Controller Design for a Delta Robot Using Lagrangian Multipliers

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Controller design for a Delta robot using Lagrangian multipliers

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The novelty of this research is that the study utilizes Lagrangian multipliers for an articulated close-chain robot mechanism as time-varying states to augment an energy-based Lagrangian dynamic model, and the strategy facilitates the system computations while satisfying the forward/inverse dynamics and mechanism constraints simultaneously for the close-chained robot. Then, this article develops a controller based upon the augmented dynamics model for the Delta robot mechanism and drives its moving platform to track prescribed trajectories in space. As a result, the simulation results validate the effectiveness of the augmented system model in undertaking complex dynamics of close-chained robot mechanisms.

The dynamics of articulated closed-chain mechanisms, such as Delta robots, are changeling to undertake due to their complexity in mechanism constraints. This research uses Lagrangian multipliers in an energy-based approach as augmented time-varying states and effectively develops a controller to track prescribed trajectories in space.

Nowadays, Delta robots handle various objects because their parallel or articulated design facilitates the mechanisms to move rapidly and accurately, and their robust structures assist deployments for various highspeed tasks in the manufacturing and food processing industries. Delta robot structures are complex electromechanical systems due to their parallel/articulated mechanisms and nonlinear characteristics. Conventional controller design usually attempts various structural geometries, mechanism materials, and controller means to achieve acceptable Delta robot performances, such as stability and accuracy. For prescribing acceptable tracking error bounds for a Delta robot manipulator, some researchers developed hybrid nonlinear controllers to asymptotically converge the tracking errors to zero[1]. Others proposed nonlinear PD (proportional-derivative) controllers to manage coupling effects and external disturbances for high-precision trajectory tracking[2].

Researchers typically investigate Delta robot performances by studying forward/inverse kinematics, forward/inverse dynamics, energy-based analyses, and other theoretical approaches[3]. For example, researchers derived an energy-based dynamic model using kinematic relations for the Delta robot mechanisms and solved the inverse dynamics equations to investigate different serial-mechanism influences[4]. Other researchers transformed a Delta highspeed manipulator model into a SimMechanics development platform, and that study simulated the manipulator's motion to design the corresponding controllers[5].

Since the last decade, applications in the automation field have demanded high-speed positioning accuracy for Delta robot manipulators, and disturbance rejections and error compensations have become critical issues[6]. Some researchers proposed sliding mode control approaches based on Delta robot dynamics, and the object is to track given reference trajectories with uncertain model parameters for that manipulator[7]. By employing kinematics and dynamics with feedback for a Delta robot manipulator, others presented simple model-based control schemes to evaluate applied driving torques, and their schemes compensated the errors to fulfill the closed-loop control requirements[8]. Besides, other researchers developed nonlinear controllers using structural optimization methods and symbolic adaptation algorithms to track prescribed trajectories[9].

Delta robot applications also demand compact mechanisms with lightweight controllers for the industry[10], and embedded electronics using wireless interfacing become mandatory. Some researchers utilized an FPGA (Field Programmable Gate Array) device as dedicated hardware to provide a closed-loop servo control system, which exercises a Delta robot's kinematics equations to perform fast and accurate positioning[11]. In addition, other researchers designed two controllers to ensure stability and performance of a Delta robot closed-loop system, and that study embedded the control algorithms into a myRIO hardware device through the LabVIEW deployments[12].

Researchers recently employed an adaptive neural network controller for a Delta robot to track prescribed trajectories, and that study implemented an adaptive artificial neural network for online training[13]. In addition, other researchers applied a model-free iterative learning control strategy (ILC) to a Delta robot manipulator, which subjects to external disturbances for tracking nonrepetitive trajectories[14].

Literatures typically formulate equations of motion for simple multi-body systems using the energy-based Lagrangian formulation, and employing generalized coordinate variables is sufficient to describe the dynamics of the closed systems completely[15,16]. For example, researchers established a singularity-free dynamic model using an augmented Lagrangian or Hamiltonian formulation to derive the inverse dynamic equations for a Delta robot within an object-oriented modeling framework [17]. Others also presented a closed-form dynamics model using a Lagrangian formulation for a Delta mechanism in verifications[18].

Investigating physical cooperation with multiple manipulators also needs to invite additional Lagrangian multipliers to satisfy constraint conditions[19]. Researchers developed a control scheme using a constrained Lagrangian dynamics and Lagrangian multipliers formulation for cooperative mechanisms, and the exploited controller assumes both motion and force demands throughout the derived inverse dynamics models[20]. Similarly, researchers formulated inverse kinematics with position and Jacobian analyses for a Delta robot and analyzed its inverse dynamics with Lagrangian equations to satisfy constraint conditions[21,22].

Note that commercial Delta parallel or articulated robots achieve more than 10m/s velocity and 15g acceleration. Therefore, accuracy and fast dynamics calculation are essential in torque control for a Delta robot manipulator in high-speed applications. Hence, some researchers presented studies that simplified dynamics equation matches its dynamic behaviors very well with ADAMS modeling and simulations[23].

