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Abstract

Massive MIMO (multiple input multiple output) systems are best suitable for mmWave communications improving throughput and spectral efficiency in 5G. Beamforming is a wireless technique adopted by massive MIMO and used in 4G & 5G to increase the directivity and energy efficiency by focusing the signal in specific direction supporting only single user transmission with one data stream. Precoding is a generalized form of beamforming to support multi-stream transmission in multi-antenna wireless communication, such that combination of analog and digital precoding forms the hybrid precoding which shows good performance with less complexity. Successive Interference Cancellation (SIC) is a technique in which optimization of different antenna arrays will be done one by one such that while optimizing the capacity of specific array contribution of earlier optimized array is removed from the total capacity and precoder of that specific optimizing array will be computed. In this paper we have designed the SIC based hybrid precoding using sub-connected and fully-connected structures for multi-user (MU) case and compared them with the optimal precoding for mmWave massive MIMO systems in a 3D scenario, where both azimuth and elevation angles are taken into account in the channel. The proposed algorithms are simulated in the MATLAB and compared their performance with different parameters and shown that SIC-based scheme is near-optimal for multi-user case.

Index Terms

Hybrid precoding, Fully-connected, sub-connected, massive MIMO, multi-user, 3D beamforming.

I. INTRODUCTION

Millimeter waves (mmWaves) are EM waves typically defined to lie within the frequency range of 30 to 300 GHz and corresponding wavelength range of 10 mm to 1 mm respectively. MIMO uses multiple antennas and multiple signal paths to achieves higher data rates. MIMO systems are well suited for mmWave wireless communications where large antennas can be placed in small area due to tiny wavelengths, hence provides high array gains supporting spatial multiplexing, beamforming or antenna diversity. Standard MIMO uses 2 to 4 antennas while massive MIMO uses high number of antennas (more than 64 antennas).

Beamforming is a wireless technology which sends the signal in a particular required direction by forming beam in that direction, which increases signal quality and it is one of the key enabling technology for 5G. There are 2 types of beamforming ie., digital and analog beamforming. In digital beamforming each antenna is equipped with one dedicated RF chain which results into high cost and high energy consumption but gives good performance. Analog beamforming uses low cost phase shifters to send the same signal from multiple antennas but with different phases with less number of RF chains resulting into low cost but suffers from performance. Precoding supports multi-stream transmission, and combination of analog and digital precoding forms hybrid precoding. In mmWave communications there are more number of antennas at the BS and mmWave massive MIMO uses hybrid precoding to overcome signal attenuation, where Digital precoder controls the amplitude while analog precoder adjusts the phase.

A. Hybrid precoding architectures

The 2 typical architectures of the hybrid precoding ie., sub-connected and fully-connected, are shown in Figure 1.

(a) Sub-connected structure:

- Each RF chain connected to only subset of antennas.
- Requires less number of phase shifters(NM) without performance loss, hence more energy-efficient.

(b) Fully-connected structure:

- Each RF chain drives all antennas via PSs, which leads to energy-intensive but near optimal.
- Requires large number of phase shifters(N^2M), which leads to high energy consumption and hardware complexity.

Details about the HBF architecture, achievable uplink sum rate, HBF design for Multi-user (MU) mmWave MIMO systems can be found in [1], where as the paper [2] describes the two stage hybrid precoding for downlink MU MIMO system i.e. designs HP in 2 stages, such that in first stage analog precoder and analog combiners are jointly designed to maximize the

Fig. 1: (a)sub connected structure and (b)fully connected structure

desired signal power of each user, where resulting interference among users will be neglected, and in second stage the digital precoder at the BS is designed to manage the MU interference. Here the authors used HBF at the Tx side but at the Rx side only ABF used (DBF not used) because as they need cheaper hardware with low power consumption, and simulated this two stage HP in 3D scenario. The detailed information on HBF in massive MIMO can be found in [3], [4], where the authors made survey on several other related sources and discussed about their design, architecture, working mechanism etc.

The papers [5], [6], [7] describes the about the SIC based hybrid precoding for downlink mmWave massive MIMO systems using sub connected structure at the transmitter side with single user in 2D scenario, such that analog and digital precoding are considered at BS only. Here capacity of each sub array connected to single RF chain is optimized separately, as it becomes complicated by considering jointly i.e. precoding is considered separately for each sub array, where each sub-array rate is separately optimized using SVD [8]. To optimize the capacity of next sub array, SIC is used to eliminate/remove the contribution of the optimized array from total capacity. As it uses SVD and matrix inversion, so it shows high complexity. Hence in [9] the complexity of this SIC based scheme for sub connected is reduced by using power iteration algorithm whose convergence rate is increased with aitken's acceleration method (without complicated SVD and matrix inversion). In [10] described the SIC based baseband block diagonalization method for mmWave MU massive MIMO system by using fully connected architecture at both BS and user sides, such that hybrid precoding is considered at both Tx and Rx sides.

The papers [11] and [12] describes the design of hybrid precoding and combining for mmWave MU MIMO systems. [13] and [14] describes the design of hybrid beamforming in multi-cell scenario without using SIC method where each cell has one BS and some users, and compared them in 3GPP and NYUSIM channel models, where NYUSIM and 3GPP are 2 popular channel models for 5G [15], [16], such that both are stochastic channel models. [17] presents the multi-layer precoding which is a case of multi-cell scenario in which it decouples the precoding matrix of each BS as a multiplication of 3 precoding matrices which are called as layers, such that first layer manages inter cell interference, second layer is responsible for desired signal BF and third layer cancels the intra-cell MU interference. Other than SVD, the GMD based hybrid precoding can be found in the paper [18], in which the authors used OMP algorithm to find the analog precoder, while digital precoder is calculated using GMD [19].

