Modeling of a Mobile Spatial Cable Robot With Flexible Cables and Investigating the Effect of Its Nonlinear Vibrations on the System Dynamics

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Modeling of a mobile spatial cable robot with flexible cables and investigating the effect of its nonlinear vibrations on the system dynamics

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Abstract

In this paper the modeling of a novel moving cable robot is conducted considering the vibration of the cables in its nonlinear format. The robot has 6 DOFs while the controlling input number is 12. Considering the fact that the elasticity of the cables is coupled with the dynamic model of the system, their vibration effects on the robot performance and accuracy. The target of this paper is to model the robot considering the cables' elasticity and study its effect on the robot performance. This study can be considered in designing the controller of tower cranes and decrease the swing of the cables and increasing their stability. In order to cover the mentioned aim, the continuous vibration of the cables are modeled as a nonlinear system and it is added to the moving platform dynamics. Moreover the differences between the nonlinear modeling of the cables' vibration and estimating them as a linear system is studied and their related results are compared and analyzed. The correctness of modeling is shown by comparing the results with previous research and the superiority of modeling the cables' vibration in its nonlinear format is verified by the aid of a series of simulation scenarios in MATLAB. Moreover, by conducting some experimental test on the manufactured moving cable robot of IUST, it is illustrated that, modeling the cables in these robots as a nonlinear system results in more accurate results. It is shown that not only considering the cables' vibration is significant in analyzing the robot dynamic, but also it is shown that promoting the mentioned model into nonlinear one increase the accuracy of the robot modeling which sequentially can provide a stronger controller for stabilizing and controlling the end-effector within a predefined trajectory.

Keywords: Mobile base parallel cable robot, cable nonlinear vibrations, Rayleigh-Ritz method, null-space method

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1. Introduction

Cable robots as the new version of parallel robot have some advantages compared to linkage parallel ones. However, because of the nature of the cables as the related power transmission media, and its special characteristics such as its nonlinear dynamics, limitation of transferring compressional forces, low mass and also high flexibility, the control and accuracy of the end-effector is more challenging. Considering the fact that one of the most important applications of these kind of robots is their load carrying capability as a substitution of tower cranes, the mentioned characters and specially the flexibility one have a significant effect on the load capacity of the robot. Moreover, taking into account that usually the mass of the cables is ignorable compared to the mass of its load, the transversal vibrations of the cables can be ignored compared to its longitudinal one [1]. It should be also considered that, most of the present cable robots have a stationary chassis which extremely limits their related workspace. Promoting them to mobile cable robot can significantly increase the workspace of the robot but this promotion also increases the effect of the cables' elasticity on the performance of the robot as a result of higher engaged accelerations. Most of the available flexible cable analysis are performed with a big approximation i.e. considering the flexibility as a set of linear vibrating equations while we know that the cables' dynamic is nonlinear. Extracting the exact nonlinear model of the cable and coupling them with the dynamics of the mobile cable robot can significantly increase the accuracy of its modeling and provides an efficient feedforward signal for their related controlling systems.

Regarding a reduction in the complexity of modeling and dynamic analysis of cable robots, some studies have neglected the flexibility of the cables and assumed them as an ideal solid system. Zi et al. evaluated a fixed base cable robot with ideal cables and six degrees of freedom and obtained the dynamic equation of the robot end-effector using the Euler Lagrange method [2]. Torajizadeh et al. extracted the dynamic equations of a fixed base cable robot with ideal cables manufactured at Iran University of Science and Technology utilizing the Lagrange method [1]. In the mentioned studies, the chassis of the cable robot is stationary and thus their related workspace is limited. Therefore, Yousefzadeh et al. obtained the dynamic equations of the cable robot with mobile chassis using the Gibbs-Appell method [3].

Previous research has mainly assumed the cables as ideal solid system while cables and moving parts of the robot have elasticity, and consequently the length of the cables can be changed due to their vibration. Therefore, some research have considered the effect of cables elasticity in the performance of the cable robot. Cable vibrations in cable robots can be monitored longitudinally, transversely, or both. Diao et al. examined a type of parallel cable robot with flexible linear cables. In this robot, the vibrations of the end-effector connected to the cables, which are caused by the longitudinal and transverse vibrations of the cables, were simulated. The results revealed that the effect of transverse and longitudinal vibrations of the cables on the vibrations of the end-effector was 1.4 and 98.6%, respectively. As a result, the end-effector vibrations due to the

transverse vibrations of the cables are usually ignored and the cables are assumed as a linear system with longitudinal springs [4].

Cable vibration in cable robots is analyzed in both linear and nonlinear ways. Considering the simplicity of analysis and increasing the simulation speed in some studies, the vibrations of the cables were investigated linearly. Taghirad et al. extracted the dynamic equations of a fixed base cable robot with flexible linear cables using the Lagrange method [5]. By applying the Lagrange method, Torajizadeh et al. derived the dynamic equations of a fixed load-carrying cable robot, where the cables have linear vibration [6]. Yousefzadeh et al. obtained the dynamic equations of a mobile base load-carrying cable robot with flexible linear cables using Gibbs-Appell method [7]. Du et al. investigated a fixed base cable robot with a large work space and four long cables and analyzed the cables as distributed masses, using the finite element method. Then, they calculated the dynamic equation and vibration of each cable utilizing the Hooke's law [8]. In these papers, the elasticity of the cables are not considered and thus the delivered model is inaccurate. Following research have extracted the cables' model as a nonlinear system.

Yuan et al. examined a fixed base cable robot with six cables and showed that the effect of the dynamics of the cables due to the effect of mass and flexibility on the dynamic stiffness of the cable robot is significant, and it can change the amount of natural frequencies of the robot or add new resonances to the robot [9]. Sudden changes in the end-effector speed, friction of the cables around pulleys, turbulence related to wind [10], mass of the cables, flexibility of the cable material, dynamic behavior of the cables including low or high tensile, resonance caused by external or internal excitations, and their combination are among the effective factors in the cables nonlinear vibration in the cable robots [11]. Lee et al. studied the nonlinear and transverse vibrations of a cable with simple supports, considering the mass of the cables as one of the nonlinear factors. They used a mathematical method of perturbation analysis or multiple scales method to analyze the nonlinear vibration equations [12]. Pakdemirli et al. addressed the transverse vibrations of the cables with simple supports by the multiple scales method. Here, they ignored the effect of the mass of the cables and the resulting gravitational force [13]. Ferravante et al. investigated a fixed base cable robot with 8 cables by ignoring the nonlinear effects of the cables' vibration. Then, they extracted the nonlinear equations of the vibrating cables using the finite element method and cable discretization [14].

Caverly et al. evaluated a parallel cable robot with 4 cables in the case planar motion of the end-effector. In this study, the lumped-mass method was used to analyze the nonlinear vibration of cables. Here the cable was continuously discretized in different places as lumped-masses. After modeling the flexibility and damping properties of the cables by springs and dampers between lumped-masses, the Lagrange method was used to extract the dynamic equations of the robot, and then, the null space method was applied to eliminate the Lagrange coefficients. The extracted equations have nonlinear terms indicating the effects of changing the mass and stiffness in the cable initial area close to the pulley, which results in a more realistic modeling of the cable and is one of the advantages of the lumped-mass method [15]. Godbole et al.

investigated a parallel base cable robot with a transitional degree of freedom for the end-effector. To this aim, the Rayleigh-Ritz method was used to analyze the longitudinal and nonlinear vibrations of cables. Then, the robot dynamic equations were derived using the Lagrangian method and the null space method was utilized to remove the Lagrangian coefficients. The nonlinear terms of these equations are related to the change of the mass and stiffness matrix of the vibrating cables respect to time due to the changes in the cables' length by motors. Comparing the frequency responses induced by Rayleigh-Ritz method and lumped-mass method showed the superiority of the Rayleigh-Ritz method [16]. However in the above mentioned studies, the presented model is not coupled with the dynamic of a cable robot to investigate the effect of cables' elasticity on the robot performance.

