0}{t_1} \right\rceil$  in the worst case. Continuing the chain in the similar fashion, the process will be terminated after some finite steps, say  $s$ , such that  $t_{s-1} = 1$  and  $t_{s-1} < t_{s-2} \dots < t_0$ . At the culminating step, pool size becomes 1 indicating that the remaining individuals can be tested for COVID19 infections. Thus the problem reduces to finding the group sizes  $t_0, t_1, \dots, t_{s-2}, t_{s-1}$  so as to minimize the tests required for each level. Mathematically,

$$\begin{aligned} & \min \left( \left\lceil \frac{n}{t_0} \right\rceil + \left\lceil \frac{pt_0}{t_1} \right\rceil + \dots + \left\lceil \frac{pt_{s-2}}{t_{s-1}} \right\rceil \right) \\ & \text{subject to } t_{s-1} < t_{s-2} \dots < t_0 \text{ and } p < \left\lceil \frac{n}{t_0} \right\rceil \end{aligned} \quad (5)$$

Finding an exact minimum value to the problem stated in Eqn. (5) is difficult and therefore an approximate solution procedure is proposed utilizing binary trees.

Let us consider a binary tree with height  $h = \lceil \log_2 n \rceil$  and form pools of sizes that are power of two at each level say these are represented by  $t_i = 2^{s-1-i}, i = 0, 1, 2, \dots, s-1$ . Using this, the minimization problem given in Eqn. 4 reduces to following

$$\begin{aligned} & \min_s \left( \left\lceil \frac{n}{2^{s-1}} \right\rceil + 2p(s-1) \right) \\ & \text{s.t. } t_{s-1} < t_{s-2} \dots < t_0 \text{ and } s \leq h+1 - \lceil \log_2 p \rceil \end{aligned} \quad (6)$$

Owing to the fact that number of steps are finite in nature and are represented by natural numbers, the possible values of  $s$  lie in the set  $S = \{1, 2, \dots, h+1 - \lceil \log_2 p \rceil\}$ . Now, the remaining task is to find the optimal value of  $s \in S$  for which minima will occur. To do so,

$$\begin{aligned}
f(1) &= n \\
f(2) &= \left\lceil \frac{n}{2} \right\rceil + 2p \\
&\vdots \\
f(h - \lceil \log_2 p \rceil) &= \left\lceil \frac{n}{2^{h - \lceil \log_2 p \rceil - 1}} \right\rceil + 2p(h - \lceil \log_2 p \rceil - 1) \\
f(h + 1 - \lceil \log_2 p \rceil) &= \left\lceil \frac{n}{2^{h - \lceil \log_2 p \rceil}} \right\rceil + 2p(h - \lceil \log_2 p \rceil)
\end{aligned} \tag{7}$$

It can be easily observed that whenever  $n$  is a perfect power of 2 then  $h = \log_2 n$  is the height of binary tree and in such a case minima is attained at  $s = h - \lceil \log_2 p \rceil$ . Thus the minimum value is

$$\left\lceil \frac{n}{2^{h - \lceil \log_2 p \rceil - 1}} \right\rceil + 2p(h - \lceil \log_2 p \rceil - 1) = \left\lceil \frac{n}{2^{s-1}} \right\rceil + 2p(s - 1) = f(s) \tag{8}$$

and the corresponding pool sizes are  $\{t_0, t_1, \dots, t_{s-1}\} = \{2^{h - \lceil \log_2 p \rceil - 1}, 2^{h - \lceil \log_2 p \rceil}, \dots, 1\}$ . In other cases, when  $n$  is not a perfect power of 2, minimum value can be identified by comparing values of  $f(s) : \forall s = 1, 2, \dots, h + 1 - \lceil \log_2 p \rceil$ . Let us validate the proposed solution in view of an example:

Consider a city wherein the population is  $n = 128$  and percentage of corona cases are  $P = 4.68\%$ , which leads to total infected individuals  $p = \left\lceil \frac{128 \times 4.68}{100} \right\rceil = 5.99 \approx 6$  and height of binary tree is  $h = \lceil \log_2 n \rceil = \lceil \log_2 128 \rceil = 7$ , which provides the steps  $s = h - \lceil \log_2 p \rceil = 7 - 3 = 4$  and therefore the pool sizes at each level are  $\{t_0, t_1, t_2, t_3\} = \{2^3, 2^2, 2^1, 1\} = \{8, 4, 2, 1\}$ . This has led to the minimum number of tests (using Eqn. 7) required  $f(4) = 2^{3+1} + 2 \times 6 \times (7 - 3 - 1) = 52$ . Therefore, it is concluded that there is the requirement of at most 52 tests for the entire population of the city. As it is mentioned that the total number of tests to be done as obtained from the proposed solution are minimum then considering any other value of  $t_0$  will lead to more tests to be done in totality.

## 5. Results and Discussion

In this section, the efficiency of the solutions of the proposed pool tests are explored. The main intent is to compare these two solutions and try to see which tests could be used in which situation and also appreciate the effectiveness of them. For this purpose, the data for different countries has been taken and the minimum number of COVID19 tests using both the proposed strategies are obtained and then

compared. The results can be seen in the table I, where in  $n$  denotes total number of persons on which test is administered,  $p$  denotes individuals who tested positive, in other words, number of tests that resulted into positive and  $P$  be the percentage of COVID19 infected individuals. The results listed in the columns with heading SPT denote the minimum number of tests to be done in the worst case when applying the Single COVID19 pool test and GPT denotes the minimum number of tests to be done in the worst case when applying the general COVID19 pool test. A general trend that is visible is that for a small value of  $P$ , general COVID19 pool test gives the better solution while as the value of  $P$  becomes significant the Single COVID19 pool test gives better results.

