Cosmic Gas Thermodynamics at $z = 1090$ and $z = 0$

Martin R. Johnson (✉ oakman1-cosmo@usa.net)

Trinapco, Inc. <https://orcid.org/0000-0002-5096-3206>

Short Report

Keywords: dark energy, intergalactic medium, gas thermodynamics

Posted Date: September 7th, 2021

DOI: <https://doi.org/10.21203/rs.3.rs-883925/v1>

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Cosmic Gas Thermodynamics at $z = 1090$ and $z = 0$

By Martin Reid Johnson

<https://orcid.org/0000-0002-5096-3206>

When it comes to general public awareness, the term “dark energy” is second only to Einstein’s $E = Mc^2$ as the most recognized concept in physics, and first on the list of unsolved problems. This paper purports to solve that problem, with methods more commonly used in power plant design: gas thermodynamics. At one time, the baryonic matter (protons and helium nuclei) in our Universe was a homogenous neutral gas, expanding as always, and where we begin. We will derive a simple linear relation between rate of expansion and distance between two points, and carry this relation to the present day as a volume-proportional scalar field of baryonic internal energy (U_i). Adjustment to some interpretations of the Friedman equation may be needed.

One such interpretation, the minimal flat-universe Λ CDM Model, is given by (1):

$$H^2(a) = H_0^2 [\Omega_{rad} a^{-4} + \Omega_m a^{-3} + \Omega_{cdm} a^{-3} + \Omega_\Lambda a^{-3(1+w)}] \quad (1)$$

Where $H(a)$ is the Hubble parameter at scale factor $a = 1/(1+z)$, z is the cosmic redshift $(\lambda_e - \lambda_0)/\lambda_e$ of an emitted photon of known λ_0 , w is the ratio of internal energy to overall energy density, and H_0 is the present-day Hubble constant, $2.2 \times 10^{-18} \text{ sec}^{-1}$. The Ω values or *density parameters* add up to one and relate their energy density to the present-day *critical density* $\epsilon_{crit} = 3c^2(H_0)^2/8\pi G$:

$$1 = \sum \Omega_x = \sum \frac{\epsilon_x}{\epsilon_{crit}} \quad (2)$$

In eq. (1), $\Omega_m = 0.0486$ is baryonic matter, $\Omega_{cdm} = 0.0259$ is cold dark matter, and Ω_{rad} is relativistic energy. These are treated as attractive terms that pull the universe together with gravity. The remaining term, Ω_Λ , is repulsive. In this paper, (1) gives us Ω_m and Ω_{cdm} .

In all the cosmos, density reduction (“expansion”) must be adiabatic. In cosmology, adiabatic expansion is treated as isentropic. We use that here but it’s not really true. In a classic setting, there are two kinds of adiabatic gas expansion: reversible and free. Reversible expansion is isentropic by definition. When a gas expands reversibly, U_i decreases, the gas performs work, and the temperature and pressure drop. When a gas expands freely, internal energy is not lost and only the pressure drops. The temperature stays the same:

$$P\partial V = P(\partial V_S + \partial V_T) \quad (3)$$

Both happen cosmically, but with differences. In the first, *cosmic isentropic expansion*, the internal energy lost isn’t equal to the energy stored by gravity. The excess becomes vectored

kinetic energy. The second, *cosmic free expansion*, must also be included. There's an additional entropic component of expansion, from the second law. This gives more volume to accommodate the same amount of gas, just like classic free expansion. Unlike classic free expansion, internal energy *is* lost, to gravity.

First, we select a time: $z = 1089$ - 1090 , the time of recombination, when baryonic matter was almost all neutral gas and acoustic oscillation was minimal. Baryons were then present as a mixture of 75% monatomic hydrogen (H_1) : 25% helium (He) by weight, or about 92 mole % H_1 : 8 mole % He. This gives a mean molecular weight $\mathcal{K} = 1.2475 \times 10^{-3} \text{ Kg/mol}$.¹ The baryon density ρ_{1090} was $(\Omega_m \rho_{crit} / a^3) = (0.0486)(8.62 \times 10^{-27})(1090)^3 = 5.43 \times 10^{-19} \text{ Kg/m}^3$, or 2.6×10^8 atoms per cubic meter. The background radiation had just decoupled so the baryon temperature was 2971 K (CMB = 2.7255 K)($1/a = 1090$).

