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Extractable quantum work from a two-mode Gaussian state in a noisy channel

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ABSTRACT

We study a Szilard engine based on a Gaussian state of a system consisting of two bosonic modes placed in a noisy channel. As the initial state of the system is taken an entangled squeezed thermal state, and the quantum work is extracted by performing a measurement on one of the two modes. We use the Markovian Kossakowski-Lindblad master equation for describing the time evolution of the open system and the quantum work definition based on the second order Rényi entropy to simulate the engine. We show that the extractable quantum work increases with the temperature of the reservoir and the squeezing between the modes, average numbers of thermal photons and frequencies of the modes. The work increases also with the strength of the measurement, attaining the maximal values in the case of a heterodyne detection. As well the extractable work is decreasing by increasing the squeezing parameter of the noisy channel and it oscillates with the phase of the squeezed thermal reservoir.

Introduction

The key problem of thermodynamics at its foundation was to design devices for work production using internal energy resources. The first phenomenological description of thermodynamical engines was provided by Sadi Carnot in 1824¹. On the basis of the first and second laws of thermodynamics it was deduced that any engine requires a heat flux and should be connected to a hot and a cold thermal reservoirs. According to Sadi Carnot, the maximal efficiency of heat to work conversion is determined by the temperatures of the hot and cold reservoirs and it is always lower than the efficiency of an ideal thermal machine².

The end of the XIX century was characterised by the fast gain of knowledge about the microscopical structure of matter. In particular, it was elaborated the kinetic molecular theory and made connection between the statistical physics and thermodynamics. Using the statistical physics approach it was shown the presence of fluctuations in temperature, density and other physical parameters in systems composed of a small amount of molecules³. If a fluctuation can be characterised by larger values of thermodynamical parameters than surrounding matter, then it stores a finite amount of energy^{4,5}. Scientific debate about the absolute character of the second law of thermodynamics and physics of fluctuations gave rise to a novel concept of hypothetical devices for energy fluctuation harvesting^{6,7}.

In 1929 Leo Szilard proposed a model of a single molecular engine connected to a single heat reservoir⁸. The principal component of a Szilard engine is a cavity with a single molecule inside. At first stage one introduces a mobile impenetrable membrane in the middle of the cavity. After that, a measurement is performed to determine in which part of the cavity the molecule is localised. Using the measurement result, a load is attached to the membrane to extract a work via an isothermal expansion at a constant temperature^{7,9}.

To put his engine functionality in agreement with the second law of thermodynamics, Szilard assumed that getting and using information about the molecule position implies energy costs¹⁰. He deduced that any information dependent machine requires energy for its functionality. This idea was developed by Landauer, who asserted that by erasing the information about the system one loses the ability to extract work from it. In addition, Landauer calculated the maximal value of extractable work per bit of information about the system structure. From the Landauer principle, it follows that any classical Szilard engine cannot produce work, because all extracted work will be used to write the measurement results and it will be dissipated as heat after erasing the memory¹¹. The discussions about the possibility of work extraction using a quantum Szilard are crucial for study of quantum thermodynamics¹².

Until the last decades of the XX century no experimental implementations of Szilard engine were possible, due to the weak development of micro-technologies and low precision of measurement techniques^{13,14}. However, with the fast development of

nanotechnologies^{15–17}, enhancement of ion traps technology and implementation of the first quantum computers¹⁸, the design of the Szilard engine became a practical challenge. Besides the classical schemes of Szilard engine, associated with Maxwell demon, there were proposed memoryless models, where the measurement is used only to localise the particle in its part of the box^{19,20}.

Another way to maximise the extractable work is by using quantum correlations²¹. According to Refs.^{22,23} two correlated Szilard engines allow to extract more work than a single engine. Likewise, in Ref.⁹ it is proven that a two particle quantum Szilard engine allows to extract a larger quantity of work than a classical one in the case of bosons, however less work is extracted in the case of fermions. These facts made attractive the study of bosonic multi-particle Szilard engines for nanoscale energy harvesting²⁴.