In another experiment, the effect of varied  $P$  (in %) is observed on required minimum tests to be administered. For the same, the size of the population has made fixed to  $n = 6400$  in the experiment and the corresponding results are depicted in table II. It is clear from the table that the similar trend is observed here as well. In general, general pool test gives better results whenever the value of  $P$  is less than 7%. In contrast, single pool test provides better results whenever the value of  $P$  is less than or equal to 25% and more than or equal to 7%. Situations for which value of  $P$  are beyond 25%, it is suggested to perform COVID19 test on every individual to identify the infected individuals. This is due to the fact that such situations violate the underlying assumptions on which single and general pool test problems are formulated. If at all, the single or general pool test will be conducted in this situation, the total number of tests to be administered will always be equal to the size of the population and hence same as administering tests on all individuals. Hence, it can be concluded that the pool test is very effective whenever the percentage of COVID19 infected individuals is less.

Before moving on it is worth noting that unless the tests results for each pool at a level are evaluated, we cannot move to the next level of pool testing. Thus, there is a continuous trade off between the number of levels ( $i$ ) and the time constraint. In simple language, if time taken per test is too long then the number of levels has to be reduced. If time is not a constraint then the proposed solution using a binary tree is the best solution. However, if time is the major thrust point, then the proposed solution using the binary tree may not be appropriate in the sense that it adaptively calculates the optimal number of levels followed by the optimal pool sizes. To deal with this situation, another solution is proposed here using genetic algorithm (GA). This solution provides the freedom to choose the number of levels and thus enabling us to choose the best possible minima. GA is a heuristic algorithm for optimization. It has the ability of exploration and exploitation, so ability to give the best possible solution, though no guarantee of convergence [16].

To solve the integer optimization problem Eqn. (4), a real coded genetic algorithm presented in [17] is used and its stepwise procedure is illustrated in Fig. 6. It essentially consists of tournament selection

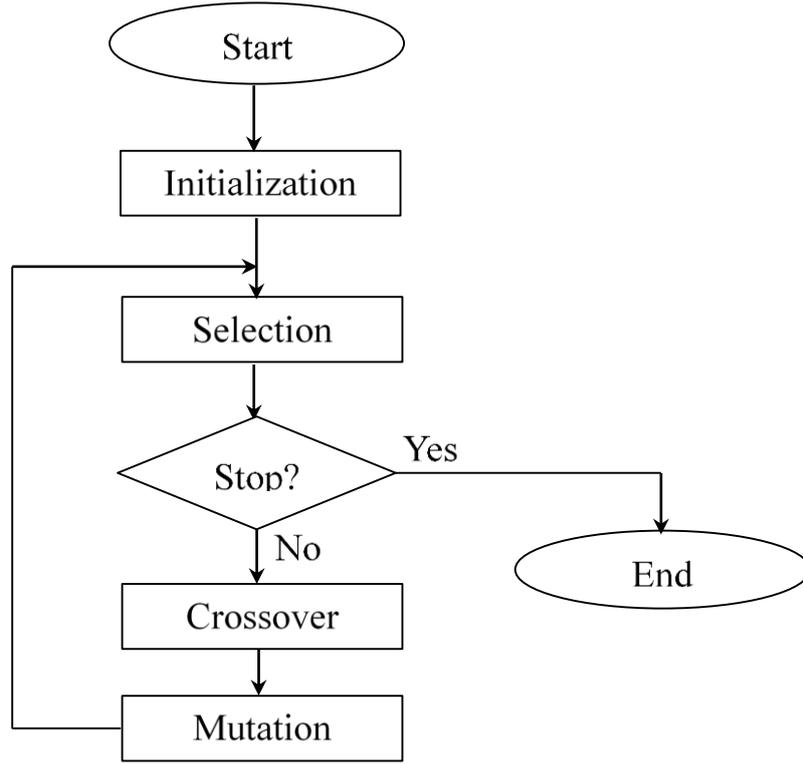


Fig. 6. Flow Chart of used real coded Genetic Algorithm.

operator, Laplace crossover and power mutation. In the experiments, selection size (2), probability of crossover (0.8) and mutation probability (0.01) is used. For handling the integer restrictions on variables, each one is randomly converted to the nearest lower or upper value, then after fitness function is computed to perform selection. This process is repeated as per Fig 6 until the optimized solution to Eqn. 4 is received. The minimization problem given in Eqn (4) can be written in the following in view of structure of real coded GA.

$$\min \left( \left\lceil \frac{n}{x_0} \right\rceil + \left\lceil \frac{Px_0}{x_1} \right\rceil + \dots + \left\lceil \frac{Px_{l-2}}{1} \right\rceil \right) \quad (9)$$

$$\text{subject to } x_{j-2} - x_{j-3} + 1 \leq 0 \forall j = 3, 4, \dots, l \quad (10)$$

