

Vehicular Trajectory Estimation Utilizing Slip Angle Based on GNSS Doppler/IMU

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Vehicular Trajectory Estimation utilizing Slip Angle based on GNSS Doppler/IMU

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Abstract— Accurate vehicular trajectory estimation is important for the recently developed autonomous driving systems. As the accuracy of the vehicular trajectory estimation is reduced with the slippage that occurs during turning, we propose a method in this study to accurately estimate the trajectory of a vehicle, focusing on the slip angle estimation. Although the two-wheel model is used as a general concept slip angle estimation, the accurate estimation of the parameters was difficult using the conventional methods. Therefore, a global navigation satellite system (GNSS) Doppler was used for parameter estimation. In addition, the roll angle was estimated as it occurs during turning and affects the slip angle of the vehicle. Specifically, we verified the improvement in accuracy of the vehicular trajectory estimation using the cost-effective GNSS Doppler/inertial measurement unit.

I. INTRODUCTION

Highly accurate vehicular trajectory estimation has become imminently important for automated vehicles and advanced driver assistance systems that have been developed in the recent years [1]. Accurate trajectory

information has been used in various studies such as constraints for position estimation, 3-D mapping for automated vehicles, route planning, and vehicular control [2-7]. The trajectory estimation accuracy of an automated vehicle should be accurate to one tire with a 2-D error of 0.30 m/100 m. Moreover, conventional automated vehicles often use global navigation satellite systems/inertial measurement unit (GNSS/IMU) systems with expensive sensors, such as fiber-optic gyros, to achieve the required accuracy. However, the cost of using these expensive sensors is an issue. Therefore, the method proposed in this study aims to achieve accurate trajectory estimation using inexpensive sensors. A difference is known to occur between the directions of the velocity vector and the attitude angle (slip angle) when the vehicle turns. Therefore, the slip angle estimation is generally carried out using a two-wheeled model to improve the accuracy of vehicular trajectory estimations [8]. However, the two-wheeled models require one-by-one estimation of multiple parameters, which imparts error biases and

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54 makes the accurate estimation of vehicular trajectory
55 difficult.

56 Therefore, the proposed method uses a GNSS
57 Doppler to ensure the high-accuracy estimation of
58 vehicular trajectories by compensating for slip angles
59 occurring in a motorcycle model. In order to confirm
60 the effectiveness of the correction of the slip angle on
61 the trajectory, the accuracy of the trajectory was
62 determined in the direction straight down the road with
63 data acquired and evaluated from two different
64 environments.

65

66 II. RELATED PRIOR RESEARCH

67 A. Overview of conventional trajectory estimation

68 Generally, conventional vehicle trajectory estimation utilizes
69 IMUs and wheel speedometers [1,9,10]. First, the trajectory
70 can be broken into its east and north components, and the
71 trajectory at time t can be expressed using Equations (1) and
72 (2) as follows.

73

$$74 \quad T_{\text{East}}^t = T_{\text{East}}^{t-1} + V \cdot \cos(\psi + \beta) \cdot dt \quad (1)$$

$$75 \quad T_{\text{North}}^t = T_{\text{North}}^{t-1} + V \cdot \sin(\psi + \beta) \cdot dt \quad (2)$$

76 V : velocity, ψ : heading angle, β : slip angle.

77

78 Here, the heading angle ψ is expressed in the following
79 equation (3) using the yaw rate $\dot{\psi}$:

$$80 \quad \psi^t = \psi^{t-1} + (\dot{\psi}^t + \delta\dot{\psi}) \cdot \frac{1}{\cos(\theta) \cdot \cos(\varphi)} \cdot dt \quad (3)$$

81 $\dot{\psi}$: yaw rate, θ : pitch angle, φ : roll angle.

82

83 Equation (3) considers the error in the IMU yaw rate $\dot{\psi}$ with
84 an offset of $\delta\dot{\psi}$. Furthermore, Equation (4) presents the
85 consideration of including the scale factor of wheel speed and
86 the effect of the longitudinal slope of the road with respect to
87 V .

$$88 \quad V = SF \cdot V_{\text{can}} \cos(\theta) \quad (4)$$

89 V_{can} : CAN-bus Velocity, SF : Scale Factor Error.

90 Using Equations (1)–(4), the vehicle trajectory can be
91 calculated from the output of the IMU using the wheel
92 speed. Therefore, the elements of trajectory that need to
93 be estimated are ψ , β , $\delta\dot{\psi}$, SF , φ , and θ . Among them,
94 a highly accurate estimation method has been proposed
95 for ψ , $\delta\dot{\psi}$, SF , and θ [1]; therefore, the estimation of
96 the roll angle φ and slip angle β remains unfulfilled.
97 Thus, this study focuses to estimate the slip angle,
98 which is more accurate along the direction of the road
99 and has a greater correction effect.

100

101 B. Previous studies on slip angle estimation

102 The equipment required to directly measure the slip
103 angle of a vehicle is expensive and difficult to install in
104 a vehicle [11]. Consequently, estimation is often
105 preferred and performed, where the direct integration
106 method [8] and the linear observer estimation method
107 [12] are used as typical methods for conventional
108 estimation of the slip angle. However, the direct
109 integration method poses a problem of accumulation of
110 noise and offset in the sensor detections owing to the
111 inclusion of an integrator, and the error increases with
112 time. On the contrary, the linear two-degree-of-

113 freedom model (two-wheel model) of the linear
 114 observer estimation method is used to construct the
 115 settings for an observer to conduct the estimation
 116 (Figure 1). In the two-wheel model, both the left and
 117 right front tires of a four-wheeler vehicle are assumed
 118 to have identical slip angles and are treated as
 119 equivalent to those of the two wheels. Equation (5)
 120 depicts the relationship between the slip angle at the
 121 center of gravity β and the parameters of the vehicle.

122

$$123 \quad \beta = \left(\frac{1 - \frac{mL_f}{2LL_rK_r} V^2}{1 - \frac{m}{2L^2} \left(\frac{L_fK_f - L_rK_r}{K_fK_r} \right) V^2} \right) \frac{L_r \delta}{L} \quad (5)$$

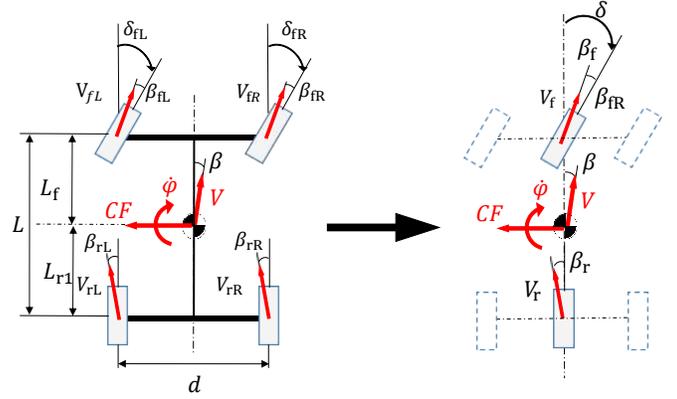
124 m : vehicle weight, L : distance between front and rear
 125 wheels, L_f : distance between center of gravity of front
 126 wheel, L_r : distance between center of gravity and the
 127 rear wheels of the vehicle, V : velocity, K_f : front
 128 wheel cornering power, K_r : rear wheel cornering power
 129 , δ : steering angle.

130

131 Here, the linear observer estimation method measures
 132 the parameters required for the estimation of the slip
 133 angle from the inertial sensor measurements [11-12].
 134 However, individual measurements of these parameters
 135 lead to a problem of error bias. In particular, the
 136 coefficients K_f and K_r are difficult to measure
 137 accurately owing to various factors such as the effect of
 138 deformed tires. In addition, there are limitations such as

139 the need for accurate estimation of the steering angle
 140 and the requirement for tuning for each vehicle [13-15].

141



142

143

Figure 1 Two-wheel model

144

145

III. OUR PROPOSAL

146

A. Overview of the slip angle estimation

147

The proposed method aims to resolve the limitation
 148 of determining the parameters of the two-wheeled
 149 model for the estimation of the slip angle β by the
 150 estimating parameters automatically using GNSS
 151 Doppler. In addition, the proposed method eliminates
 152 the term of steering angle in the two-wheel motorcycle
 153 model, and thus does the estimation of steering angle is
 154 not required. The proposed method is expected to
 155 increase the accuracy of the slip angle estimation to an
 156 extent of validating the two-wheel model. Figure 2
 157 shows an overview of the method.

158

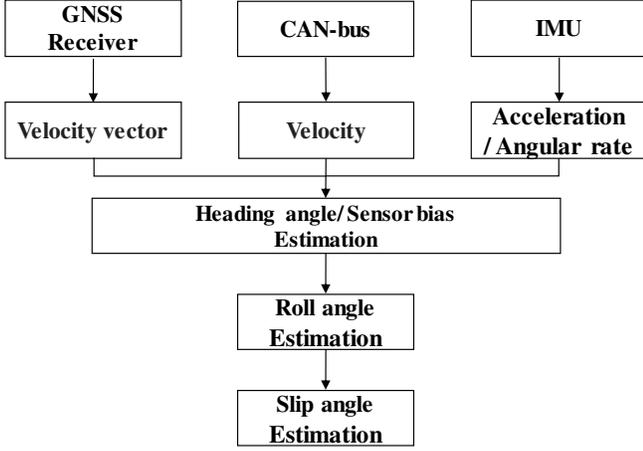


Figure 2 Overview of the proposal

In the two-wheel model, the yaw rate $\dot{\psi}$ can be expressed by Equation (6) [12] as follows.

$$\dot{\psi} = \left(\frac{1}{1 - \frac{m}{2L^2} \left(\frac{L_f K_f - L_r K_r}{K_f K_r} \right) V^2} \right) \frac{V \delta}{L} \quad (6)$$

Therefore, Equations (5) and (6) can be combined to obtain the relationship between β and $\dot{\psi}$ as given by Equation (7).

$$\beta = \dot{\psi} \left(\frac{L_r}{V} - \frac{m L_f}{2 L K_r} V \right) \quad (7)$$

However, the above equation represent the slip angle β at the center of gravity. The slip angle β_r at the center of the rear wheel axle is given by equation (8).

$$\beta_r = \beta - \frac{L_r}{V} \dot{\psi} \quad (8)$$

Combining Equations (7) and (8), we can obtain equation (9) as

$$\beta_r = -\frac{m L_f}{2 L K_r} \dot{\psi} V \quad (9)$$

As all the parameters in Equation (9) are fixed and constant parameters, except for $\dot{\psi}$ and V , they can be comprehensively represented as the gain K in the following equation (10):

$$\beta_r = -K \dot{\psi} V \quad (10)$$

where $\dot{\psi} V$ is the product of the yaw rate and velocity and refers to the centrifugal acceleration, so it can be summarized as G_y in Equation (11).

$$\dot{\psi} V = G_y \quad (11)$$

Therefore, Equation (11) simplifies to Equation (12).

$$\beta_r = -K \cdot G_y \quad (12)$$

Furthermore, during motion of the vehicle, the component of gravitational acceleration g for the roll angle needs to be considered for Equation (12) because the component of gravity is included at the time of roll angle generation. Figure 3 shows the relationship between the roll angle and its occurrence. The

207 relationship between the roll angle φ and the lateral
 208 acceleration G_y is expressed in Equation (13).

209

$$210 \quad G_y = \dot{\psi} \cdot V - g \cdot \sin \varphi \quad (13)$$

211

212 Substituting G_y from Equation (13) to Equation (12),
 213 we obtain Equation (14) as:

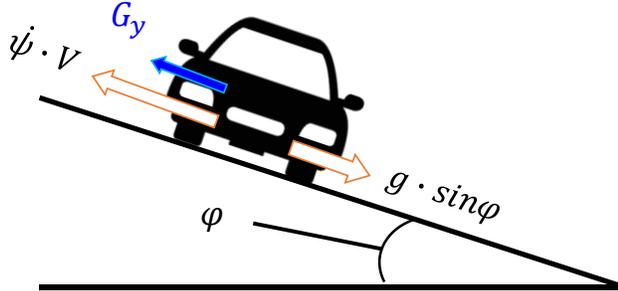
214

$$215 \quad \beta_r = -K(\dot{\psi} \cdot V - g \cdot \sin \varphi) \quad (14)$$

216

217 Therefore, the sine component of the roll angle is
 218 necessary for the estimation of the slip angle from
 219 Equation (14), which is estimated in the next
 220 subsection.

221



222

223 **Figure 3 Effect of road lateral gradient**

224

225 B. Roll angle estimation

226 The roll angle can be estimated with the formula
 227 presented by Tseng et al. [16]:

228

$$229 \quad \varphi = \arcsin\left(\frac{V}{g} \cdot \dot{\psi} - \frac{G_y}{g}\right) \quad (15)$$

230

231 Moreover, the transverse acceleration G_y can be
 232 measured from the IMU, and for the error in
 233 acceleration of the IMU as δG_y^{imu} , Equation (15) can
 234 be expressed as Equation (16).

235

$$236 \quad \varphi = \arcsin\left(\frac{V}{g} \cdot \dot{\psi} - \frac{G_y^{\text{imu}} + \delta G_y^{\text{imu}}}{g}\right) \quad (16)$$

237

238 Assuming that Equation (16) holds true, the roll
 239 angle φ can be estimated upon the correction of
 240 δG_y^{imu} .

241 Therefore, this study proposes a method for
 242 estimating the lateral acceleration error using the
 243 relationship between the variation in the heading angle
 244 ψ at certain intervals and the integration of the yaw rate
 245 $\dot{\psi}$ with the roll angle φ . The relationship between the
 246 variation of heading angle at certain intervals and the
 247 integration of the yaw rate $\dot{\psi}^{2D}$ as viewed from the
 248 horizontal plane can be expressed by equation (17).

249

$$250 \quad \psi^{t+k} - \psi^t = \int_t^{t+k} \dot{\psi}^{2D} dt \quad (17)$$

251

252 However, the roll angle φ needs to be considered with
 253 respect to the yaw rate in case of a lateral slope on the
 254 road surface.

255

$$256 \quad \psi^{t+k} - \psi^t = \int_t^{t+k} \dot{\psi} \cdot \frac{1}{\cos(\varphi)} dt \quad (18)$$

257

258 Therefore, the expression of roll angle in Equation (16)
 259 can be substituted into the above Equation (18) to
 260 obtain the following Equation (19).

