Robustness of Majorana Zero-energy State in a Spin-orbit Coupled Semiconductor/superconductor Heterostructure

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Robustness of Majorana zero-energy state in a spin-orbit coupled semiconductor/superconductor heterostructure

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Abstract

Based on the principle of linearized stability proposed by Lyapounov, we investigate the robustness of Majorana zero energy state (MZES), which plays an important role in topological quantum computation. Our study is different from previous works that usually explore the stability of MZES by the numerical test of some special perturbations, our treatment is suitable for arbitrary perturbations. Since our method follows the stability theory of differential equation, the results we obtained are reliable. As an example, we demonstrate it by the stability of MZES in the spin-orbit coupled semiconductor/ superconductor junction, the analytical and numerical results indicate that the MZES is unstable in this system.

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I. Introduction

In 1996, Shor presented a Fault-tolerant quantum computation^[1] to overcome the difficulty caused by the decoherence of quantum bits. However, it is difficult to achieve due to its small threshold value for those errors in quantum logical gates, so some treatments of topological quantum computation^[2,3] were proposed to avoid the problem of decoherence, in which a decoherence free subspace is exploited to process the quantum information. The decoherence free subspace of degenerate states can exist in some two-dimensional system, such as chiral p-wave superconductor^[4], the $\nu = \frac{5}{2}$ fractional quantum Hall system^[5], and some heterostructures, i.e., s-wave superconductor/topological insulator^[6], spin-orbit coupled semiconductor/ superconductor system^[7], where the Majorana bound states--the elementary excitations obey the non-Abelian statistics can be used to carry quantum information, it is robust against the noise.

Topological stability of MZES had been investigated extensively^[8], especially in superconductor/topological system^[9], s-wave superconductivity on the surface of a spin-orbit coupled semiconductor^[10,11], and superconductor/ferromagnet junction on a strong topological insulator surface^[12]. However, S. Das Sarma et al. pointed out that the stability of MZES can be destroyed by the quantum tunneling process of Majorana Fermion between vortex^[13-15], in which

the ground state degeneracy is split. Cheng et al. calculated the splitting of the zero-energy ground state in a two-dimensional p-wave superconductor topological insulator/ superconductor structure and superconductor/semiconductor hybrid topological system^[13-14]. An experiment in InAs nanowire segment with epitaxial aluminum had observed this splitting^[16]. These works tell us that the robustness of MZES is worthwhile to be further explored.

Since the bulk gap can protect the ground state degeneracy from perturbation and keep it's stability, people usually concentrate on the calculation of gap of quasiparticle excitation in the vortex^[10,11]. There exist nontopological Caroli-de Gennes-Matricon (CdGM) states localized inside the core of vortex on the MZES, in order to suppress these states Rakhmanov et al. ^[17-19] proposed a topological insulator/ superconductor heterostructure with a hole, in the hole the MZES is very robust because of the large energy gap from the single-particle nontopological excitations. However, in some other systems, i.e., the spin-orbit coupled semiconductor/ superconductor junction, the minigap between the Majorana modes and the band of single particle nontopological excitation is very small, the MZES may be easily disturbed by the thermal fluctuations. Despite of the argument that the MZES is still robust by the authors, we should reexamine this by another way.

As we know, people often discuss the topological stability of the

system by investigating the topological characters of energy bands or the phase transition, they seldom directly discuss the robustness of wavefunction, since the quantum information is carried by the wavefunction, i.e., the MZES in the system, and the wavefunction is directly influenced by the decoherence, so we had better directly explore the stability of the wavefunction instead of the minigap. It should be noted that the MZES is a solution of Bogoliubov-de Gennes equation, while the latter is a differential equation. Mathematically, it is concerning with the stability analysis on the solution of a differential equation, whose theory had been well established by Lyapounov^[20,21]. In this manuscript, we will study the stability of MZES by the method of linear stability analysis developed by Lyapounov^[20,21]. Under this stability analysis, the MZES in some systems may be robust, while in other systems may be unstable^[22], in this manuscript we will investigate the stability in the spin-orbit coupled semiconductor/ superconductor junction.