Here  $l$  is the number of level, which need to be decided a-priori and the bounds are given by the following equation:

$$2 \leq x_j \leq \left( \frac{N}{P} - 1 \right) \text{ if } P | N \text{ else } 2 \leq x_j \leq \left\lceil \frac{N}{P} \right\rceil \text{ if } P \nmid N \quad \forall j = 0, 1, \dots, l-2 \quad (11)$$

The same example which was used in assessing the binary tree based solution to general pool test is considered for aforementioned GA based solution. The corresponding results are depicted in Table 3 where the effect of number of levels ( $l$ ) with respect to the value of  $p$  (in%), i.e., percentage of infected

individuals in a population increases is given. It can be observed that for some values  $T \geq N$  and so it is advised that in such cases the tests should be administered individually rather than following the General COVID19 pool test. From the table it can be seen that as the value of  $p$  increases the best minima is obtained for a lower value of  $l$ . Therefore, the proposed solution using GA is an efficient way for the situations where the freedom of choosing levels is with the administration/user.

## 6. Conclusion

In these difficult times, the scarce available resource of COVID19 testing kits needs to be used efficiently. In this paper, to address this issue, two strategies based on pool testing are proposed. These strategies are termed as single and general pool tests. Both the strategies are formulated mathematically and the minimization function to both is being proposed. For the prior problem, an exact solution is given and two approximate solutions using binary trees and genetic algorithm are given for the later. A bound on the percentage of infected individuals is also identified by which it would be easy to decide for the authorities whether to adopt pool testing strategies or not. In essence, the bounds and mathematical expressions proposed can be used to determine when to use single pool testing, when to use general pool testing and when to use none of these and administer the COVID19 test on each and every individual of the population. Further, both the pool testing strategies are formulated considering the worst case scenario and thus on an average pool testing will provide much better results within the bounds obtained. Finally, the detailed comparison between the proposed strategies has been done and it is concluded that for small percentage of COVID19 infection, general pool testing should be done whereas for a significant percentage of COVID19 infection single pool testing should be done.

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Table 1. Country wise comparisons of Single Pool Test (SPT) and General Pool Test (GPT).

<b>Continent</b>	<b>Country</b>	<b><i>n</i></b>	<b><i>p</i></b>	<b><i>P</i></b>	<b>SPT</b>	<b>GPT</b>
Africa	Ghana	68591	1042	1.52	16910	12564
	South Africa	114711	3158	2.57	38067	32434
	Senegal	5213	367	7.04	2772	2772
	Kenya	1387	270	19.47	1234	1234
Asia	Vietnam	163377	276	0.17	13432	5055
	Taiwan	53632	420	.78	9496	5878
	South Korea	563035	10674	1.9	155152	120582
	Bahran	90307	1873	2.07	26012	20629
	India	401586	12765	4.30	166643	126789
	Malaysia	108216	5389	4.98	48589	45861
	Israil	240303	13491	5.61	114040	110984
	Thiland	40897	2765	2.76	21285	21285
	Pakistan	104302	8418	8.07	59748	59748
	Japan	116725	10751	9.21	71162	72186
	Bangladesh	26604	2456	9.23	16236	16475
	Turkey	673980	86306	12.80	483578	509602
	Indonesia	43749	6575	15.03	34308	35025
	Euorpe	Italy	14000000	178972	1.28	3166304
Latvia		36668	727	1.98	10328	8108
Lithuania		66352	1326	2.00	18761	14755
Russia		2050000	42853	2.09	592829	470949
Slovakia		46734	1161	2.48	14755	12209
Slovenia		41802	1330	3.18	14947	13206
Estonia		40938	1528	3.73	15828	14286
Czech Republic		172123	6787	3.94	68360	62238
Island		43102	1771	4.11	17476	16014
Hungary		48057	1984	4.13	19532	17912
Poland		214236	9287	4.33	89283	82502
Norway		143255	7068	4.93	63991	60315
Finland		58727	3783	6.44	29814	29314
Denmark		96244	7384	7.67	53597	56335
Austria		182949	14710	8.04	104578	104578
Romania		98491	8746	8.88	59069	59607
Portugal		193614	20206	10.44	125156	129228
Luxembourg		33798	3550	10.50	21916	22650
Serbia		41812	6318	15.11	32892	33542
Netherland		171415	32655	19.05	151018	151018
United Kingdom	386044	120067	31.10	386044	386044	
North-America	El Salvador	12210	201	1.65	3135	2372
	Quba	28598	1035	3.62	10895	9785
	Canada	555551	34777	6.26	277996	277996
	Costa Rica	7826	660	8.43	4589	4597
	United States	4000000	759687	18.99	3519374	3519374
	Mexico	39431	8261	20.95	36238	36238
	Panama	20137	4467	22.18	19003	19003

	New Zeland	86305	1105	1.28	19535	13748
	Australia	431754	6612	1.53	106866	79631
South America	Paraguay	5486	206	3.75	2128	1922
	Uruguay	12826	592	4.62	5526	5156
	Argentina	34568	2930	8.48	20313	20362
	Chile	11827	10088	8.49	69873	11827
	Peru	148011	15628	10.56	96221	99515
	Bolivia	4135	564	13.64	3071	3196