With the relevant literature reports, this paper systematically reformulates the equations of motions for a Delta robot, and the research develops a novel approach to design a controller for evaluating the Delta robot performances. Then, this paper presents a simplified system modeling for the Delta robot complex dynamics, in which the model employs the Lagrangian equations for the Delta mechanism and manages the inverse dynamic equations and mechanical constraint equations. Finally, the research utilizes the developed Delta robot dynamics model to apply PID weight parameters and torque control techniques for evaluating the controllers' robust performance. The strategy employs time-varying Lagrangian multipliers iteratively to satisfy the mechanical constraints and realize the tracking objects, while the dynamics model is iteratively refurnishing the time-varying controller. As a result, the developed approach verifies the controller's effectiveness in tracking given circular trajectories in space.

This paper organizes the article as section one for introduction, section two for system descriptions on the Delta robot mechanism and augmented dynamics model formulation using the Lagrangian multipliers, section three for a nonlinear tracking controller development with the Lagrangian multipliers embedded, section four for simulations and discussions using the augmented dynamics model, and finally section 5 for conclusions of the study.

Method

This paper builds a Delta robot model instead of an actual prototype by incorporating the Lagrangian multipliers into the controller development. It conducts various analyses and diverse designs to investigate the Delta robot's kinematics and dynamics' physical limits.

Delta Robot mechanisms. The Delta robot comprises a fixed base, three identical driving links, three parallelograms branched chains, and a moving platform. The mechanism employs hinges to connect the driving links and the driven branch chains. Another set of hinges connect the driven branch chains and the moving platform. Three motors fixed on the upper base drive their corresponding driving links and control the input angles θ_1, θ_2, and θ_3, respectively. Figure 1 illustrates the articulated Delta robot mechanism.

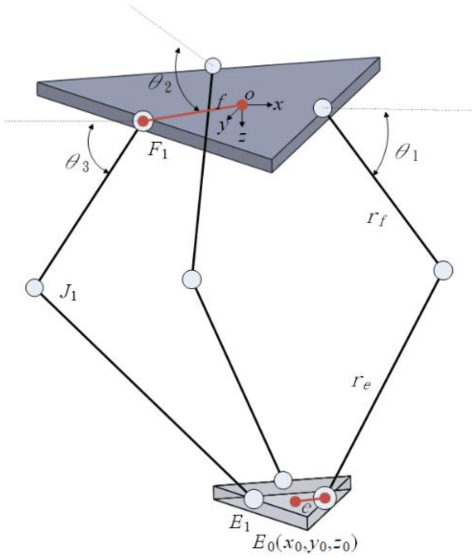


Figure 1. Delta robot mechanism illustration.

r_f : the length of the driving link

θ_i : the angular displacement of the i^{th} driving link

r_e : the length of the driven link

f : the distance from O to F_i (the radius of the fixed platform)

e : the distance from E_o to E_i (the radius of the movable platform)

m_a : the mass of the driving link

m_b : the mass of the driven link

m_p : the mass of the movable platform

$E_0(x_0, y_0, z_0)$: coordinates of the movable platform

Precisely, the Delta robot mechanism in Fig. 1 consists of three subsystems: a movable platform, a fixed platform, and a set of two-link mechanisms. A driving link and a driven link form one two-link chain mechanism. Three independent DC brushless motors on the fixed platform drive the three driving links, respectively, with their associated three-driven links, as shown in Fig. 2[24]. Hence, the three-driven links cooperatively displace the movable platform. For investigating the movable platform's motions, the research assumes: (i) the movable platform displaces smoothly; (ii) no slippage occurs between the driving links and the corresponding driven links. In the following section, this paper uses notations to formulate constraint equations and dynamic equations regarding the Delta robot.

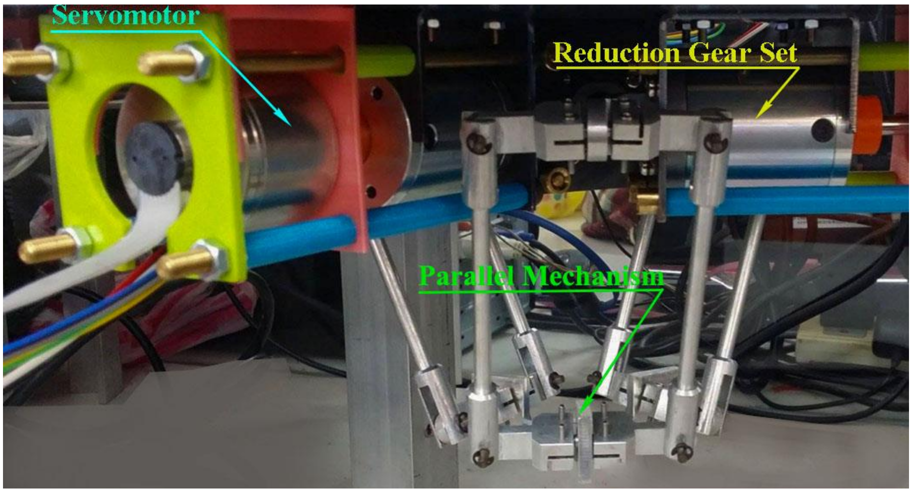


Figure 2. Delta robot prototype mechanism.