II. Theoretical formalism

Consider a system with Majorana bound states, such as the s-wave superconductor/topological insulator or the semiconductor with strong spin orbital coupling in proximity to superconductors, the MZES can occur on the interfaces of the above systems, which can be described by the following Bogoliubov de-Gennes Hamiltonian

$$\hat{H} = \begin{pmatrix} \hat{h} & \Delta_{sc} \\ \Delta_{sc}^+ & -\hat{h} \end{pmatrix}, \quad (1)$$

where the kinetic energy, potential, spin orbital coupling as well as Zeeman term are all included in the single particle Hamiltonian $\hat{h}$, and Δ_{SC} represents the superconductor pair potential. The requirement of particle-hole symmetry for the Majorana bound states demands that the wavefunction must adopt the Nambu notation $\psi = ((\psi_{\uparrow}, \psi_{\downarrow}), (\psi_{\downarrow}^{\dagger}, -\psi_{\uparrow}^{\dagger}))$. According to the Bogoliubov de-Gennes eigen-equation $\hat{H}\psi = E\psi$, the MZES satisfies

$$\hat{H}\psi_{\text{MZES}} = 0, \quad (2)$$

which corresponds to the zero energy $E = 0$. Mathematically, ψ_{MZES} is just an equilibrium solution for the above differential equation, we can study its robustness by the well established theory of stability given by Lyapounov^[20,21].

Let the solution of Eq. (2) to be written in the form

$$\psi = \psi_{\text{MZES}} + \delta\psi, \quad (3)$$

where $\delta\psi$ is the perturbation from equilibrium. Substituting it into the time dependent Bogoliubov de-Gennes equation $i\hbar \frac{\partial\psi}{\partial t} = \hat{H}\psi$, we have the time dependent equation for the perturbation

$$i\hbar \frac{\partial\delta\psi}{\partial t} = \hat{H}\delta\psi, \quad (4)$$

which is a linear equation about $\delta\psi$, its special solution has the form like

$$\delta\psi(\vec{r}, t) = \delta\psi_0 e^{\lambda_n t + i\vec{k}_n \cdot \vec{r}}, \quad (5)$$

where $\delta\psi_0 = ((\delta\psi_{\uparrow 0}, \delta\psi_{\downarrow 0}), (\delta\psi_{\downarrow 0}^{\dagger}, -\delta\psi_{\uparrow 0}^{\dagger}))$ is the amplitude of the perturbation, λ_n and $\vec{k}_n$ are the frequency and wavevector,

respectively. Inserting Eq. (5) into Eq. (4), we have

$$i\hbar\lambda_n(\delta\psi_0 e^{\lambda_n t + i\vec{k}_n \cdot \vec{r}}) = \hat{H}(\delta\psi_0 e^{\lambda_n t + i\vec{k}_n \cdot \vec{r}}), \quad (6)$$

it is a linear equations group about $\delta\psi_{\uparrow 0}, \delta\psi_{\downarrow 0}, \delta\psi_{\downarrow 0}^\dagger, \delta\psi_{\uparrow 0}^\dagger$. To obtain its nonzero solutions, one should require the determinant of its coefficient to be zero, which is just the characteristic equation for λ_n at certain $\vec{k}_n$.

According to the principle of linearized stability proposed by Lyapounov^[20,21], if all the eigen-values λ_n have negative real parts, then ψ_{MZES} is a stable equilibrium solution. If some eigen-values λ_n have positive real parts, then ψ_{MZES} is unstable. Therefore we can judge the robustness of MZES ψ_{MZES} by the principle of linearized stability instead of the minigap calculated by prior works. We will demonstrate this by an example of spin-orbit coupled semiconductor/superconductor junction in the next section.