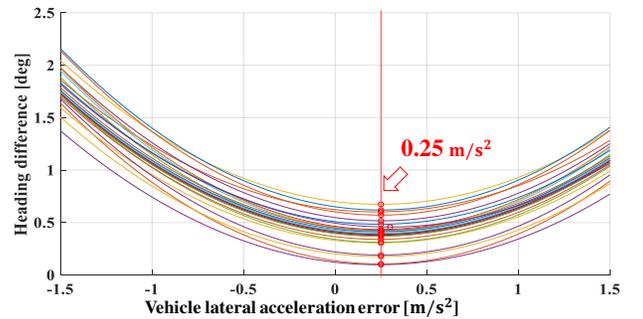
261
 262 $\psi^{t+k} - \psi^t =$
 263
$$\int_t^{t+k} \psi \cdot \frac{1}{\cos\left\{\arcsin\left(\frac{V}{g} \cdot \dot{\psi} - \frac{G_y^{\text{imu}} + \delta G_y^{\text{imu}}}{g}\right)\right\}} dt (19)$$

264
 265 Equation (19) shows the relationship between the
 266 azimuthal angle and the transverse acceleration error at
 267 certain intervals. Therefore, the accurate evaluation of
 268 the lateral acceleration error through Equation (19) can
 269 be used to accurately estimate the roll angle through
 270 Equation (16).

271 In this study, we determined δG_y^{imu} by utilizing the
 272 heading difference between the left and right sides of
 273 Equation (19). By varying the value of δG_y^{imu} in
 274 Equation (19), we search for cases where the heading
 275 difference is zero or closest to zero, and thus
 276 analytically determine δG_y^{imu} . In Equation (19), the
 277 heading angle ψ can be estimated by using the GNSS
 278 Doppler/IMU [1,17,18], the velocity V can be
 279 measured by the wheel speed sensor, and G_y^{imu} can be
 280 measured by the IMU, so only δG_y^{imu} remains unknown,
 281 for which the estimation can be made. The transverse
 282 acceleration error δG_y^{imu} cannot be estimated using
 283 Equation (19) when entire right-hand side of the
 284 equation becomes zero for 0 rad/s yaw rate $\dot{\psi}$.
 285 Therefore, the computation was performed only at the

286 point where the curve turned once. In this study, we
 287 used data for 100 s with $k = 100$. Figure 4 portrays the
 288 variation in the heading difference between the left and
 289 right sides for different values of δG_y^{imu} within -1.5 to
 290 1.5 m/s^2 according to Equation (19). As shown, the
 291 variation of δG_y^{imu} produced quadratic curves with
 292 inflection points indicating minimum differences
 293 between the left and right sides. As each of these
 294 inflection points was calculated at a different position
 295 on a curve, the average of these values were considered
 296 to neutralize the transverse acceleration error δG_y^{imu} .
 297 This estimate did not considerably vary over time so
 298 determining sequence was not necessary. Therefore,
 299 the real-time position estimation can be performed right
 300 after conducting the calibration run estimating the
 301 initial position. For this dataset, we considered
 302 0.25 m/s^2 as the estimated value.

303



304

305 **Figure 4 Estimation results of the lateral acceleration**
 306 **error**

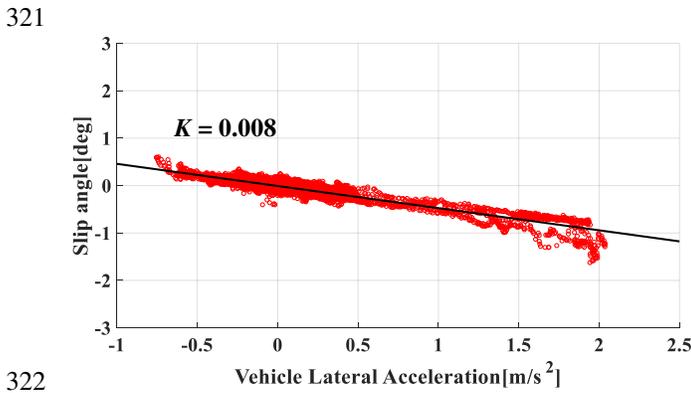
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308

309 *C. Parameter estimation for two-wheel model using*
 310 *GNSS Doppler*

311 Figure 5 presents the relationship between the slip
 312 angle and lateral acceleration according to Equation

312 (14) for a distance of approximately 5 km around the
 313 urban area (Odaiba, Tokyo, Japan). The slip angle and
 314 the lateral acceleration were measured by Applanix
 315 POSLV220, which is known as a high-precision
 316 GNSS/IMU system [19]. The correlation between the
 317 slip angles with lateral acceleration can be observed
 318 from Figure 5, where the slope represents the gain K
 319 from equation (14). Therefore, the slip angle can be
 320 estimated by evaluating this relationship.



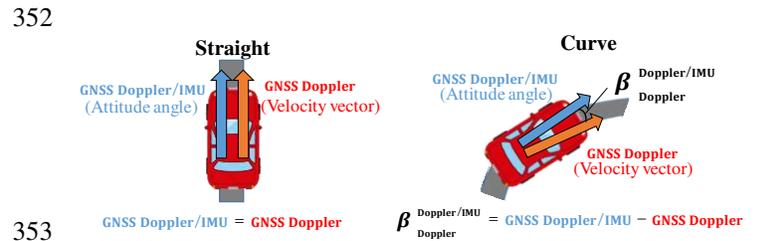
323 **Figure 5 Correlation between slip angle and lateral**
 324 **acceleration (POSLV220)**

325 On the contrary, the velocity vector and attitude
 326 angle can be estimated by utilizing the GNSS Doppler
 327 angle can be estimated by utilizing the GNSS Doppler
 328 [20] and combining the IMU with it [1], respectively.
 329 Therefore, this study proposes a method to approximate
 330 the variation in attitude angles using the GNSS
 331 Doppler/IMU and evaluate the first-order
 332 approximation of velocity vectors using the GNSS
 333 Doppler with the least-squares method, where the slope
 334 of the first-order equation is represented by the gain K .
 335 An overview of the proposed method has been
 336 presented in Figure 6, where β_r was accumulated
 337 according to Equation (20) by calculating the difference

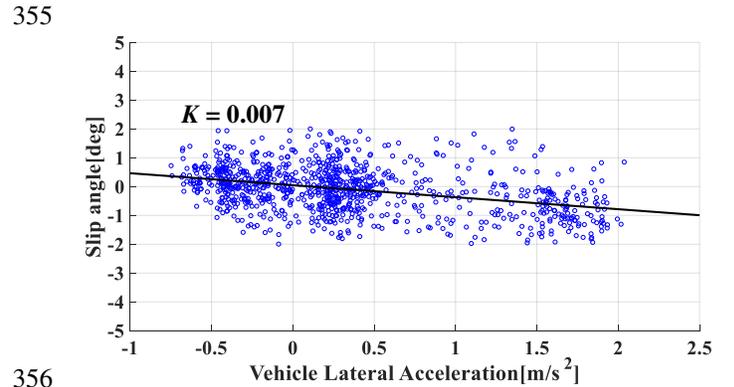
338 between the directions obtained from the GNSS
 339 Doppler/IMU $H_{\text{Doppler/IMU}}$ and the GNSS Doppler
 340 H_{Doppler} at the curve. Moreover, thresholds were set for
 341 speed and storage of data beyond a certain speed owing
 342 to the higher speed range of the moving object and the
 343 higher accuracy of the GNSS Doppler [1,17,18,20].

$$345 \beta_r^{\text{Doppler/IMU}} = H_{\text{Doppler/IMU}} - H_{\text{Doppler}} \quad (20)$$

346
 347 The gain K was estimated using the above equation
 348 with the relationship between the accumulated slip
 349 angle and lateral acceleration; the gain K shown in
 350 Figure 7 can be estimated without using a high-
 351 precision GNSS/IMU.



353
 354 **Figure 6 Overview of the slip angle**



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 356
 357 **Figure 7 Relationship between slip angle and lateral**
 358 **acceleration using GNSS/Doppler**

360

IV. EVALUATION TESTS

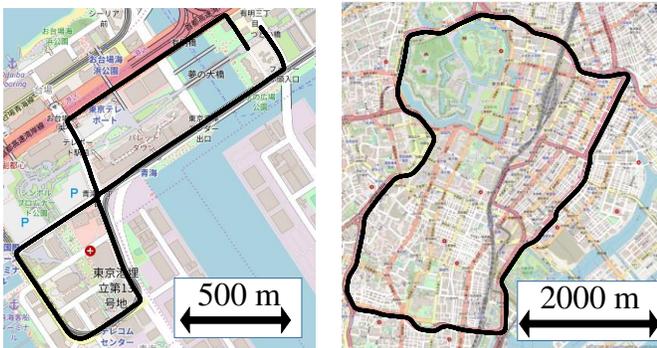
361 The evaluation tests were conducted on a 5-km
 362 course in an urban street (Odaiba, Tokyo, Japan) and a
 363 14-km course on an urban highway (Tokyo
 364 Metropolitan Expressway) with a lateral gradient; the
 365 courses are shown in Figure 8. The U-blox M8T with a
 366 GPS+BeiDou+Galileo+QZSS satellite system was
 367 used as a GNSS receiver with a reception period of 10
 368 Hz. The MEMS IMU on the TAG264 of Tamagawa
 369 Seiki was used with an acquisition period of 50 Hz. An
 370 Applanix POSLV220 was used as the reference
 371 equipment for an accurate evaluation. Table 1 lists the
 372 sensors and the reference POSLV220 used in this
 373 evaluation, where a significant difference can be seen
 374 in the cost of our proposed sensor scheme with that of
 375 the reference model.

376
377

Table 1 Equipment used for evaluation

	Sensor	Product name	Cost (US\$)
Proposal	GNSS Receiver	Ublox M8T(10Hz)	100
	GNSS Antenna	Tallysman TW2710	100
	MEMS IMU	Tamagawa AU7554(50Hz)	500
Reference	GNSS Receiver, GNSS Antenna, 3-axis FOG	Applanix POSLV220	100000

378



379

380

a) Urban street

b) Urban highway

381

Figure 8 Evaluation fields [“© OpenStreetMaps”]

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A. Roll angle estimation results

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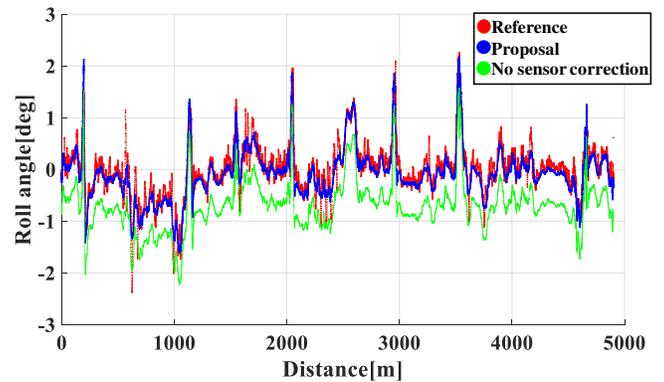
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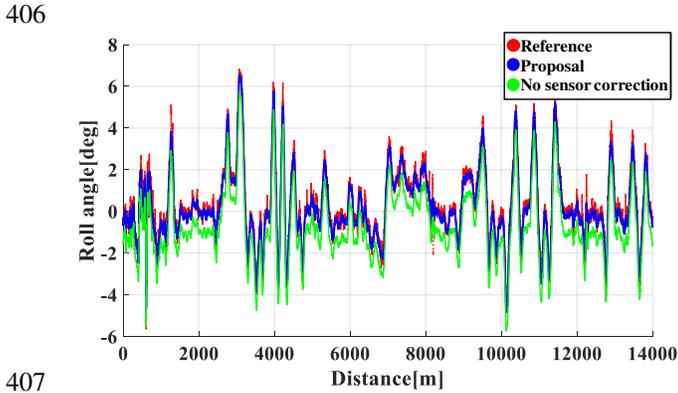
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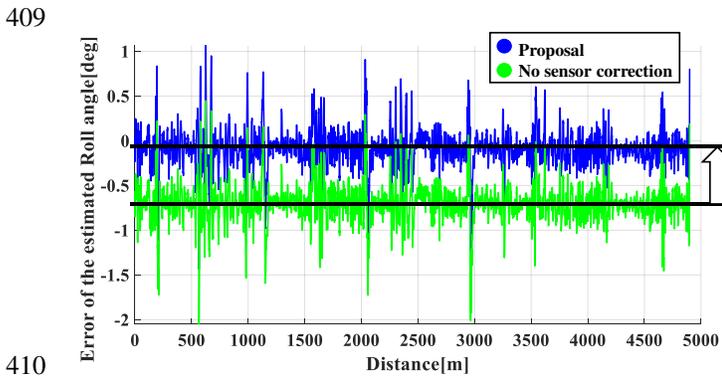
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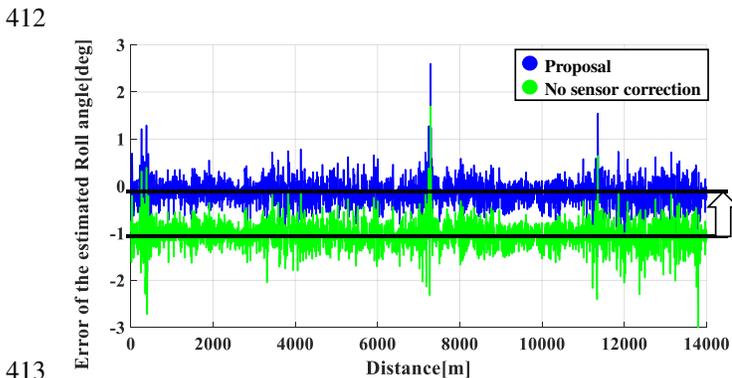
405 **Figure 9 Estimated roll angle (Urban street)**



408 **Figure 10 Estimated roll angle (Urban highway)**



411 **Figure 11 Error of estimated roll angle (Urban street)**



414 **Figure 12 Error of estimated roll angle (Urban highway)**

415 **Table 2 Error of estimated roll angle (Urban street)**

Error	Average[deg]	Standard deviation[deg]
Proposal	-0.09	0.22
No sensor correction	-0.71	0.22