In this paper an attempt is made to design the SIC-based hybrid precoding scheme and optimal precoding for multi-user case by using sub-connected and fully-connected structures separately in 3D scenario and compared among them in a mmWave massive MIMO system, and shown that proposed SIC based scheme is near optimal. Here for the case of sub-connected structure, capacity of each antenna array connected to one RF chain is optimized separately as it becomes complicated by considering jointly [5], [9], in other words the precoding is considered separately for each antenna array. To optimize the capacity of next antennna array, Successive Interference Cancellation (SIC) [6] is used to eliminate the contribution of the optimized array from total capacity. Where as in case of fully-connected structure, capacity of the array connected to all RF chains is optimized simultaneously [10].

The rest of the paper is organized as follows. Section II describes the considered system model and channel model. The proposed schemes for multi-user are specified in Section III and their capacity performance is shown in Section IV. Finally, conclusions of the work are given in Section V.

Notation: Lower case and upper case letters denotes vectors or matrices. $(\cdot)^{-1}$, $(\cdot)^T$, $(\cdot)^H$ and $|\cdot|$ denote inverse, transpose, hermitian(conjugate transpose) and determinant of a matrix respectively. $|\cdot|_1$, $|\cdot|_2$, and $|\cdot|_F$ represents 1-norm, 2-norm and frobenius norm of a vector respectively, I_N denotes the $N \times N$ identity matrix. If M is a matrix, then $M(i,j)$, $M(:,k)$ and $M(k,:)$ represents the (i,j) -th element, k -th column vector and k -th row vector of matrix M respectively, while $M(1:N,:)$ and $M(:,1:N)$

denotes the first N rows and first N columns of matrix M .

II. SYSTEM DESCRIPTION

Fig. 2: Schematic illustration of considered hybrid precoding using (a) sub connected architecture, (b) fully connected architecture

A. Channel model

The mmwave channel matrix of u^{th} user $H_u \in \mathbb{C}^{Nr \times Nt}$ is given as,

$$H_u = \gamma \sum_{l=1}^L \alpha_l \Lambda_r(\phi_l^r, \theta_l^r) \Lambda_t(\phi_l^t, \theta_l^t) f_r(\phi_l^r, \theta_l^r) f_t^H(\phi_l^t, \theta_l^t) \quad (1)$$

where $\gamma = \sqrt{\frac{NtNr}{L}}$ and $\alpha_l \in \mathbb{C}$ denotes l^{th} path gain.

ϕ_l^t, ϕ_l^r are azimuth AoDs, AoAs while θ_l^t, θ_l^r are elevation AoDs, AoAs respectively.

$\Lambda_t(\phi_l^t, \theta_l^t), \Lambda_r(\phi_l^r, \theta_l^r)$ represents Tx, Rx antenna array gain at specific AoD, AoA (for simplicity gains are set to 1). The $f_t(\phi_l^t, \theta_l^t), f_r(\phi_l^r, \theta_l^r)$ are the Tx, Rx antenna array response vectors respectively.

For 3D precoding scheme, Uniform Planar Array (UPA) antenna with $W=W_1 \times W_2$ elements are considered.

$$f_{UPA}(\phi, \theta) = \frac{1}{\sqrt{W}} \left[1, \dots, e^{j \frac{2\pi}{\lambda} d(x \sin(\phi) \sin(\theta) + y \cos(\theta))}, \dots, e^{j \frac{2\pi}{\lambda} d((W_1-1) \sin(\phi) \sin(\theta) + (W_2-1) \cos(\theta))} \right]^T \quad (2)$$

where, λ is the wavelength of the signal, W is the total number of antennas and d is antenna spacing such that $0 \leq x \leq (W_1-1)$ and $0 \leq y \leq (W_2-1)$. The angles ϕ and θ are assumed to have uniform distribution over $[0, 2\pi]$ and $[-\pi/2, \pi/2]$ respectively.

The effective channel matrix H_{eff} between transmitter and receiver is given by,

$$H_{eff} = \begin{bmatrix} H_1 \\ H_2 \\ \vdots \\ H_U \end{bmatrix}_{U N_r \times N_t} \quad (3)$$

B. System model

The schematic diagram of hybrid precoding structure for downlink mmWave massive MU-MIMO system for sub-connected and fully-connected architecture are shown in Fig. 2a and Fig. 2b. In both the cases BS has N_t number of antennas with N_{t_RF} number of RF chains to simultaneously transmit N_{t_RF} independent data streams to U users. At receiver side each user is equipped N_r antennas with N_{r_RF} RF chains to support N_{r_RF} data streams.

1) Sub-connected structure: In sub-connected structure ie., in Fig. 2a, at the Tx side there are N_{t_RF} sub-arrays with each array having $M = N_t/N_{t_RF}$ number of antennas connected to one RF chain. The number of RF chains are chosen in such a way $N_{t_RF} = U * N_{r_RF}$. In BS, data streams are first processed in digital precoder D_{bs} , and pass through N_{t_RF} RF chains and then precoded by analog precoding vector a_{bs_i} , $i \in \{1, \dots, N_{t_RF}\}$. In the Rx side each user has N_{r_RF} sub-arrays with each array having $M_u = N_r/N_{r_RF}$ number of antennas connected to one RF chain. The received symbols are first processed by analog combining vector $a_{ms_j}^u$ connected to j^{th} RF chain of user u ($j \in \{1, \dots, N_{r_RF}\}$). Then symbols will pass through the digital combiner d_{ms}^u of user u to obtain data streams.