Equations of motion for the Delta robot. Based upon Lagrange's equations of the first type, this work employs three generalized coordinates, p_x, p_y, and p_z, for the movable platform and systematically formulates equations of motion for the Delta robot[1]. Together with three angular displacements, θ_1, θ_2, and θ_3 of the three corresponding driving links, this paper uses a generalized coordinate vector, $\boldsymbol{q}$, as in equation (1) to describe the overall Delta robot states in motion:

$$\boldsymbol{q} = [\theta_1, \theta_2, \theta_3, p_x, p_y, p_z]^T. \tag{1}$$

Express the moving platform kinetic energy as

$$K_p = \frac{1}{2} m_p (\dot{p}_x^2 + \dot{p}_y^2 + \dot{p}_z^2). \tag{2}$$

Likewise, express the kinetic energy for one of the driving links as

$$K_{ai} = \frac{1}{2} (\frac{1}{3} m_a r_f^2) \dot{\theta}_f^2. \tag{3}$$

Similarly, write the kinetic energy for one of the driven links as

$$K_{bi} = \frac{1}{2} m_b (\dot{p}_x^2 + \dot{p}_y^2 + \dot{p}_z^2) + \frac{1}{2} m_b r_f^2 \dot{\theta}_i^2. \tag{4}$$

Consequently, the articulated close-chain manipulator total kinetic energy is

$$K = K_p + \sum_{i=1}^{3} (K_{ai} + K_{bi}). \tag{5}$$

The moving platform potential energy is

$$U_p = m_p g_p p_z. \tag{6}$$

The potential energy for one of the driving links is

$$U_{ai} = \frac{1}{2} m_a g_c r_f \sin\theta_i. \tag{7}$$

Similarly, the potential energy for one of the driven links is

$$U_{bi} = m_b g_c (p_z + r_f \sin\theta_i). \tag{8}$$

As references to the upper fixed x-y plane, the total potential energy of the movable linkages of the Delta Robot is

$$U = U_p + \sum_{i=1}^{3} (U_{ai} + U_{bi}). \tag{9}$$

Note that defining Lagrangian as $L = K - U$, it is simple to write the Lagrange's equations of the first type as

$$\frac{d}{dt}(\frac{\partial L}{\partial \dot{q}_j}) - \frac{\partial L}{\partial q_j} = Q_j + \sum_{i=1}^{k} \lambda_j \frac{\partial \Gamma_i}{\partial q_j} \quad for \quad j = 1 \quad to \quad n. \tag{10}$$

In equation (10), Γ_i denotes the i^{th} constraint function, k is the number of constraint functions, λ_i is the i^{th} Lagrangian multiplier, n is the number of coordinates, and Q_i

represents the i^{th} generalized force. Consequently, equation (10) results in n equations of motions in the following general form:

$$\mathbf{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \mathbf{C}(\boldsymbol{q}, \dot{\boldsymbol{q}}) + \mathbf{G}(\boldsymbol{q}) + \mathbf{J}^{\boldsymbol{T}}(\boldsymbol{q})\boldsymbol{\lambda} = \mathbf{Q}. \tag{11}$$

where $\boldsymbol{\lambda}$ is a vector consisting of the Lagrangian multipliers.

For deriving the Lagrangian multipliers, rewrite the above equation as:

$$2\sum_{i=1}^{3} \lambda_i (p_x + e\cos\varphi_i - f\cos\varphi_i - r_f\cos\varphi_i\cos\theta_i) = (m_p + 3m_b)\ddot{p}_x. \tag{12}$$

$$2\sum_{i=1}^{3} \lambda_i (p_y + e\sin\varphi_i - f\sin\varphi_i - r_f\sin\varphi_i\cos\theta_i) = (m_p + 3m_b)\ddot{p}_y. \tag{13}$$

$$2\sum_{i=1}^{3} \lambda_i (p_z - r_f\sin\theta_i) = (m_p + 3m_b)\ddot{p}_z + (m_p + 3m_b)g_c. \tag{14}$$

The constraint Jacobian matrix $\mathbf{J}$ is a $k \times n$ matrix, which consists of partial derivatives of the constraint equations concerning the generalized coordinates[25].