Consider a finite sphere around a single atom of H_1 , of radius about Earth size (1 au = 6.3781×10^6 m), at 2971 K, which at ρ_{1090} and $\mathcal{K}$ has mass $M = 455 \text{ Kg}$, or 2.02×10^8 atoms/m³. This sphere is still in thermal equilibrium, a major but necessary departure from reality. Nonequilibrium thermodynamics must be set aside so that the underlying transfer of conserved energy is more clearly described. The sphere's gravitational potential energy (U) is:²

$$U = \frac{-3GM^2}{5r} \quad (4)$$

Where G is the gravitational constant ($6.67408 \times 10^{-11} \text{ m}^3\text{kg}^{-1}\text{sec}^{-2}$). Cold dark matter, with $\Omega_{cdm} = 0.259$, has $\approx 84\%$ of all cold mass in the Universe, and doesn't act as a gas. Its only influence is gravitational.³ This is included by dividing the baryon mass by 0.16:

$$U = \frac{-3G(\frac{M}{0.16})^2}{5r} \quad (5)$$

The ideal gas law is:

$$PV = nRT = \frac{MRT}{\mathcal{K}} \quad (6)$$

Where R is the universal gas constant ($=8.31446 \text{ kg-m}^2\text{sec}^{-2}\text{mole}^{-1}\text{K}^{-1}$). The volume of a sphere is:

$$V = \frac{4}{3}\pi r^3 \quad (7)$$

When (6) and (7) are combined we get the *internal pressure* (P_i):

¹ Isotopes and heavier elements can be neglected. Diatomic hydrogen is neglected in this treatment, but may have a significant presence. Photon energy, although quite high, is also not included.

² The subscripts U_i , U_2 , and U_r refer to gravitational potential energy. The term U_i is used to denote the internal kinetic energy of a gas or plasma.

³ The weight M of gas, or *thermodynamic mass*, remains unchanged, and is used to find E .

$$P_1 = \frac{nRT}{V} = \frac{3nRT}{4\pi r_1^3} = \frac{3MRT}{4\pi r_1^3} \quad (8)$$

Entering our values for M , T , and r , we obtain $P_1 = 1.07 \times 10^{-11}$ Pa. We will also suppose that the sphere isn't getting any bigger over time. It is but for now we'll say it isn't. We increase the sphere's radius by $\sqrt[3]{1.01}$, giving a volume increase of one percent.⁴ Work is performed against gravity:

$$U_r = U_1 - U_2 = \frac{-3G(\frac{M}{0.16})^2}{5} \left(\frac{1}{r_1} - \frac{1}{r_2} \right) \quad (9)$$

Where U_1 and U_2 are the gravitational potential energies at radii r_1 and r_2 respectively. Entering the values for M , r_1 and r_2 we find that $U_r = 8.53 \times 10^{-11}$ J. The internal energy loss ($-E$) is, however, much greater than U_r :

$$W = -E = \left(\frac{3}{2}\right) P_1 V_1 \left(\left(\frac{V_2}{V_1}\right)^{\frac{2}{3}} - 1 \right) \quad (10)$$

Where W has the classic meaning of work performed by the gas, P_1 is the internal pressure before expansion, and V_1 and V_2 are the before and after volumes of the sphere respectively. V and P can be calculated from (7) and (8). Entering these into (10) gives $E = 1.16 \times 10^8$ J. This is 10^{20} times as much energy released as absorbed. The excess (E_k) is now outward, *radial kinetic energy*:

$$E_k = E + U_r = E + (U_1 - U_2) = \left(\frac{3}{2}\right) P_1 V_1 \left(\left(\frac{V_2}{V_1}\right)^{\frac{2}{3}} - 1 \right) - \frac{3G(\frac{M}{0.16})^2}{5} \left(\frac{1}{r_1} - \frac{1}{r_2} \right) \quad (11)$$

Gravity loss is negligible and $E_k = E \approx 10^8$ J. The internal pressure drops to a new value, P_2 :

$$P_2 = P_1 \left(\frac{V_2}{V_1} \right)^{-\frac{5}{3}} \quad (12)$$

Eq. (12) gives $P_2 = 1.06 \times 10^{-11}$ Pa. Dividing E_k by V_2 gives the *expansion pressure* (P_E):

$$P_E = \frac{E_{k2} - E_{k1}}{V_2} \quad (13)$$

Our sphere was static to start so $E_{k1} = 0$. Our expanded sphere has $P_E = 1.06 \times 10^{-13}$ Pa, or 1% of P_2 . It's important to emphasize that P_E does not add to P_2 ,⁵ but is instead a vector quantity which results in radial increase only. Each atom is moving in a straight line away from the center, like a bunch of tiny rockets blasting away from their despoiled planet. It helps to ignore

⁴ We define the *increment* as $\frac{r_2 - r_1}{r_1} = \frac{\Delta r_i}{r}$. This is different from the *step* (Δr_s), used for numeric integration.