417 **Table 3 Error of estimated roll angle (Urban highway)**

Error	Average[deg]	Standard deviation[deg]
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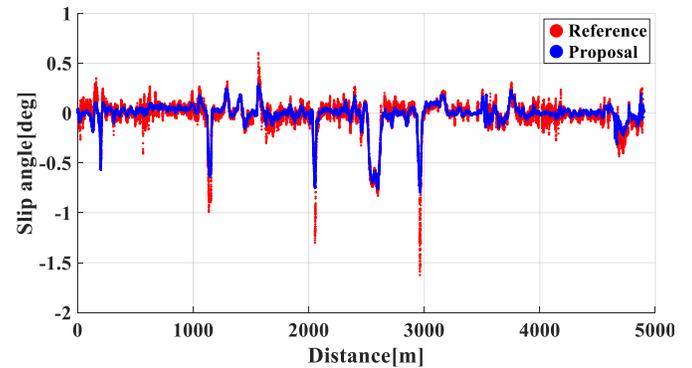
Proposal	-0.17	0.29
No sensor correction	-1.06	0.29

419

420 **B. Slip angle estimation results**

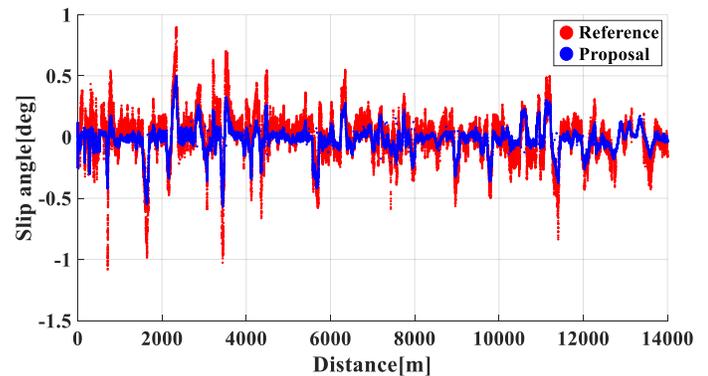
421 The slip angles estimated using the proposed method
 422 are shown in Figures 13 and 14 for the urban street and
 423 highway, respectively. These figures prove that the
 424 estimated slip angles were close to the reference model
 425 for both the courses. Furthermore, the difference
 426 between the reference and estimated slip angles are
 427 presented in Figures 15 and 16; the error in the
 428 estimated slip angle is shown in Table 4.

429



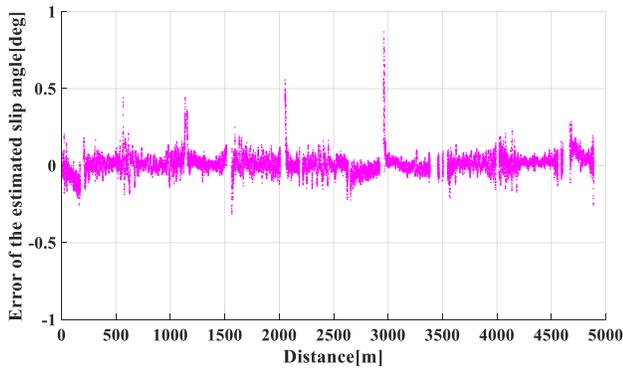
431 **Figure 13 Estimated slip angle (Urban street)**

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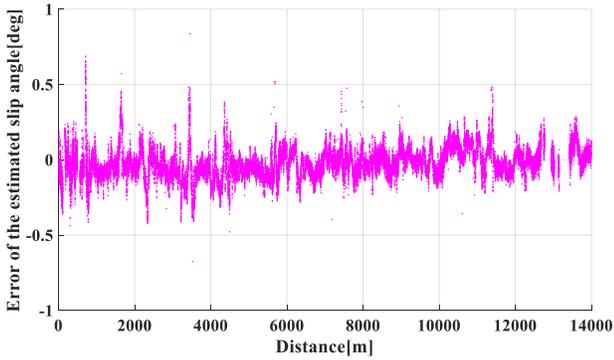


434 **Figure 14 Estimated slip angle (Urban highway)**

435



436
437 **Figure 15 Error of estimated slip angle (Urban street)**



439
440 **Figure 16 Error of estimated slip angle (Urban highway)**

442 **Table 4 Error of estimated slip angle**

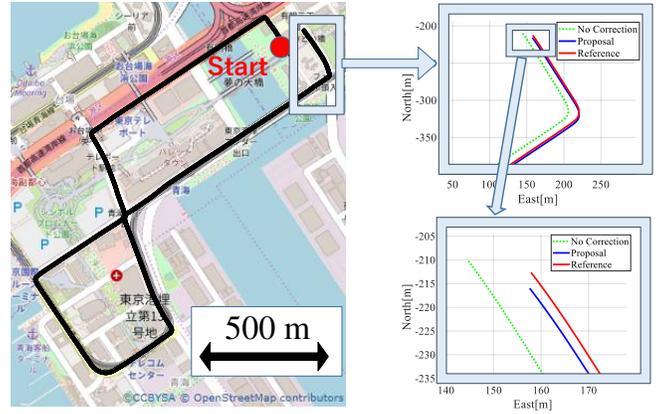
Error	Average[deg]	Standard deviation[deg]
Urban street	0.01	0.07
Urban highway	-0.01	0.10

443
444 *C. Trajectory estimation results*

445 Thereafter, the effect of the slip angle correction on
446 the trajectory was evaluated. Figure 17 shows the
447 results of using the corrected slip angle estimation for
448 the trajectory obtained from the conventional method
449 [1,4] at the urban street shown in Figure 8a. The
450 trajectory before and after the slip angle correction is
451 shown in the inset of Figure 17, compared with the
452 trajectory drawn from the starting point aligned to the
453 reference. Figure 17 further shows that the trajectory

454 beyond the turn of curve approached the reference
455 model by correcting the slip angle, which improved the
456 accuracy of the proposed model.

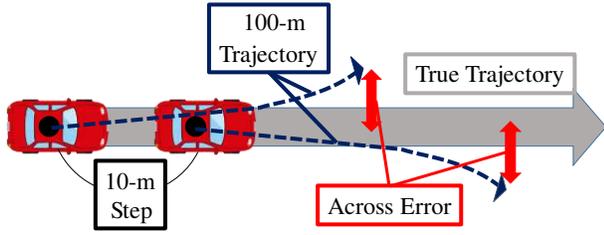
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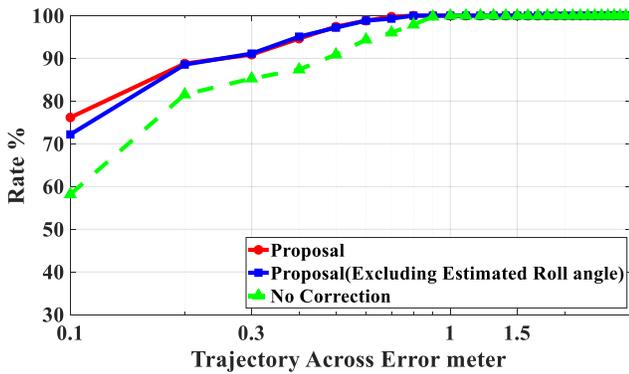
458
459 **Figure 17 Effect of slip angle correction (Urban street)**

460 In this study, we evaluated the error every 10 m in
461 the trajectory for every 100 m in the direct direction of
462 the road on the urban street and highway. The trajectory
463 was set according to the wheel speed, IMU, and slip
464 angle after aligning the POSLV220 only to the base
465 point. Figure 18 shows an overview of the evaluation
466 test; Figures 19 and 20 show the results of the
467 evaluation, where the accuracy of the trajectory was
468 improved on both the urban street and the highway. In
469 particular, the accuracy of the trajectory—at an error of
470 30 cm—was approximately 10% higher than that
471 before the correction (no correction). Moreover, the
472 estimation accuracy was improved by considering the
473 roll angle in Equation (14), as compared to excluding
474 the roll angle φ in the given equation. The effect of
475 considering the roll angle was particularly pronounced

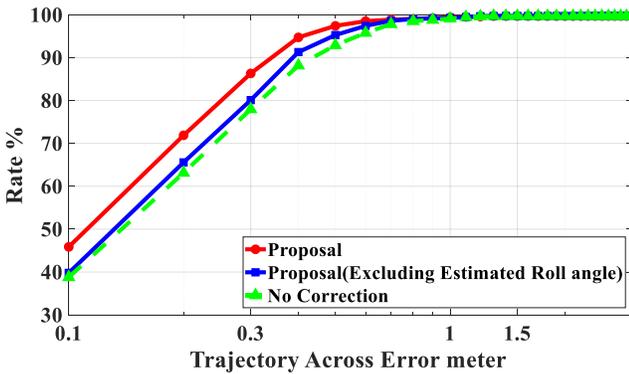
477 on the urban highway that has a large number of lateral
 478 slopes.



481 **Figure 18 Overview of trajectory evaluation**



484 **Figure 19 Evaluation of vehicle trajectory (Urban street)**



487 **Figure 20 Evaluation of vehicle trajectory (Urban
 488 highway)**

489
 490 **V. CONCLUSIONS**

491 The purpose of this study was to improve the accuracy
 492 of vehicular trajectory, which is an important factor of
 493 automatic vehicle driving. Thus, we focused on the

494 accurate estimation of the slip angle, which is a
 495 particular challenge in vehicular trajectory estimation.
 496 A linear observer estimation method using a two-
 497 wheeled model constitutes a typical conventional
 498 method for slip angle estimation. These methods
 499 facilitate one-by-one estimation of the parameters of
 500 the two-wheeled model, thus creating a problem of
 501 error bias.

502 Therefore, in this study, the two-wheel motorcycle
 503 model equation for the position of center of gravity was
 504 replaced by that for the center of the rear axle slip angle,
 505 and the parameters of the motorcycle model were
 506 collectively estimated as a gain K . This allowed the
 507 automatic estimation of parameters and ensured the
 508 removal of the steering angle term that was difficult to
 509 measure. We utilized the GNSS Doppler/IMU and
 510 GNSS Doppler to estimate the gain K . The difference
 511 between the attitude angles measured by the GNSS
 512 Doppler/IMU and the velocity vectors measured by the
 513 GNSS Doppler from the motorcycle model was first-
 514 order approximated by the least-squares method, and
 515 the slope of the first-order equation was defined as the
 516 gain K . By accounting the effect of the roll angle for the
 517 estimation of the gain K , a highly accurate slip angle
 518 estimation was performed. The roll angle estimation
 519 was based on the relationship of the integrated yaw-rate
 520 value and the variation of the attitude angle during a
 521 certain time period. This study enabled the estimation
 522 of the slip angle without the need for the observer and
 523 vehicle-specific tuning, and allowed the correction of

524 slip to be easily incorporated into the trajectory
525 estimation.

526 The effectiveness of the slip angle correction for the
527 vehicular trajectory was confirmed in the evaluation
528 test. The experiments were conducted on a standard
529 urban (street) course and a high-speed (highway)
530 course that was prone to lateral gradients; the error rates
531 of the 100-m trajectory for both courses were improved
532 by about 10% to cover within 30 cm. The consideration
533 of roll angle improved the accuracy of the estimation,
534 and its effectiveness was especially significant on a
535 course with numerous lateral slopes such as the urban
536 highway.

537 In this study, an off-line processing was used to
538 improve the accuracy of the trajectory estimation by
539 estimating a highly accurate slip angle. This was
540 effective for off-line use of trajectories such as the
541 creation of 3-D maps for automated driving. As the
542 theoretical capability of the proposed model has been
543 verified for real-time use, we will continue to improve
544 the algorithm for real-time implementation to further
545 improve the usability of the proposed model.

546

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549

550 *Authors' contributions*

551 KT made a central contribution to the writing of the manuscript by
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557

