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Optical solitons to the Ginzburg–Landau equation including the parabolic nonlinearity

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Abstract: The major goal of the present paper is to construct optical solitons of the Ginzburg–Landau (GL) equation including the parabolic nonlinearity. Such an ultimate goal is formally achieved with the aid of symbolic computation, a complex transformation, and Kudryashov and exponential methods. Several numerical simulations are given to explore the influence of the coefficients of nonlinear terms on the dynamical features of the obtained optical solitons. To the best of the authors' knowledge, the results reported in the current study, classified as bright and kink solitons, are new and have been acquired for the first time.

Keywords: Ginzburg–Landau equation; Parabolic nonlinearity; Complex transformation; Kudryashov and exponential method; Optical solitons

1. Introduction

Nonlinear partial differential (NLPD) equations have long been regarded as useful tools for describing a wide range of phenomena in a variety of scientific disciplines. As models for exploring real-world phenomena, NLPD equations play a fundamental role in the development of the contemporary world. Over the last several decades, one of the most significant challenges has been the development of new methods to construct exact solutions for NLPD equations. In recent years, several new, more powerful, and effective approaches have been established to retrieve exact solutions of NLPD equations, the sine-Gordon expansion method [1-4], the (G'/G)-expansion method [5-8], the Sardar sub-equation method [9-12], the Kudryashov method [13-18], and the exponential method [19-25], are examples to mention.

As it is evident, the nonlinear Schrödinger equation is often used to simulate soliton dynamics in nonlinear optics. Many additional models, in contrast to the nonlinear Schrödinger equation, can be used as an alternative to such a classical model, for example, the Schrödinger–Hirota equation, the Chen–Lee–Liu equation, and many more. In the present study, the authors aim to conduct a study on the following Ginzburg–Landau equation including the parabolic nonlinearity [26-28]

$$i\frac{\partial u(x,t)}{\partial t}+\alpha_1\frac{\partial^2 u(x,t)}{\partial x^2}+(\alpha_2|u(x,t)|^2+\alpha_3|u(x,t)|^4)u(x,t)-\frac{\alpha_4}{|u(x,t)|^2u^*(x,t)}\left(2|u(x,t)|^2\frac{\partial^2|u(x,t)|^2}{\partial x^2}-\left(\frac{\partial|u(x,t)|^2}{\partial x}\right)^2\right)-\alpha_5 u(x,t)=0, \tag{1}$$

and acquire its optical solitons using Kudryashov and exponential methods. In Eq. (1), $u(x,t)$ indicates the wave profile, and x and t denote spatial and temporal coordinates, respectively. Besides, α_1 is the GVD while α_4 is the coefficient of nonlinear terms, α_5 is the coefficient of detuning, and α_2 and α_3 relate to the parabolic nonlinearity. Optical solitons of the GL equation including the parabolic nonlinearity were derived by Biswas in [26] with the help of the semi-inverse method. Arshed et al. [27] employed the exponential method to obtain a series of optical soliton of the GL equation including the parabolic nonlinearity. Elboree [28] used the $\exp(-\phi(\xi))$ method to report optical solitons of the GL equation including the parabolic nonlinearity. More works are found in [29-34].

Kudryashov and exponential methods have been designed as newly well-established methods to derive solitons of NLPD equations. In recent years, these methods have achieved much attention, especially from mathematicians and physicists. Akinyemi et al. [35] used the Kudryashov method to derive solitons of a Schrödinger equation involving spatio-temporal dispersions. Nisar et al. [36] extracted solitons of a population equation with the beta-time derivative using the exponential method.

The structure of the present paper is as follows: In Section 2, a full description of Kudryashov and exponential methods are provided. In Section 3, the GL equation including the parabolic nonlinearity is reduced in a 1D regime using a complex transformation. In Section 4, Kudryashov and exponential methods are used to retrieve optical solitons of the GL equation including the parabolic nonlinearity. Furthermore, section 4 presents several numerical simulations to explore the influence of the coefficients of nonlinear terms on the dynamical features of the obtained optical solitons. The article is concluded in Section 5.