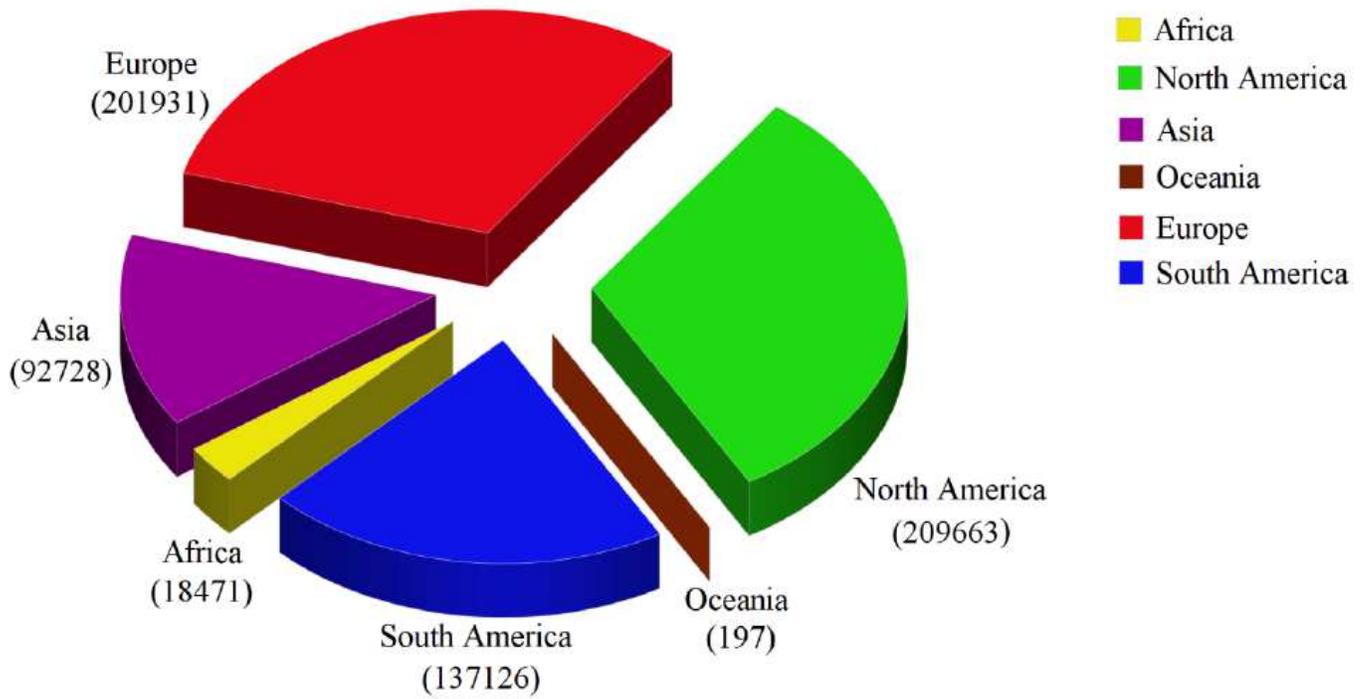
Table 2. Effect of varied values of  $P$ (in %) observed on minimum tests required for a fixed population using Single Pool Test (SPT) and General Pool Test (GPT)

S.No.	$P$	$P$	SPT	GPT
1	0.0001	1	160	26
2	0.001	1	160	26
3	0.01	1	160	26
4	0.05	4	320	87
5	0.1	7	424	135
6	0.5	32	906	484
7	1	64	1280	840
8	2	128	1811	1424
9	3	192	2219	1936
10	4	256	2560	2336
11	5	320	2880	2720
12	6	384	3136	3104
13	7	448	3392	3392
14	8	512	3648	3648
15	9	576	3862	3904
16	10	640	4054	4160
17	12	768	4438	4672
18	13	832	4630	4864
19	14	896	4822	4992
20	15	960	5014	5120
21	16	1024	5206	5248
22	17	1088	5376	5376
23	18	1152	5504	5504
24	19	1216	5632	5632
25	20	1280	5760	5760
26	21	1344	5888	5888
27	22	1408	6016	6016
28	23	1472	6144	6144
29	24	1536	6272	6272
30	25	1600	6400	6400
31	26	1664	6400	6400
32	27	1728	6400	6400

Table 3. Effect of varied values of  $p$  (in %) observed on minimum tests required for a fixed population using General COVID19 pool test (GPT) based on Genetic Algorithm ( $G$  represents the corresponding pool sizes at different levels,  $l$  represents the pre-chosen number of levels, and  $T$  is the corresponding number of pool tests to be performed).