In this paper we describe a two-particle quantum Szilard engine and estimate the maximal work quantity that can be extracted if the initial state is partially destroyed during its temporal evolution. We simulate the engine by two bosonic modes placed in a noisy channel, characterised by temperature and squeezing parameters. The time evolution of the extractable work is presented as a function of the parameters characterising the bosonic modes and the squeezed thermal environment. In Sec. "Dynamics of two bosonic modes in a Gaussian noisy channel" we describe the dynamics of two bosonic modes in a Gaussian noisy channel, and in Sec. "Quantum work extraction" we describe the work extraction protocol and derive the expression of the extractable work²¹. Then in Sec. "Results and discussions" we describe and discuss the obtained results, and in Sec. "Conclusions" we present the main results and ideas of this manuscript.

Dynamics of two bosonic modes in a Gaussian noisy channel

We study the evolution of two bosonic modes in Gaussian noisy channels in the framework of theory of open quantum systems, by means of the Markovian Kossakovski-Lindblad master equation in the interaction picture for the density operator ρ in natural units^{25–28}:

$$\frac{d\rho}{dt} = \sum_{k=1}^2 \frac{\lambda}{2} \left\{ (N_k + 1) \mathcal{L}[\hat{a}_k] + N_k \mathcal{L}[\hat{a}_k^\dagger] - M_k^* \mathcal{D}[\hat{a}_k] - M_k \mathcal{D}[\hat{a}_k^\dagger] \right\} \rho, \quad (1)$$

with $\hat{a}_k^\dagger$ and $\hat{a}_k, k = 1, 2$, being the creation and annihilation operators of the two bosonic modes. λ is the overall damping rate, while N_k and $M_k, k = 1, 2$, represent the effective photon numbers and the squeezing parameters of the squeezed (phase sensitive) baths, respectively. At thermal equilibrium, i.e. for $M_k = 0$, the N_k is the average number of thermal photons in the reservoir. Lindblad superoperators are $\mathcal{L}[\hat{O}]\rho = 2\hat{O}\rho\hat{O}^\dagger - \rho\hat{O}^\dagger\hat{O} - \hat{O}^\dagger\hat{O}\rho$ and $\mathcal{D}[\hat{O}]\rho = 2\hat{O}\rho\hat{O} - \hat{O}\hat{O}\rho - \rho\hat{O}\hat{O}$. The positivity of the density matrix imposes the constraints $|M_k|^2 \leq N_k(N_k + 1)$.

The evolution imposed by the master equation preserves the Gaussian character of the states and the temporal evolution for the covariance matrix is the following^{28–30}:

$$\sigma(t) = e^{-\lambda t} \sigma(0) + (1 - e^{-\lambda t}) \sigma(\infty), \quad (2)$$

where $\sigma(0)$ is the covariance matrix of the initial Gaussian state and $\sigma(\infty)$ is the asymptotic covariance matrix (diffusion matrix), which is determined only by the bath parameters³¹:

$$\sigma(\infty) = \bigoplus_{k=1,2} \sigma_k(\infty), \quad (3)$$

with (we put $\hbar = 1$)

$$\sigma_k(\infty) = \begin{pmatrix} (\frac{1}{2} + N_k + M_{kR})/\omega_k & M_{kI} \\ M_{kI} & (\frac{1}{2} + N_k - M_{kR})\omega_k \end{pmatrix}, \quad (4)$$

where M_{kR} and $M_{kI}, k = 1, 2$, denote the real and imaginary parts of M_k , respectively, with

$$N_k = n_{th,k}(\cosh^2 R + \sinh^2 R) + \sinh^2 R, \quad (5)$$

$$M_k = -(2n_{th,k} + 1) \cosh R \sinh R \exp i\phi, \quad (6)$$

$n_{th,k} = \frac{1}{2} (\coth(\frac{\omega_k}{2T}) - 1), k = 1, 2$, are the average numbers of thermal photons and ω_k are the frequencies of the two modes. For the two reservoirs we take the same temperature T , squeezing parameter R and squeezing phase ϕ .