Constraint equations. Because constraints exist within the Delta robot mechanism, the distances between joints J and E, $\overline{\boldsymbol{J}_i\boldsymbol{E}_i}$, must be equal to the lengths of the corresponding driven links, $\boldsymbol{r_e}$,

$$\begin{aligned} \Gamma_i &= \overline{J_iE_i}^2 - r_e^2 = (p_x + e\cos\varphi_i - f\cos\varphi_i - r_f\cos\varphi_i\cos\theta_i)^2 \\ &\quad + (p_y + e\sin\varphi_i - f\sin\varphi_i - r_f\sin\varphi_i\cos\theta_i)^2 + (p_z - r_f\sin\theta_i)^2 \\ &\quad - r_e^2 \\ &= 0 \qquad f \\ &= 1,\ 2,\ and\ 3. \end{aligned} \tag{15}$$

Differentiating equation (15) to time once obtains

$$\mathbf{J}\dot{\boldsymbol{q}} = \mathbf{0}. \tag{16}$$

Similarly, differentiating equation (16) twice obtains

$$\mathbf{J}\ddot{\boldsymbol{q}} + \dot{\mathbf{J}}\dot{\boldsymbol{q}} = \mathbf{0}. \tag{17}$$

Combining equations (11) and (17) gives the equations of motion with associated constraint equations for the Delta robot mechanism:

$$\begin{bmatrix} \mathbf{M}(\boldsymbol{q}) & \mathbf{J}^{\mathrm{T}}(\boldsymbol{q}) \\ \mathbf{J}(\boldsymbol{q}) & \mathbf{0} \end{bmatrix} \begin{bmatrix} \ddot{\boldsymbol{q}} \\ \ddot{\boldsymbol{\lambda}} \end{bmatrix} = \begin{bmatrix} \mathbf{Q} - \mathbf{C}(\boldsymbol{q}, \dot{\boldsymbol{q}}) - \mathbf{G}(\boldsymbol{q}) \\ -\dot{\mathbf{J}}\dot{\boldsymbol{q}} \end{bmatrix}. \tag{18}$$

Note that it is achievable now to derive the Lagrangian multipliers in their more straightforward form:

$$\begin{bmatrix} \ddot{\boldsymbol{q}} \\ \ddot{\boldsymbol{\lambda}} \end{bmatrix} = \begin{bmatrix} \mathbf{M}(\boldsymbol{q}) & \mathbf{J}^{\mathrm{T}}(\boldsymbol{q}) \\ \mathbf{J}(\boldsymbol{q}) & \mathbf{0} \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{Q} - \mathbf{C}(\boldsymbol{q}, \dot{\boldsymbol{q}}) + \mathbf{G}(\boldsymbol{q}) \\ -\dot{\mathbf{J}}\dot{\boldsymbol{q}} \end{bmatrix}. \tag{19}$$

With equation (19) and a given set of initial conditions, $\boldsymbol{q}(0)$ and $\dot{\boldsymbol{q}}(0)$, it is ready to simulate the Delta robot motions to investigate the movable platform's motions.

Substitution equations (5) and (9) into equation (10) yields a set of three 2nd-order equations, as shown in equations (20), (21), and (22), in which six state variables are the three Lagrangian multipliers, λ_i, for i = 1 to 3, and the three motor torques, Q_j, for j = 4 to 6.

$$\begin{aligned} \tau_1 &= (\frac{1}{3}m_a r_f^2 + m_b r_f^2)\ddot{\theta}_1 + (\frac{1}{2}m_a + m_b)g_c r_f\cos\theta_1 \\ &\quad -2r_f\lambda_1\left[(p_x\cos\varphi_1 + p_y\sin\varphi_1 + e - f)\sin\theta_1 - p_z\cos\theta_1)\right]. \end{aligned} \tag{20}$$

$$\begin{aligned} \tau_2 &= (\frac{1}{3}m_a r_f^2 + m_b r_f^2)\ddot{\theta}_2 + (\frac{1}{2}m_a + m_b)g_c r_f\cos\theta_2 \\ &\quad -2r_f\lambda_2\left[(p_x\cos\varphi_2 + p_y\sin\varphi_2 + e - f)\sin\theta_2 - p_z\cos\theta_2)\right]. \end{aligned} \tag{21}$$

$$\begin{aligned} \tau_3 &= (\frac{1}{3}m_a r_f^2 + m_b r_f^2)\ddot{\theta}_3 + (\frac{1}{2}m_a + m_b)g_c r_f\cos\theta_3 \\ &\quad -2r_f\lambda_3\left[(p_x\cos\varphi_3 + p_y\sin\varphi_3 + e - f)\sin\theta_3 - p_z\cos\theta_3)\right]. \end{aligned} \tag{22}$$

Solving equations (20), (21), and (22) obtain the displacements of the Delta robot links with given driving torques, and specifying the movable platform motions realize the torque controllers to track its prescribed targets.

Model and controller verifications. For examining the controller performance using the developed model for the Delta robot mechanism, this research specifies a circular trajectory in the spatial workspace for the controlled movable platform to follow:

$$\begin{aligned} x &= -0.01\, sin(4\pi t) \\ y &= \quad 0.01\, cos(4\pi t) \\ z &= -0.1 + 0.01\, sin(2\pi t) \end{aligned} \tag{23}$$

where t denotes the time variable and x, y and z are centre coordinates (mm) of the movable platform. Table 1 lists geometries and material properties of the Delta Robot mechanism for simulations.