$p$ (in %)	$P$	$l = 2$		$l = 3$		$l = 4$		$l = 5$	
		G	T	G	T	G	T	G	T
0.0001	1	(82,1)	161	(320,19,1)	56	(804,81,9,1)	36	(1067,216,36,6,1)	26
0.001	1	(82,1)	161	(320,19,1)	56	(804,81,9,1)	36	(1067,216,36,6,1)	26
0.01	1	(41,1)	161	(320,19,1)	56	(804,81,9,1)	36	(1067,216,36,6,1)	26
0.05	4	(30,1)	320	(143,13,1)	141	(271,42,7,1)	102	(380,90,19,4,1)	88
0.1	7	(14,1)	424	(97,10,1)	204	(160,28,5,1)	155	(256,64,16,4,1)	137
0.5	32	(10,1)	906	(33,6,1)	562	(56,14,4,1)	483	(74,25,9,3,1)	463
1	64	(7,1)	1280	(22,5,1)	893	(32,10,3,1)	811	(43,18,7,3,1)	809
2	128	(6,1)	1811	(14,4,1)	1418	(20,8,3,1)	1366	(22,9,4,2,1)	1404
3	192	(5,1)	2219	(10,3,1)	1856	(13,5,2,1)	1857	(16,8,4,2,1)	1936
4	256	(5,1)	2560	(9,3,1)	2248	(11,5,2,1)	2298	(13,7,4,2,1)	2441
5	320	(4,1)	2880	(8,3,1)	2614	(9,4,2,1)	2712	(12,7,4,2,1)	2923
6	384	(4,1)	3136	(7,3,1)	2963	(8,4,2,1)	3104	(9,5,3,2,1)	3388
7	448	(4,1)	3392	(5,2,1)	3296	(8,4,2,1)	3488	(8,5,3,2,1)	3832
8	512	(3,1)	3648	(5,2,1)	3584	(7,4,2,1)	3859	(8,5,3,2,1)	4266
9	576	(3,1)	3862	(5,2,1)	3872	(7,4,2,1)	4227	(7,5,3,2,1)	4698
10	640	(3,1)	4054	(5,2,1)	4160	(6,3,2,1)	4587	(7,5,3,2,1)	5118
12	768	(3,1)	4438	(4,2,1)	4672	(5,3,2,1)	5248	(6,4,3,2,1)	5931
13	832	(3,1)	4630	(4,2,1)	4928	(5,3,2,1)	5579	(7,7,6,4,1)	7294
14	896	(3,1)	4822	(4,2,1)	5184	(5,3,2,1)	5910	(7,7,6,4,1)	7785
15	960	(3,1)	5014	(4,2,1)	5440	(5,3,2,1)	6240	(5,4,4,3,1)	7600
16	1024	(2,1)	5206	(4,2,1)	5696	(4,3,2,1)	6550	(5,4,4,3,1)	8022
17	1088	(2,1)	5376	(3,2,1)	5942	(4,3,2,1)	6859	(5,5,4,2,1)	8080
18	1152	(2,1)	5504	(3,2,1)	6166	(4,3,2,1)	7168	(5,5,4,2,1)	8480
19	1216	(2,1)	5632	(3,2,1)	6390	(4,3,2,1)	7478	(5,5,4,2,1)	8880
20	1280	(2,1)	5760	(3,2,1)	6614	(4,3,2,1)	7787	(4,4,3,2,1)	9067
21	1344	(2,1)	5888	(3,2,1)	6838	(4,3,2,1)	8096	(4,4,3,2,1)	9440
22	1408	(2,1)	6016	(3,2,1)	7062	(4,3,2,1)	8406	(4,4,3,2,1)	9814
23	1472	(2,1)	6144	(3,2,1)	7286	(4,3,2,1)	8715	(4,4,3,2,1)	10187
24	1536	(2,1)	6272	(3,2,1)	7510	(4,3,2,1)	9024	(4,4,3,2,1)	10560
25	1600	(2,1)	6400	(3,2,1)	7734	(3,3,2,1)	9334	(3,3,3,2,1)	10934

# Figures



**Figure 1**

Region wise total death toll due to COVID 19 as on July 31, 2020 [15].