Effective analog precoder matrix A_{bs} at BS is given by,

$$A_{bs} = \begin{bmatrix} a_{bs_1} & 0 & \dots & 0 \\ 0 & a_{bs_2} & & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \dots & a_{bs_N_{t_RF}} \end{bmatrix}_{N_t \times N_{t_RF}} \quad (4)$$

and digital precoder matrix D_{bs} is,

$$D_{bs} = \begin{bmatrix} d_{bs_1} & 0 & \dots & 0 \\ 0 & d_{bs_2} & & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \dots & d_{bs_N_{t_RF}} \end{bmatrix}_{N_{t_RF} \times N_{t_RF}} \quad (5)$$

The hybrid precoder matrix P , which satisfies the power constraint, $\|P\|_F^2 \leq N_{t_RF}$ is given by.

$$P = A_{bs} \cdot D_{bs} = \begin{bmatrix} a_{bs_1} \cdot d_{bs_1} & 0 & \dots & 0 \\ 0 & a_{bs_2} \cdot d_{bs_2} & & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \dots & a_{bs_N_{t_RF}} \cdot d_{bs_N_{t_RF}} \end{bmatrix}_{N_t \times N_{t_RF}} \quad (6)$$

Similarly at Rx side, analog combiner matrix a_{ms}^u of user u is given by,

$$a_{ms}^u = \begin{bmatrix} a_{ms_1}^u & 0 & \dots & 0 \\ 0 & a_{ms_2}^u & & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \dots & a_{ms_N_{r_RF}}^u \end{bmatrix}_{N_r \times N_{r_RF}} \quad (7)$$

and digital combiner matrix d_{ms}^u of user u is,

$$d_{ms}^u = \begin{bmatrix} d_{ms_1}^u & 0 & \dots & 0 \\ 0 & d_{ms_2}^u & & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \dots & d_{ms_N_{r_RF}}^u \end{bmatrix}_{N_{r_RF} \times N_{r_RF}} \quad (8)$$

Combiner matrix of user u is $C_u = a_{ms}^u \cdot d_{ms}^u$ of size $N_r \times N_{r_RF}$, which satisfies the power constraint $\|C_u\|_F^2 \leq N_{r_RF}$.

Now hybrid combiner matrix C of all users at the Rx side is given by,

$$C = \begin{bmatrix} C_1 & 0 & \dots & 0 \\ 0 & C_2 & & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \dots & C_U \end{bmatrix}_{UN_r \times UN_{r_RF}} \quad (9)$$

2) **Fully-connected structure:** In BS data streams are first precoded by digital precoder $D_{bs} \in \mathbb{C}^{N_{t_RF} \times N_{t_RF}}$ followed by analog precoder $A_{bs} \in \mathbb{C}^{N_t \times N_{t_RF}}$. At Rx side, at u^{th} user received symbols are first processed by analog combiner $a_{ms}^u \in \mathbb{C}^{N_{r_RF} \times N_{r_RF}}$, then passes through the digital combiner $d_{ms}^u \in \mathbb{C}^{N_{r_RF} \times N_{r_RF}}$ as in Fig. 2b.

The hybrid precoder and combiner matrix at BS and at the user u is given by

$$P = A_{bs} \cdot D_{bs} \in \mathbb{C}^{N_t \times N_{t_RF}}$$

and

$$C_u = a_{ms}^u \cdot d_{ms}^u \in \mathbb{C}^{N_{r_RF} \times N_{r_RF}}$$

Hybrid combiner matrix C is same as sub connected structure.

For both sub connected and fully connected structures, the received signal at u^{th} user is given by,

$$y_u = \rho C_u^H H_u P s + C_u^H n_u \quad (10)$$

where ρ is a average received power, $H_u \in \mathbb{C}^{N_r \times N_t}$ is a Channel matrix between BS and user u .

The transmitted vector $s = [s_1, s_2, \dots, s_{N_{t_RF}}]^T$ satisfy the constraint $\mathbb{E}(ss^H) = \frac{1}{N_{t_RF}} I_{N_{t_RF}}$ and the element in the noise vector is chosen as $n_u = [n_1, n_2, \dots, n_{N_{t_RF}}]^T \sim \mathcal{CN}(0, \sigma^2)$.

Sum rate achieved by the multi-user mmWave MIMO system is given by

$$R = \log_2 \left(\left| I_{(UN_{r_RF})} + \frac{\rho}{(U * N_{r_RF}) \sigma^2} (C^H C)^{-1} C^H H_{eff} P P^H H_{eff}^H C \right| \right) \quad (11)$$

III. HYBRID BEAMFORMING DESIGN

A. Using sub connected architecture

In this section the design of hybrid precoding for sub connected structure is discussed.

1) **SIC based hybrid precoding design:** In the SIC based scheme for sub-connected structure, the precoder or combiner of each sub-array connected to one RF chain computed separately and each sub-array rate is optimized separately using the singular value decomposition. The detail derivation of the design equations are given in Appendix A.

To design SIC based hybrid precoding, first C_u is initialized as $N_r \times N_{r_RF}$ matrix of zeros and P is initialized as $N_t \times N_{t_RF}$ matrix of zeros. Then to find C_u at the user u , at each RF chain ie., for $j = 1$ to N_{r_RF} , G_u of size $N_r \times N_r$ is calculated, which is given by,

$$G_u = H_u \left(I_{N_t} + \frac{\rho}{N_{t_RF} * \sigma^2} H_u^H C_u(:, 1:j-1) C_u(:, 1:j-1)^H H_u \right)^{-1} H_u^H \quad (12)$$

Where $C_u(:, 1:j-1)$ indicates the first $(j-1)$ columns of C_u matrix.

S_u is a $M_u \times M_u$ sub-matrix of G_u such that

$$S_u = R_u G_u R_u^H \quad (13)$$

where, $R_u = [0_{M_u \times M_u(j-1)} \quad I_{M_u} \quad 0_{M_u \times M_u(N_{r_RF}-j)}]$ is a selection matrix. In other words, S_u only keeps the rows and columns from the $(M_u(j-1) + 1)$ to $(M_u j)$ of G_u matrix.