III. Stability analysis on spin-orbit coupled Semiconductor/Superconductor junction

It is shown that the interaction between s-wave superconductor and spin-orbit coupled semiconductor by the proximity effect can induce the superconductivity in the topological surface states, so we consider a s-wave superconductor/spin-orbit coupled semiconductor/magnetic insulator junction as shown in Fig.1, it can be described by the following Hamiltonian

$$\hat{H} = \begin{pmatrix} H_0 & \Delta_{SC}(r) \\ \Delta_{SC}^\dagger(r) & -\sigma_y H_0^\dagger \sigma_y \end{pmatrix}, \quad (7)$$

where

$$H_0 = \begin{pmatrix} \frac{p^2}{2m^*} + V_z - \mu & i\alpha p_x + \alpha p_y \\ -i\alpha p_x + \alpha p_y & \frac{p^2}{2m^*} - V_z - \mu \end{pmatrix}, \quad (8)$$

in which V_z denotes the perpendicular Zeeman field induced by the proximity contact with the magnetic insulator, μ is the chemical potential, and α is the strength of Rashba spin orbital coupling in the semiconductor. As pointed out by Mao et al.^[11], the MZES can exist at the interface of superconductor and spin-orbit coupled semiconductor.

Fig.1 Schematic structure of a Superconductor/spin-orbit coupled Semiconductor/Magnetic Insulator junction.

To demonstrate the stability of MZES in the s-wave superconductor/spin-orbit coupled semiconductor/magnetic insulator junction, we should follow the same procedure as described in Sec. II. According to Eq. (6), the amplitude $\delta\psi_0 = ((\delta\psi_{\uparrow 0}, \delta\psi_{\downarrow 0}), (\delta\psi_{\downarrow 0}^\dagger, -\delta\psi_{\uparrow 0}^\dagger))$ satisfies

$$i\hbar\lambda_n \begin{pmatrix} \delta\psi_{\uparrow 0} \\ \delta\psi_{\downarrow 0} \\ \delta\psi_{\downarrow 0}^\dagger \\ -\delta\psi_{\uparrow 0}^\dagger \end{pmatrix} e^{\lambda_n t + i\vec{k}_n \cdot \vec{r}} = \begin{pmatrix} \frac{p^2}{2m^*} + V_z - \mu & i\alpha p_x + \alpha p_y & \Delta_{SC}(r) & 0 \\ -i\alpha p_x + \alpha p_y & \frac{p^2}{2m^*} - V_z - \mu & 0 & \Delta_{SC}(r) \\ \Delta_{SC}^*(r) & 0 & -\frac{p^2}{2m^*} + V_z + \mu & i\alpha p_x + \alpha p_y \\ 0 & \Delta_{SC}^*(r) & -i\alpha p_x + \alpha p_y & -\frac{p^2}{2m^*} - V_z + \mu \end{pmatrix} \begin{pmatrix} \delta\psi_{\uparrow 0} \\ \delta\psi_{\downarrow 0} \\ \delta\psi_{\downarrow 0}^\dagger \\ -\delta\psi_{\uparrow 0}^\dagger \end{pmatrix} e^{\lambda_n t + i\vec{k}_n \cdot \vec{r}}, \quad (9)$$

which is a linear equations group about $\delta\psi_{\uparrow 0}$, $\delta\psi_{\downarrow 0}$, $\delta\psi_{\downarrow 0}^\dagger$, $\delta\psi_{\uparrow 0}^\dagger$ as

$$\begin{cases} \left(\frac{\hbar^2 k_x^2 + \hbar^2 k_y^2}{2m} + V_z - \mu - i\hbar\lambda_n \right) \delta\psi_{\uparrow 0} + (i\alpha\hbar k_x + \alpha\hbar k_y) \delta\psi_{\downarrow 0} + \Delta_{SC} \delta\psi_{\downarrow 0}^\dagger = 0 \\ (-i\alpha\hbar k_x + \alpha\hbar k_y) \delta\psi_{\uparrow 0} + \left(\frac{\hbar^2 k_x^2 + \hbar^2 k_y^2}{2m} - V_z - \mu - i\hbar\lambda_n \right) \delta\psi_{\downarrow 0} - \Delta_{SC} \delta\psi_{\uparrow 0}^\dagger = 0 \\ \Delta_{SC}^* \delta\psi_{\uparrow 0} + \left(\frac{-\hbar^2 k_x^2 - \hbar^2 k_y^2}{2m} + V_z - \mu - i\hbar\lambda_n \right) \delta\psi_{\downarrow 0}^\dagger - (i\alpha\hbar k_x + \alpha\hbar k_y) \delta\psi_{\uparrow 0}^\dagger = 0 \\ \Delta_{SC}^* \delta\psi_{\downarrow 0} + (-i\alpha\hbar k_x + \alpha\hbar k_y) \delta\psi_{\uparrow 0}^\dagger + \left(\frac{\hbar^2 k_x^2 + \hbar^2 k_y^2}{2m} + V_z - \mu + i\hbar\lambda_n \right) \delta\psi_{\downarrow 0}^\dagger = 0 \end{cases}, \quad (10)$$