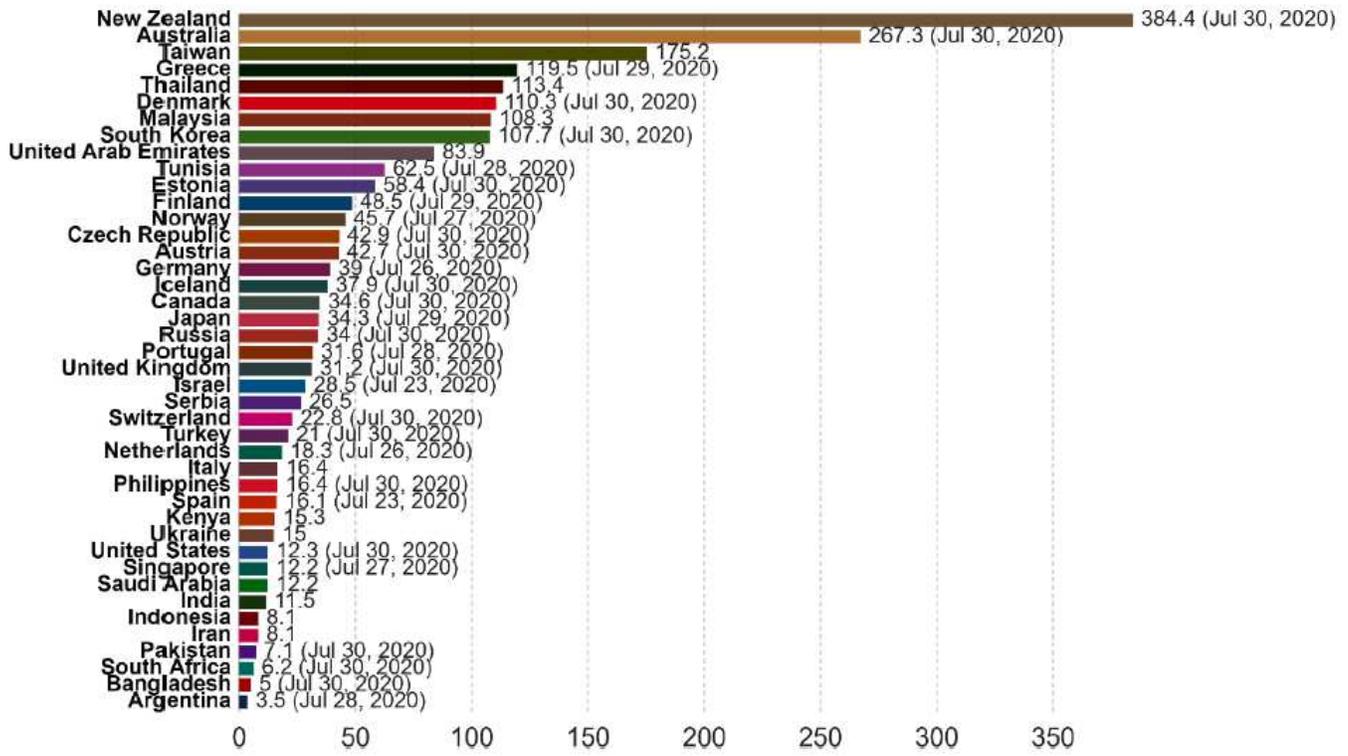


Figure 2

Total average number of COVID-19 test per confirmed case as on July 31, 2020 [15].

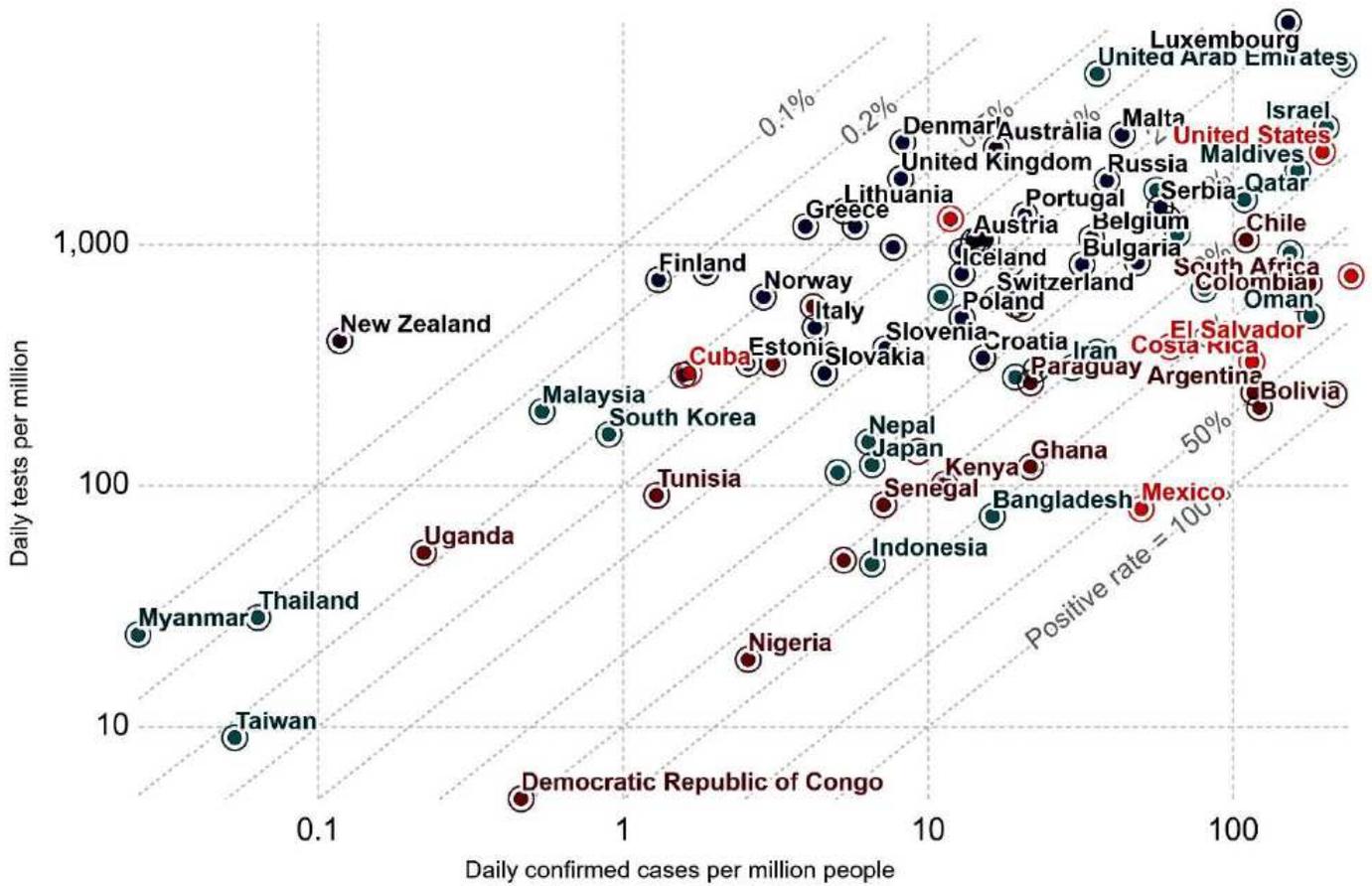


Figure 3

Daily tests against daily new confirmed cases per million as on July 31, 2020 [15].