Quantum work extraction

We consider a bipartite system AB evolving in time in Gaussian noisy channels, as described in the previous Section, in a Gaussian state of the modes $\hat{a} \equiv \hat{a}_1$ and $\hat{b} \equiv \hat{a}_2$, characterised by a covariance matrix of the block form^{32,33}:

$$\sigma(t) = \begin{pmatrix} \sigma_a(t) & \sigma_{ab}(t) \\ \sigma_{ab}^T(t) & \sigma_b(t) \end{pmatrix}, \quad (7)$$

where σ_a and σ_b are the covariance matrices of the two individual modes, and σ_{ab} contains the correlations between the modes.

The first moments of the canonical variables of the system are set to zero, since they are irrelevant for our purposes. We consider Gaussian measurements, performed by party B on his mode, of the form $\pi_b(X) = \pi^{-1} D_b(X) \rho^{\pi_b} D_b^\dagger(X)$ ²¹, where $D_b(X) = \exp(X\hat{b}^\dagger - X^*\hat{b})$ represents the displacement operator and ρ^{π_b} is the density operator describing a pure Gaussian state whose covariance matrix is $\gamma^{\pi_b} = R(\theta) \text{diag}(\mu/2, \mu^{-1}/2) R(\theta)^T$, with μ taking values in the interval $[0, \infty)$ and the rotation matrix expressed by using the y-Pauli matrix σ_y in the following way: $R(\theta) = \cos \theta I - i \sin \theta \sigma_y$. $\mu = 0$ corresponds to a homodyne measurement and $\mu = 1$ corresponds to a heterodyne one.

The conditional state of the mode $\hat{a}$ does not depend on the measurement result $\pi_b(X)$, i.e. $\sigma_{a|X}^{\pi_b} \equiv \sigma_a^{\pi_b}$. Its covariance matrix expression is given by^{21,34}:

$$\sigma_a^{\pi_b} = \sigma_a - \sigma_{ab}(\sigma_b + \gamma^{\pi_b})^{-1} \sigma_{ab}^T. \quad (8)$$

After the measurement the reduced state of the first mode $\hat{a}$ is out of equilibrium and part A can extract work from the thermal bath by letting his state to diffuse in the phase space²¹. The system prepared in the post-measurement state is in contact with the thermal reservoir and, for simplicity, we take the reference state as the time-local state $\sigma_a(t)$, therefore the extracted work is due to the measurement backreaction²¹. As the state is independent of the outcome, its average entropy is $\int dX p_X S(\sigma_{a|X}^{\pi_b}) = S(\sigma_a^{\pi_b})$. Then, the extractable work can be defined by²¹:

$$W = k_B T [S(\sigma_a) - S(\sigma_a^{\pi_b})], \quad (9)$$

where k_B is Boltzmann constant and T the temperature of the thermal reservoir.

To quantify the entropy of the conditional state (8), we use the Rényi entropy of order 2, $S_2(\rho) = -\ln \text{Tr}[\rho^2]$ ³⁵, which in case of Gaussian states becomes a fully legitimate entropy functional given by the expression

$$S_2(\sigma_{ab}) = \frac{1}{2} \ln(\det \sigma_{ab}). \quad (10)$$

Then the expression of the work (9) becomes^{21,36}

$$W = \frac{k_B T}{2} \ln \left(\frac{\det \sigma_a}{\det \sigma_a^{\pi_b}} \right). \quad (11)$$

We emphasize that in our case work W has to be addressed as the output of a suitable work-extraction protocol. The existence of a nonzero W proves the presence of classical correlations between the two parts A and B²¹.