2. Kudryashov and exponential methods

The current section gives a full description of Kudryashov and exponential methods. The Kudryashov method recommends a series as follows

$$U(\epsilon) = a_0 + a_1 K(\epsilon) + a_2 K^2(\epsilon) + \cdots + a_N K^N(\epsilon), \quad a_N \neq 0, \tag{2}$$

as the solution of

$$O(U(\epsilon), U'(\epsilon), U''(\epsilon), \dots) = 0. \tag{3}$$

In series (2), $a_i, i = 0,1,\dots,N$ are derived later, N is found through the balance principle, and $K(\epsilon)$ is

$$K(\epsilon) = \frac{1}{(A-B)\sinh(\epsilon) + (A+B)\cosh(\epsilon)},$$

satisfying

$$\left(K'(\epsilon)\right)^2 = K^2(\epsilon)\left(1 - 4ABK^2(\epsilon)\right).$$

From Eqs. (2) and (3), we reach a consistent nonlinear system whose solution leads to solitons of Eq. (3).

Compared to the Kudryashov method, the exponential method seeks the following nontrivial solution

$$U(\epsilon) = \frac{a_0 + a_1 a^{\epsilon} + a_2 a^{2\epsilon} + \cdots + a_N a^{N\epsilon}}{b_0 + b_1 a^{\epsilon} + b_2 a^{2\epsilon} + \cdots + b_N a^{N\epsilon}}, \quad a_N \neq 0, \quad b_N \neq 0, \tag{4}$$

as the solution of Eq. (3). In Eq. (4), the coefficients are acquired later and $N \in \mathbb{N}$.

As before, from Eqs. (4) and (3), we arrive at a consistent nonlinear system whose solution yields solitons of Eq. (3).

3. The model in its 1D regime

To reduce the governing model in a 1D regime, it is assumed that the model solution has the form

$$u(x,t) = U(\epsilon)e^{i\phi(x,t)},$$

where $U(\epsilon)$ indicates the shape of the pulse and

$$\epsilon = x - ct, \quad \phi(x,t) = -kx + wt.$$

The above transformation causes

$$i(2k\alpha_1 + c)\frac{dU(\epsilon)}{d\epsilon} + (-\alpha_1 + 4\alpha_4)\frac{d^2U(\epsilon)}{d\epsilon^2} + (k^2\alpha_1 + w + \alpha_5)U(\epsilon) - \alpha_2 U^3(\epsilon) - \alpha_3 U^5(\epsilon) = 0. \quad (5)$$

From Eq. (5), one can acquire

$$(2k\alpha_1 + c)\frac{dU(\epsilon)}{d\epsilon} = 0,$$

$$(-\alpha_1 + 4\alpha_4)\frac{d^2U(\epsilon)}{d\epsilon^2} + (k^2\alpha_1 + w + \alpha_5)U(\epsilon) - \alpha_2 U^3(\epsilon) - \alpha_3 U^5(\epsilon) = 0. \quad (6)$$

Due to the first equation, one can find the soliton speed as

$$c = -2k\alpha_1.$$

Now, considering Eq. (6) and the transformation $U(\epsilon) = \sqrt{V(\epsilon)}$ results in

$$(\alpha_1 - 4\alpha_4)\left(\frac{dV(\epsilon)}{d\epsilon}\right)^2 + (-2\alpha_1 + 8\alpha_4)\left(\frac{d^2V(\epsilon)}{d\epsilon^2}\right)V(\epsilon) + (4k^2\alpha_1 + 4w + 4\alpha_5)V^2(\epsilon) - 4\alpha_2 V^3(\epsilon) - 4\alpha_3 V^4(\epsilon) = 0. \quad (7)$$

Using the balance principle, from $\left(\frac{dV(\epsilon)}{d\epsilon}\right)^2$ and $V^4(\epsilon)$ in Eq. (7), it is found that

$$2N + 2 = 4N \Rightarrow N = 1. \quad (8)$$

4. The model and its solitons

In the current section, Kudryashov and exponential methods are applied to acquire optical solitons of the GL equation including the parabolic nonlinearity. Furthermore, the present section gives several numerical simulations to explore the influence of the coefficients of nonlinear terms on the dynamical features of the obtained optical solitons.