Parameter	Value	Unit
Driving link mass: m_a	11.1	g
Driven link mass: m_b	9.3	g
Movable platform mass: m_p	22.3	g
Fixed base radius: f	0.02	m
Movable platform radius: e	0.03	m
Driving link length: r_f	0.04	m
Driven link length: r_e	0.10	m

Table 1. Delta Robot mechanism geometries and material properties

Conventionally, nonholonomic and holonomic constraints existing in system models elaborate designing their controllers and performing simulations. This research proposed a tracking control strategy using the Lagrangian multipliers for Delta robots, and the controller implements a systematic approach to apply the schemed strategy during trajectory tracking. In the following section, this study confirms that the Lagrangian formulation-based strategy with associated multipliers achieves an effective tracking controller for Delta robot mechanisms.

Tuned PID controller design. Figure 3 illustrates a simple scheme that controls the Delta robot on the controller design platform using a built-in PID module. The simulation is time-consuming because it is hard to tune the parameters (*P*, *I*, and *D*) for the PID module. However, an automatic tuning process in the PID module obtains appropriate parameters, and the Delta robot's movable platform fast tracks a prescribed trajectory.

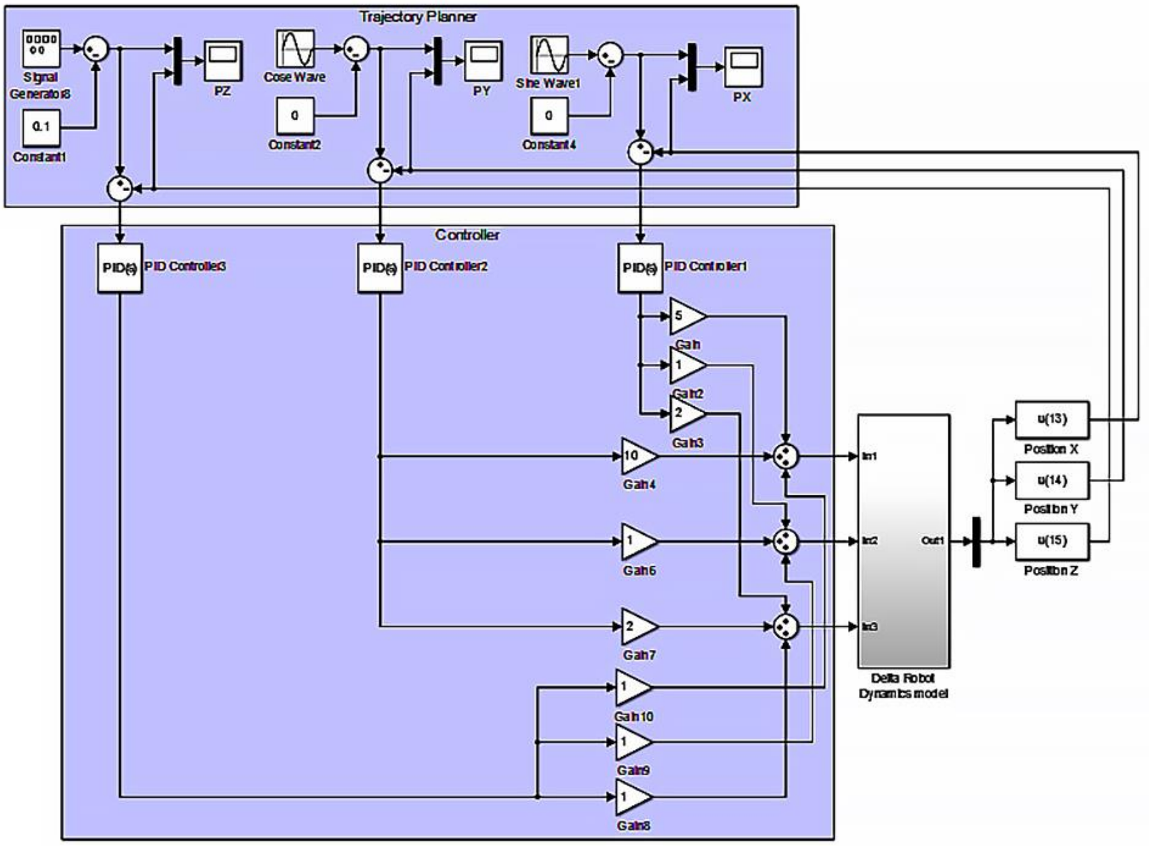


Figure 3. Scheme to control the Delta robot using a built-in PID module.

Through adaptive adjustment of PID parameters for the dynamic Delta robot model in Fig. 3, this study uses the obtained PID parameters as the reference weights for subsequent control strategy to shorten the simulation and verification time. Then, the system imports the parameters into the PID control scheme, as shown in Fig. 4.