it is a homogeneous equations group, if we want obtain a nonzero

solution about $\delta\psi_{\uparrow 0}$, $\delta\psi_{\downarrow 0}$, $\delta\psi_{\downarrow 0}^\dagger$, $\delta\psi_{\uparrow 0}^\dagger$, the determinant of its

coefficient should be zero:

$$\begin{vmatrix} \frac{\hbar^2 k_x^2 + \hbar^2 k_y^2}{2m} + V_z - \mu - i\hbar\lambda_n & i\alpha\hbar k_x + \alpha\hbar k_y & \Delta_{SC} & 0 \\ -i\alpha\hbar k_x + \alpha\hbar k_y & \frac{\hbar^2 k_x^2 + \hbar^2 k_y^2}{2m} - V_z - \mu - i\hbar\lambda_n & 0 & -\Delta_{SC} \\ \Delta_{SC}^* & 0 & \frac{-\hbar^2 k_x^2 - \hbar^2 k_y^2}{2m} + V_z - \mu - i\hbar\lambda_n & -i\alpha\hbar k_x - \alpha\hbar k_y \\ 0 & \Delta_{SC}^* & -i\alpha\hbar k_x + \alpha\hbar k_y & \frac{\hbar^2 k_x^2 + \hbar^2 k_y^2}{2m} + V_z - \mu + i\hbar\lambda_n \end{vmatrix} = 0. \quad (11)$$

Then we can obtain the characteristic equation for the eigen-value λ_n , it is

$$a\lambda_n^4 + b\lambda_n^3 + c\lambda_n^2 + d\lambda_n + e = 0, \quad (12)$$

where the coefficient $a = -\hbar^4$, $b = 2i\mu\hbar^3$, $c = \hbar^2 \left[\left(\frac{\hbar^2 k_x^2 + \hbar^2 k_y^2}{2m} - V_z - \mu \right) \left(\frac{-\hbar^2 k_x^2 - \hbar^2 k_y^2}{2m} + V_z - \mu \right) - \left(\frac{\hbar^2 k_x^2 + \hbar^2 k_y^2}{2m} + V_z - \mu \right)^2 - 2\Delta_{SC}\Delta_{SC}^* - 2(\hbar^2\alpha^2 k_x^2 + \hbar^2\alpha^2 k_y^2) \right]$, $d = i\hbar \left[2\mu \left(\frac{\hbar^2 k_x^2 + \hbar^2 k_y^2}{2m} + V_z - \mu \right)^2 + 2\mu\Delta_{SC}\Delta_{SC}^* + 2\mu(\hbar^2\alpha^2 k_x^2 + \hbar^2\alpha^2 k_y^2) \right]$ and $e = \left(\frac{\hbar^2 k_x^2 + \hbar^2 k_y^2}{2m} + V_z - \mu \right)^2 \left(\frac{\hbar^2 k_x^2 + \hbar^2 k_y^2}{2m} - V_z - \mu \right) \left(\frac{-\hbar^2 k_x^2 - \hbar^2 k_y^2}{2m} + V_z - \mu \right) + \Delta_{SC}\Delta_{SC}^* \left(\frac{\hbar^2 k_x^2 + \hbar^2 k_y^2}{2m} - V_z - \mu \right) \left(\frac{-\hbar^2 k_x^2 - \hbar^2 k_y^2}{2m} + 2V_z \right) + (\hbar^2\alpha^2 k_x^2 + \hbar^2\alpha^2 k_y^2) \left(\frac{\hbar^2 k_x^2 + \hbar^2 k_y^2}{2m} + V_z - \mu \right) \left(\frac{\hbar^2 k_x^2 + \hbar^2 k_y^2}{2m} - 2V_z \right) + 2\Delta_{SC}\Delta_{SC}^* (\hbar^2\alpha^2 k_x^2 + \hbar^2\alpha^2 k_y^2) - (\hbar^2\alpha^2 k_x^2 + \hbar^2\alpha^2 k_y^2)^2 - \Delta_{SC}^2 \Delta_{SC}^{*2}$. There are four roots for the characteristic equation (12), they are