⁵ Since one of our assumptions is thermal equilibrium, P_2 remains a state function.

U_i to properly visualize this. Expansion pressure already existed in the sphere since the Universe has been expanding all along.

The linear rate of expansion, or *radial velocity* (v_s) of the sphere is:

$$v_s = \sqrt{\frac{2E_k}{M}} \quad (14)$$

which is addressed below. The temperature drop is given as:

$$T_2 = T_1 \left(\frac{P_2}{P_1} \right)^{2/5} \quad (15)$$

The temperature drops from 2971 to 2951 K or 0.7%. This can be compared to the CMB-derived temperature: $(2.7255)(1090/\sqrt[3]{1.01}) = 2961$ K, or 0.3% for $r = 1 \rightarrow \sqrt[3]{1.01}$ au. The gas is cooling faster than the photons.

We can get a value for E_{k1} and v_s at an instant in time ∂t by modeling (11), using a small increment $\Delta r_i/r = 10^{-10}$ and increasing r independently.

First we define the *gravity ratio* (X):

$$X = \frac{U_2 - U_1}{E} \quad (16)$$

As we increase r_l , the mass rises and v_s falls until the *endpoint* ($r_e = 1.409 \times 10^{17}$ m), or *adiabatic radius*, is reached, where $X = 1$ and $\frac{\partial E_k}{\partial r} = 0$. The sphere at r_e is the *adiabatic sphere*, and its surface is the *adiabatic surface*.

The *cutoff ratio* (X') is defined as:

$$X' = \frac{-U_r}{E_k} = \frac{U_2 - U_1}{E_k} \leq 0.00001 \quad (17)$$

Below the *cutoff radius* (r_c), gravity can be neglected and (11) simplifies to (10). The choice of $X' < 10^{-5}$ is best seen graphically. Figure 1 shows a semilog map of v_s vs. r from 10^{12} to 10^{16} m.⁶ Below r_c ($= 4 \times 10^{14}$ m) v_s is constant, giving the same (E/M) :⁷

$$\frac{E}{M} = \frac{\partial E}{\partial M} = \frac{\partial V}{\partial M} \frac{\partial E}{\partial V} = \left(\frac{RT}{\mathcal{K}P} \right) \frac{\partial E}{\partial V} = \frac{RT}{\mathcal{K}} \left(\frac{\partial E}{P \partial V} \right) = \frac{RT}{\mathcal{K}} \quad (18)$$

This gives the *initial radial velocity* (v_i):

⁶ Both r_c and r_e are independent of $\Delta r_i/r$ over a wide range.

⁷ For isentropic adiabatic expansion, $\partial E = P \partial V$.

$$v_i = \sqrt{\frac{2E}{M}} = \sqrt{\frac{2RT}{\mathcal{M}}} = \sqrt{\frac{2(8.31446)(2971)}{0.0012475}} = 6293 \text{ m/sec} \quad (19)$$

Above r_c , gravity takes its toll, and v_s drops, reaching zero at r_e . The radial velocity (v) of the adiabatic sphere is the sum of the contained shells:

$$v = (v_i) \left(\frac{r_c}{r_e} \right) + \sum_{r_c}^{r_e} v_{s'} \frac{\Delta r_s}{r_e} \rightarrow (v_i) \left(\frac{r_c}{r_e} \right) + \frac{1}{r_e} \int_{r_c}^{r_e} v_{s'} \partial r \quad (20)$$

Where Δr_s is the shell thickness or *step*, $v_{s'} = v_i \left(\frac{v_s}{v_{s_0}} \right)$, and v_{s_0} is the constant $v_s < r_c$. For all $r < r_c$, $v = 24.3 \text{ m/sec}$. That leaves the remaining 99.6% of v to be found. Integration of (11) is problematic so we resort to a map (Figure 2) whose cumulative value at $r/r_e = 1$ is 4956 m/sec, or $0.79v_i$.⁸

For any two atoms separated by a distance r , their recession rate v_r is:

$$v_r = 0.79 \frac{r}{r_e} \sqrt{\frac{2RT}{\mathcal{M}}} \quad (21)$$

And the comoving Hubble value is:

$$H = \frac{v_r}{r} = \frac{0.79}{r_e} \sqrt{\frac{2RT}{\mathcal{M}}} \quad (22)$$

Which at $z = 1090$ gives $H = 3.83 \times 10^{-14}/\text{sec}$.