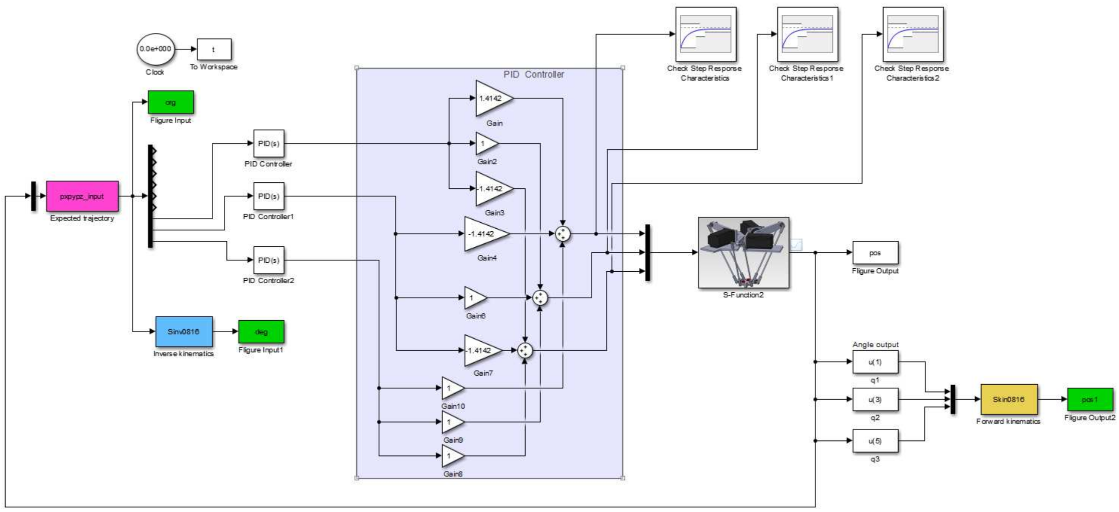


Figure 4. The scheme controls the moving platform to track a prescribed trajectory.

Using the developed control scheme with the tuned parameters, Fig. 4 illustrates how the study designs the control strategy, aiming to track a prescribed trajectory, and Fig. 5 illustrates the block diagrams of the tracking system with a reformatted S-function coding approach.

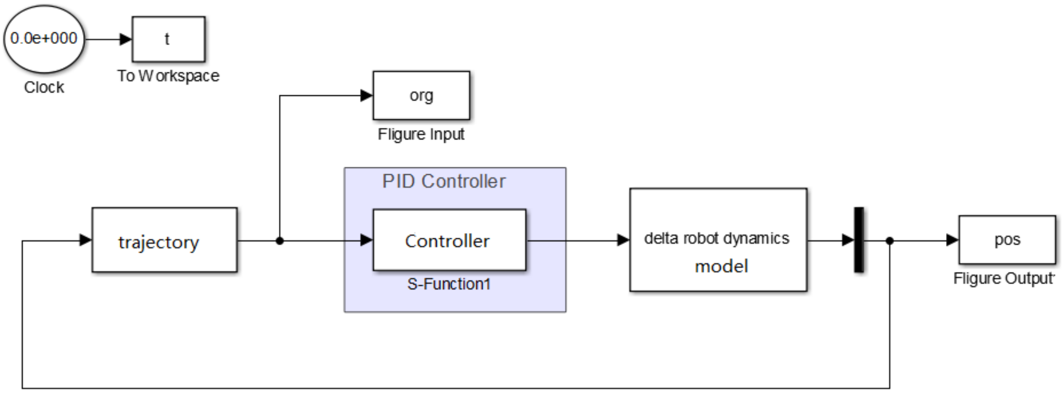


Figure 5. System dynamics model using an S-Function coded controller.

System augmented dynamics model with Lagrangian multipliers. Because the nonlinear Lagrangian equations with associated Lagrangian multipliers to satisfy the mechanical constraints are challenging to solve, especially the nonlinear equations own time-varying characteristics while the Delta robot mechanism is in motion. Moreover, conventional programming language definitions consume vast computation time and resources and often lead to divergence. This study directly employs the augmented dynamics model for the Delta robot mechanism in equation (19) to solve the time-varying nonlinear Lagrangian equations with associated Lagrangian multipliers. The approach successfully obtains the controlled Delta robot augmented dynamics through iteratively solving the Lagrangian multipliers parameters, as shown in the red box of Fig. 6, for subsequent dynamics computations.