4.1. Employing the Kudryashov method

Based on Eqs. (2) and (8), the solution of Eq. (7) takes the following form

$$V(\epsilon) = a_0 + a_1 K(\epsilon), \quad a_1 \neq 0, \quad (9)$$

where a_0 and a_1 are unknown, and $K(\epsilon)$ has been defined in Section 2. After inserting Eq. (9) into Eq. (7), we reach a consistent nonlinear system as follows

$$12ABa_1^2\alpha_1 - 48ABa_1^2\alpha_4 - 4a_1^4\alpha_3 = 0,$$

$$16ABa_0a_1\alpha_1 - 64ABa_0a_1\alpha_4 - 16a_0a_1^3\alpha_3 - 4a_1^3\alpha_2 = 0,$$

$$4k^2a_1^2\alpha_1 - 24a_0^2a_1^2\alpha_3 - 12a_0a_1^2\alpha_2 + 4wa_1^2 - a_1^2\alpha_1 + 4a_1^2\alpha_4 + 4a_1^2\alpha_5 = 0,$$

$$8k^2a_0a_1\alpha_1 - 16a_0^3a_1\alpha_3 - 12a_0^2a_1\alpha_2 + 8wa_0a_1 - 2a_0a_1\alpha_1 + 8a_0a_1\alpha_4 + 8a_0a_1\alpha_5 = 0,$$

$$4k^2a_0^2\alpha_1 - 4a_0^4\alpha_3 - 4a_0^3\alpha_2 + 4wa_0^2 + 4a_0^2\alpha_5 = 0,$$

whose solution gives

Case 1:

$$a_0 = \frac{1}{4}\sqrt{4}\sqrt{\frac{1}{AB}}a_1,$$

$$w = -\frac{48ABk^2\alpha_4 + 4k^2a_1^2\alpha_3 + 12AB\alpha_5 + 5a_1^2\alpha_3}{12AB},$$

$$\alpha_1 = \frac{12AB\alpha_4 + a_1^2\alpha_3}{3AB},$$

$$\alpha_2 = -\frac{2a_1\alpha_3\sqrt{4}}{3AB\sqrt{\frac{1}{AB}}}.$$

Thus, the following optical soliton to the GL equation including the parabolic nonlinearity is derived

$$u_1(x,t) = \sqrt{\frac{1}{4}\sqrt{4}\sqrt{\frac{1}{AB}}a_1 + a_1\frac{1}{(A-B)\sinh(x+2k\alpha_1 t)+(A+B)\cosh(x+2k\alpha_1 t)}}$$
$$\times e^{i\left(-kx-\frac{48ABk^2\alpha_4+4k^2a_1^2\alpha_3+12AB\alpha_5+5a_1^2\alpha_3}{12AB}t\right)},$$

where

$$\alpha_1 = \frac{12AB\alpha_4 + a_1^2\alpha_3}{3AB},$$

$$\alpha_2 = -\frac{2a_1\alpha_3\sqrt{4}}{3AB\sqrt{\frac{1}{AB}}}.$$

Case 2:

$$a_0 = -\frac{1}{4}\sqrt{4}\sqrt{\frac{1}{AB}}a_1,$$

$$w = -\frac{48ABk^2\alpha_4 + 4k^2a_1^2\alpha_3 + 12AB\alpha_5 + 5a_1^2\alpha_3}{12AB},$$