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560

561 *Competing interests*

562 The authors declare that they have no competing interests.

563

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Figures

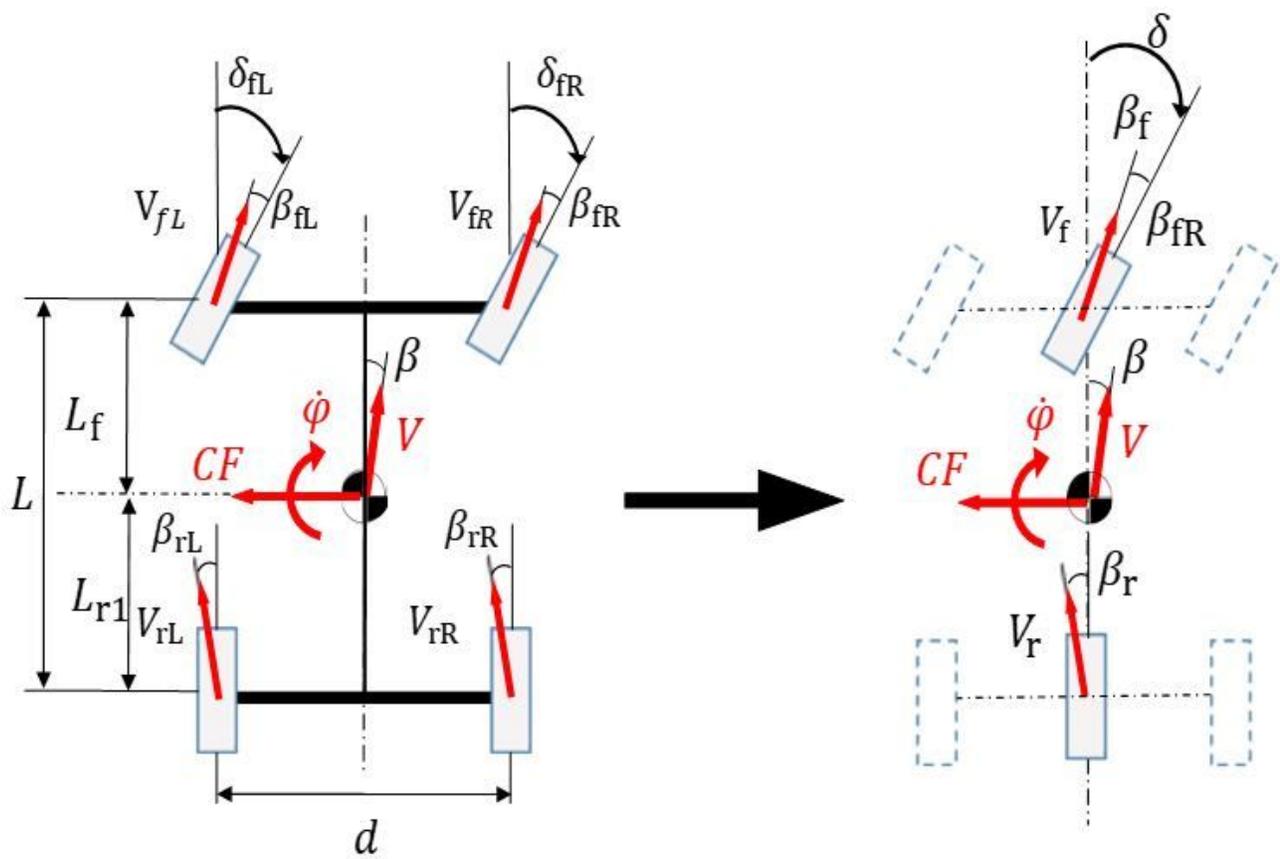


Figure 1

Two-wheel model

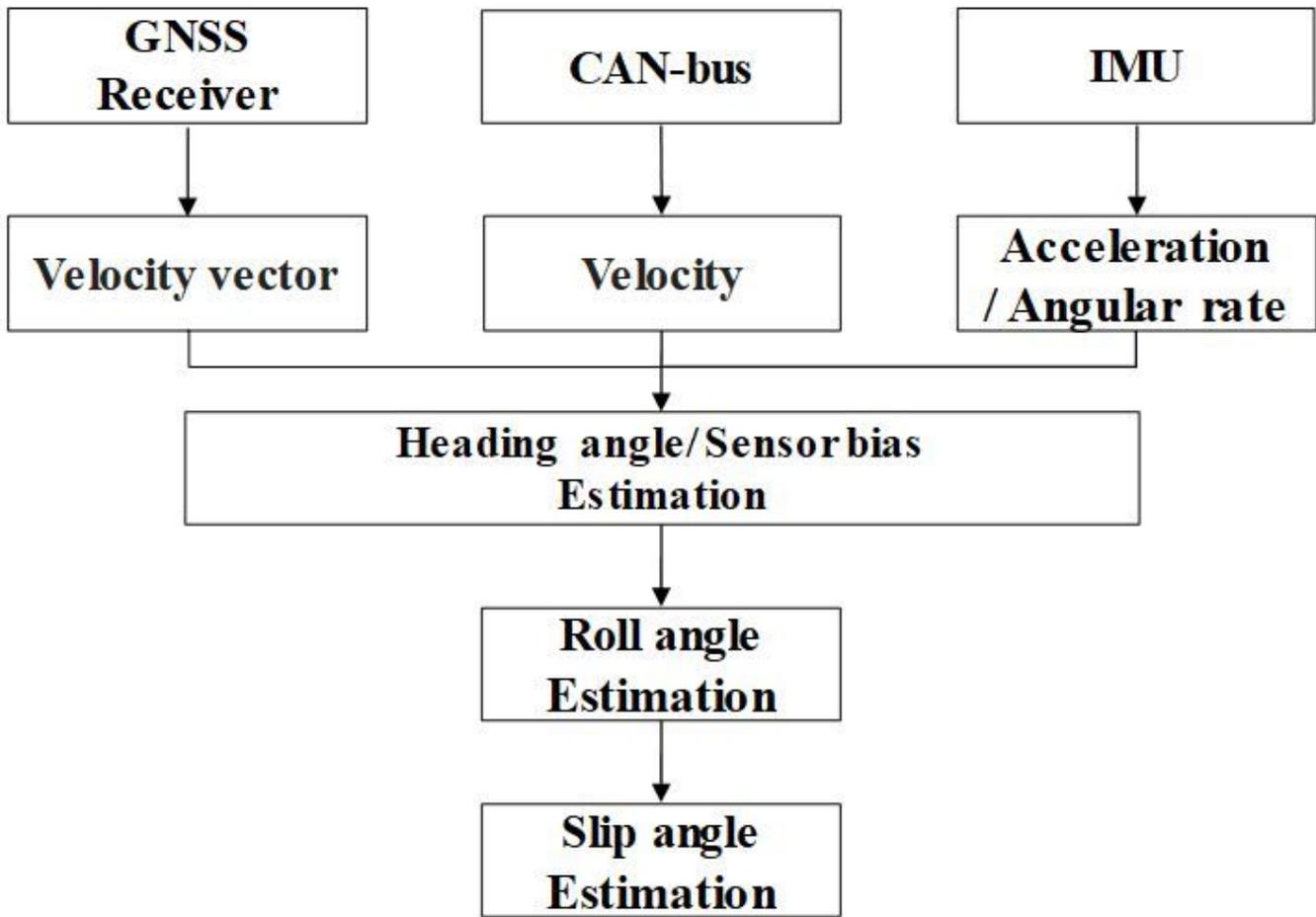


Figure 2

Overview of the proposal

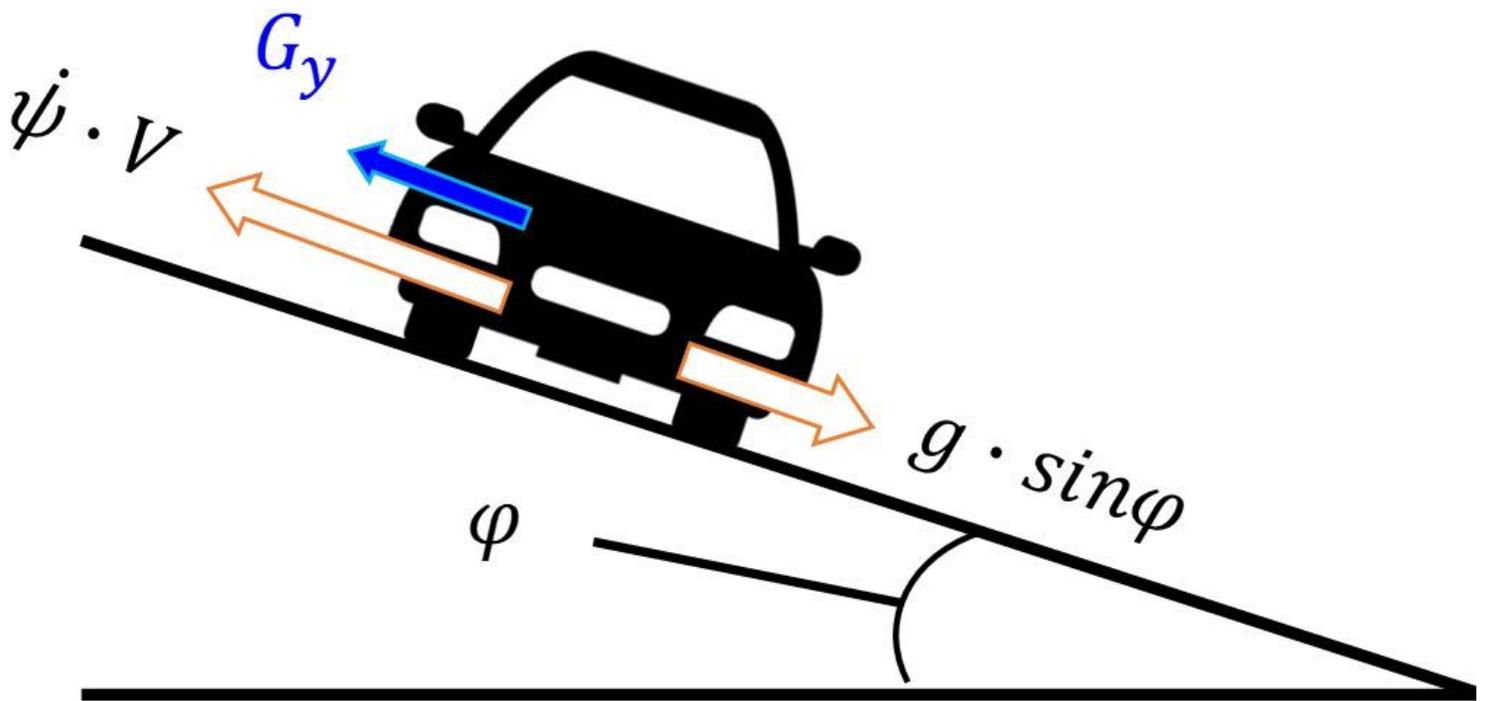


Figure 3

Effect of road lateral gradient

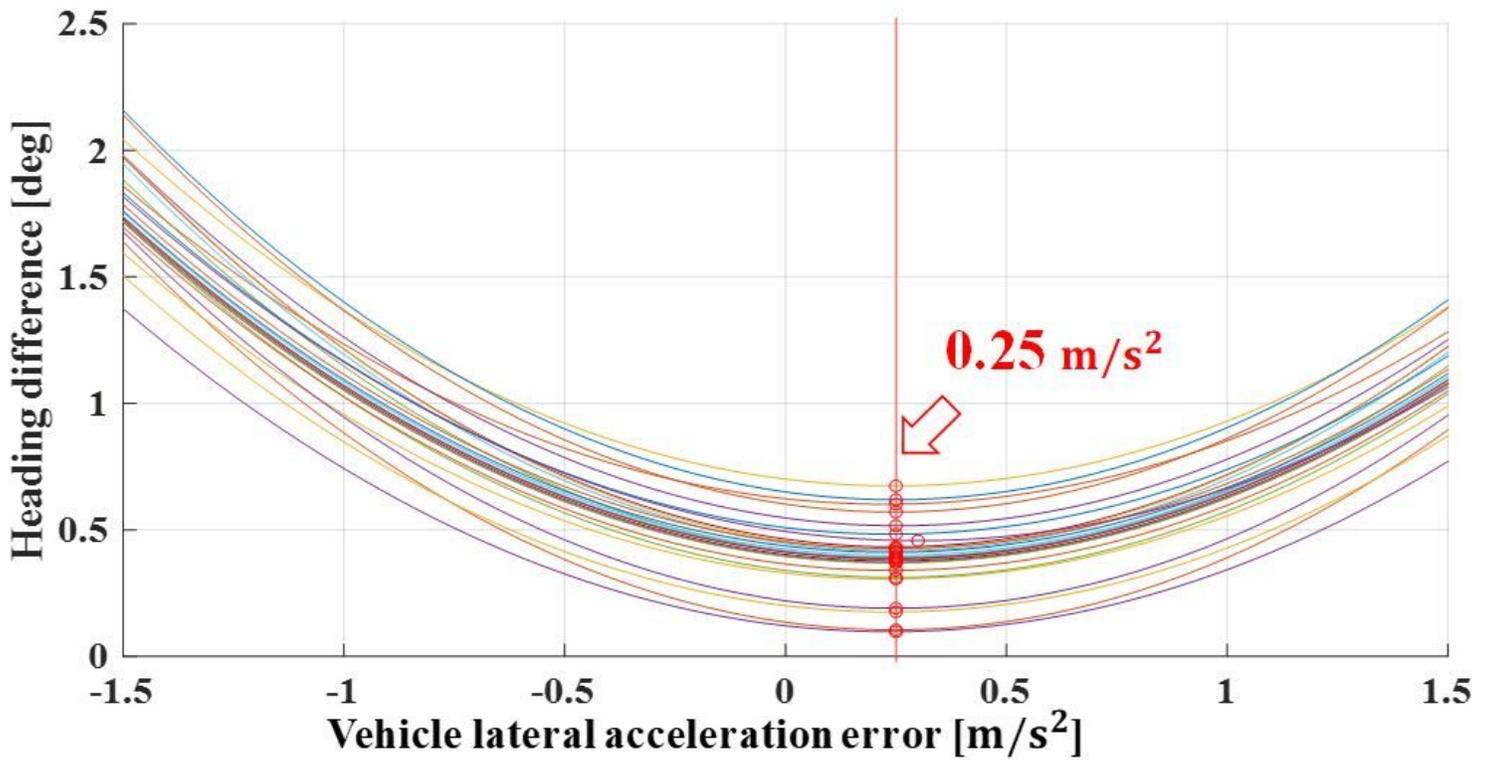


Figure 4

Estimation results of the lateral acceleration error

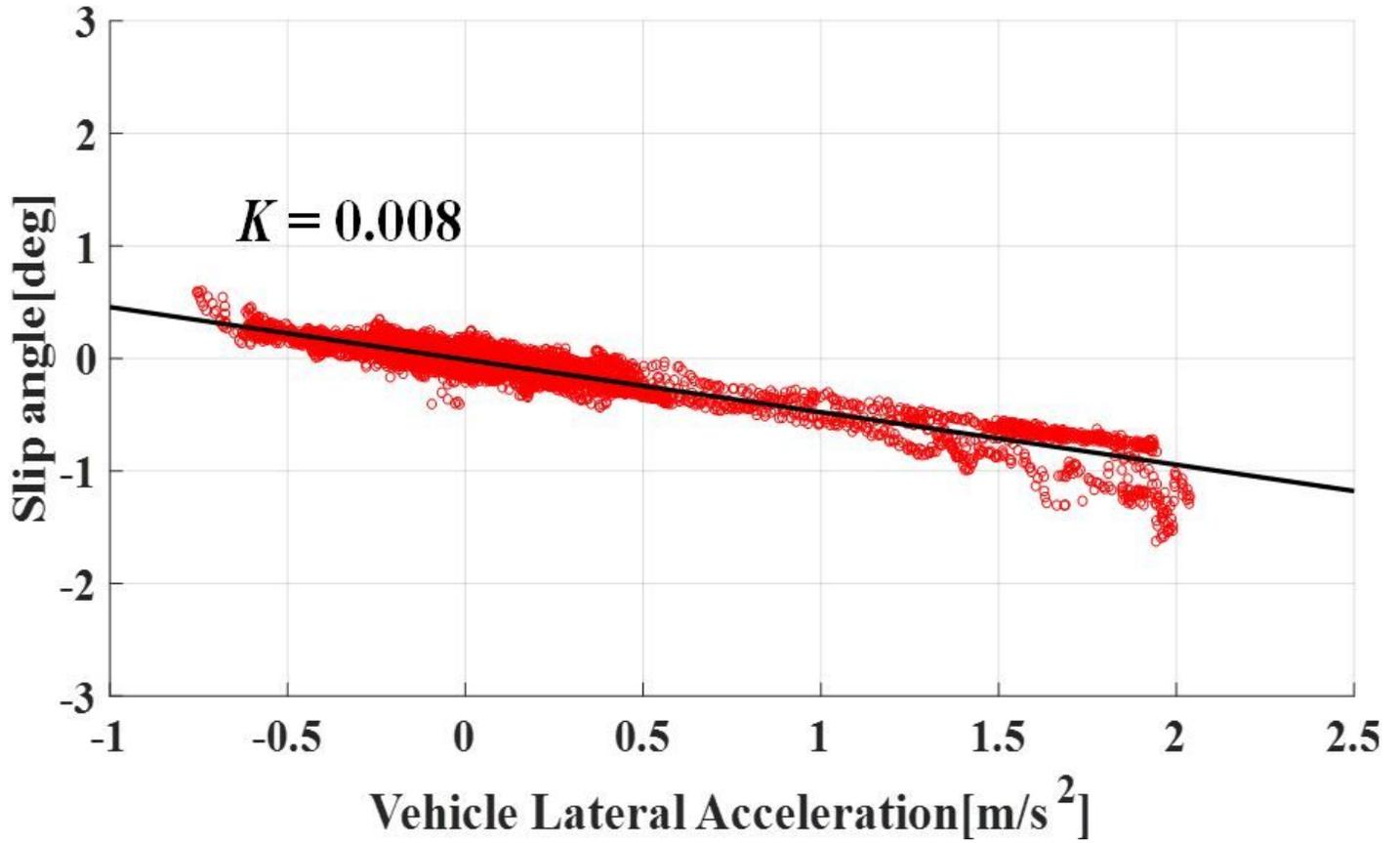


Figure 5

Correlation between slip angle and lateral acceleration (POSLV220)