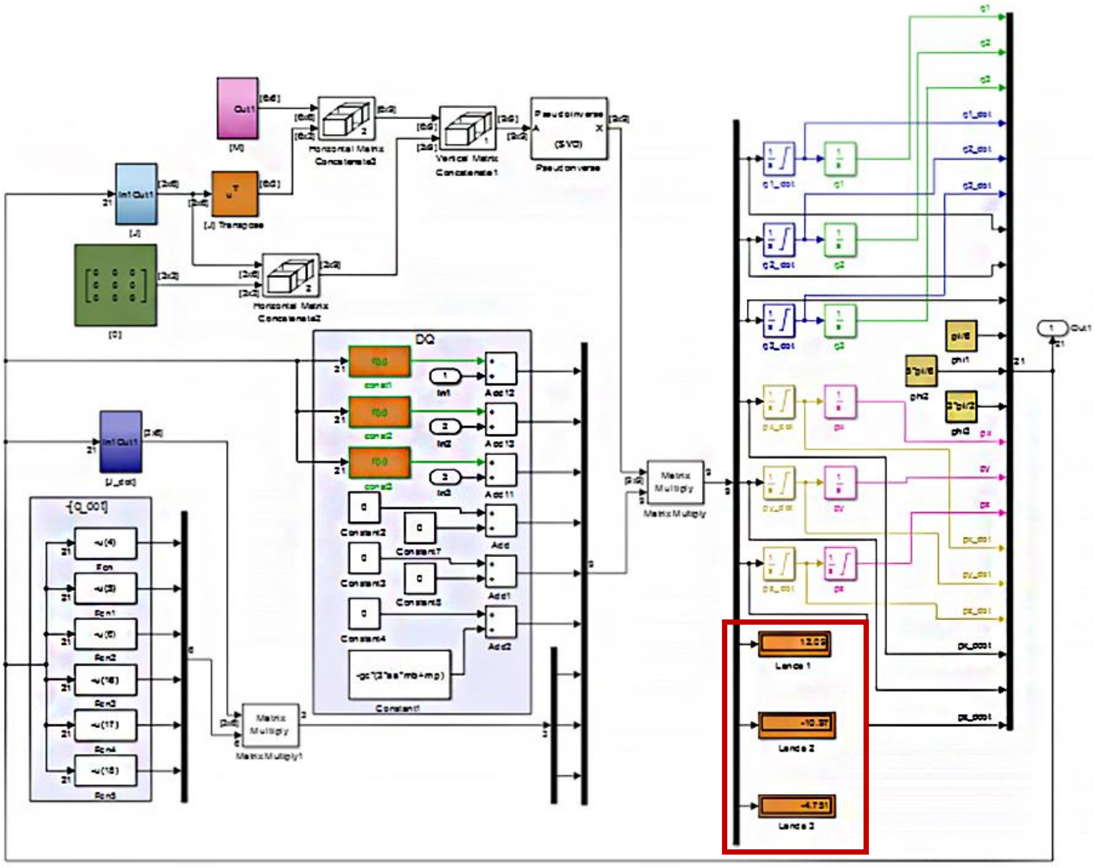


Figure 6. Augmented system dynamics model using a controller for the Delta robot model with the Lagrangian multipliers considered.

Figure 6 shows the redesigned system dynamics with the Lagrangian multipliers employed to satisfy the constraints in equation (11). The augmented controller successfully achieves the movable platform's commanded motions while tracking a given 3D circular trajectory and avoiding singularities within a limited workspace range.

Result

By deriving the forward/inverse dynamic equations and considering the articulated closed-chain mechanism's constraints, this study performs simulations to evaluate the augmented dynamic equations and applies the developed controller for the Delta robot mechanism.

Simulation. The work employs the reconstructed model for the Delta robot with the geometries and material properties listed in Table 1. In addition, it implements the augmented PID controller with the Lagrangian multipliers incorporated to simulate the movable platform motions while tracking a specified trajectory. The simulation results demonstrate that the controlled movable platform closely tracks a 3D circular trajectory in the spatial workspace, as shown in Fig. 7, with the Lagrangian multipliers constraint equations considered.

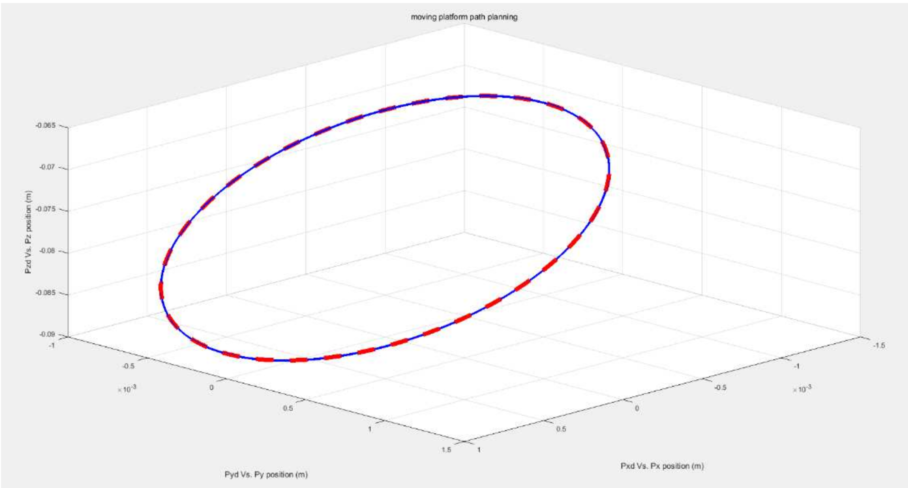
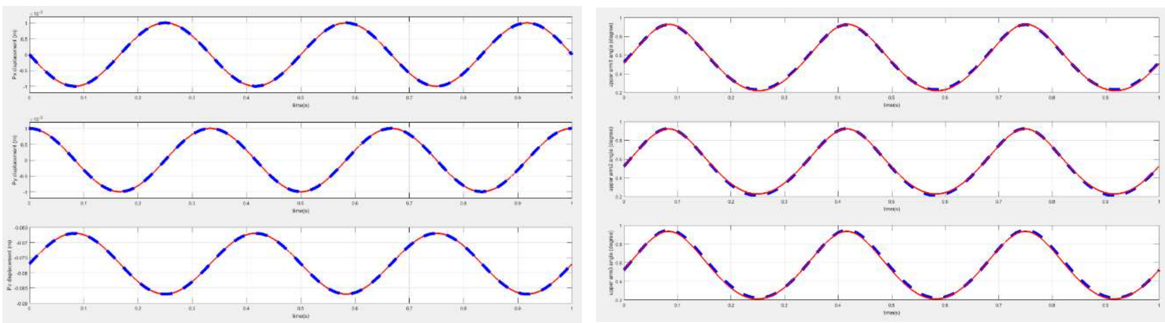