$$\alpha_1 = \frac{12AB\alpha_4 + a_1^2\alpha_3}{3AB},$$

$$\alpha_2 = \frac{2a_1\alpha_3\sqrt{4}}{3AB\sqrt{\frac{1}{AB}}}.$$

Accordingly, the following optical soliton to the GL equation including the parabolic nonlinearity is acquired

$$u_2(x,t) = \sqrt{-\frac{1}{4}\sqrt{4}\sqrt{\frac{1}{AB}}a_1 + a_1\frac{1}{(A-B)\sinh(x+2k\alpha_1 t)+(A+B)\cosh(x+2k\alpha_1 t)}}$$
$$\times e^{i\left(-kx-\frac{48ABk^2\alpha_4+4k^2a_1^2\alpha_3+12AB\alpha_5+5a_1^2\alpha_3}{12AB}t\right)},$$

where

$$\alpha_1 = \frac{12AB\alpha_4+a_1^2\alpha_3}{3AB},$$

$$\alpha_2 = \frac{2a_1\alpha_3\sqrt{4}}{3AB\sqrt{\frac{1}{AB}}}.$$

Several numerical simulations are presented in Figure 1 to explore the influence of the coefficients of nonlinear terms on the dynamical features of $|u_1(x,t)|$. The following sets

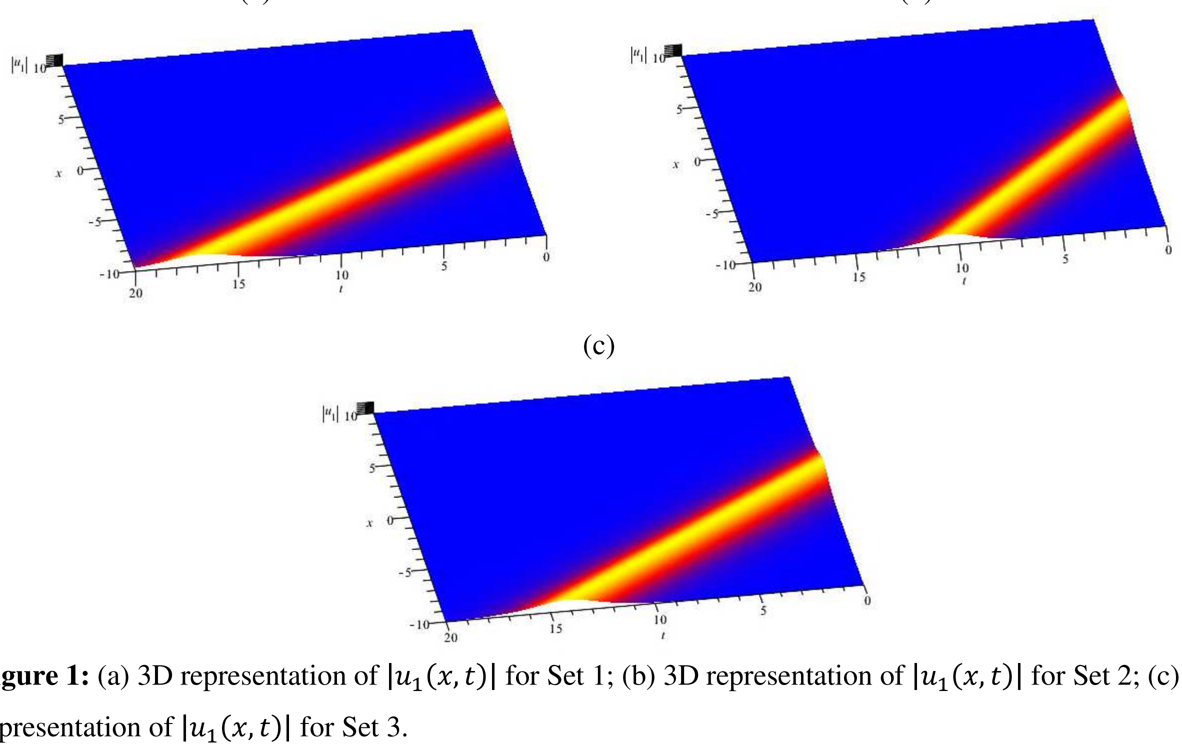
Set 1: $\{A = 0.1, B = 0.5, a_1 = 1, \alpha_3 = 1, \alpha_4 = 1, \alpha_5 = 1, k = 0.03\}$,