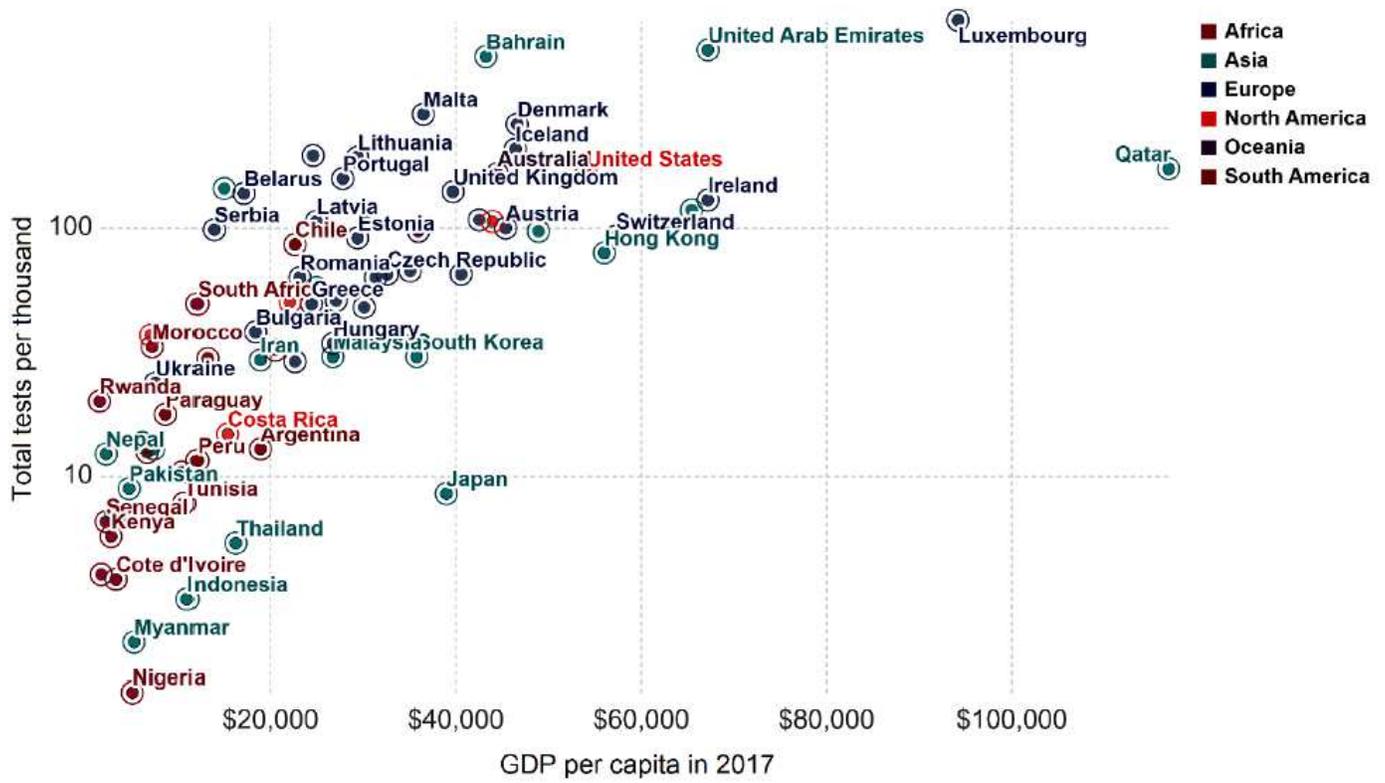


Figure 4

Total COVID19 tests per thousand people vs. GDP per capita as on July 31, 2020 [15].