As a result, in this paper, the nonlinear dynamics of the cables are extracted and the related vibrating equations are coupled with the dynamic equations of a mobile cable robot. The effect of elasticity module is investigated and the superiority of the modeling the cables as a nonlinear system is illustrated. The correctness of the presented modeling is investigated by experimental tests conducted on the mobile cable robot of IUST. In the next section, the modeling of the mobile cable robot with nonlinear cables is extracted including its kinematics and kinetics and afterwards, the related state space is delivered. In section 3, by the aid of a series of simulation scenarios, the effect of elasticity module of the cables on the robot performance and also the superiority of modeling the cable as a nonlinear system is shown. Finally to verify the modeling, the results of MATLAB simulations are compared with the experimental data obtained from testing on the IUST mobile cable robot. It is shown that, the longitudinal vibration of the cables can decrease the accuracy of the robot model and moreover estimating it as a linear system cannot compensate the error completely. Thus it is concluded here that by providing the exact nonlinear vibrating equations of the cables and coupling them to the robot dynamic, it is possible to cancel the parametric uncertainties of the cables' parameters in the controlling parameters and provide a more accurate feedforward signal for the mentioned controlling system.

2. Modeling and formulas

2.1. Kinematic modeling

Figures 1 and 2 illustrate a schematic view of a mobile cable robot manufactured in the Robotics Laboratory of the Iran University of Science and Technology.

Fig. 1 a) Wheeled mobile cable robot, b) Robot mobile base

Fig. 2 a) Robot base and b) end-effector from top view

Figures 1 and 2 show reference (O_N), base (O_A), and end-effector (O_B) coordinate systems. Points P show cables connection to the pulleys on the base while points B indicate the connection of the cables to the end-effector. Further, r_{wh} represents the radius of the base right and left wheels, b indicates the half distance between the wheels and, d is the distance between the base coordinate system and the line connecting the right and left wheels. Based on the research [3] and Figs. 1 and 2, when robot cables are assumed ideal, there are two constrained equations between the speeds of different parts of the robot. The first constraint equation is related to the mobile base of the robot and is expressed according to Eq. (1).

$$\begin{bmatrix} \dot{x}_{AN} & \dot{y}_{AN} & \dot{\phi}_A \end{bmatrix}^T = U_w \dot{\theta}_w \quad (1)$$

where

$$U_w = \frac{r_{wh}}{2} \begin{bmatrix} \frac{d}{b} \sin \varphi_A + \cos \varphi_A & -\frac{d}{b} \sin \varphi_A + \cos \varphi_A \\ -\frac{d}{b} \cos \varphi_A + \sin \varphi_A & \frac{d}{b} \cos \varphi_A + \sin \varphi_A \\ \frac{-1}{b} & \frac{1}{b} \end{bmatrix}, \quad \dot{\theta}_w = [\dot{\theta}_r \quad \dot{\theta}_l]^T$$

where $\dot{x}_{AN}$ and $\dot{y}_{AN}$ show the transitional velocities of the center of the base coordinate system relative to the reference coordinate system, φ_A represents the rotation angle of the coordinate system attached to the base (O_A) relative to the reference coordinate system, $\dot{\theta}_r$ indicates the rotational speed of the base right wheel, $\dot{\theta}_l$ gives the rotational speed of the base left wheel, and U_w is the Jacobin matrix related to the kinematic constraints of the mobile base. The second constraint equation is defines the connection between the angular velocity of the cable motors and the change in the cables' length according to Eq. (2).

$$\begin{aligned} \dot{q}_i &= -r\dot{\theta}_i \quad i = 1, \dots, 6 \\ \dot{\theta}_l &= U_a \dot{x}_A + U_b \dot{x}_B \end{aligned} \quad (2)$$

where

$$\begin{aligned} \dot{\theta}_l &= [\dot{\theta}_1 \quad \dot{\theta}_2 \quad \dot{\theta}_3 \quad \dot{\theta}_4 \quad \dot{\theta}_5 \quad \dot{\theta}_6]^T, \quad \dot{x}_A = [\dot{x}_{AN} \quad \dot{y}_{AN} \quad 0 \quad 0 \quad 0 \quad \dot{\varphi}_A]^T, \\ \dot{x}_B &= [\dot{x}_{BN} \quad \dot{y}_{BN} \quad \dot{z}_{BN} \quad \dot{\alpha} \quad \dot{\beta} \quad \dot{\gamma}]^T, \quad U_a = \frac{\Delta^T {}^A \bar{R}_B C_1}{-r}, \quad U_b = \frac{\Delta^T {}^A \bar{R}_B C_2 C_3}{-r}, \\ \Delta &= \begin{bmatrix} \dots & {}^N R_A^T {}^N \hat{q}_i & \dots \\ \dots & {}^N R_A^T ({}^N R_B^B r_{Bi} \times {}^N \hat{q}_i) & \dots \end{bmatrix}_{6 \times 6} \quad i = 1, \dots, 6, \quad {}^A \bar{R}_B = \begin{bmatrix} I_3 & 0_3 \\ 0_3 & {}^N R_A^T {}^N R_B \end{bmatrix}, \\ C_1 &= \begin{bmatrix} -{}^N R_A^T & {}^A \bar{r}_{BA} \\ 0_3 & -{}^N R_B^T {}^N R_A \end{bmatrix}, C_2 = \begin{bmatrix} {}^N R_A^T & 0_3 \\ 0_3 & I_3 \end{bmatrix}, C_3 = \begin{bmatrix} I_3 & 0_3 \\ 0_3 & P_{BN} \end{bmatrix}, P_{BN} = \begin{bmatrix} 1 & 0 & -\sin \beta \\ 0 & \cos \alpha & \sin \alpha \cos \beta \\ 0 & -\sin \alpha & \cos \alpha \cos \beta \end{bmatrix} \end{aligned}$$

Also q_i represents the cables longitudinal vector, ${}^N \hat{q}_i$ indicates the cables' unit vector, r shows the radius of pulleys connected to the cables, $\dot{x}_A$ displays the base velocity vector relative to the reference coordinate system, $\dot{x}_B$ demonstrates the end-effector velocity relative to the reference coordinate system, $\dot{\theta}_l$ depicts the angular velocity vector of the motors connected to the cables, P_{BN} presents the Euler angle rate matrix of the end-effector relative to the reference coordinate

system, and U_a and U_b are Jacobin matrices. Further, ${}^N R_A$ and ${}^N R_B$ are the base and end-effector rotational matrices relative to the reference coordinate system, respectively. Ultimately, ${}^A r_{BA}$ represents the position vector of the end-effector relative to the base in the base coordinate system, as well as the dual vector of the asymmetric matrix ${}^A \bar{r}_{BA}$. As shown in Fig. 3(a), Yousefzadeh studied the longitudinal vibrations of the cables for a mobile robot linearly and considered each cable as a spring [7].