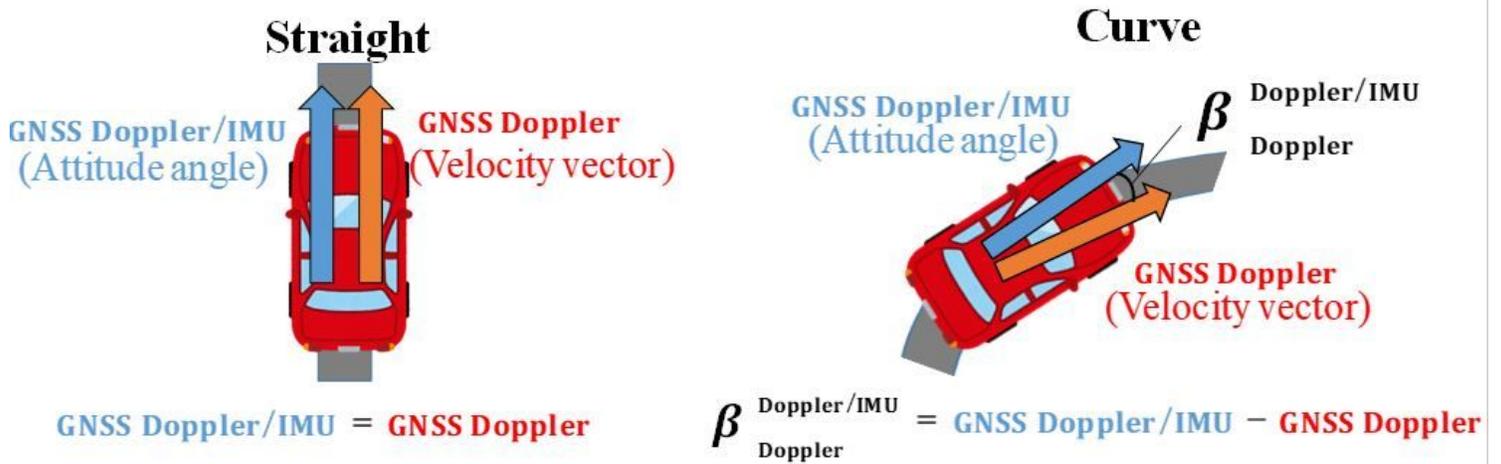


Figure 6

Overview of the slip angle

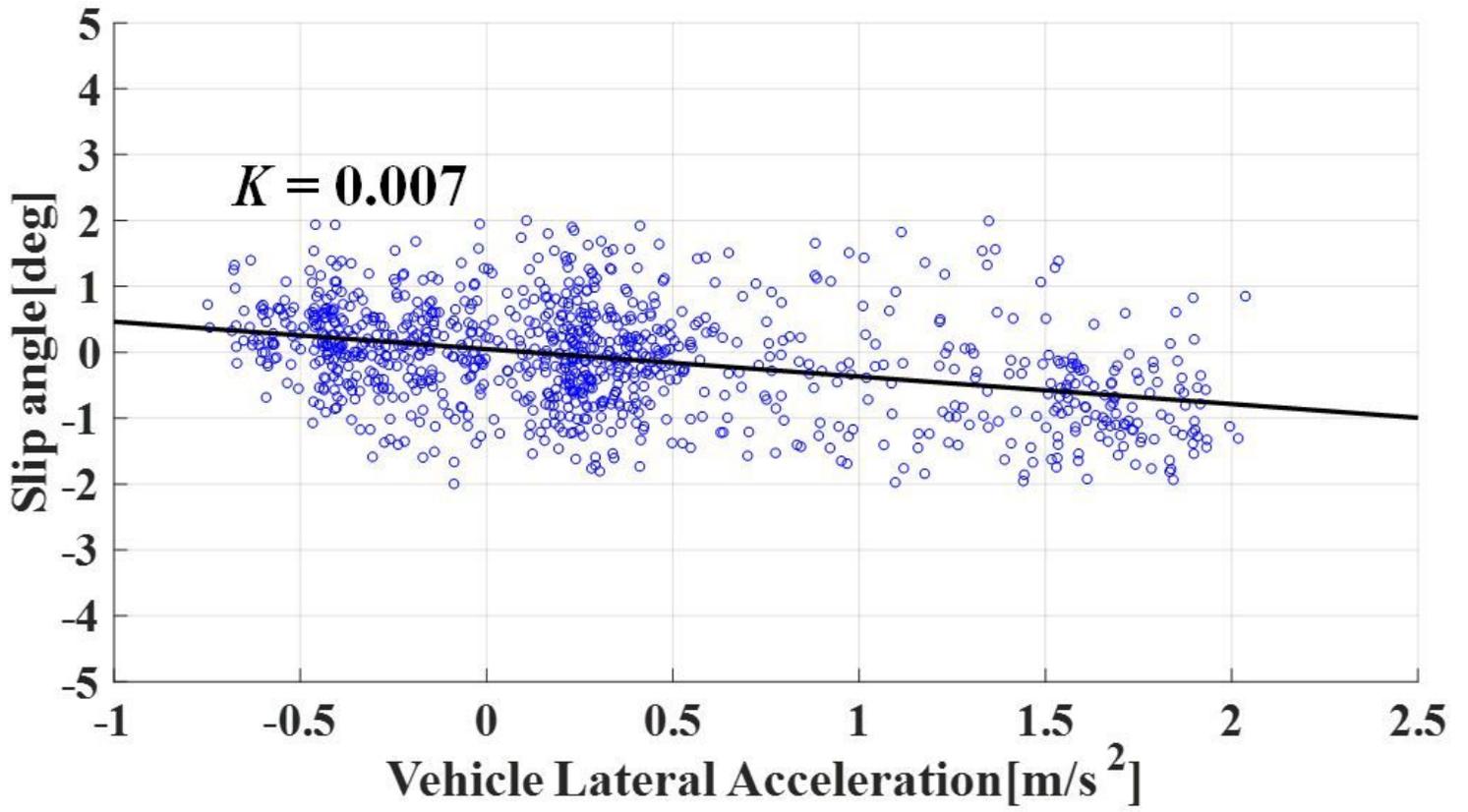


Figure 7

Relationship between slip angle and lateral acceleration using GNSS/Doppler

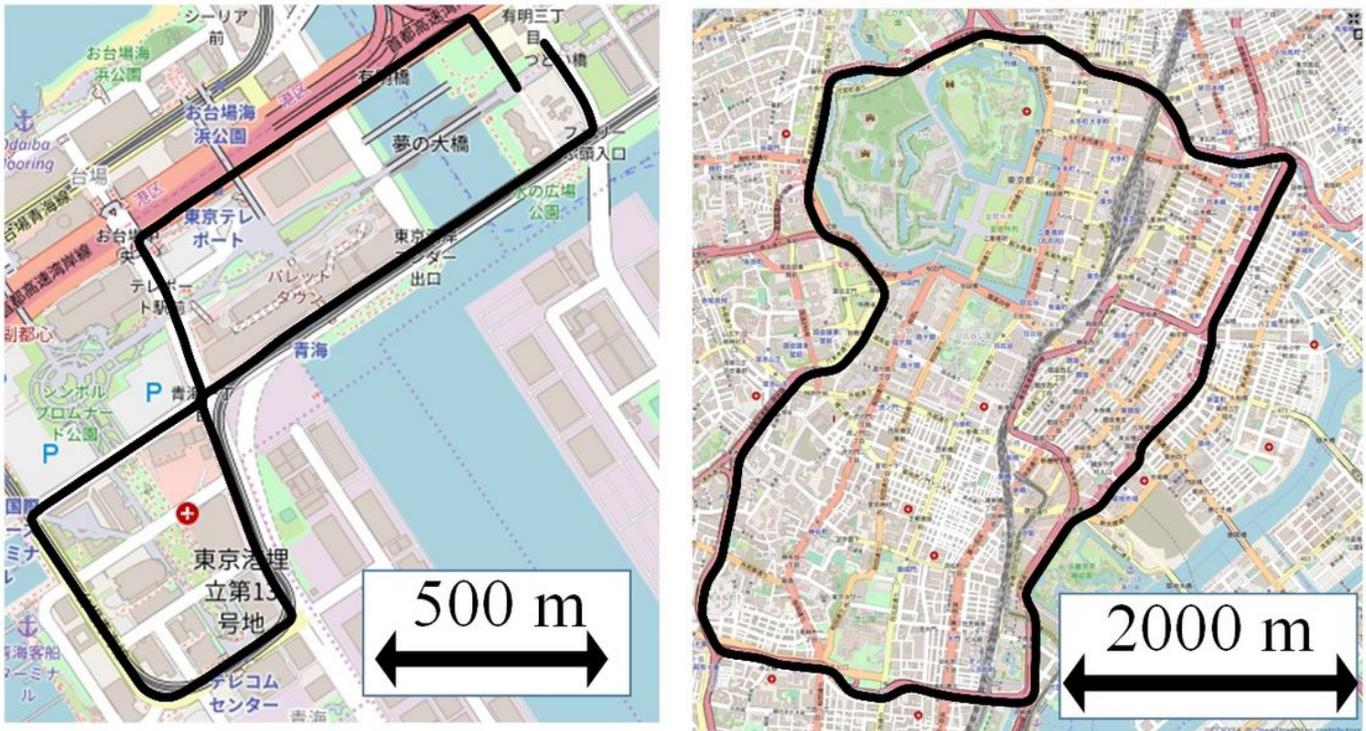


Figure 8

Evaluation fields [“© OpenStreetMaps”] Note: The designations employed and the presentation of the material on this map do not imply the expression of any opinion whatsoever on the part of Research Square concerning the legal status of any country, territory, city or area or of its authorities, or concerning the delimitation of its frontiers or boundaries. This map has been provided by the authors.

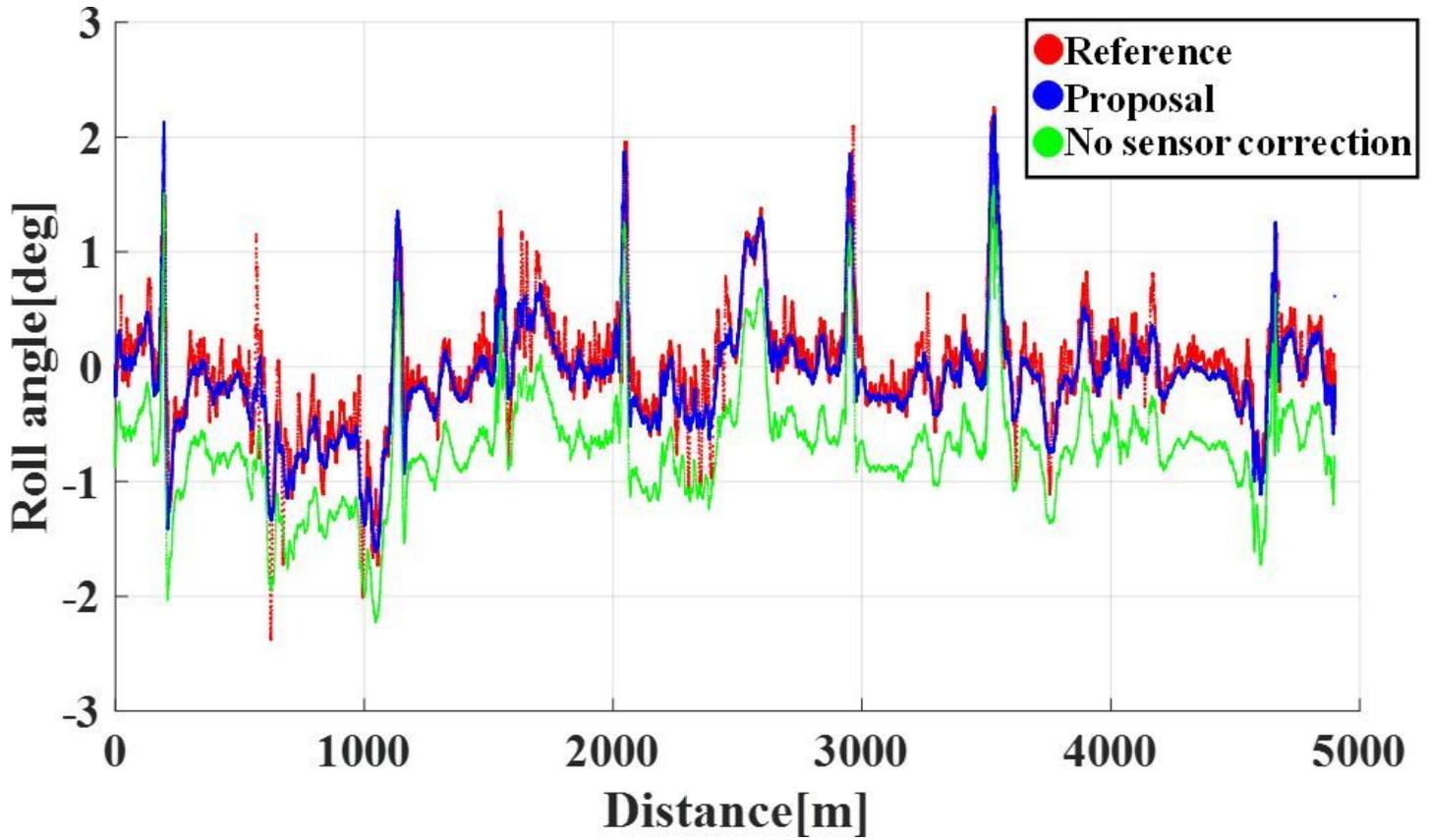


Figure 9

Estimated roll angle (Urban street)