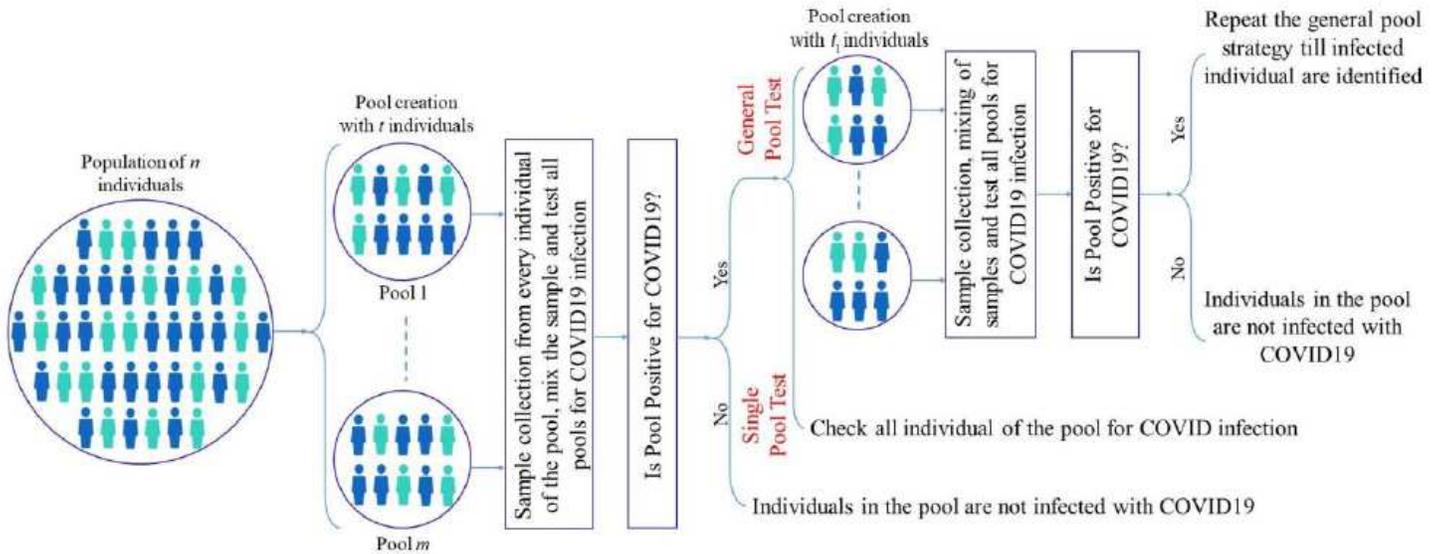
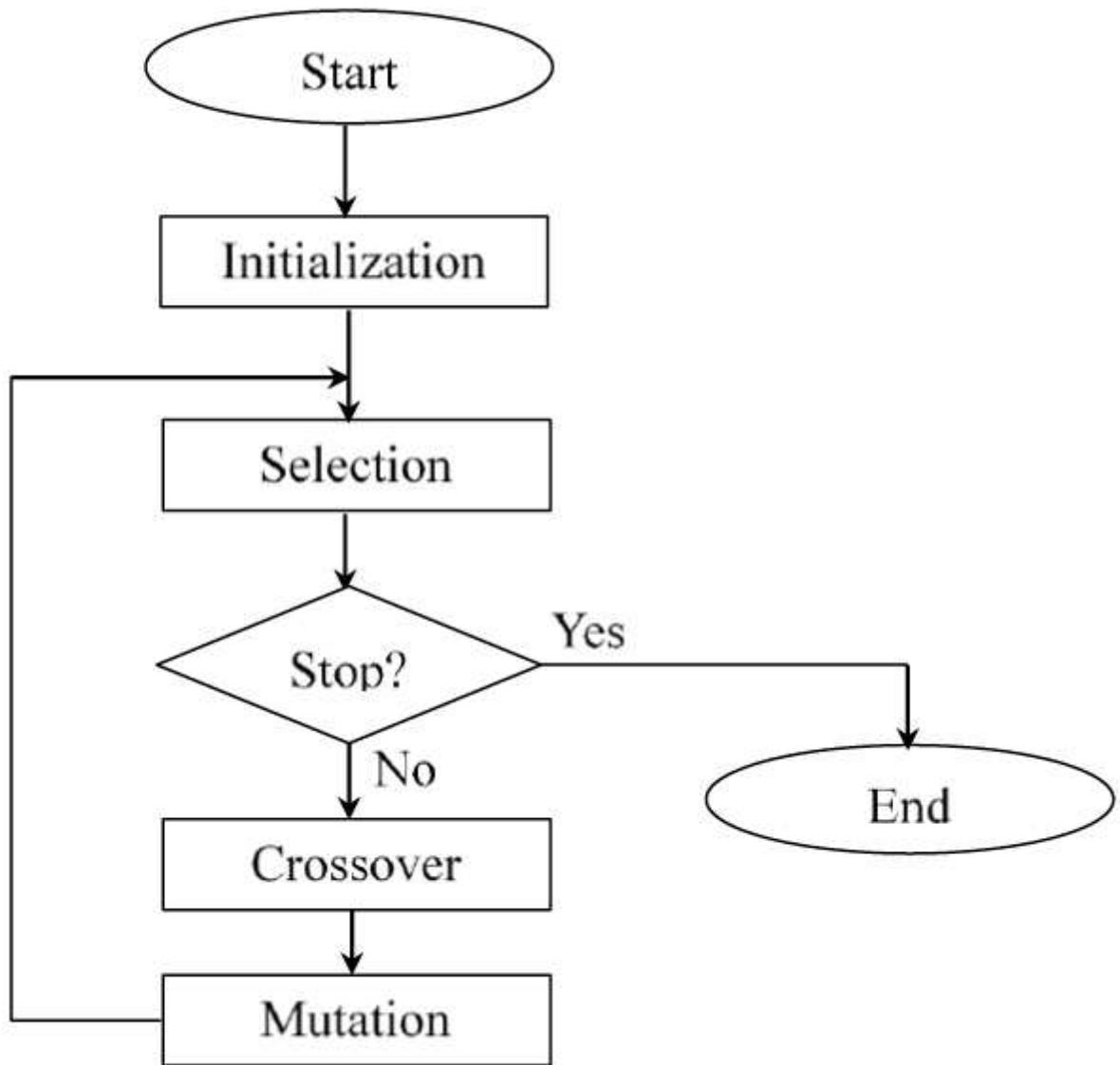


Figure 5

Basic structure of COVID19 Pool Test problem.



**Figure 6**

Flow Chart of used real coded Genetic Algorithm.