Fig. 3 a) Robot with linear flexible cables b) Robot with nonlinear flexible cables

In this case, the change in the length of each spring shows the change in the length of the cables, due to linear vibrations. Consequently, the constraint Eq. (2) is modified as Eq. (3).

$$\begin{aligned} \dot{q}_i &= -r\dot{\theta}_i + \dot{q}_{li} \quad i = 1, \dots, 6 \\ \dot{\theta}_i &= U_a \dot{x}_A + U_b \dot{x}_B + U_e \dot{q}_{li} \end{aligned} \quad (3)$$

where

$$U_e = \frac{I_6}{r}, \quad \dot{q}_{li} = [\dot{q}_{l1} \quad \dot{q}_{l2} \quad \dot{q}_{l3} \quad \dot{q}_{l4} \quad \dot{q}_{l5} \quad \dot{q}_{l6}]^T$$

Here q_{li} represents the vibration dynamic coordinates related to linear flexible cables and U_e indicates the Jacobin matrix for vibration cables. In this paper, the Rayleigh-Ritz method is used to analyze the vibration of the cables in a nonlinear way. As illustrated in Fig. 3(b), the amount of longitudinal vibration of the mass element in each cable is expressed as follows [16]:

$$w(x_i, t) = \Gamma(x_i, t) q_{ei}(t) \quad i = 1, \dots, 6 \quad (4)$$

where

$$\Gamma(x_i, t) = \sin\left(\frac{\pi(x_i - r\theta_i)}{2(L_i - r\theta_i)}\right) \quad i = 1, \dots, 6$$

Also $q_{ei}(t)$ represents the dynamic coordinates of the vibration related to the nonlinear vibration cables, $\Gamma(x_i, t)$ indicates the vibration mode shape, L_i shows the initial length of the cables at the x_i direction, and θ_i is the rotation angle of the pulleys connected to the cables. As shown in Fig. 3(b) and Eqs. (4)-(5) is used to obtain the longitudinal vector of the cables with nonlinear vibration.

$$q_i = L_i - r\theta_i + \Gamma_{(L_i, t)} q_{ei}(t) \quad i = 1, \dots, 6 \quad (5)$$

The change in the cable length in this case is expressed by deriving from Eq. (5) and based on Eq. (6).

$$\dot{q}_i = -r\dot{\theta}_i + \dot{q}_{ei}\Gamma_{(L_i, t)} + q_{ei}\dot{\Gamma}_{(L_i, t)} \quad i = 1, \dots, 6 \quad (6)$$

where

$$\dot{\Gamma}(x_i, t) = \frac{\pi r \dot{\theta}_i (x_i - L_i)}{2(L_i - r\theta_i)^2} \cos\left(\frac{\pi(x_i - r\theta_i)}{2(L_i - r\theta_i)}\right), \quad \Gamma_{(L_i, t)} = 1, \quad \dot{\Gamma}_{(L_i, t)} = 0$$

By using Eq. (6) and in case the cables have longitudinal and nonlinear vibration, the constraint Equation (2) is modified as Eq. (7).

$$\begin{aligned} \dot{q}_i &= -r\dot{\theta}_i + \dot{q}_{ei} \quad i = 1, \dots, 6, \\ \dot{\theta}_i &= U_a \dot{x}_A + U_b \dot{x}_B + U_e \dot{q}_{ei} \end{aligned} \quad (7)$$

where

$$\dot{q}_{ei} = [\dot{q}_{e1} \quad \dot{q}_{e2} \quad \dot{q}_{e3} \quad \dot{q}_{e4} \quad \dot{q}_{e5} \quad \dot{q}_{e6}]^T, \quad U_e = \frac{L_6}{r}$$

As displayed in Fig. 3(b), the velocity of each mass element of cables 1 to 6 relative to the reference system is obtained according to Eq. (8).

$${}^N v_{dmi} = {}^N v_{pi} + {}^N v_{\frac{dmi}{pi}} \quad i = 1, \dots, 6 \quad (8)$$

where ${}^N v_{pi}$ represents the velocity of the P points corresponding to the pulleys in the reference coordinate system and is obtained from Eq. (9).

$${}^N v_{pi} = {}^N v_A + {}^N \dot{R}_A {}^A r_{pi} \quad (9)$$

where ${}^N v_{\frac{dmi}{pi}}$ is the relative velocity of the mass element of each cable relative to its corresponding pulley in the reference coordinate system, which is obtained from Eq. (10).

$${}^N v_{\frac{dmi}{pi}} = {}^N R_A ({}^A v_{\frac{dmi}{pi}} - {}^N \dot{R}_A^T {}^N r_{\frac{dmi}{pi}}) \quad (10)$$

In which, ${}^N r_{\frac{dmi}{pi}}$ and ${}^A v_{\frac{dmi}{pi}}$ represent the position vector and the relative velocity of the element of each cable relative to the corresponding pulley in the reference coordinate system, which are obtained from Eqs. (11)-(12), as shown in Fig. 3(b).

$${}^N r_{\frac{dmi}{pi}} = (x_i - r\theta_i + q_{ei}\Gamma_i) {}^N \hat{q}_i \quad i = 1, \dots, 6 \quad (11)$$

$${}^A v_{\frac{dmi}{pi}} = (-r\dot{\theta}_i + \dot{q}_{ei}\Gamma_i + q_{ei}\dot{\Gamma}_i) {}^A \hat{q}_i \quad i = 1, \dots, 6 \quad (12)$$

where

$${}^A \hat{q}_i = {}^N R_A^T {}^N \hat{q}_i$$

where ${}^A \hat{q}_i$ represents the unit vector of the cables relative to the base coordinate system. By substituting Eqs. (9)-(10) into Eq. (8), the mass element velocity of the cables in the reference system (${}^N v_{dmi}$) is obtained from Eq. (13).

$${}^N v_{dmi} = {}^N v_A + {}^A \bar{\omega}_{AN} {}^N r_{\frac{pi}{A}} + {}^N R_A v_{mti} {}^N R_A^T {}^N \hat{q}_i + {}^N R_A {}^A \bar{\omega}_{AN} {}^N R_A^T r_{mti} {}^N \hat{q}_i \quad i = 1, \dots, 6 \quad (13)$$

where

$$\begin{aligned} r_{mti} &= \begin{bmatrix} r_{mi} & 0 & 0 \\ 0 & r_{mi} & 0 \\ 0 & 0 & r_{mi} \end{bmatrix}, \quad v_{mti} = \begin{bmatrix} v_{mi} & 0 & 0 \\ 0 & v_{mi} & 0 \\ 0 & 0 & v_{mi} \end{bmatrix}, \quad {}^N v_A = [\dot{x}_{AN} \quad \dot{y}_{AN} \quad 0]^T, \\ {}^A \omega_{AN} &= [0 \quad 0 \quad \dot{\phi}_A]^T, \quad {}^N r_{\frac{pi}{A}} = {}^N R_A {}^A r_{pi} = \begin{bmatrix} {}^N r_{piA,x} & {}^N r_{piA,y} & 0 \end{bmatrix}^T, \\ r_{mi} &= x_i - r\theta_i + q_{ei}\Gamma_i, \quad v_{mi} = -r\dot{\theta}_i + \dot{q}_{ei}\Gamma_i + q_{ei}\frac{\partial \Gamma_i}{\partial \theta_i} \dot{\theta}_i, \end{aligned}$$

where ${}^N v_A$ indicates the transitional velocity vector of the base center of mass, ${}^A \bar{\omega}_{AN}$ shows the antisymmetric matrix of the dual vector, and ${}^N r_{pi_A}$ is the position vector of the pulleys relative to the base center of mass in the reference system.

2.2. Dynamic modeling

This section addresses the dynamic modeling of mobile base cable robot in three modes including ideal cables, cables with linear vibration, and cables with nonlinear vibration. The modeling is conducted using Lagrange method. The Lagrangian dynamic relation is generally expressed according to Eq. (14) [18].