Set 2: $\{A = 0.1, B = 0.5, a_1 = 1, \alpha_3 = 2, \alpha_4 = 1, \alpha_5 = 1, k = 0.03\}$,

Set 3: $\{A = 0.1, B = 0.5, a_1 = 1, \alpha_3 = 1, \alpha_4 = 1.5, \alpha_5 = 1, k = 0.03\}$,

have been used to carry out this goal. A series of bright solitons are observed in Figure 1.

(a) (b)



(c)

Figure 1: (a) 3D representation of $|u_1(x,t)|$ for Set 1; (b) 3D representation of $|u_1(x,t)|$ for Set 2; (c) 3D representation of $|u_1(x,t)|$ for Set 3.

4.2. Employing the exponential method

Since $N \in \mathbb{N}$, we choose $N = 1$. Such a selection leads to

$$V(\epsilon) = \frac{a_0 + a_1 a^{\epsilon}}{b_0 + b_1 a^{\epsilon}}, \quad a_1 \neq 0, \quad b_1 \neq 0, \tag{10}$$

where a_0, a_1, b_0, and b_1 are unknown. After setting Eq. (10) in Eq. (7), the following consistent nonlinear system is acquired

$$4k^2 a_0^2 \alpha_1 b_0^2 + 4w a_0^2 b_0^2 - 4a_0^4 \alpha_3 - 4a_0^3 \alpha_2 b_0 + 4a_0^2 \alpha_5 b_0^2 = 0,$$

$$2(\ln(a))^2 a_0^2 \alpha_1 b_0 b_1 - 8(\ln(a))^2 a_0^2 \alpha_4 b_0 b_1 - 2(\ln(a))^2 a_0 a_1 \alpha_1 b_0^2 + 8(\ln(a))^2 a_0 a_1 \alpha_4 b_0^2 + 8k^2 a_0^2 \alpha_1 b_0 b_1 + 8k^2 a_0 a_1 \alpha_1 b_0^2 + 8w a_0^2 b_0 b_1 + 8w a_0 a_1 b_0^2 - 16a_0^3 a_1 \alpha_3 - 4a_0^3 \alpha_2 b_1 - 12a_0^2 a_1 \alpha_2 b_0 + 8a_0^2 \alpha_5 b_0 b_1 + 8a_0 a_1 \alpha_5 b_0^2 = 0,$$

$$-(\ln(a))^2 a_0^2 \alpha_1 b_1^2 + 4(\ln(a))^2 a_0^2 \alpha_4 b_1^2 + 2(\ln(a))^2 a_0 a_1 \alpha_1 b_0 b_1 - 8(\ln(a))^2 a_0 a_1 \alpha_4 b_0 b_1 - (\ln(a))^2 a_1^2 \alpha_1 b_0^2 + 4(\ln(a))^2 a_1^2 \alpha_4 b_0^2 + 4k^2 a_0^2 \alpha_1 b_1^2 + 16k^2 a_0 a_1 \alpha_1 b_0 b_1 + 4k^2 a_1^2 \alpha_1 b_0^2 + 4w a_0^2 b_1^2 + 16w a_0 a_1 b_0 b_1 + 4w a_1^2 b_0^2 - 24a_0^2 a_1^2 \alpha_3 - 12a_0^2 a_1 \alpha_2 b_1 + 4a_0^2 \alpha_5 b_1^2 - 12a_0 a_1^2 \alpha_2 b_0 + 16a_0 a_1 \alpha_5 b_0 b_1 + 4a_1^2 \alpha_5 b_0^2 = 0,$$