While expansion pressure P_E may be universal, it's very much a local phenomenon, dependent on local density, temperature, massive objects, etc. Furthermore, for all z post-nucleosynthesis, most Universal volume had a positive P_E . When $z > 1090$ or $z < 100$, U_i was replenished by irradiation. By $z < 8$, the Universe was fully reionized, resulting in steady-state U_i in regions of low ρ . These ionized, low-density regions dominate Universal volume today, and define its expansion as a volume-proportional scalar energy field U_i of protons and ionized helium nuclei.⁹ For example, we can define a region as having $\rho_{\text{crit}}/10 = 8.62 \times 10^{-28} \text{ Kg/m}^3$ at 10^4 K . For this region, $v_i = 11,545.5 \text{ m/sec}$, $r_c = 1 \times 10^{19} \text{ m}$, $r_e = 5.95 \times 10^{21} \text{ m}$, $v/v_i = 0.790$, and $H = 1.5 \times 10^{-18}/\text{sec}$, or $0.70H_0$. Raising the temperature to 10^5 K gives $v_i = 36,510 \text{ m/sec}$ and $H = 0.69H_0$. Raising the density to ρ_{crit} gives $H = 0.70H_0$.

⁸ In Figure 2, there are three sets of 100 points, each of exponentially increasing Δr_s and concentrated near r_e . The integral was calculated by the plotting program, Dplot, giving a third-order correlation of 0.999989 and $v/v_i = 0.78746$ at $r/r_e = 1$. The v/v_i value floats around 0.79 depending on the input.

⁹ It's worth mentioning that the repulsion of any two naked protons in the void follows an inverse-square law at distances many times that of neutral gas, creating dynamic pressure at very low densities.

To summarize: *For large gaseous regions of low and homogenous density, every proton in the region is surrounded by a finite sphere of radius r_e , with the internal kinetic energy of the protons in that sphere exerting outward radial force on those protons.*

For inhomogenous regions, the adiabatic surface around a proton isn't spherical. Of interest are the regions near the plane of the spiral arms of galaxies. The endpoint in these regions varies, can exceed the interstellar distance, and the regional composition of baryonic matter defies a simple expression of r_e .

Entropic volume gain, or cosmic free expansion, occurs simultaneously with cosmic reversible expansion and is more difficult to treat. Internal energy values of the 92/8 plasma are needed for calculation at $z = 0$. We can, however, get an expression for v_i with added entropy, by looking at the increase in $P\partial V$. Free expansion of the sphere $r < r_c$ has no energy cost and occurs simultaneously with reversible expansion. From (3):

$$P\partial V' = P(\partial V_S + \partial V_T) = P(4\pi r^2 \partial r + 4\pi r^2 \partial r) = 2P\partial V \quad (23)$$

And from (19):

$$\frac{E}{M} = \frac{\partial E}{\partial M} = \frac{\partial V}{\partial M} \frac{\partial E}{\partial V'} = \left(\frac{RT}{\mathcal{K}P}\right) \frac{\partial E}{2\partial V} = \frac{RT}{\mathcal{K}} \left(\frac{\partial E}{2P\partial V}\right) = \frac{RT}{2\mathcal{K}} \quad (24)$$

So with entropic expansion added:

$$v_i = \sqrt{\frac{2E}{M}} = \sqrt{\frac{RT}{\mathcal{K}}} \quad (25)$$

This suggests a version of (21):

$$v_r = K \frac{r}{r_e} \sqrt{\frac{RT}{\mathcal{K}}} \quad (26)$$

Where $K \approx 0.7 - 0.8$ is a constant to be found. We can set $K = 0.79$ as before, while keeping the same endpoint value. This gives $H = 0.49H_0$, about half. We know the endpoint will shrink from entropic gravity loss; this would appear to be around 50% in order to give $H = H_0$. A more precise determination of both K and r_e is in order.

In summary, we have found a result consistent with what is seen through the telescope. The recession of galaxies is mostly caused by expanding hot intergalactic plasma. If there is some other form of dark energy, the conclusions of this paper should be taken into account.

The Author declares no competing interest.

Figures

Figure 1

Uncorrected sphere radial velocity vs. radius at $z = 1090$ $T = 2971 \text{ K}$, $d = 5.43 \times 10^{-19} \text{ Kg/increment } dr/r = 10^{-10}$

FIGURE 2.

Corrected sphere radial velocity (v_s') vs. r/r_e at $z = 1090$

$T = 2971 \text{ K}$, $d = 5.43 \times 10^{-19} \text{ Kg/m}^3$

$r_e = 1.409 \times 10^{17} \text{ m}$

Figure 2

Corrected sphere radial velocity (v_s') vs. r/r_e at $z = 1090$ $T = 2971 \text{ K}$, $d = 5.43 \times 10^{-19} \text{ Kg/m}^3$ $r_e = 1.409 \times 10^{17} \text{ m}$