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{q}} \right)^T - \left(\frac{\partial L}{\partial q} \right)^T + \frac{\partial R}{\partial \dot{q}} = f + \Xi^T \lambda \quad (14)$$

The kinetic energy of a mobile base consists of two rotational and translational kinetic energies and can be obtained from Eq. (15).

$$T_A = \frac{1}{2} m_A (\dot{x}_{AN}^2 + \dot{y}_{AN}^2) + \frac{1}{2} {}^A \omega_{AN}^T {}^N R_A I_{OA} {}^N R_A^T {}^A \omega_{AN} = \frac{1}{2} \dot{q}_A^T M_A \dot{q}_A \quad (15)$$

where

$$I_{oA} = \begin{bmatrix} I_{A1} & 0 & I_{A4} \\ 0 & I_{A2} & 0 \\ I_{A5} & 0 & I_{A3} \end{bmatrix}, \quad \dot{q}_A = [\dot{x}_{AN} \quad \dot{y}_{AN} \quad \dot{\phi}_A]^T, \quad M_A = \text{diag}\{m_A, m_A, I_{A3}\}$$

Here m_A represents the base mass, M_A indicates the base mass matrix, $\dot{q}_A$ shows the generalized velocity vector related to the base, and I_{oA} is the base inertia moment matrix relative to the system attached to the base. The kinetic energy of the end-effector is expressed according to Eq. (16).

$$T_B = \frac{1}{2} m_B v_B^T v_B + \frac{1}{2} \dot{\Psi}_{BN}^T P_{BN}^T I_{OB} P_{BN} \dot{\Psi}_{BN} = \frac{1}{2} \dot{q}_B^T M_B \dot{q}_B \quad (16)$$

where

$$v_B = [\dot{x}_{BN} \quad \dot{y}_{BN} \quad \dot{z}_{BN}]^T, \quad \dot{\Psi}_{BN} = [\dot{\alpha} \quad \dot{\beta} \quad \dot{\gamma}]^T, \quad I_{OB} = \begin{bmatrix} I_{B1} & 0 & 0 \\ 0 & I_{B2} & 0 \\ 0 & 0 & I_{B3} \end{bmatrix},$$

$$\dot{q}_B = [\dot{x}_{BN} \quad \dot{y}_{BN} \quad \dot{z}_{BN} \quad \dot{\alpha} \quad \dot{\beta} \quad \dot{\gamma}]^T, \quad M_B = \begin{bmatrix} m_B I_3 & 0_3 \\ 0_3 & P_{BN}^T I_{OB} P_{BN} \end{bmatrix}$$

Also m_B demonstrates the end-effector mass, M_B indicates the end-effector mass matrix, v_B shows the end-effector velocity vector in the reference coordinate system, $\dot{\Psi}_{BN}$ depicts the operator angular velocity vector in the reference system, I_{OB} gives the operator inertial moment matrix relative to the coordinate system attached to the end-effector and $\dot{q}_B$ is the generalized velocity vector of the end-effector. Since the base wheels have a rotational speed, the kinetic energy of the wheels is expressed according to Eq. (17).

$$T_w = \frac{1}{2} J_l \dot{\theta}_l^2 + \frac{1}{2} J_r \dot{\theta}_r^2 = \frac{1}{2} \dot{q}_w^T M_w \dot{q}_w \quad (17)$$

where

$$\dot{q}_w = \begin{bmatrix} \dot{\theta}_r & \dot{\theta}_l \end{bmatrix}^T, \quad M_w = \text{diag} \{J_r, J_l\}$$

In which $\dot{q}_w$ represents the generalized velocity vectors related to the mobile base wheels, M_w shows the mass matrix of the base wheels, J_r presents the inertial moment of the base right wheel, and J_l is the inertial moment of the base left wheel. The kinetic energy of the motors connected to the cables is expressed in Eq. (18).

$$T_m = \frac{1}{2} (\dot{\theta}_i + C_4 \dot{x}_A)^T I_m (\dot{\theta}_i + C_4 \dot{x}_A) \quad (18)$$

where

$$C_4 = \begin{bmatrix} 0_{6 \times 5} & 1_{6 \times 1} \end{bmatrix}, \quad I_m = J_m I_6$$

Here J_m defines the inertial moment of each motor around its rotation axis. As displayed in Fig. 3(b) and Eq. (13), in order to obtain the kinetic energy of each vibrating cable, it is necessary to first calculate the kinetic energy of the mass element of each cable using the mass element velocity of the cables in the reference coordinate system (${}^N v_{dmi}$). Then, by integrating the kinetic energy of the mass elements in the direction x_i , the kinetic energy of the nonlinear vibrating cables can be obtained from Eq. (19) [16].

$$T_{ci} = \frac{1}{2} \int_{r\theta_i}^{L_i} {}^N v_{dmi}^T {}^N v_{dmi} \rho A dx_i = \frac{1}{2} \dot{q}_{ci}^T M_{ci} \dot{q}_{ci} \quad i = 1, \dots, 6 \quad (19)$$

where

$$\dot{q}_{ci} = \begin{bmatrix} \dot{x}_{AN} & \dot{y}_{AN} & \dot{\phi}_A & \dot{\theta}_i & \dot{q}_{ei} \end{bmatrix}^T$$

Here ρ demonstrates the density of the cables, A shows the cables cross section, and $\dot{q}_{ci}$ is the generalized velocity vector for the cables. According to Eq. (19), the mass matrices of cables 1 to 6 are extracted according to Eq. (20).

$$M_{ci} = \begin{bmatrix} m_i & m_{i+6} \\ 0_{2,3} & m_{i+12} \end{bmatrix} \quad i = 1, \dots, 6 \quad (20)$$

where

$$m_i = \begin{bmatrix} 1 & 0 & 2r\theta_i^N \hat{q}_{iy} - 2\Lambda_i^N \hat{q}_{iy} - 2\Lambda_{i+6} q_{ei}^N \hat{q}_{iy} - 2({}^N r_{p_i A, y}) \\ 0 & 1 & 2({}^N r_{p_i A, x}) + 2^N \hat{q}_{ix} \Lambda_i + 2^N \hat{q}_{ix} \Lambda_{i+6} q_{ei} - 2r^N \hat{q}_{ix} \theta_i \\ 0 & 0 & m_{i(3,3)} \end{bmatrix},$$

$$\begin{aligned} m_{i(3,3)} = & ({}^N \hat{q}_{ix}^2 \Lambda_{i+18} + {}^N \hat{q}_{iy}^2 \Lambda_{i+18} + ({}^N r_{p_i A, x})^2 + ({}^N r_{p_i A, y})^2 + {}^N \hat{q}_{ix}^2 r^2 \theta_i^2 + {}^N \hat{q}_{iy}^2 r^2 \theta_i^2 \\ & + 2^N \hat{q}_{ix} ({}^N r_{p_i A, x}) \Lambda_i + 2^N \hat{q}_{iy} ({}^N r_{p_i A, y}) \Lambda_i + {}^N \hat{q}_{ix}^2 \Lambda_{i+24} q_{ei}^2 + {}^N \hat{q}_{iy}^2 \Lambda_{i+24} q_{ei}^2 \\ & + 2^N \hat{q}_{ix} ({}^N r_{p_i A, x}) \Lambda_{i+6} q_{ei} - 2r^N \hat{q}_{ix} ({}^N r_{p_i A, x}) \theta_i - 2r^N \hat{q}_{iy} ({}^N r_{p_i A, y}) \theta_i + 2^N \hat{q}_{ix}^2 \Lambda_{i+36} q_{ei} \\ & + 2^N \hat{q}_{iy}^2 \Lambda_{i+36} q_{ei} - 2r^N \hat{q}_{iy}^2 \Lambda_i \theta_i - 2r^N \hat{q}_{ix}^2 \Lambda_{i+6} q_{ei} \theta_i - 2r^N \hat{q}_{iy}^2 \Lambda_{i+6} q_{ei} \theta_i), \end{aligned}$$

$$m_{i+6} = \begin{bmatrix} 2^N \hat{q}_{ix} \Lambda_{i+12} q_{ei} - 2r^N \hat{q}_{ix} & 2^N \hat{q}_{ix} \Lambda_{i+6} \\ 2^N \hat{q}_{iy} \Lambda_{i+12} q_{ei} - 2r^N \hat{q}_{iy} & 2^N \hat{q}_{iy} \Lambda_{i+6} \\ 2(r - \Lambda_{i+12} q_{ei})({}^N \hat{q}_{ix} ({}^N r_{p_i A, y}) - {}^N \hat{q}_{iy} ({}^N r_{p_i A, x})) & -2\Lambda_{i+6} ({}^N \hat{q}_{ix} ({}^N r_{p_i A, y}) - {}^N \hat{q}_{iy} ({}^N r_{p_i A, x})) \end{bmatrix},$$

$$m_{i+12} = \begin{bmatrix} \Lambda_{i+30} q_{ei}^2 + r^2 - 2r \Lambda_{i+12} q_{ei} & -2\Lambda_{i+6} r + 2\Lambda_{i+42} q_{ei} \\ 0 & \Lambda_{i+24} \end{bmatrix}, \quad {}^N \hat{q}_i = \begin{bmatrix} {}^N \hat{q}_{ix} & {}^N \hat{q}_{iy} & {}^N \hat{q}_{iz} \end{bmatrix}^T$$