$$-2(\ln(a))^2 a_0 a_1 \alpha_1 b_1^2 + 8(\ln(a))^2 a_0 a_1 \alpha_4 b_1^2 + 2(\ln(a))^2 a_1^2 \alpha_1 b_0 b_1 - 8(\ln(a))^2 a_1^2 \alpha_4 b_0 b_1 + 8k^2 a_0 a_1 \alpha_1 b_1^2 + 8k^2 a_1^2 \alpha_1 b_0 b_1 + 8w a_0 a_1 b_1^2 + 8w a_1^2 b_0 b_1 - 16a_0 a_1^3 \alpha_3 - 12a_0 a_1^2 \alpha_2 b_1 + 8a_0 a_1 \alpha_5 b_1^2 - 4a_1^3 \alpha_2 b_0 + 8a_1^2 \alpha_5 b_0 b_1 = 0,$$

$$4k^2 a_1^2 \alpha_1 b_1^2 + 4w a_1^2 b_1^2 - 4a_1^4 \alpha_3 - 4a_1^3 \alpha_2 b_1 + 4a_1^2 \alpha_5 b_1^2 = 0,$$

whose solution gives

$$a_0 = 0,$$

$$w = -\frac{12(\ln(a))^2 k^2 \alpha_4 b_1^2 + (\ln(a))^2 a_1^2 \alpha_3 + 3\alpha_5 (\ln(a))^2 b_1^2 - 4k^2 a_1^2 \alpha_3}{3(\ln(a))^2 b_1^2},$$

$$\alpha_1 = \frac{4\left(3b_1^2 (\ln(a))^2 \alpha_4 - a_1^2 \alpha_3\right)}{3(\ln(a))^2 b_1^2},$$

$$\alpha_2 = -\frac{4a_1 \alpha_3}{3b_1}.$$

Consequently, the following optical soliton to the GL equation including the parabolic nonlinearity is obtained

$$u_3(x,t) = \sqrt{\frac{a_1 a^{x+2k\alpha_1 t}}{b_0 + b_1 a^{x+2k\alpha_1 t}}}\, e^{i\left(-kx - \frac{12(\ln(a))^2 k^2 \alpha_4 b_1^2 + (\ln(a))^2 a_1^2 \alpha_3 + 3\alpha_5 (\ln(a))^2 b_1^2 - 4k^2 a_1^2 \alpha_3}{3(\ln(a))^2 b_1^2} t\right)},$$

where

$$\alpha_1 = \frac{4\left(3b_1^2(\ln(a))^2\alpha_4 - a_1^2\alpha_3\right)}{3(\ln(a))^2 b_1^2},$$

$$\alpha_2 = -\frac{4a_1\alpha_3}{3b_1}.$$

Figure 2 presents some numerical simulations to show the influence of the coefficients of nonlinear terms on the dynamical features of $|u_3(x,t)|$. The following sets

Set 1: $\{a_1 = 1, b_0 = 1, b_1 = 1, \alpha_3 = 1, \alpha_4 = 1, \alpha_5 = 1, k = 0.05, a = 2.7\}$,

Set 2: $\{a_1 = 1, b_0 = 1, b_1 = 1, \alpha_3 = 2, \alpha_4 = 1, \alpha_5 = 1, k = 0.05, a = 2.7\}$,

Set 3: $\{a_1 = 1, b_0 = 1, b_1 = 1, \alpha_3 = 1, \alpha_4 = 1.5, \alpha_5 = 1, k = 0.05, a = 2.7\}$,

have been applied to achieve this aim. Several kink solitons are seen in Figure 2.