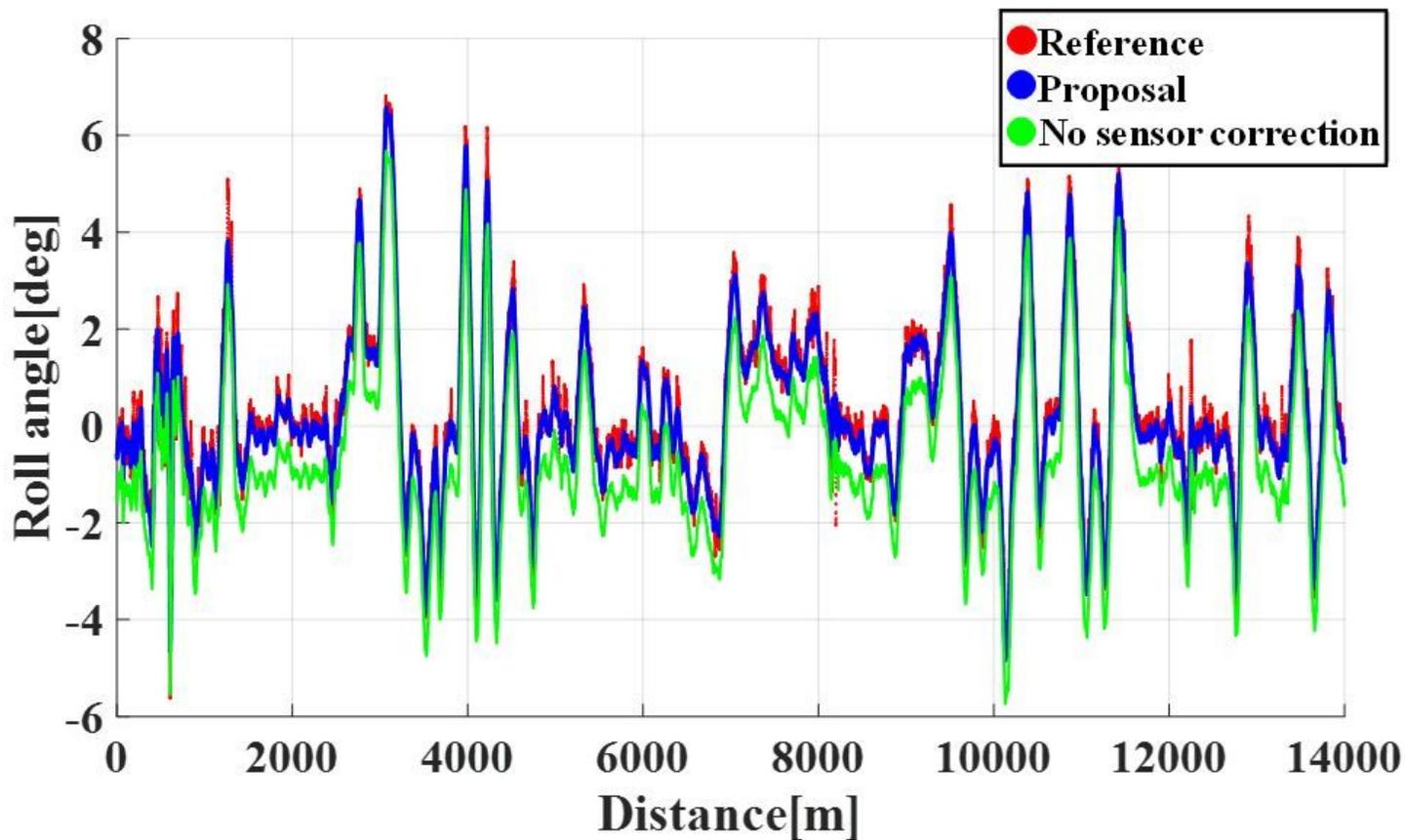


Figure 10

Estimated roll angle (Urban highway)

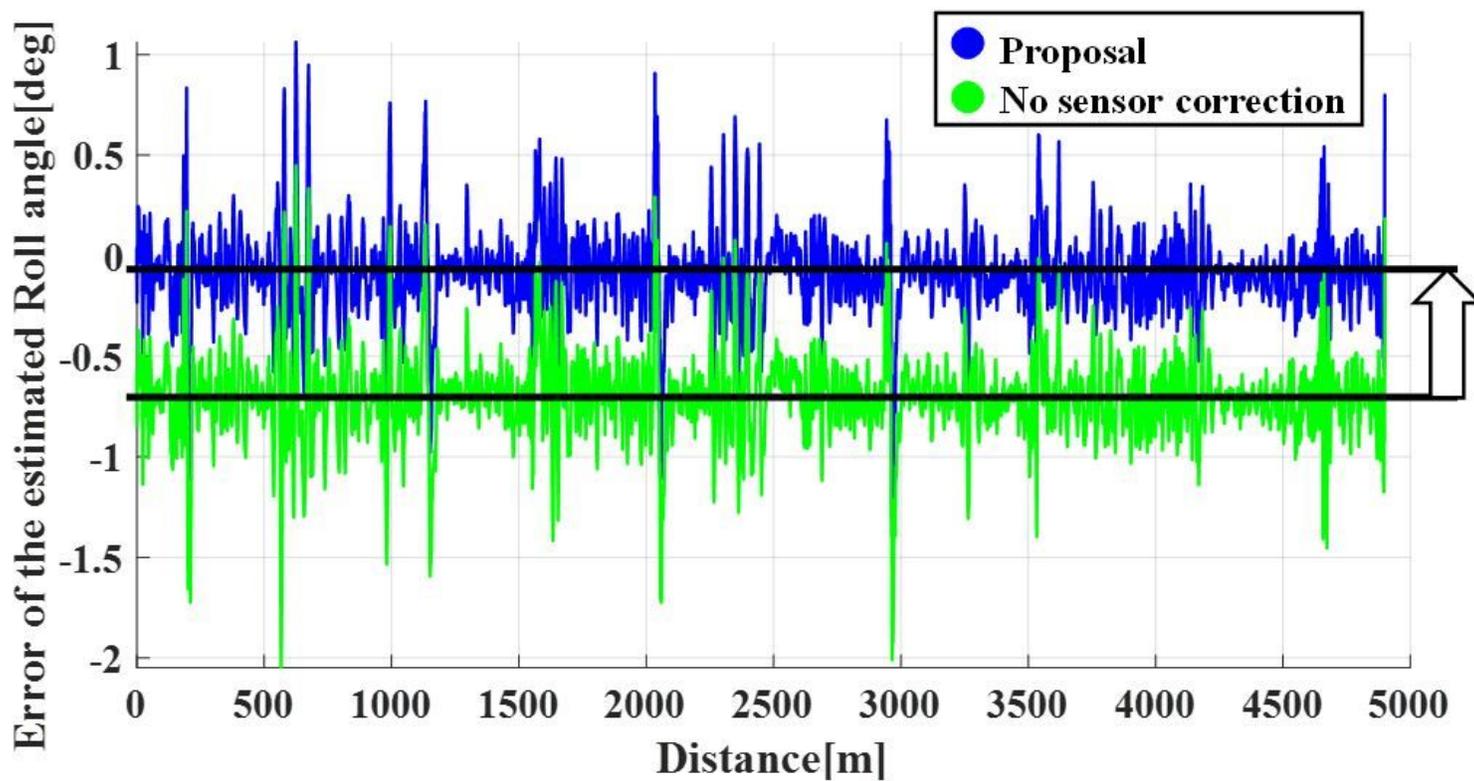


Figure 11

Error of estimated roll angle (Urban street)

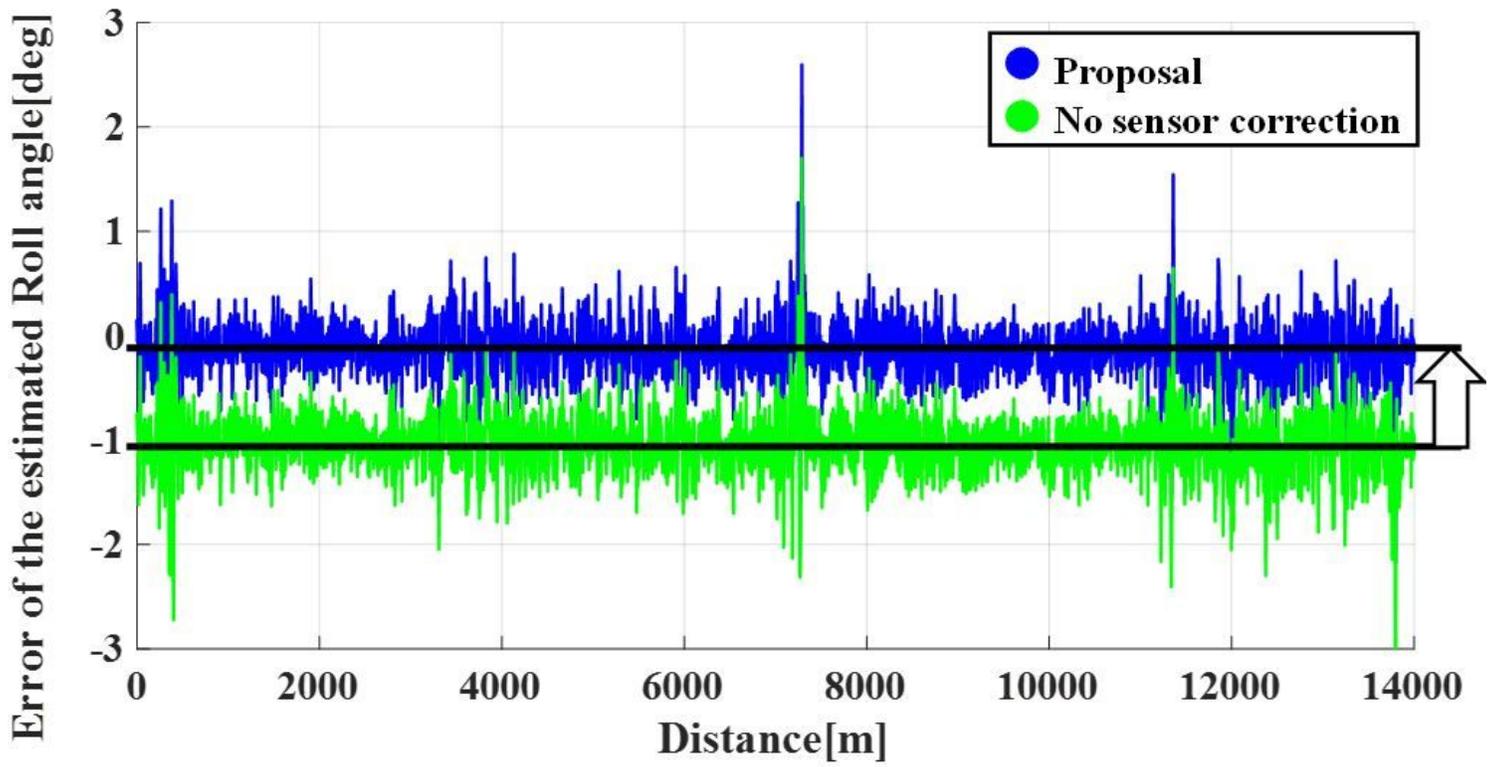


Figure 12

Error of estimated roll angle (Urban highway)