In the above expressions, the integral values are expressed as follows:

$$\Lambda_i = \int_{r\theta_i}^{L_i} x_i \, dx_i, \quad \Lambda_{i+6} = \int_{r\theta_i}^{L_i} \Gamma_i \, dx_i, \quad \Lambda_{i+12} = \int_{r\theta_i}^{L_i} \frac{\partial \Gamma_i}{\partial \theta_i} \, dx_i, \quad \Lambda_{i+18} = \int_{r\theta_i}^{L_i} x_i^2 \, dx_i, \quad i = 1, \dots, 6$$

$$\Lambda_{i+24} = \int_{r\theta_i}^{L_i} \Gamma_i^2 \, dx_i, \quad \Lambda_{i+30} = \int_{r\theta_i}^{L_i} \left(\frac{\partial \Gamma_i}{\partial \theta_i} \right)^2 \, dx_i, \quad \Lambda_{i+36} = \int_{r\theta_i}^{L_i} x_i \Gamma_i \, dx_i, \quad \Lambda_{i+42} = \int_{r\theta_i}^{L_i} \Gamma_i \left(\frac{\partial \Gamma_i}{\partial \theta_i} \right) \, dx_i$$

The potential energy of a mobile base robot with vibrating cables is corresponding to the strain energy of the cables with longitudinal vibration. The strain energy stored in the cables can be obtained from Eq. (21) [16].

$$V_{ci} = \frac{1}{2} \int_{r\theta_i}^{L_i} EA \left(\frac{\partial (\Gamma_i q_{ei})}{\partial x_i} \right)^2 \, dx_i = \frac{1}{2} K_{ci} q_{ei}^2 \quad i = 1, \dots, 6 \quad (21)$$

where

$$K_{ci} = \int_{r\theta_i}^{L_i} EA \left(\frac{\partial \Gamma_i}{\partial x_i} \right)^2 dx_i$$

Also, K_{ci} shows the stiffness related to each cable and E is the Young's modulus related to the cables. Furthermore, the Rayleigh dissipation function is defined for each vibrating cable according to Eq. (22).

$$R_{ci} = \frac{1}{2} \dot{q}_{ci}^T D_{ci} \dot{q}_{ci} \quad i = 1, \dots, 6 \quad (22)$$

where

$$D_{ci} = \text{diag} \{0, 0, 0, 0, 0, c_i\} \quad , \quad \dot{q}_{ci} = \begin{bmatrix} \dot{x}_{AN} & \dot{y}_{AN} & \dot{\phi}_A & \dot{\theta}_i & \dot{q}_{ei} \end{bmatrix}^T$$

In which D_{ci} represents the damping matrix and c_i shows damping coefficients for vibrating cables 1 to 6. Finally, the kinetic energy, potential energy, and Rayleigh dissipation function related to a mobile base cable robot with nonlinear vibration are obtained from Eq. (23).

$$T_e = T_A + T_B + T_w + T_m + \sum_{i=1}^6 T_{ci} \quad , \quad V_e = \sum_{i=1}^6 V_{ci} \quad , \quad R_e = \sum_{i=1}^6 R_{ci} \quad (23)$$

By substituting Eq. (23) in Lagrange equation (14), the dynamic equations of the mobile base robot with nonlinear vibrating cables can be extracted according to Eq. (24) [16].

$$M_{t1} \ddot{q}_{t1} + D_{t1} \dot{q}_{t1} + K_{t1} q_{t1} = B_{t1} \tau + f_{tg1} + f_{m1} + \Xi^T \lambda \quad (24)$$

where

$$\begin{aligned} \dot{q}_{t1} &= \begin{bmatrix} \dot{\theta}_w^T & \dot{x}_{AN} & \dot{y}_{AN} & \dot{\phi}_A & \dot{\theta}_1 & \dot{q}_{e1} & \dot{\theta}_2 & \dot{q}_{e2} & \dot{\theta}_3 & \dot{q}_{e3} & \dot{\theta}_4 & \dot{q}_{e4} & \dot{\theta}_5 & \dot{q}_{e5} & \dot{\theta}_6 & \dot{q}_{e6} & \dot{x}_B^T \end{bmatrix}^T, \\ D_{t1} &= \text{diag} \{0, 0, 0, 0, 0, 0, c_1, 0, c_2, 0, c_3, 0, c_4, 0, c_5, 0, c_6, 0, 0, 0, 0, 0, 0\}_{23 \times 23}, \\ K_{t1} &= \text{diag} \{0, 0, 0, 0, 0, 0, K_{c1}, 0, K_{c2}, 0, K_{c3}, 0, K_{c4}, 0, K_{c5}, 0, K_{c6}, 0, 0, 0, 0, 0, 0\}_{23 \times 23}, \\ B_{t1} \tau &= \begin{bmatrix} \tau_r & \tau_l & 0 & 0 & 0 & \tau_1 & 0 & \tau_2 & 0 & \tau_3 & 0 & \tau_4 & 0 & \tau_5 & 0 & \tau_6 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T, \\ f_{tg1} &= \begin{bmatrix} 0_{1 \times 19} & m_B \times g & 0_{1 \times 3} \end{bmatrix}^T, \quad f_{m1} = -\dot{M}_{t1} \dot{q}_{t1} + \frac{1}{2} \dot{q}_{t1}^T \left(\frac{\partial M_{t1}}{\partial q_{t1}} \right)^T \dot{q}_{t1} - \frac{1}{2} q_{t1}^T \left(\frac{\partial K_{t1}}{\partial q_{t1}} \right)^T q_{t1}, \end{aligned}$$

$$M_{t1} = \begin{bmatrix} M_w & 0_{2 \times 3} & 0_{2 \times 2} & 0_{2 \times 2} & 0_{2 \times 2} & 0_{2 \times 2} & 0_{2 \times 2} & 0_{2 \times 2} & 0_{2 \times 6} \\ 0_{3 \times 2} & H_1 + M_A + \sum_{i=1}^6 m_i & H_2 + m_7 & H_2 + m_8 & H_2 + m_9 & H_2 + m_{10} & H_2 + m_{11} & H_2 + m_{12} & 0_{3 \times 6} \\ 0_{2 \times 2} & 0_{2 \times 3} & H_3 + m_{13} & 0_{2 \times 2} & 0_{2 \times 2} & 0_{2 \times 2} & 0_{2 \times 2} & 0_{2 \times 2} & 0_{2 \times 6} \\ 0_{2 \times 2} & 0_{2 \times 3} & 0_{2 \times 2} & H_3 + m_{14} & 0_{2 \times 2} & 0_{2 \times 2} & 0_{2 \times 2} & 0_{2 \times 2} & 0_{2 \times 6} \\ 0_{2 \times 2} & 0_{2 \times 3} & 0_{2 \times 2} & 0_{2 \times 2} & H_3 + m_{15} & 0_{2 \times 2} & 0_{2 \times 2} & 0_{2 \times 2} & 0_{2 \times 6} \\ 0_{2 \times 2} & 0_{2 \times 3} & 0_{2 \times 2} & 0_{2 \times 2} & 0_{2 \times 2} & H_3 + m_{16} & 0_{2 \times 2} & 0_{2 \times 2} & 0_{2 \times 6} \\ 0_{2 \times 2} & 0_{2 \times 3} & 0_{2 \times 2} & 0_{2 \times 2} & 0_{2 \times 2} & 0_{2 \times 2} & H_3 + m_{17} & 0_{2 \times 2} & 0_{2 \times 6} \\ 0_{2 \times 2} & 0_{2 \times 3} & 0_{2 \times 2} & 0_{2 \times 2} & 0_{2 \times 2} & 0_{2 \times 2} & 0_{2 \times 2} & H_3 + m_{18} & 0_{2 \times 6} \\ 0_{6 \times 2} & 0_{6 \times 3} & 0_{6 \times 2} & 0_{6 \times 2} & 0_{6 \times 2} & 0_{6 \times 2} & 0_{6 \times 2} & 0_{6 \times 2} & M_B \end{bmatrix}_{23 \times 23},$$

$$H_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 6J_m \end{bmatrix}, \quad H_2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 2J_m & 0 \end{bmatrix}, \quad H_3 = \begin{bmatrix} J_m & 0 \\ 0 & 0 \end{bmatrix}$$