Figure 7. Command and response traces for the moving platform while tracking a 3D circular trajectory: desired trajectory (segmented red trace) and dynamic simulated trajectory (continuous blue trace).

The study intentionally places the initial position of the moving platform away from the coordinates' origin. The tracking errors of the movable platform's center converge to zeros in all three directions, i.e., P_x, P_y, and P_z track closely to the prescribed trajectories within 10 ms, as shown in Fig. 8a, and thus validate the effectiveness of the proposed controller design strategy in this research. Fig. 8b shows the corresponding rotations of the three upper arms while the moving platform tracks the trajectory, and the augmented dynamic model's correspondingly obtained upper arm rotations satisfy the requirements.



(a) (b)

Figure 8. Delta robot motion while tracking a 3D circular trajectory. **(a)** Displacements of the moving platform, from top to bottom: P_x, P_y, and P_z, respectively; (b) Rotations of the three upper arms, from top to bottom: upper arm_1, upper arm_2, and upper arm_3, respectively.

The study also examines the moving platform's velocities in three directions. The moving platform moves from initial zero velocity and fast converges to the requested velocity within 1 ms. The moving platform's maximum velocity is 0.02 m/s in the *x*-direction and *y*-direction and 0.3 m/s in the *z*-direction, which is reasonable in the limited workspace.

Due to the promptly demanded tracking responses, there are minimal initial differences between the desired and controlled moving platform motion traces. Nevertheless, the developed controller achieves the 3D circular path tracking while experiencing certain variations for the Lagrangian multipliers, which successfully adapts the augmented controller to achieve track performance.

The simulation results verify that the developed controller can successfully command the moving platform to track prescribed trajectories with incorporated mechanical constraints. The developed strategy is beneficial to investigate other types of controllers for the Delta robot for tracking purposes. Because it is intuitive and straightforward to construct the platform using the proposed strategy, adjusting the robot's geometry and other parameters when the design requests modifications are convenient.

Discussion

This research develops an effective tracking controller for a Delta robot based upon a strategy using the Lagrangian equations of motion for the articulated closed-chain mechanism and incorporating the Lagranign multipliers for the constraints. The study reformulates the forward/inverse dynamic equations of motion and integrates the mechanism constraint equations to acquire state equations for the Delta robot dynamics. As a result, the augmented system now employs six states, including the moving platform's three center coordinates and three Lagrangian multipliers, to dominate the mechanism's dynamics in resolving the demanded motor torques and tracking positions.

The work applies a tuned PID controller for the movable platforms to track prescribed trajectories with the developed controlling strategy and the constraints considered. During the PID controller functions period, the developed system closely tracks the prescribed trajectory and feedback on the movable platform's current motion for tuning and updating control parameters. Furthermore, during all the tracking timespan, the system dynamics demand satisfying the Lagrangian equations of the motion and constraint equations for the Lagrangian multipliers.

The research results confirm that the Lagrangian formulation-based strategy can design effective tracking controllers for Delta robot mechanisms. Furthermore, the Delta robot's moving platform achieves satisfactory tracking performance using the developed controller with Lagrangian multipliers. Hence, the proposed design approach facilitates the reformulation of complex dynamics equations with mechanism constraints, improving dynamic modeling efficiency and tracking controller performance.

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Author contributions
Fu-Shin Lee and Chen-I Lin performed material preparation, data collection, and analysis. Fu-Shin Lee wrote the first draft of the manuscript and all authors commented on previous versions of the manuscript. All authors read and approved the final manuscript.

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Competing interests
Fu-Shin Lee and Chen-I Lin declare no competing interests.