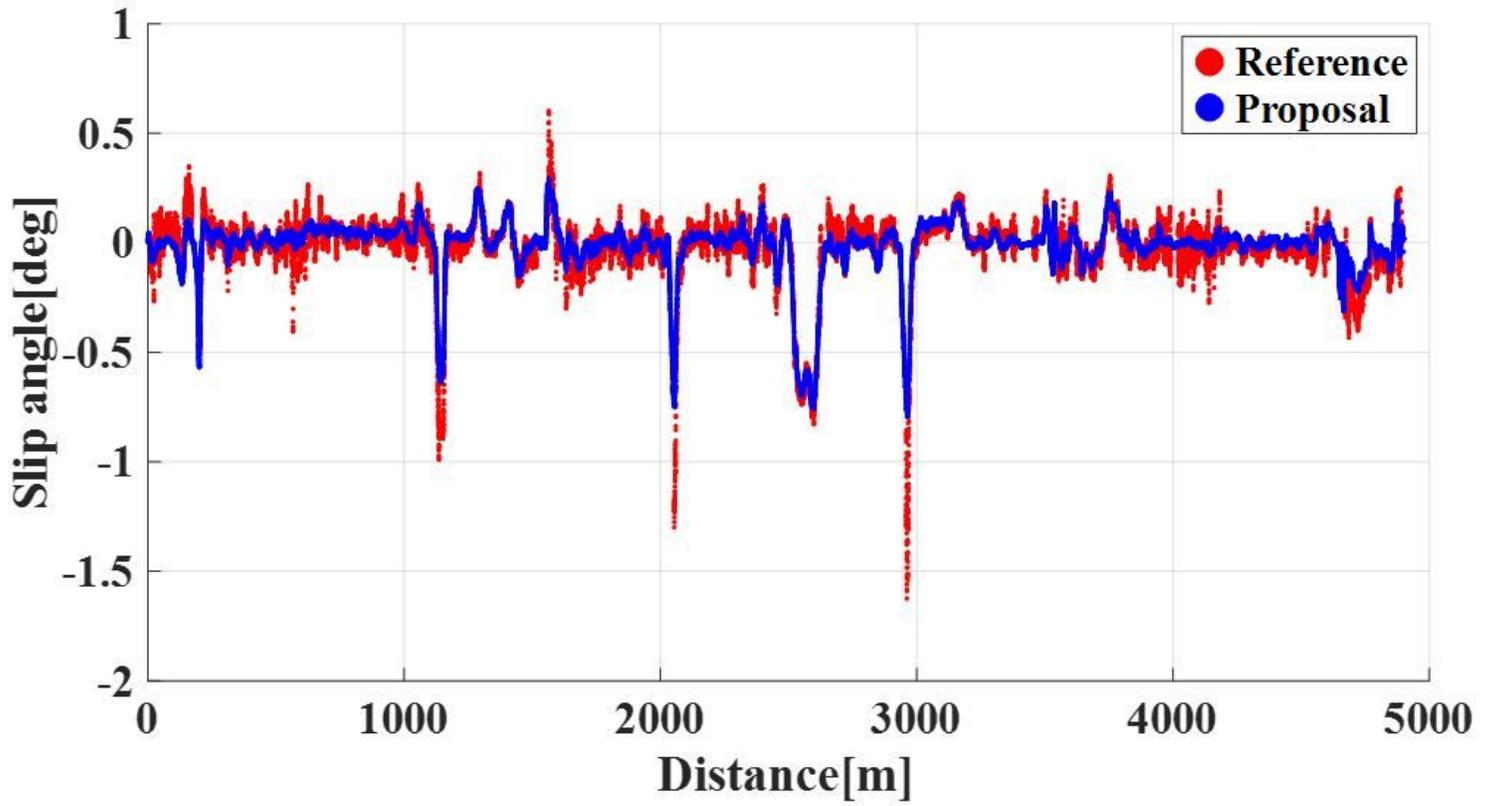


Figure 13

Estimated slip angle (Urban street)

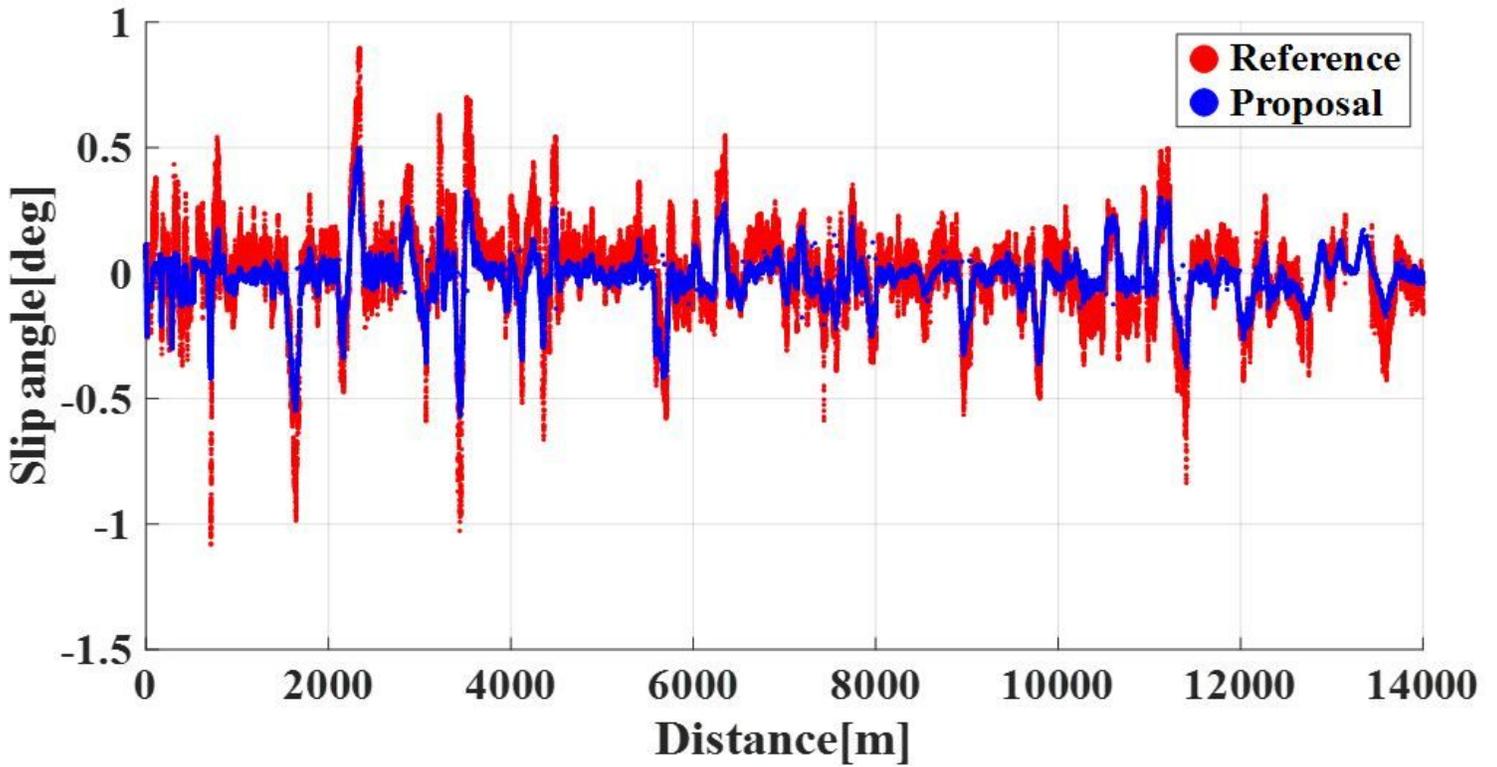


Figure 14

Estimated slip angle (Urban highway)

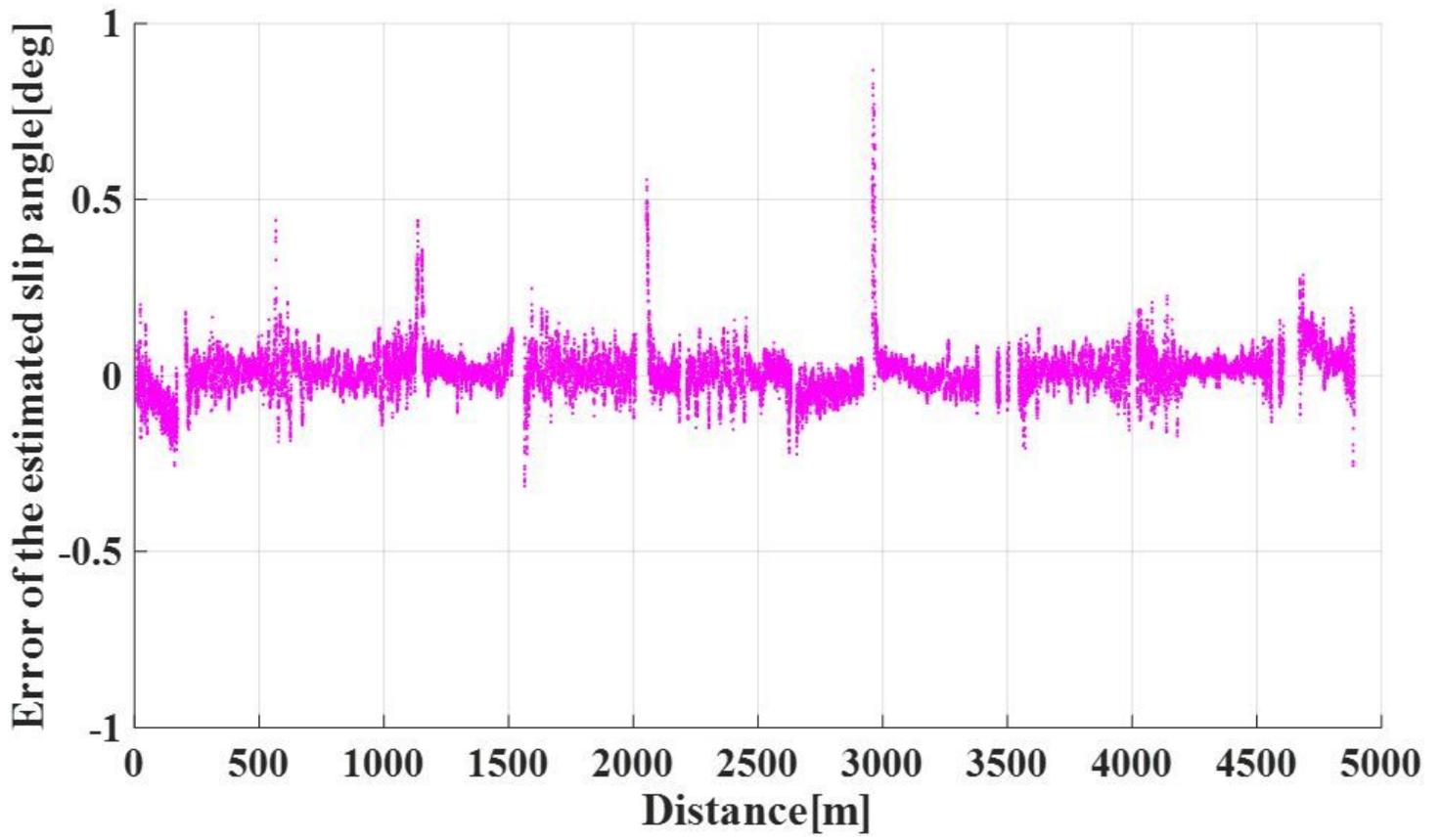


Figure 15

Error of estimated slip angle (Urban street)

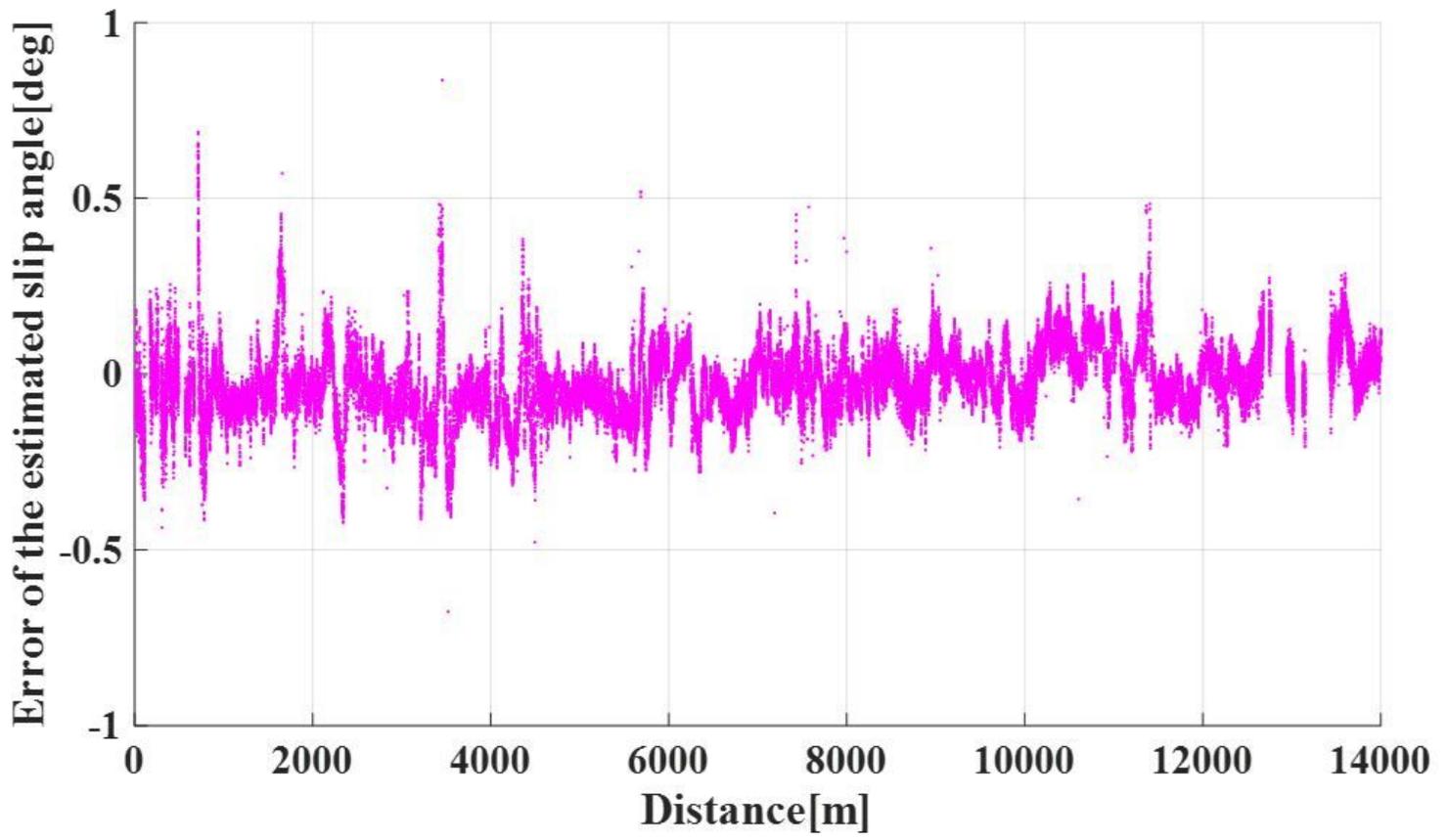


Figure 16

Error of estimated slip angle (Urban highway)