In which M_{t1} indicates the mass matrix, $\dot{q}_{t1}$ shows the generalized velocity vector, D_{t1} represents the damping matrix, K_{t1} demonstrates the stiffness matrix, $B_{t1}\tau$ displays the vector including motor torque, f_{tg1} shows the vector including gravitational force of the final actuator, f_{m1} gives the nonlinear part of the equations, Ξ depicts the Jacobin matrix related to nonholonomic constraints, and λ is the Lagrangian coefficients. Finally, by using nonholonomic constraints (1) and (7) and applying the null space method to eliminate the Lagrangian coefficients, the dynamic equations of the mobile base robot with nonlinear vibrating cable can be obtained considering the related constraints according to Eq. (25) [15, 19].

$$M_{t2}\ddot{q}_{t2} + D_{t2}\dot{q}_{t2} + K_{t2}q_{t2} = B_{t2}\tau + f_{tg2} + f_{m2} \quad (25)$$

where,

$$M_{t2} = U_2^T U_1^T M_{t1} U_1 U_2, \quad \dot{q}_{t2} = [\dot{\theta}_w^T \quad \dot{q}_{et}^T \quad \dot{x}_B^T]^T, \quad D_{t2} = U_2^T U_1^T D_{t1} U_1 U_2,$$

$$K_{t2}q_{t2} = U_2^T U_1^T K_{t1}q_{t1}, \quad B_{t2} = U_2^T U_1^T B_{t1}, \quad f_{tg2} = U_2^T U_1^T f_{tg1},$$

$$f_{m2} = U_2^T (U_1^T f_{m1} - U_1^T M_{t1} \dot{U}_1 U_2 \dot{q}_{t2}) - U_2^T U_1^T M_{t1} U_1 \dot{U}_2 \dot{q}_{t2},$$

$$U_1 = \begin{bmatrix} I_5 & 0_{5 \times 12} \\ C_5 & C_6 \\ 0_{6 \times 11} & I_6 \end{bmatrix}_{23 \times 17}, \quad U_2 = \begin{bmatrix} I_2 & 0_{2 \times 12} \\ U_w & 0_{3 \times 12} \\ 0_{12 \times 2} & I_{12} \end{bmatrix}_{17 \times 14},$$

$$C_5 = \begin{bmatrix} 0 & 0 & U_{a(1,1)} & U_{a(1,2)} & U_{a(1,6)} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & U_{a(2,1)} & U_{a(2,2)} & U_{a(2,6)} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & U_{a(3,1)} & U_{a(3,2)} & U_{a(3,6)} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & U_{a(4,1)} & U_{a(4,2)} & U_{a(4,6)} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & U_{a(5,1)} & U_{a(5,2)} & U_{a(5,6)} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & U_{a(6,1)} & U_{a(6,2)} & U_{a(6,6)} \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, C_6 = \begin{bmatrix} U_{e(1,1)} & 0 & 0 & 0 & 0 & 0 & 0 & U_{b(1,:)} \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & U_{e(2,2)} & 0 & 0 & 0 & 0 & 0 & U_{b(2,:)} \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & U_{e(3,3)} & 0 & 0 & 0 & 0 & U_{b(3,:)} \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & U_{e(4,4)} & 0 & 0 & 0 & U_{b(4,:)} \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & U_{e(5,5)} & 0 & 0 & U_{b(5,:)} \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & U_{e(6,6)} & 0 & U_{b(6,:)} \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

Similarly, by using nonholonomic constraints (1) and (3) and utilizing the null space method to eliminate the Lagrangian coefficients, the dynamic equations of the mobile base robot with linear vibration of the cables can be obtained according to Eq. (26).

$$M_{k2}\ddot{q}_{k2} + D_{k2}\dot{q}_{k2} + K_{k2}q_{k2} = B_{k2}\tau + f_{kg2} + f_{kn2} \quad (26)$$

where,

$$M_{k2} = U_2^T U_1^T M_{k1} U_1 U_2, \quad \dot{q}_{k2} = \begin{bmatrix} \dot{\theta}_w^T & \dot{q}_{lt}^T & \dot{x}_B^T \end{bmatrix}^T, \quad D_{k2} = U_2^T U_1^T D_{k1} U_1 U_2,$$

$$K_{k2}q_{k2} = U_2^T U_1^T K_{k1}q_{k1}, \quad B_{k2} = U_2^T U_1^T B_{k1}, \quad f_{kg2} = U_2^T U_1^T f_{kg1},$$

$$f_{kn2} = U_2^T (U_1^T f_{kn1} - U_1^T M_{k1} \dot{U}_1 U_2 \dot{q}_{k2}) - U_2^T U_1^T M_{k1} U_1 \dot{U}_2 \dot{q}_{k2},$$

$$M_{k1} = \begin{bmatrix} M_w & 0_{2 \times 3} & 0_{2 \times 2} & 0_{2 \times 2} & 0_{2 \times 2} & 0_{2 \times 2} & 0_{2 \times 2} & 0_{2 \times 2} & 0_{2 \times 6} \\ 0_{3 \times 2} & H_1 + M_A & H_2 & H_2 & H_2 & H_2 & H_2 & H_2 & 0_{3 \times 6} \\ 0_{2 \times 2} & 0_{2 \times 3} & H_3 & 0_{2 \times 2} & 0_{2 \times 2} & 0_{2 \times 2} & 0_{2 \times 2} & 0_{2 \times 2} & 0_{2 \times 6} \\ 0_{2 \times 2} & 0_{2 \times 3} & 0_{2 \times 2} & H_3 & 0_{2 \times 2} & 0_{2 \times 2} & 0_{2 \times 2} & 0_{2 \times 2} & 0_{2 \times 6} \\ 0_{2 \times 2} & 0_{2 \times 3} & 0_{2 \times 2} & 0_{2 \times 2} & H_3 & 0_{2 \times 2} & 0_{2 \times 2} & 0_{2 \times 2} & 0_{2 \times 6} \\ 0_{2 \times 2} & 0_{2 \times 3} & 0_{2 \times 2} & 0_{2 \times 2} & 0_{2 \times 2} & H_3 & 0_{2 \times 2} & 0_{2 \times 2} & 0_{2 \times 6} \\ 0_{2 \times 2} & 0_{2 \times 3} & 0_{2 \times 2} & 0_{2 \times 2} & 0_{2 \times 2} & 0_{2 \times 2} & H_3 & 0_{2 \times 2} & 0_{2 \times 6} \\ 0_{2 \times 2} & 0_{2 \times 3} & 0_{2 \times 2} & 0_{2 \times 2} & 0_{2 \times 2} & 0_{2 \times 2} & 0_{2 \times 2} & H_3 & 0_{2 \times 6} \\ 0_{6 \times 2} & 0_{6 \times 3} & 0_{6 \times 2} & 0_{6 \times 2} & 0_{6 \times 2} & 0_{6 \times 2} & 0_{6 \times 2} & 0_{6 \times 2} & M_B \end{bmatrix}_{23 \times 23},$$

$$D_{k1} = \text{diag}\{0, 0, 0, 0, 0, 0, c_1, 0, c_2, 0, c_3, 0, c_4, 0, c_5, 0, c_6, 0, 0, 0, 0, 0, 0\}_{23 \times 23},$$