Now decompose S_u as,

$$\text{SVD: } S_u = U_u \Sigma_u V_u^H \quad (14)$$

Analog combiner and digital combiner corresponding to j th RF chain is,

$$a_{ms_j}^u = \frac{1}{\sqrt{M_u}} e^{j\angle(U_u(:,1))} \quad (15)$$

$$d_{ms_j}^u = \frac{\|U_u(:,1)\|_1}{\sqrt{M_u}} \quad (16)$$

where $U_u(:,1)$ represents the first column of U_u . Multiplication of both these ie., $a_{ms_j}^u * d_{ms_j}^u$ will result into $M_u \times 1$ vector, which is placed in the position from $(M_u * (j-1) + 1)$ to $(M_u * (j-1) + M_u)$ in the j th column of C_u matrix.

Algorithm 1 SIC-based hybrid precoding using sub connected architecture

Input: $H_u, H_{eff}, N_t, N_{t_RF}, N_r, N_{r_RF}, M, M_u$ and U

Initialization: $C_u = 0_{N_r \times N_{r_RF}}$
 $P = 0_{N_t \times N_{t_RF}}$

At receiver side:

```

1: for  $u = 1$  to  $U$  do
2:   for  $j = 1$  to  $N_{r\_RF}$  do
3:     Compute  $S_u$  using equation (13)
4:      $[U_u, \sim, \sim] = \text{SVD}(S_u)$ 
5:      $a_{ms\_j}^u = \frac{1}{\sqrt{M_u}} e^{j\angle(U_u(:,1))}$ 
6:      $d_{ms\_j}^u = \frac{\|U_u(:,1)\|_1}{\sqrt{M_u}}$ 
7:      $C_u(:,j) = a_{ms\_j}^u * d_{ms\_j}^u$ 
8:   end for
9: end for
10:  $C = \text{diag}[C_1, C_2, \dots, C_U]$ 

```

At transmitter side:

```

11: for  $i = 1$  to  $N_{t\_RF}$  do
12:   Compute  $S$  according to (18)
13:    $[\sim, \sim, V] = \text{SVD}(S)$ 
14:    $a_{bs\_i} = \frac{1}{\sqrt{M}} e^{j\angle(V(:,1))}$ 
15:    $d_{bs\_i} = \frac{\|V(:,1)\|_1}{\sqrt{M}}$ 
16:    $P(:,i) = a_{bs\_i} * d_{bs\_i}$ 
17: end for

```

Output: P, C

Now the procedure to find precoder matrix P is almost same as mentioned above with some changes. Here the at each RF chain ie., for $i = 1$ to N_{t_RF} , G of size $N_t \times N_t$ is calculated as,

$$G = H_{eff}^H C \left(I_{(U * N_{r_RF})} + \frac{\rho}{(U * N_{r_RF}) \sigma^2} (C^H C)^{-1} C^H H_{eff} P(:, 1:i-1) P(:, 1:i-1)^H H_{eff}^H C \right)^{-1} C^H H_{eff} \quad (17)$$

such that $P(:, 1:i-1)$ indicates the first $(i-1)$ columns of P . S is a sub-matrix P of size $M \times M$ given by,

$$S = R_t G R_t^H \quad (18)$$

where $R_t = [0_{M \times M(i-1)} \ I_M \ 0_{M \times M(N_{t_RF}-i)}]$ is a matrix which helps to select the rows and columns from $(M(i-1) + 1)$ to (Mi) of G matrix.

$$\text{SVD: } S = U \Sigma V^H \quad (19)$$

Now analog precoder and digital precoder corresponding to i th RF chain are calculated as,

$$a_{bs_i} = \frac{1}{\sqrt{M}} e^{j\angle(V(:,1))} \quad (20)$$

$$d_{bs_i} = \frac{\|V(:,1)\|_1}{\sqrt{M}} \quad (21)$$

where $V(:,1)$ is the first column of V . Then $a_{bs_i} * d_{bs_i}$ is placed in the respective position of the i^{th} column of P matrix. Pseudo code of SIC based hybrid precoding design is given in the Algorithm 1.

Algorithm 2 Optimal precoding for sub connected structure

Input: $H_u, H_{eff}, N_t, N_{t_RF}, N_r, N_{r_RF}, M, M_u$ and U

Initialization: $C_u = 0_{N_r \times N_{r_RF}}$

$P = 0_{N_t \times N_{t_RF}}$

At receiver side:

```
1: for  $u = 1$  to  $U$  do
2:   for  $j = 1$  to  $N_{r\_RF}$  do
3:     Compute  $S_u$  using equation (13)
4:      $[U_u, \sim, \sim] = \text{SVD}(S_u)$ 
5:      $C_u(:, j) = U_u(:, 1)$ 
6:   end for
7: end for
8:  $C = \text{diag}[C_1, C_2, \dots, C_U]$ 
```

At transmitter side:

```
9: for  $i = 1$  to  $N_{t\_RF}$  do
10:  Compute  $S$  according to (18)
11:   $[\sim, \sim, V] = \text{SVD}(S)$ 
12:   $P(:, i) = V(:, 1)$ 
13: end for
```

Output: P, C

2) **Optimal precoding design:** The design of optimal precoding for sub-connected is almost same as that of above said SIC based design for sub-connected structure, except that analog precoder and digital precoder at both Tx and Rx sides are not calculated. Instead of that columns of C_u and P matrices are directly computed by taking first column of U_u and V matrices respectively as described in the Algorithm 2.

B. Using fully connected architecture

In section we will discuss the design of hybrid precoding schemes which uses fully connected structure at both Tx and Rx side.