Results and discussions

As the initial state we choose a squeezed thermal state given by:

$$\sigma_{STS} = \begin{pmatrix} a & 0 & c & 0 \\ 0 & a & 0 & -c \\ c & 0 & b & 0 \\ 0 & -c & 0 & b \end{pmatrix}, \quad (12)$$

with the matrix elements:

$$a = n_1 \cosh^2 r + n_2 \sinh^2 r + \frac{1}{2} \cosh 2r, \quad (13)$$

$$b = n_1 \sinh^2 r + n_2 \cosh^2 r + \frac{1}{2} \cosh 2r, \quad (14)$$

$$c = \frac{1}{2} (n_1 + n_2 + 1) \sinh 2r, \quad (15)$$

where n_1 and n_2 denote the average thermal photon numbers of the two modes and r represents the squeezing parameter. For $n_1 = n_2 = 0$ the squeezed thermal state becomes a squeezed vacuum state.

Our purpose is to determine the evolution in time of the work extracted by two local agents, or demons, A and B, who share a bipartite Gaussian state of modes $\hat{a}$ and $\hat{b}$. They check how much work party A can extract from a heat bath when only local Gaussian measurements are performed. In order to run an information engine, party A does not need to perform a measurement on his system and extract work from the recorded outcomes: A can exploit the backreaction induced by the Gaussian measurement π_b performed by B on the joint state and A extracts mechanical work by letting its conditional state $\sigma_a^{\pi_b}$ expand, that is let its state to thermalise. This expansion can be converted into mechanical work²¹.

We consider general two-mode states in the block form (Eq. (7)), with the initial covariance matrix given by Eq. (12), corresponding to squeezed thermal states. The extractable work $W(\theta)$ depends on the measurement angle, therefore we shall consider the average work value $W = \frac{1}{2\pi} \int_0^{2\pi} d\theta W(\theta)$. In all the following simulations we set the Boltzmann constant $k_B = 1$ and the dissipation constant is taken $\lambda = 0.1$.

First we consider the resonant case, when both modes have the same frequency and the initial state is a squeezed vacuum state. In Fig. 1 we illustrate the dependence of the extractable work by the Szilard engine cycle on time, squeezing between the modes and bath temperature. The extractable work is averaged over the measurement angle. These parameters have a strong influence on the quantity of the extracted work. From Figs. 1 (a) and (c) one can see that the extractable work increases with the squeezing between the modes and decreases in time. The engine has the best performance for strongly squeezed vacuum states and small times of evolution. In Figs. 1 (b) and (d) it is represented the influence of bath temperature on the engine performance. For the whole range of the evolution times the extractable work averaged over the measurement angle increases with temperature. However the best results are noticed for times very closed to the initial time. For long times, the extractable work decreases, due to the interaction with the thermal bath.

From Fig. 2 (a) we see that the extractable work is very weakly influenced by the phase of the bath and from Fig. 2 (b) we notice that it slowly decreases by increasing the squeezing parameter of the bath. As already noticed, the extractable work decreases in time. In Fig. 2 (c) we illustrate the influence of the strength of measurement on the extractable work. We observe that the extractable work averaged over the rotational angle increases with the strength of the measurement and it reaches a maximum value for $\mu = 1$, corresponding to a heterodyne detection, in accordance with the study²¹. Therefore, the heterodyne detection is optimal for work extraction, while the homodyne detection realises a minimum device performance. The extractable work is monotonically increasing for $\mu \in [0, 1]$ and monotonically decreasing for $\mu \geq 1$. We studied also the influence of the thermal photon number on the extractable work. From Fig. 2 (d) we notice that this quantity increases with the thermal photon number.

In the following we study the influence of the frequencies of the modes on the extractable work. In Fig. 3 (a) we present the dependence of the extractable work averaged over the rotational angle on the frequency of the first mode and squeezing between the modes at some finite moment of time. We observe that for a weak squeezing the extractable work practically does not depend on the frequency. However, for a stronger squeezing the extractable work increases with the frequency and then it saturates. In Fig. 3 (b) we show the dependence of the extractable work on time and the frequency of the first mode. Like previously, the extractable work quantity decreases in time and increases with the frequency. From Fig. 3 (c) we observe again that the extractable work has a weak oscillatory dependence on the phase of the squeezed thermal bath, with a period 2π . Finally, in Fig. 3 (d) we illustrate the dependence of the extractable work on the environment squeezing and the frequency of the first mode, at some definite moment of time. Like previously, the quantity of extractable work decreases with the squeezing of the reservoir, but it increases with the frequency of the first mode, especially for relatively small values of this frequency, after which it reaches a plateau.