$$K_{k1} = \text{diag}\{0,0,0,0,0,0,k_{cl1},0,k_{cl2},0,k_{cl3},0,k_{cl4},0,k_{cl5},0,k_{cl6},0,0,0,0,0,0\}_{23 \times 23},$$

$$k_{cli} = \frac{EA}{L_i - r\theta_i} \quad i=1,...,6, \quad B_{k1}\tau = [\tau_r \quad \tau_l \quad 0_{1 \times 3} \quad \tau_1 \quad 0 \quad \tau_2 \quad 0 \quad \tau_3 \quad 0 \quad \tau_4 \quad 0 \quad \tau_5 \quad 0 \quad \tau_6 \quad 0_{1 \times 7}]^T,$$

$$f_{kg1} = [0_{1 \times 19} \quad m_B \times g \quad 0_{1 \times 3}]^T, \quad f_{kn1} = -\dot{M}_{k1}\dot{q}_{k1} + \frac{1}{2}(\frac{\partial}{\partial q_{k1}}\dot{q}_{k1}^T M_{k1}\dot{q}_{k1})^T - \frac{1}{2}(\frac{\partial}{\partial q_{k1}}q_{k1}^T K_{k1}q_{k1})^T,$$

$$\dot{q}_{k1} = [\dot{\theta}_w^T \quad \dot{x}_{AN} \quad \dot{y}_{AN} \quad \dot{\phi}_A \quad \dot{\theta}_1 \quad \dot{q}_{l1} \quad \dot{\theta}_2 \quad \dot{q}_{l2} \quad \dot{\theta}_3 \quad \dot{q}_{l3} \quad \dot{\theta}_4 \quad \dot{q}_{l4} \quad \dot{\theta}_5 \quad \dot{q}_{l5} \quad \dot{\theta}_6 \quad \dot{q}_{l6} \quad \dot{x}_B^T]^T$$

Similarly, considering the constraint equations (1) and (2), the fully constrained dynamic equations related to the mobile base cable robot with ideal cables can be extracted according to Eq. (27).

$$M_{s2}\ddot{q}_{s2} = B_{s2}\tau + f_{sg2} + f_{sn2} \quad (27)$$

where

$$M_{s2} = U_4^T U_3^T M_{s1} U_3 U_4, \quad \dot{q}_{s2} = [\dot{\theta}_w^T \quad \dot{x}_B^T]^T, \quad B_{s2} = U_4^T U_3^T B_{s1},$$

$$f_{sg2} = U_4^T U_3^T f_{sg1}, \quad f_{sn2} = U_4^T (U_3^T f_{sn1} - U_3^T M_{s1} \dot{U}_3 U_4 \dot{q}_{s2}) - U_4^T U_3^T M_{s1} U_3 \dot{U}_4 \dot{q}_{s2}$$

$$M_{s1} = \begin{bmatrix} M_w & 0 & 0 & 0 \\ 0 & H_1 + M_A & H_4 & 0 \\ 0 & 0 & J_m I_6 & 0 \\ 0 & 0 & 0 & M_B \end{bmatrix}_{17 \times 17}, \quad B_{s1} = \begin{bmatrix} I_2 & 0 \\ 0 & 0 \\ 0 & I_6 \\ 0 & 0 \end{bmatrix}_{17 \times 8},$$

$$H_4 = [2J_m \quad 2J_m \quad 2J_m \quad 2J_m \quad 2J_m \quad 2J_m], \quad \tau = [\tau_r \quad \tau_l \quad \tau_1 \quad \tau_2 \quad \tau_3 \quad \tau_4 \quad \tau_5 \quad \tau_6]^T$$

$$U_3 = \begin{bmatrix} I_5 & 0_{5 \times 6} \\ 0_{6 \times 2} & U_{a(1:6,1:2)} & U_{a(1:6,1:2)} & U_b \\ & 0_{6 \times 5} & I_6 & \end{bmatrix}_{17 \times 11}, \quad U_4 = \begin{bmatrix} I_2 & 0_{2 \times 6} \\ U_w & 0_{3 \times 6} \\ 0_{6 \times 2} & I_6 \end{bmatrix}_{11 \times 8},$$

$$f_{sg1} = [0_{1 \times 14} \quad m_B g \quad 0_{1 \times 2}]^T, \quad f_{sn1} = -\dot{M}_{s1}\dot{q}_{s1} + \frac{1}{2}(\frac{\partial}{\partial q_{s1}}\dot{q}_{s1}^T M_{s1}\dot{q}_{s1})^T,$$

$$\dot{q}_{s1} = [\dot{\theta}_w^T \quad \dot{x}_{AN} \quad \dot{y}_{AN} \quad \dot{\phi}_A \quad \dot{\theta}_t \quad \dot{x}_B^T]^T$$

2.3. State equations

Next, to numerically solve the extracted dynamic equations, the robot dynamic equations are written in state space format. According to Eq. (25), the state space equations of the robot with nonlinear vibrating cables can be written according to Eq. (28).

$$\dot{y}_t = \begin{bmatrix} U_5 \dot{\theta}_w \\ \dot{q}_{et} \\ \dot{x}_B \\ \{M_{t2}^{-1}(B_{t2}\tau + f_{tg2} + f_{m2} - D_{t2}\dot{q}_{t2} - K_{t2}q_{t2})\} \end{bmatrix}_{30 \times 1} \quad (28)$$

where

$$U_5 = \begin{bmatrix} U_{w(1:2,1:2)} \\ I_2 \end{bmatrix}_{4 \times 2}, \quad y_t = [x_{AN} \quad y_{AN} \quad \theta_r \quad \theta_l \quad q_{et}^T \quad x_B^T \quad \dot{\theta}_r \quad \dot{\theta}_l \quad \dot{q}_{et}^T \quad \dot{x}_B^T]^T$$

According to Eq. (26), the state space equations of the robot with linear vibrating cables is also expressed as Eq. (29).

$$\dot{y}_k = \begin{bmatrix} U_5 \dot{\theta}_w \\ \dot{q}_{lt} \\ \dot{x}_B \\ \{M_{k2}^{-1}(B_{k2}\tau + f_{kg2} + f_{kn2} - D_{k2}\dot{q}_{k2} - K_{k2}q_{k2})\} \end{bmatrix}_{30 \times 1} \quad (29)$$

where

$$y_k = [x_{AN} \quad y_{AN} \quad \theta_r \quad \theta_l \quad q_{lt}^T \quad x_B^T \quad \dot{\theta}_r \quad \dot{\theta}_l \quad \dot{q}_{lt}^T \quad \dot{x}_B^T]^T$$

Based on Eq. (27), the state space equations of the robot with ideal cables is expressed as follows:

$$\dot{y}_s = \begin{bmatrix} U_5 \dot{\theta}_w \\ \dot{x}_B \\ \{M_{s2}^{-1}(B_{s2}\tau + f_{sg2} + f_{sn2})\} \end{bmatrix}_{18 \times 1} \quad (30)$$

where

$$y_s = [x_{AN} \quad y_{AN} \quad \theta_r \quad \theta_l \quad x_B^T \quad \dot{\theta}_r \quad \dot{\theta}_l \quad \dot{x}_B^T]^T$$

The constraint related to the relation between the base rotation and the rotation of the base right and left wheels is of the holonomic type. By having the amount of wheel rotation, the base rotation can be obtained from Eq. (31) [20].

$$\varphi_A(t) = \frac{r_{wh}}{2b} (\theta_l(t) - \theta_r(t)) \quad (31)$$

Therefore, the base rotation is not considered as a state variable.