1) **SIC based hybrid precoding design:** In SIC based hybrid precoding for multi-user case, for each user at the receiver side first channel between BS and user u ie., H_u is calculated and is decomposed into SVD as,

$$H_u = U_u \Sigma_u V_u^H \quad (22)$$

The analog precoder a_{ms}^u and digital precoder d_{ms}^u of user u are calculated as,

$$a_{ms}^u = \frac{1}{\sqrt{N_r}} e^{j\angle(U_u(:, 1:N_{r_RF}))} \quad (23)$$

$$d_{ms}^u = \frac{\|U_u(:, 1:N_{r_RF})\|_1}{\sqrt{N_r}} \quad (24)$$

where $U_u(:, 1:N_{r_RF})$ represents the first N_{r_RF} columns of U_u matrix. Now hybrid combiner of user u can be obtained by $C_u = a_{ms}^u * d_{ms}^u$ and then find C . Similarly to find hybrid precoder P at the BS, first svd of H_{eff} taken ie.,

$$H_{eff} = U \Sigma V^H \quad (25)$$

Calculate a_{bs} and d_{bs} as,

$$a_{bs} = \frac{1}{\sqrt{N_t}} e^{j\angle(V(:, 1:N_{t_RF}))} \quad (26)$$

$$d_{bs} = \frac{\|V(:, 1:N_{t_RF})\|_1}{\sqrt{N_t}} \quad (27)$$

Now obtain hybrid precoder as, $P = a_{bs} * d_{bs}$. All these above mentioned procedure is described in the Algorithm 3. The proof of above equations can be found in Appendix A.

2) **Optimal precoding design:** The difference between Optimal precoding design and above mentioned SIC based scheme for fully-connected structure is, the combiner of user u is directly calculated as, $C_u = U_u(:, 1:N_{r_RF})$ such that there is no need to find analog precoder and digital precoder separately, then C_u and C are computed. Similarly $P = V(:, 1:N_{t_RF})$ is calculated. The pseudo code for optimal precoding scheme is given in the Algorithm 4.

Algorithm 3 SIC-based hybrid precoding using fully connected architecture

Input: $H_u, H_{eff}, N_t, N_{t_RF}, N_r, N_{r_RF}$ and U *At receiver side:*

- 1: **for** $u = 1$ to U **do**
- 2: $[U_u, \sim, \sim] = \text{svd}(H_u)$
- 3: $a_{ms}^u = \frac{1}{\sqrt{N_r}} e^{j\text{angle}(U_u(:, 1:N_{r_RF}))}$
- 4: $d_{ms}^u = \frac{\|U_u(:, 1:N_{r_RF})\|_1}{\sqrt{N_r}}$
- 5: $C_u = a_{ms}^u * d_{ms}^u$
- 6: **end for**
- 7: $C = \text{diag}[C_1, C_2, \dots, C_U]$

At transmitter side:

- 8: $[\sim, \sim, V] = \text{svd}(H_{eff})$
- 9: $a_{bs} = \frac{1}{\sqrt{N_t}} e^{j\text{angle}(V(:, 1:N_{t_RF}))}$
- 10: $d_{bs} = \frac{\|V(:, 1:N_{t_RF})\|_1}{\sqrt{N_t}}$
- 11: $P = a_{bs} * d_{bs}$

Output: P, C

Algorithm 4 Optimal precoding for fully connected structure

Input: $H_u, H_{eff}, N_t, N_{t_RF}, N_r, N_{r_RF}$ and U *At receiver side:*

- 1: **for** $u = 1$ to U **do**
- 2: $[U_u, \sim, \sim] = \text{SVD}(H_u)$
- 3: $C_u = U_u(:, 1 : N_{r_RF})$
- 4: **end for**
- 5: $C = \text{diag}[C_1, C_2, \dots, C_U]$

At transmitter side:

- 6: $[\sim, \sim, V] = \text{SVD}(H_{eff})$
- 7: $P = V(:, 1 : N_{t_RF})$

Output: P, C

IV. SIMULATION RESULTS

In this section we shown the simulation results of achievable sum-rate for the above mentioned algorithms and compared their sum rate with different parameters by simulating results for 100 iterations. The title above in each figures gives the simulation parameters chosen for that result, and also inside the square bracket given their execution time. As antennas are arranged in UPA in 3D beamforming, at transmitter side $W_1 = N_{t_RF}$, $W_2 = N_t/N_{t_RF}$ and at receiver side $W_1 = N_{r_RF}$, $W_2 = N_r/N_{r_RF}$ are chosen such that $W = W_1 * W_2$. Here it is convenient to choose number of RF chains and users in such a way that, $N_{t_RF} = U * N_{r_RF}$, as number of data streams transmitted at the Tx side is equals to N_{t_RF} and total number of data streams received at the Rx side is equals to $U * N_{r_RF}$.

A. Observations

From the simulated results we can observe that,

- In Fig. 3, sum rate achieved by the proposed hybrid precoding schemes is almost near to that of rate achieved by the optimal precoding schemes, which indicates that SIC based hybrid precoding is near optimal in terms of spectral efficiency. Also algorithms using fully connected structure achieving higher rate as compared to that of sub connected structure based algorithms. Thus fully connected structure shows good performance (achieves higher spectral efficiency) than the sub connected one.
- In Fig. 4, sum rate is increasing with the number of antennas at the Tx and Rx sides, which is due to transmission of larger amount of data with the increase in the N_t & N_r resulting into increased spectral efficiency.
- In Fig. 5, sum rate is increasing with the number of users, due to increased number of antennas at the Tx & Rx. Also as mmWave channel is best suitable for large number of paths [5], so sum rate increases with the small L and becomes almost constant for large L.
- In Fig. 6, sum rate increases only for sub connected structure, where as in case of fully connected structure rate is increasing for small RF chains but decreasing for large number of RF chains due to interference.