$$\lambda_{n1,2} = \sqrt{A^2} \pm \sqrt{-A^2 - \frac{1}{2}P + 2\sqrt{A^4 + \frac{1}{2}PA^2 + \frac{1}{16}(P^2 - 4R)}}, \quad (13)$$

and

$$\lambda_{n3,4} = -\sqrt{A^2} \pm \sqrt{-A^2 - \frac{1}{2}P - 2\sqrt{A^4 + \frac{1}{2}PA^2 + \frac{1}{16}(P^2 - 4R)}} \quad (14)$$

where

$$A^2 = -\frac{1}{6}P + \sqrt[3]{-\frac{73PR-27Q^2-2P^3}{3456} + \sqrt{\left(\frac{73PR-27Q^2-2P^3}{3456}\right)^2 - \left(\frac{P^2+12R}{144}\right)^3}} + \sqrt[3]{-\frac{73PR-27Q^2-2P^3}{3456} - \sqrt{\left(\frac{73PR-27Q^2-2P^3}{3456}\right)^2 - \left(\frac{P^2+12R}{144}\right)^3}} \quad (15)$$

$$\text{with } P = \frac{8ac-3b^2}{8a^2}, \quad Q = \frac{b^3-4abc+8a^2d}{8a^3} \quad \text{and} \quad R = \frac{16ab^2c-3b^4-64a^2bd+256a^3e}{256a^4}.$$

In our calculation, we adopt the Fermi energy E_f as 0.84eV, the pair Potential $\Delta(r)$ is zero at the vortex core $r = 0$, approaches a uniform value Δ_0 in the bulk and drops to zero at the boundary of vortex^[11]. As MZES is realized in the center of vortex, so we only choose $\Delta_{sc}(0) = 0eV$ in our calculation. The numerical results for the real parts of four eigenvalues as a function of V_z with different chemical potentials are shown in Fig.2-Fig.5, respectively, here we have chosen the strength of Rashba spin orbital coupling $\alpha = 1.0$. We can see that the real parts of $\lambda_{n1,2}$ are positive, while the real parts of $\lambda_{n3,4}$ have negative eigen-values. According to the principle of linearized stability^[20,21], the positive real

parts of $\lambda_{n1,2}$ will lead to the instability of MZES in the s-wave

Fig.2 The real part of characteristic value λ_{n1} vs Zeeman field at different chemical potentials.

Fig.3 The real part of characteristic value λ_{n2} vs Zeeman field at different chemical Potentials.

Fig.4 The real part of characteristic value λ_{n3} vs Zeeman field at different chemical potentials.

Fig.5 The real part of characteristic value λ_{n4} vs Zeeman field at different chemical Potentials.

superconductor/spin-orbit coupled semiconductor/magnetic insulator junction. Since the Zeeman field V_z can be manipulated by the magnetic insulator, then V_z can be used as the control parameter. If all the real parts of the four characteristic values can be tuned to negative by manipulating

the V_z , we can employ V_z to keep the stability of the system. But one can see in the above, it is impossible, the real parts of the first and second characteristic values are positive and will violate the stability of the system. The things doesn't change when we chose other Fermi energies, the real part of the first eigen-value is always positive, even the real parts of the other eigenvalues are not negative everywhere, their figures are similar to Fig.2-5, so we don't show them here. Moreover, In Fig.6-Fig.9, we plot the real parts of four eigen-values vs Zeeman field at different strengths of spin orbital coupling, which shows that the real parts of λ_{n1} and λ_{n2} are all positive, so the MZES in this case is still unstable. Finally, we have to conclude that the MZES in this system is unstable. As pointed out by Mao et al^[11], the MZES can exist when the Zeeman field $V_z > \sqrt{\mu^2 + \Delta_0^2}$, while disappears when $V_z < \sqrt{\mu^2 + \Delta_0^2}$. Mathematically, the disappearance of MZES is a kind of unstable. According to the above study, the MZES is also unstable even $V_z > \sqrt{\mu^2 + \Delta_0^2}$ where MZES exist, so we can say that the MZES is unstable whatever $V_z > \sqrt{\mu^2 + \Delta_0^2}$ or $V_z < \sqrt{\mu^2 + \Delta_0^2}$. We plot the whole region for the Zeeman field V_z in our figures. In fact, we can understand the instability of MZES in this system from the point of energy gap. As pointed out by Rakhmanov et al. ^[17], the zero-energy state in the topological insulator/superconductor heterostructure with a hole can be separated by a large energy gap when the density of states vanishes at the Dirac point,