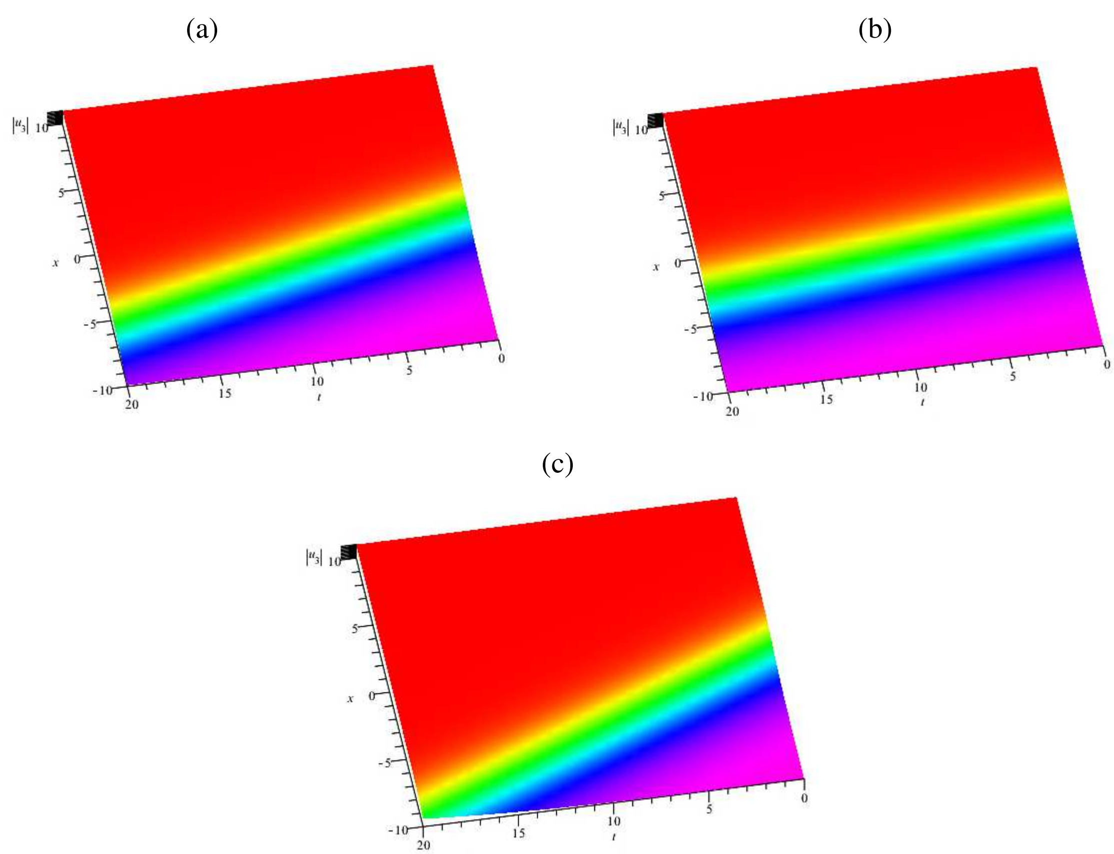


Figure 2: (a) 3D representation of $|u_3(x,t)|$ for Set 1; (b) 3D representation of $|u_3(x,t)|$ for Set 2; (c) 3D representation of $|u_3(x,t)|$ for Set 3.

5. Conclusion

In the present paper, the authors acquired optical solitons to the Ginzburg–Landau equation including the parabolic nonlinearity by employing the Kudryashov and exponential methods. As a result, a series of optical solitons, classified as bright and kink solitons, to the governing model was formally formalized. Some numerical simulations were considered to examine the influence of the coefficients of nonlinear terms on the dynamical features of the obtained optical solitons. The current study's findings proved the superior performance of Kudryashov and exponential methods in dealing with the Ginzburg–Landau equation including the parabolic nonlinearity. It is worth noting that the findings of the current research are novel and have not before been published in the literature.

Declaration of Competing Interest The authors declare no conflict of interest.

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