(a) Sum rate vs SNR

(b) Sum rate vs SNR

Fig. 3: Spectral efficiency comparison against SNR

(a) Sum-rate vs number of Rx antennas at user u

(b) Sum-rate vs number of BS antennas

Fig. 4: Spectral efficiency comparison against number of antennas

(a) Sum-rate vs number of paths/rays

(b) Sum-rate vs number of users

Fig. 5: Spectral efficiency comparison against number of paths and users

(a) Sum-rate vs number of RF chains of user u at Rx side

(b) Sum-rate vs number of RF chains at the BS

Fig. 6: Spectral efficiency comparison against number of RF chains

Here the execution time (i.e. time taken to simulate the particular result) increases with the number of antennas, users, paths and RF chains due to increased number of iterations and computations.

V. CONCLUSIONS

In this paper we proposed the SIC based hybrid precoding scheme using sub connected and fully connected structures for multi-user case in a 3D scenario, in which both azimuth and elevation angles are considered. Also designed the optimal precoding for multi-user case and compared with the SIC based scheme, where we have used svd to optimize the capacity of each sub-array. In the simulation part, the proposed schemes are simulated in MATLAB and compared with different parameters, where we can observe that sum-rate increases with the N_t , N_r , L and U . Performance of SIC based scheme is near to that of optimal precoder. Fully connected structure shows good performance than the sub connected one but with high hardware complexity. Our further work will focus on extending these schemes to multi-cell case, where there is a separate BS and users in each cell.

APPENDIX A

PROOF OF ANALOG AND DIGITAL PRECODERS

The sum rate from equation (11) is given by

$$R = \log_2 \left(\left| I_{(UN_{r_RF})} + \frac{\rho}{(U * N_{r_RF})\sigma^2} (C^H C)^{-1} C^H H_{eff} P P^H H_{eff}^H C \right| \right)$$

For simplicity, choose $\frac{\rho}{(U * N_{r_RF})\sigma^2} (C^H C)^{-1} = A$, $N_{t_RF} = N$ and $I_{(UN_{r_RF})} = I$, then

$$\begin{aligned} R &= \log_2 \left(\left| I + A C^H H_{eff} P P^H H_{eff}^H C \right| \right) \\ R &= \log_2 \left(\left| I + A C^H H_{eff} [P_{N-1} p_N] [P_{N-1} p_N]^H H_{eff}^H C \right| \right) \end{aligned}$$

where, P_{N-1} is first $N-1$ columns of P and p_N is N th column of P

$$R = \log_2 \left(\left| I + A C^H H_{eff} P_{N-1} P_{N-1}^H H_{eff}^H C + A C^H H_{eff} p_N p_N^H H_{eff}^H C \right| \right)$$

$$\text{Let } T_{N-1} = I + A C^H H_{eff} P_{N-1} P_{N-1}^H H_{eff}^H C \quad (28)$$

$$\begin{aligned} R &= \log_2 \left(\left| T_{N-1} + A C^H H_{eff} p_N p_N^H H_{eff}^H C \right| \right) \\ &= \log_2 \left(\left| T_{N-1} (I + A T_{N-1}^{-1} C^H H_{eff} p_N p_N^H H_{eff}^H C) \right| \right) \end{aligned}$$

$$\therefore \log_2(mn) = \log_2(m) + \log_2(n)$$

$$R = \log_2(|T_{N-1}|) + \log_2 \left(I + A T_{N-1}^{-1} C^H H_{eff} p_N p_N^H H_{eff}^H C \right)$$

Let $X = T_{N-1}^{-1} C^H H_{eff} p_N$ and $Y = p_N^H H_{eff}^H C$,

$\therefore |I + XY| = |1 + YX|$, so

$$R = \log_2(|T_{N-1}|) + \log_2(1 + Ap_N^H H_{eff}^H CT_{N-1}^{-1} C^H H_{eff} p_N)$$

where the term $\log_2(1 + Ap_N^H H_{eff}^H CT_{N-1}^{-1} C^H H_{eff} p_N)$ indicates the rate achieved by Nth sub-antenna array. $\log_2(|T_{N-1}|)$ from equation (28) is similar to R, hence $\log_2(|T_{N-1}|)$ can be further decomposed as,

$$\log_2(|T_{N-1}|) = \log_2(|T_{N-2}|) + \log_2(1 + Ap_{N-1}^H H_{eff}^H CT_{N-2}^{-1} C^H H_{eff} p_{N-1})$$

After N decompositions, total achievable rate is,

$$\begin{aligned} R &= \sum_{n=1}^N \log_2(1 + Ap_n^H H_{eff}^H CT_{n-1}^{-1} C^H H_{eff} p_n) \\ &= \log_2(1 + Ap_1^H H_{eff}^H CT_0^{-1} C^H H_{eff} p_1) + \log_2(1 + Ap_2^H H_{eff}^H CT_1^{-1} C^H H_{eff} p_2) + \\ &\quad \log_2(1 + Ap_3^H H_{eff}^H CT_2^{-1} C^H H_{eff} p_3) + \dots \end{aligned} \quad (29)$$

where, $T_n = I + AC^H H_{eff} P_n P_n^H H_{eff}^H C$ and $T_0 = I$

A. nth sub-array rate optimization:

Consider Nth sub-array rate, ie., $\log_2(1 + Ap_N^H H_{eff}^H CT_{N-1}^{-1} C^H H_{eff} p_N)$,

$$\text{Let } G_{n-1} = H_{eff}^H CT_{n-1}^{-1} C^H H_{eff} \quad (30)$$

Then for nth precoding vector p_n corresponding to nth sub-array, the optimized sub-rate is given by,

$$p_n^{opt} = \underset{p_n \in F}{argmax} \log_2(1 + Ap_n^H G_{n-1} p_n) \quad (31)$$