which assures the robustness of Majorana modes in their system. However, in Mao's system^[11], there only exist a very small minigap between MZES and nontopological excitations, it is not robust against the thermal fluctuation, so we conclude that the MZS in Mao's system is unstable.

Fig.6 The real part of characteristic value λ_{n1} vs Zeeman field at different spin orbital coupling strengths.

Fig.7 The real part of characteristic value λ_{n2} vs Zeeman field at different spin orbital coupling strengths.

Fig.8 The real part of characteristic value λ_{n3} vs Zeeman field at different spin orbital coupling strengths.

Fig.9 The real part of characteristic value λ_{n4} vs Zeeman field at different spin orbital coupling strengths.

IV. Summary and discussion

Although our conclusion in the above is different from the result

presented by Mao et al.^[11], it is conceded with the points given by Cheng et al.^[12-14]. Mao's work only investigated several special perturbations during their numerical test, such as the spin-independent Gaussian impurity, a magnetic impurity potential $U(r)\sigma_z$, and a random impurity potentials, the verification of robustness for MZES in their work inevitably depends on the special perturbations adopted by them. Maybe MZES is robustness to some special perturbations, but it is unstable to other perturbations. For example, the stability of MZES can be destroyed by choosing the perturbation as the intervortex tunneling which induces the splitting of ground state degeneracy as pointed by Cheng et al.. We can't only discuss the stability of MZES by this kind of numerical test, because some special perturbations must be adopted in the model. It is necessary to study the stability of MZES by other methods, such as the stability theory of differential equation as we shown above, it is a more general way and suitable for arbitrary perturbations.

For simplicity we only choose the pair potential as coordinate independent in our numerical calculation, in fact they are coordinate dependent in realistic systems. The coordinate dependence of the superconductivity order parameter is crucial in the study of the MZES, but as shown by the numerical results given by Mao et al.^[11], the superconductivity order parameter can be assumed as coordinate independent away from the edge, it is a kind of

approximation. To better control the ferromagnet layer, it's magnetization can also be chosen as coordinate independent, which is similar to the uniform magnetic field adopted by Akzyanov et al.^[17]. How to consider the coordinate dependence is an important task to do.

It should be emphasized that we only keep the first order term of perturbation and neglect higher order terms in Eq. (4) and (9), Lyapounov's theorem- the principle of linearized stability tells us that it is enough to look at the linearized problem to determine the stability. Incidentally, if some of the eigen-values have zero real parts, which corresponds to the critical case, then one must keep the nonlinear terms in order to determine stability or instability of the equilibrium solution.

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References:

1. P. Shor, in Proceedings of the symposium on the foundations of computer science, IEEE press, los Alamitos, CA, 1996, e-print quant-ph: 9605011.