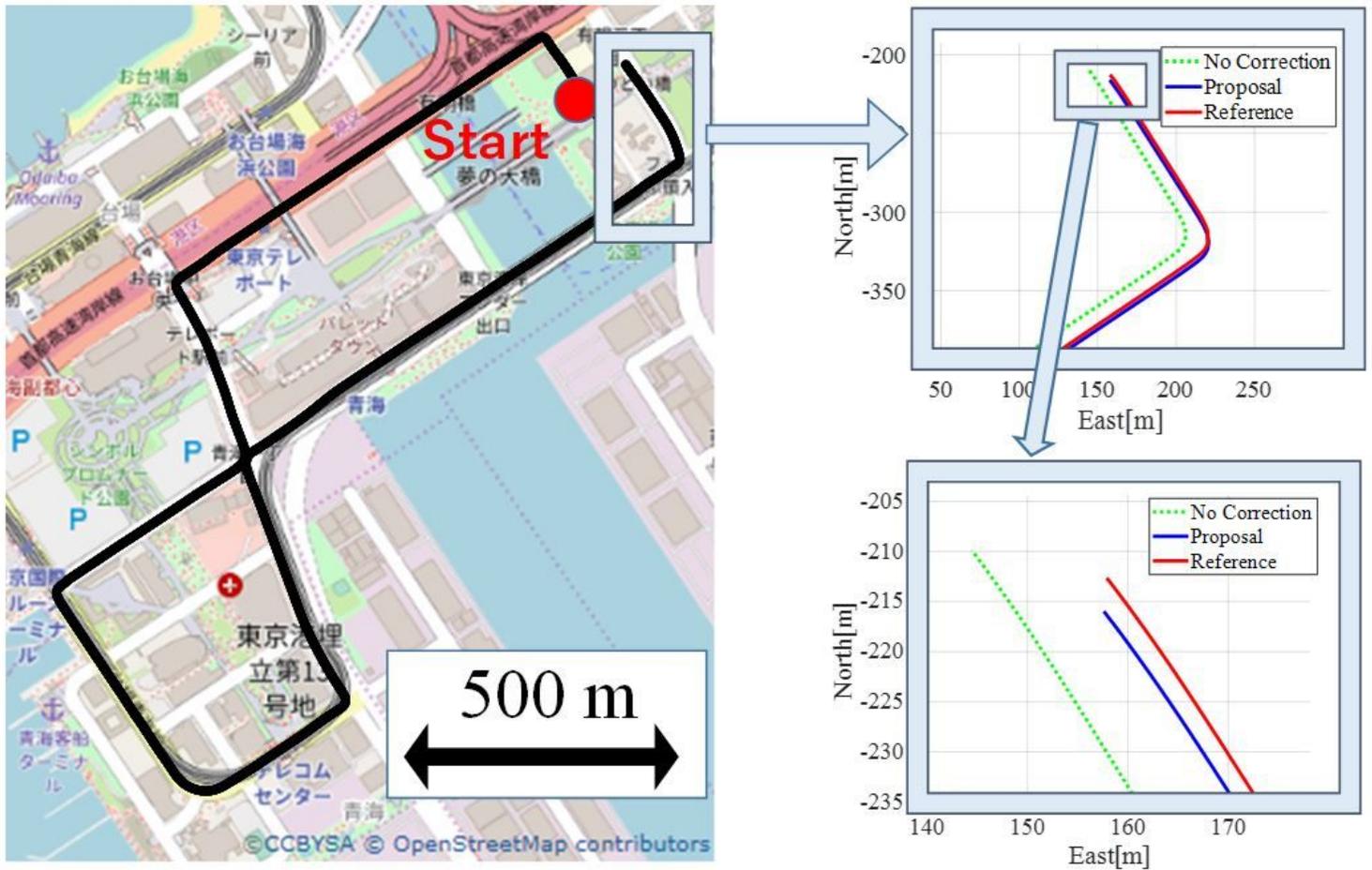


Figure 17

Effect of slip angle correction (Urban street) Note: The designations employed and the presentation of the material on this map do not imply the expression of any opinion whatsoever on the part of Research Square concerning the legal status of any country, territory, city or area or of its authorities, or concerning the delimitation of its frontiers or boundaries. This map has been provided by the authors.

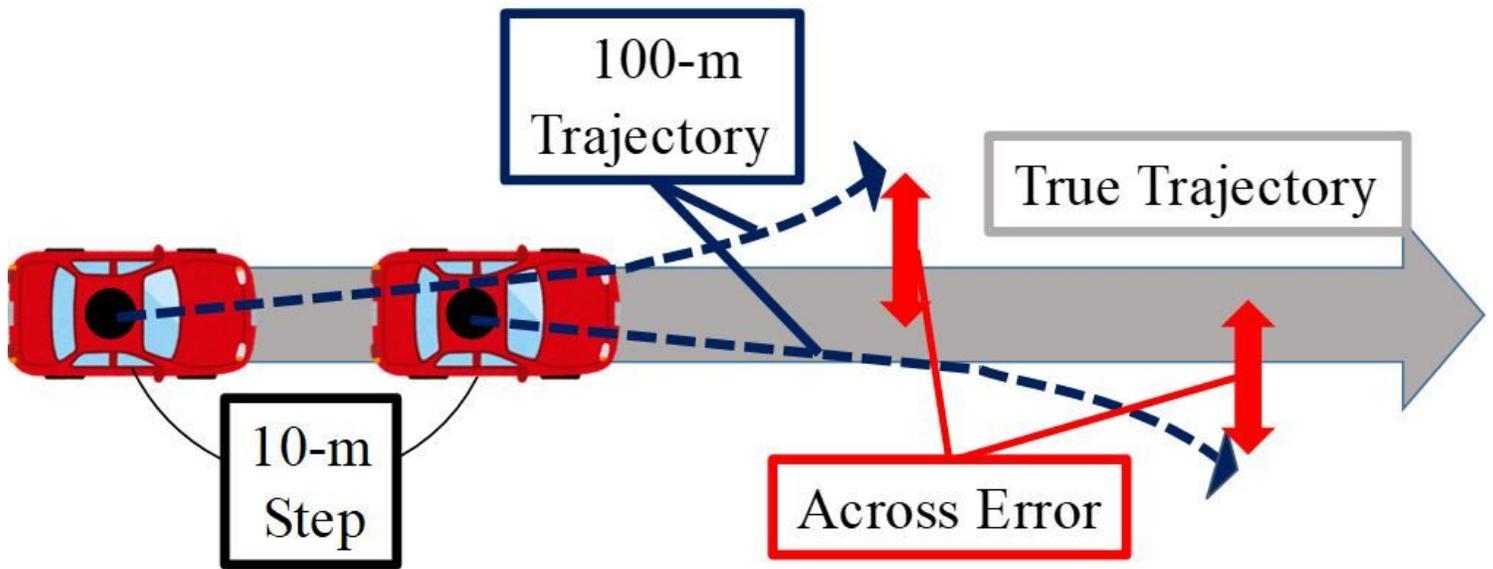


Figure 18

Overview of trajectory evaluation

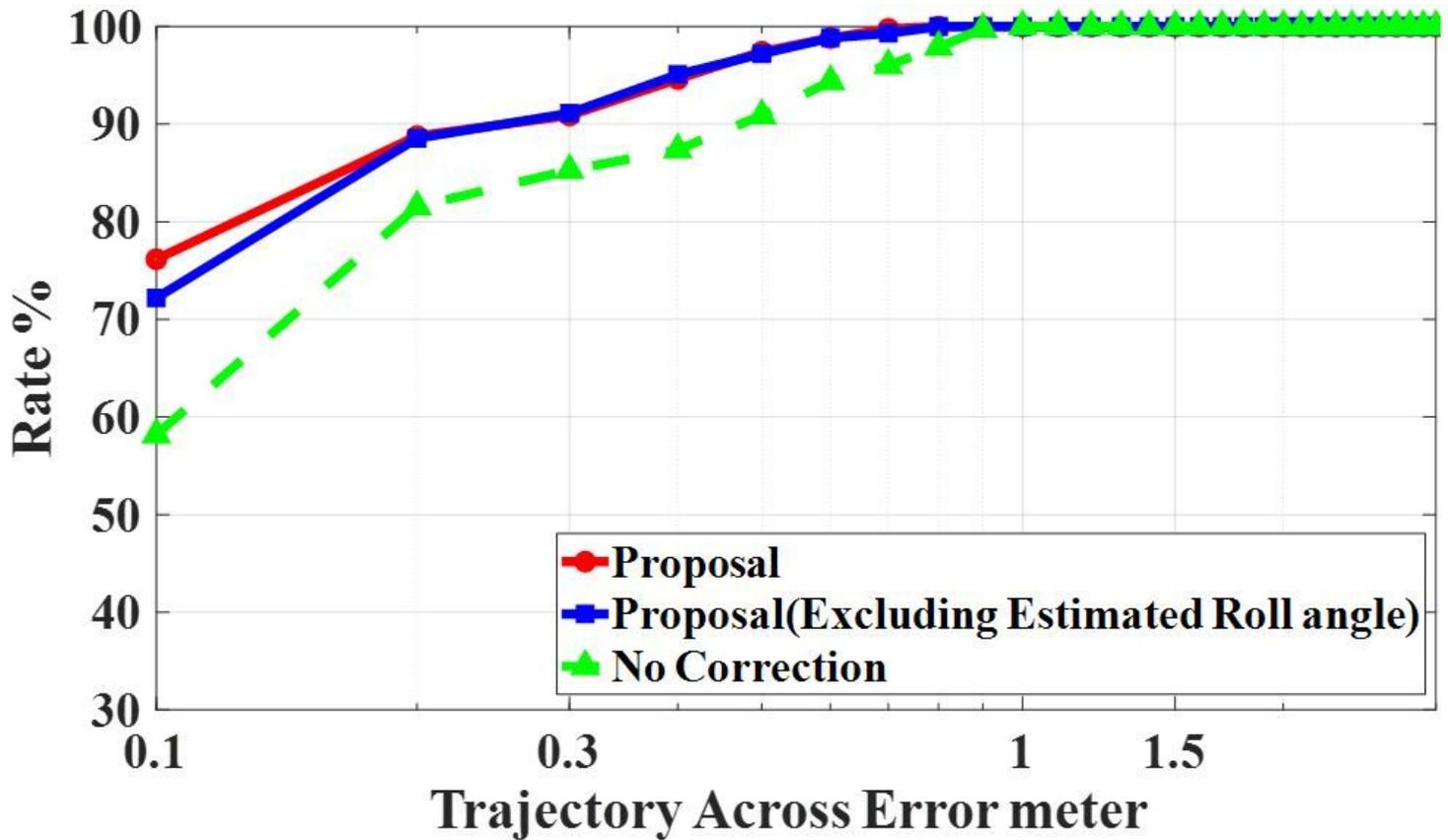


Figure 19

Evaluation of vehicle trajectory (Urban street)

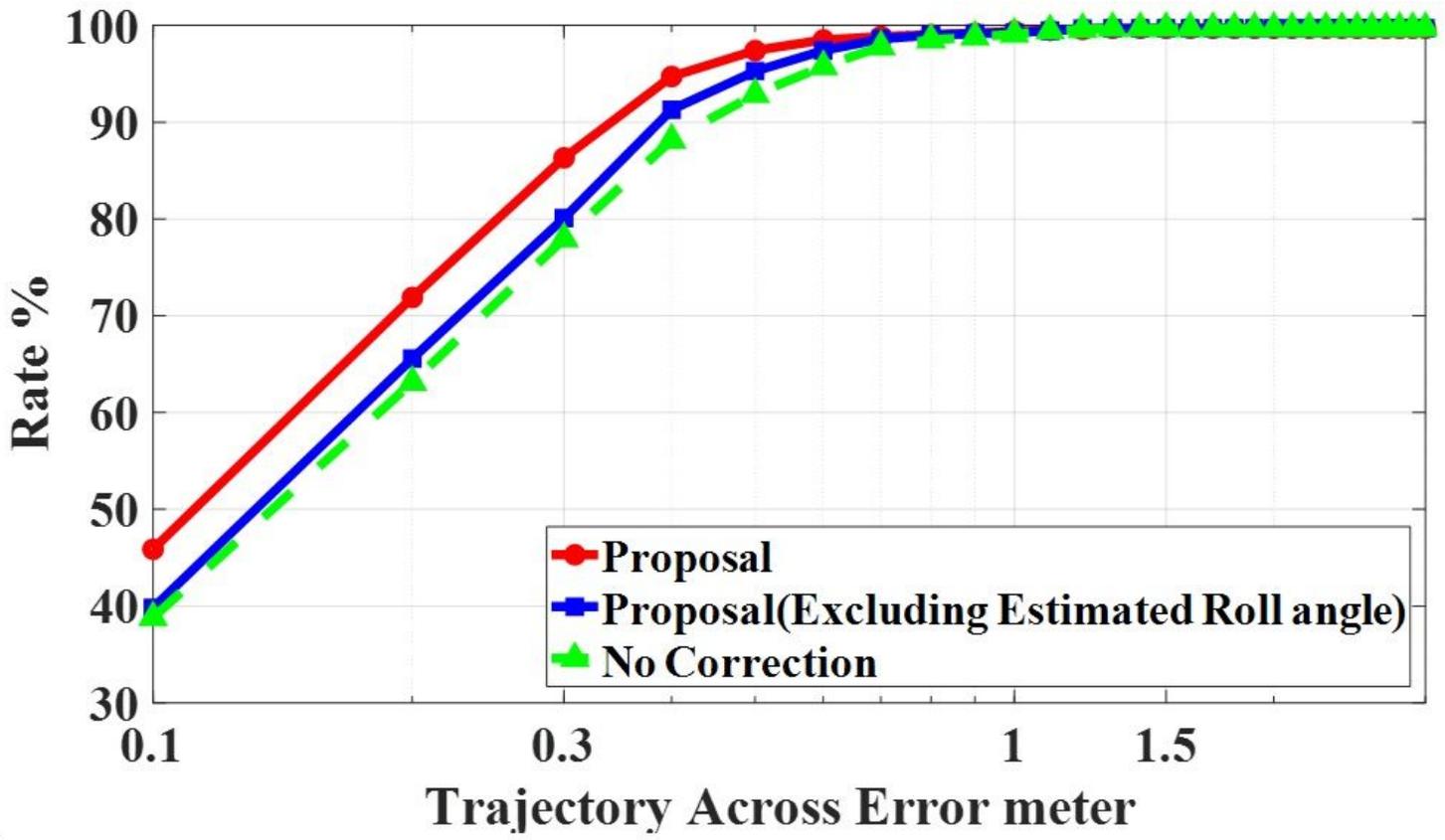


Figure 20

Evaluation of vehicle trajectory (Urban highway)