$p_n \in \mathbb{C}^{N_t \times 1}$ and F is a set of all vectors satisfying 3 constraints

- 1) Structure constraint: $P = AD = \text{diag}\{\bar{p}_1, \bar{p}_2, \dots, \bar{p}_N\}$, such that $\bar{p}_n$ is a $M \times 1$ non-zero vector
- 2) Amplitude constraint: non-zero elements of each columns of p should have same amplitude
- 3) Power constraint: $\|P\|_F^2 \leq N$

$\therefore p_n$ has M non-zero elements form $M(n-1) + 1$ to Mn , where $1 \leq n \leq N$,

$$\bar{p}_n^{opt} = \underset{\bar{p}_n \in \bar{F}}{argmax} \log_2(1 + A\bar{p}_n^H S_{n-1} \bar{p}_n) \quad (32)$$

$\bar{F}$ - set of $M \times 1$ vectors satisfying constraints (2) and (3).

S_{n-1} - $M \times M$ sub-matrix of G_{n-1} can be written as

$$S_{n-1} = R_t G_{n-1} R_t^H \quad (33)$$

$R_t = [0_{M \times M(n-1)} \ I_M \ 0_{M \times M(N-n)}]$ is a selection matrix

$$\text{SVD: } S_{n-1} = V \Sigma V^H \quad (34)$$

Note that $\bar{p}_n^{opt}$ is first column V , but v_1 does not have same amplitude elements.

The rate achieved by nth array,

$$R_n = \log_2(1 + A\bar{p}_n^H S_{n-1} \bar{p}_n)$$

$$\begin{aligned} R_n &= \log_2(1 + (A)\bar{p}_n^H V \Sigma V^H \bar{p}_n) \\ &= \log_2\left(1 + (A)\bar{p}_n^H \begin{bmatrix} v_1 & v_2 \end{bmatrix} \begin{bmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{bmatrix} \begin{bmatrix} v_1 & v_2 \end{bmatrix}^H \bar{p}_n\right) \\ &= \log_2(1 + A\bar{p}_n^H v_1 \sigma_1 v_1^H \bar{p}_n + A\bar{p}_n^H v_2 \sigma_2 v_2^H \bar{p}_n) \end{aligned}$$

To find $\bar{p}_n$ close to v_1 , assume $\bar{p}_n^H v_2 \approx 0$, then

$$\begin{aligned}
R_n &= \log_2 \left(1 + A \bar{p}_n^H v_1 \sigma_1 v_1^H \bar{p}_n \right) \\
&= \log_2 \left(1 + \frac{\rho}{(U * N_{r_RF}) \sigma^2} (C^H C)^{-1} \bar{p}_n^H v_1 \sigma_1 v_1^H \bar{p}_n \right) \\
&= \log_2 \left(1 + \frac{\rho \sigma_1}{(U * N_{r_RF}) \sigma^2} - \frac{\rho \sigma_1}{(U * N_{r_RF}) \sigma^2} + \frac{\rho \sigma_1}{(U * N_{r_RF}) \sigma^2} (C^H C)^{-1} \bar{p}_n^H v_1 v_1^H \bar{p}_n \right) \\
&= \log_2 \left(1 + \frac{\rho \sigma_1}{(U * N_{r_RF}) \sigma^2} - \frac{\rho \sigma_1}{(U * N_{r_RF}) \sigma^2} \left(1 - (C^H C)^{-1} \bar{p}_n^H v_1 v_1^H \bar{p}_n \right) \right) \\
&= \log_2 \left[\left(1 + \frac{\rho \sigma_1}{(U * N_{r_RF}) \sigma^2} \right) \left[1 - \left(1 + \frac{\rho \sigma_1}{(U * N_{r_RF}) \sigma^2} \right)^{-1} \frac{\rho \sigma_1}{(U * N_{r_RF}) \sigma^2} \left(1 - (C^H C)^{-1} \bar{p}_n^H v_1 v_1^H \bar{p}_n \right) \right] \right] \\
&= \log_2 \left(1 + \frac{\rho \sigma_1}{(U * N_{r_RF}) \sigma^2} \right) + \log_2 \left[1 - \left(1 + \frac{\rho \sigma_1}{(U * N_{r_RF}) \sigma^2} \right)^{-1} \frac{\rho \sigma_1}{(U * N_{r_RF}) \sigma^2} \left(1 - (C^H C)^{-1} \bar{p}_n^H v_1 v_1^H \bar{p}_n \right) \right]
\end{aligned}$$

at high SNR $(\frac{\rho}{\sigma^2})$, $\left(1 + \frac{\rho \sigma_1}{(U * N_{r_RF}) \sigma^2} \right)^{-1} \frac{\rho \sigma_1}{(U * N_{r_RF}) \sigma^2} \approx 1$

$$\begin{aligned}
R_n &= \log_2 \left(1 + \frac{\rho \sigma_1}{(U * N_{r_RF}) \sigma^2} \right) + \log_2 \left((C^H C)^{-1} \bar{p}_n^H v_1 v_1^H \bar{p}_n \right) \\
&= \log_2 \left(1 + \frac{\rho \sigma_1}{(U * N_{r_RF}) \sigma^2} \right) + \log_2 ((C^H C)^{-1}) + \log_2 (\bar{p}_n^H v_1 v_1^H \bar{p}_n) \\
R_n &= \log_2 ((C^H C)^{-1} + A \sigma_1) + \log_2 (\|v_1^H \bar{p}_n\|_2^2)
\end{aligned} \tag{35}$$

Maximization of R_n can be done by maximizing $\|v_1^H \bar{p}_n\|_2^2$

where, $\|v_1^H \bar{p}_n\|_2$ is shortest distance between v_1 and $\bar{p}_n$ (ie., inner product between $\bar{p}_n$ and v_1).