2. A. Yu Kitaev, Fault-tolerant quantum computation by anyons, Ann. Phys. 303, 2 (2003).
3. A. Stem and N. H. Lindner, Topological quantum computation- From basic concepts to first experiment, Science, 339, 1179 (2013).
4. D. A. Ivanov, Phys. Rev. Lett. 86, 268 (2001); A. Stern, F. von Oppen, E. Mariann, Phys. Rev. B70, 205338 (2004); S. Das Sarma, C. Mayakans, S. Tewari, Phys. Rev. B73, 220502 (2006); Y. Gurarie, L. Radzihovsky and A. V. Andreev, Phys. Rev. Lett. 94, 230403 (2005).
5. G. Moore and N. Read, Nucl. Phys. B360, 362 (1991); S. Das Sarma, M. Freedman and C. Nayak, Phys. Rev. Lett. 94, 166802 (2005); A. Stern, B. I. Halperin, Phys. Rev. Lett. 96, 016802 (2006); P. Bonderson, A. Kitaev and K. Shtengel, Phys. Rev. Lett. 96, 016803 (2006).
6. L. Fu and C. L. Kane, Superconducting proximity effect and Majorana fermions at the surface of a topological insulator, Phys. Rev. Lett. 100, 096407 (2008); R. Jackiw and P. Rossi, Nucl. Phys. B190, 681 (1981).
7. J. D. Sau, R. M. Lutchyn, S. Tewari and S. Das Sarma, Generic new platform for topological quantum computation using semiconductor heterostructures, Phys. Rev. Lett. 104, 040502 (2010); A. Chrestin, T. Matsuyama and U. Merkt, Phys. Rev. B55, 8457 (1997); F. Giazotto et al. J. Supercond, 17, 317 (2004).

8. L. Mazza, M. Rizzi, M. D. Lukin and J. I. Cirac, Phys. Rev. B88, 205142 (2013); B. Bauer, R. M. Lutchyn, M. B. Hastings and M. Troyer, Phys. Rev. B87, 014503 (2013).
9. T. Fukui and T. Fujiwara, Topological stability of Majorana zero modes superconductor topological insulator system, J. Phys. Soc. Jpn. 79, 033701 (2010).
10. J. D. Sau, R. M. Lutchyn, S. Tewari, S. Das Sarma, Robustness of Majorana fermions in proximity-induced superconductors, Phys. Rev. B 82, 094522 (2010).
11. L. Mao and C. Zhang, Robustness of Majorana modes and minigaps in a spin-orbit-coupled semiconductor-superconductor heterostructure, Phys. Rev. B82, 174506 (2010).
12. X. J. Luo, Y. P. He, T. F. J. Poon, X. Liu and X. J. Liu, Braiding Majorana modes in spin space: from worldline to worldribbon, arXiv: 1803.02173, con-mat.
13. M. Cheng, R. M. Lutchyn, V. Galitski and S. Das Sarma, Splitting of Majorana modes due to intervortex tunneling in a p_x+ip_y superconductor, Phys. Rev. Lett. 103, 107001 (2009).
14. M. Cheng, R. M. Lutchyn, V. Galitski and S. Das Sarma, Tunneling of anionic Majorana excitations in topological superconductors, Phys. Rev. B82, 094504 (2010).
15. S. Das Sarma, J. D. Sau and T. D. Stanescu, Splitting of the zero-bias

- conductance peak as smoking gun evidence for the existence of the Majorana mode in a superconductor-semiconductor nanowire, *Phys. Rev. B* **86**, 220506 (R) (2012).
16. S. M. Albrecht, A. P. Higginbotham, M. Madsen and F. Kuemmeth, Exponential protection of zero modes in Majorana islands, *Nature* **531**, 206 (2016).
 17. A. L. Rakhmanov, A. V. Rozhkov and Franco Nori, Majorana fermions in pinned vortices, *Phys. Rev. B* **84**, 075141 (2011).
 18. R. S. Akzyanov, A. V. Rozhkov, A. L. Rakhmanov, and F. Nori, Tunneling spectrum of a pinned vortex with a robust Majorana state. *Phys. Rev. B* **89**, 085409 (2014).
 19. R. S. Akzyanov, A. L. Rakhmanov, A. V. Rozhkov, and F. Nori, Tunable Majorana Fermion from Landau Quantization in 2D Topological Superconductors, *Phys. Rev. B* **94**, 125428 (2016).
 20. R. Bellman, *Stability theory of Differential equations*, McGraw-Hill, New York 1953.
 21. D. H. Sattinger, *Topics in stability and Bifurcation theory*, Springer, Berlin, 1973.
 22. Z. C. Wang, Stability analysis on Majorana zero-energy state in the superconductor/topological insulator/ferromagnet insulator junction, *Journal of superconductivity and novel magnetism* **34**, 1 (2021).