Conclusions

By using as a Szilard engine a system composed of two entangled bosonic modes in a noisy channel, we investigated the evolution in time of the extractable work, as a function of the squeezing between the two modes, frequencies of the modes, their average numbers of thermal photons, temperature, squeezing and phase of the squeezed thermal reservoir, and strength of the measurement. The studies have been performed based on the covariance matrix formalism.

The main topic studied in this article is the evolution of maximal extracted work in the time prior to the measurement. We show that the maximal extracted work decreases with the evolution time for all range of discussed parameters.

Moreover, we have shown that the quantity of extractable quantum work increases with the temperature of the reservoir and the squeezing between the modes, average numbers of thermal photons and frequencies of the modes. The work increases also with the strength of the measurement, attaining the maximal values in the case of a heterodyne detection. At the same time the extractable work is decreasing by increasing the squeezing parameter of the noisy channel and it oscillates with the phase of the squeezed thermal reservoir. The obtained results might find practical applications in correlation detection in the system, such as quantum steering and entanglement.

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Author contributions statement

All authors contributed equally to this work.

Additional information

The authors declare no competing interests.

Figure 1. Evolution of the extractable work W averaged over the detection angle θ with: (a) squeezing r between modes at fixed moments of time $t = 0$ (red), $t = 1$ (green), $t = 2$ (blue), for $T = 0.1$, $\phi = \frac{\pi}{4}$, $\mu = 0$; (b) time t for temperatures $T = 0.1$ (red), $T = 1$ (green), $T = 2$ (blue), for $r = 1.8$, $\phi = \frac{\pi}{4}$, $\mu = 0$; (c) time and squeezing between the modes, for $T = 3$, $\phi = 0$, $\mu = 1$; (d) time t and temperature of the reservoir T , for $r = 1.8$, $\phi = \frac{\pi}{4}$, $\mu = 0$. We consider the resonant case with $\omega_1 = \omega_2 = 1$, an initial squeezed vacuum state ($n_1 = n_2 = 0$) and $R = 0.2$. This figure was obtained using Wolfram Mathematica 11.0.

Figure 2. Evolution of the extractable work W averaged over the detection angle θ with: (a) time t and phase ϕ of the channel, for $n_1 = n_2 = 0$, $R = 0.2$, $\mu = 0$; (b) time and squeezing R of the bath, for $n_1 = n_2 = 0$, $\phi = 0$, $\mu = 1$; (c) time and strength of the measurement μ , for $n_1 = n_2 = 0$, $R = 0.2$, $\phi = 0$; (d) time and the thermal photon number of the first mode n_1 , for $R = 0.2$, $\phi = \frac{\pi}{4}$, $n_2 = 0$, $\mu = 0$. We consider the resonant case with $\omega_1 = \omega_2 = 1$, $r = 1.8$ and $T = 3$. This figure was obtained using Wolfram Mathematica 11.0.

Figure 3. Dependence of the extractable work W averaged over the rotational angle θ on: (a) squeezing between modes r and frequency of the first mode ω_1 , for $t = 2$, $R = 0.2$, $\phi = \frac{\pi}{4}$; (b) time t and frequency of the first mode, for $r = 1.8$, $R = 0.2$, $\phi = \frac{\pi}{4}$; (c) phase ϕ of the reservoir and frequency of the first mode, for $t = 2$, $r = 1.8$, $R = 0.2$; (d) squeezing parameter R of the bath and frequency of the first mode, for $t = 2$, $r = 1.8$, $\phi = \frac{\pi}{4}$. We consider an initial squeezed vacuum state ($n_1 = n_2 = 0$), $\omega_2 = 1$, $\mu = 0$, $T = 1$. This figure was obtained using Wolfram Mathematica 11.0.