$\because v_1$ is fixed, maximization of $R_n \implies$ maximization of square of L2-norm (ie., smallest euclidean distance between $\bar{p}_n$ and v_1).

Thus,

$$\bar{p}_n^{opt} = \underset{\bar{p}_n \in \bar{F}}{\operatorname{argmin}} \|v_1 - \bar{p}_n\|_2^2 \tag{36}$$

if d_n and $\bar{a}_n$ are digital precoder and analog precoder of nth sub-array respectively, then $\bar{p}_n = d_n \bar{a}_n$

$$\begin{aligned}
\|v_1 - \bar{p}_n\|_2^2 &= \|v_1 - d_n \bar{a}_n\|_2^2 \\
&= v_1^H v_1 - d_n (v_1^H \bar{a}_n + \bar{a}_n^H v_1) + d_n^2 \bar{a}_n^H \bar{a}_n
\end{aligned}$$

$\because d_n$ is scalar and real, $d_n = d_n^H$

$\bar{a}_1^H v_1 = v_1^H \bar{a}_1$ is 1x1 with imaginary part ≈ 0

so, $\bar{a}_1^H v_1 = \operatorname{Re}[\bar{a}_1^H v_1]$

$$\|v_1 - \bar{p}_n\|_2^2 = v_1^H v_1 + d_n^2 \bar{a}_n^H \bar{a}_n - 2d_n \operatorname{Re}(v_1^H \bar{a}_n)$$

Each elements of $\bar{a}_n$ is $= 1/\sqrt{M}$ so, $\bar{a}_n^H \bar{a}_n = 1$

$$\begin{aligned}
\|v_1 - \bar{p}_n\|_2^2 &= 1 + d_n^2 - 2d_n \operatorname{Re}(v_1^H \bar{a}_n) \\
&= 1 + d_n^2 - 2d_n \operatorname{Re}(v_1^H \bar{a}_n) + [\operatorname{Re}(v_1^H \bar{a}_n)]^2 - [\operatorname{Re}(v_1^H \bar{a}_n)]^2 \\
&= [d_n^2 + [\operatorname{Re}(v_1^H \bar{a}_n)]^2 - 2d_n \operatorname{Re}(v_1^H \bar{a}_n)] + [1 - (\operatorname{Re}(v_1^H \bar{a}_n))^2] \\
\|v_1 - \bar{p}_n\|_2^2 &= [d_n - \operatorname{Re}(v_1^H \bar{a}_n)]^2 + [1 - [\operatorname{Re}(v_1^H \bar{a}_n)]^2]
\end{aligned}$$

Thus, distance between $\bar{p}_n$ and v_1 depends on 2 terms. The first term can be minimized to zero by taking $d_n = \operatorname{Re}(v_1^H \bar{a}_n)$. The second term can be minimized by maximizing $|\operatorname{Re}(v_1^H \bar{a}_n)|$

$$\therefore \boxed{\bar{a}_n^{opt} = \frac{1}{\sqrt{M}} e^{j\angle(v_1)}} \tag{37}$$

v_1 has same value for all n. Hence

$$d_n^{opt} = \operatorname{Re}(v_1^H \bar{a}_1) = \frac{1}{\sqrt{M}} \operatorname{Re} \left(v_1^H e^{j\angle(v_1)} \right) = \frac{\|v_1\|_1}{\sqrt{M}}$$

$$\therefore d_n^{opt} = \frac{\|v_1\|_1}{\sqrt{M}} \quad (38)$$

Now optimal precoder of nth sub-array can be calculated as, $\bar{p}_n^{opt} = d_n^{opt} \bar{a}_n^{opt} = \frac{1}{M} \|v_1\|_1 e^{jangle(v_1)}$
 $\therefore \|v_1\|_2 = 1$ and $e^{jangle(v_1^H)} * e^{jangle(v_1)} = M$ also $e^{jangle(v_1)} \leq 1$, so $\|\bar{p}_n^{opt}\|_2^2 \leq 1$.

For all sub-arrays (n=1,2,...,N), $\bar{p}_n^{opt}$ is similar. so,

$$\|P^{opt}\|_F^2 = \|\text{diag}\{\bar{p}_1^{opt}, \bar{p}_2^{opt}, \dots, \bar{p}_N^{opt}\}\|_F^2 \leq N \quad (39)$$

ie., power constraint is satisfied.

Note that the equations (37), (38) are derived for sub-connected structure. Similarly the analog and digital precoders for fully-connected structure is given by,

$$\bar{a}^{opt} = \frac{1}{\sqrt{N_t}} e^{jangle(v(:,1:N))} \quad (40)$$

$$d^{opt} = \frac{\|v(:,1:N)\|_1}{\sqrt{N_t}} \quad (41)$$

where, $v(:,1:N)$ indicates the first N columns of v , since antennas driven by all RF chains simultaneously as each RF chain connected to all antennas.

DECLARATIONS

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Competing Interests

The authors declare that they have no competing interests.

Data availability

This is not applicable as no datasets are used during the study.

Code availability

The simulation codes are written in MATLAB. The GitHub repository for the codes are -

🔗 SIC_based_HP_MU_3D

(https://github.com/Rajashekhar8877/SIC_based_HP_MU_3D)

Author contributions

Both authors are involved in the developing the algorithms and simulations. Rajashekhar involved in writing the paper. Vanidevi is responsible for editing and supervision. Both authors have read and agreed to the manuscript.

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