2.4. Investigating the cable's tension with nonlinear flexibility in mobile cable robot

In order to investigate the effect of nonlinear flexibility of the cables in the kinetics of the moving cable robot, the inverse dynamics is solved using the following equation:

$$\tau = B_{t2}^{-1} (M_{t2} \ddot{q}_{t2} + D_{t2} \dot{q}_{t2} + K_{t2} q_{t2} - f_{tg2} - f_{m2}) \quad (32)$$

Since in this robot, the mass of the end-effector is significantly higher than the mass of the cables, thus during the robot motion, it is possible to ignore the velocity and acceleration of the cable's elements in their dynamic performance [21].

$$q_{ei} \neq 0, \quad \dot{q}_{ei} = \ddot{q}_{ei} = 0 \quad i = 1, \dots, 6 \quad (33)$$

Substituting Eq. (33) in Eq. (32), one can conclude the following equation:

$$\tau = B_{t3}^{-1} (M_{t3} \ddot{q}_{t3} + D_{t3} \dot{q}_{t3} + K_{t3} q_{t3} - f_{tg3} - f_{m3}) \quad (34)$$

where

$$\dot{q}_{t3} = \begin{bmatrix} \dot{\theta}_w^T & \dot{x}_B^T \end{bmatrix}^T, \quad q_{ei} \neq 0 \quad i = 1, \dots, 6$$

Consequently, the related cable's tension with nonlinear flexibility can also be calculated through the following equation:

$$T_i = \frac{\tau_i}{r} \quad i = 1, \dots, 6 \quad (35)$$

3. Simulation and validation

3.1. Dynamic simulation

Comparing the results of solving the inverse and direct dynamics is performed firstly in order to validate the extracted dynamic equations. The optimal path of the bases and the end-effector is considered as Eq. (36).

$$\begin{aligned} x_A &= \begin{bmatrix} 2 \sin(\frac{2\pi}{200}t + 0.2068)m & -2 \cos(\frac{2\pi}{200}t + 0.2068)m & 0_{1 \times 4} & \frac{2\pi}{200}t \text{ rad} \end{bmatrix}^T \\ x_B &= x_A + \begin{bmatrix} 0.02 \sin(\frac{2\pi}{20}t)m & 0.02 \cos(\frac{2\pi}{20}t)m & 0.8 + 0.1 \sin(\frac{2\pi}{20}t)m & 0_{1 \times 3} \end{bmatrix}^T \end{aligned} \quad (36)$$

Considering the path (36) and solving the Eq. (27) related to the mobile base cable robot with ideal cables, the motor torque required for the mentioned path is obtained from Eq. (37).

$$\tau = B_{s2}^{-1}(M_{s2}\ddot{q}_{s2} - f_{sg2} - f_{sn2}) \quad (37)$$

As shown in Eq. (37), the velocity and acceleration vectors of the base wheels angles should be first calculated to solve the related inverse dynamics. Through the investigating the mobile base robot cable with ideal cables, Yousefzadeh showed that the robot base could not be controlled with feed forward term without feedback signals because of the existence of disturbances and significant uncertainties [7]. The present study aims to investigate the effect of nonlinear vibrations of cables on the system dynamics. It should be considered that when the cables have nonlinear vibrations, the modeling and simulating complexity increase significantly. Eq. (38) is used to obtain more accurate values of the base wheel rotational velocity, as Fig. 1(b) [17].

$$\dot{\theta}_r = \frac{1}{r_{wh}}(\dot{x}_{AN} \cos \varphi_A + \dot{y}_{AN} \sin \varphi_A - b\dot{\varphi}_A) , \quad \dot{\theta}_l = \frac{1}{r_{wh}}(\dot{x}_{AN} \cos \varphi_A + \dot{y}_{AN} \sin \varphi_A + b\dot{\varphi}_A) \quad (38)$$

Table 1 presents the values of the employed geometric and dynamic parameters of the robot.

Table 1 Geometric and dynamic parameters of the robot

Name	Symbol	Value	Unit
Cable initial length	$L_i \ i = 1, \dots, 6$	1	M
Side length of the triangular upper plate of the platform	--	1.19	M
Side length of the triangular end-effector	--	0.17	M
Distance between Z axis of the platform and the line connecting the driving wheels	d	0.314	M
Half of the distance between the wheels	b	0.54	M
Radius of the driving wheels	r_{wh}	0.05	M
Radius of the cable drums	r	1	m
Platform mass	m_A	170	kg
End-effector mass	m_B	3.1	kg
Moment of inertia tensor of the platform	I_A	$\begin{bmatrix} 62.4 & 0 & 2.2 \\ 0 & 61.9 & 0 \\ 2.2 & 0 & 33.4 \end{bmatrix}$	kg.m ²
Moment of inertia tensor of the end-effector	I_B	$\begin{bmatrix} 0.001 & 0 & 0 \\ 0 & 0.001 & 0 \\ 0 & 0 & 0.002 \end{bmatrix}$	kg.m ²
Equivalent moment of inertia of the platform motors	I_w	$0.005I_2$	kg.m ²
Equivalent moment of inertia of the CDPR motors	I_m	$0.0003I_6$	kg.m ²
Multiplication of the density by the cross section of the cables	ρA	0.002	kg/m
Multiplication of the Young's modulus by the cross section of the cables	EA	1500 , 15000	N
Damping coefficient of the vibrating cables	$c_i \ i = 1, \dots, 6$	0.001	$\frac{N.s}{m^2}$

The motors' torque resulting from the inverse dynamics of Eq. (37) are considered as the inputs for the state equations (28) to (30). The motion diagrams related to the robot's base and the end-effector are obtained in Figs. 4 and 5 based on Table 1 and the numerical solution of the equations with ode45 solver in Matlab software. Then, the results are compared in five modes consisting the desired path, ideal cable (cr), linear vibrating cable (cl), and nonlinear vibrating cable (ce) with high and low Young modulus.

Fig. 4 Transitional and rotational movements of the robot mobile base

As shown in Fig. 4, the two-dimensional path of the mobile base is not harmonic when the cables are vibrating and the vibration of the cables has little effect on the base motion. This is contributed to the fact that the mass of the cables are too small compared to the 170 kg mass of the mobile base which neutralizes the coupled effect of the cables and base states.

Fig. 5 Transitional and rotational motion diagrams of the end-effector

According to the first validation method and Figs. 4 and 5, when the cables are vibrating, it is observed that the transitional and rotational motions of the base and the end-effector, follow the desired paths with a small error, showing the accuracy of the extracted dynamic equations. As shown in Figs. 4 and 5, the difference between the two modes of the ideal cable and the vibrating cable has two main reasons. The first one is related to numerical solution errors and the lack of implementation of a closed loop controller for the mobile base robot. The second reason refers to the effect of longitudinal and nonlinear vibrations of cables with sinusoidal mode shapes, which can be observed as a harmonic motion in the diagrams. As shown in Fig. 5, the diagrams z_{BN} , α , and β of the end-effector are harmonic in the vibrating cable case while the diagrams x_{BN} ,

y_{BN} , and γ of the end-effector have a small harmonic response. This is contributed to the fact that the nonlinear and longitudinal vibrations of the cables are in the direction x_i of the cables and thus its main effect can be observed along the translational movements z_{BN} and rotational movements α and β of the end-effector, as illustrated in Fig. 3(b). Therefore, the translational movements x_{BN} and y_{BN} , as well as rotational movements γ of the end-effector, are located in a plane of space that receive the least effect from the longitudinal vibration of the cables while they receive the highest effect from the mobile base motion instead. Figure 6 and Table 2 depict the differences of the diagrams z_{BN} , α , and β of the end-effector between the vibrating cable and the ideal cable.

Fig. 6 Diagram of the difference between the vibrating cable and the ideal cable related to α , β , and z_{BN} of the end-effector

Based on Table 2, it is observed that the longitudinal vibrations of the cables has the highest effect on the movement z_{BN} for the end-effector movement, and the maximum error of load-carrying along the z_{BN} direction reaches to 1.42 cm in the case of nonlinear vibrating cable. The maximum error related to α and β is 0.18 and 0.16 degrees, respectively.

Table 2 Maximum differences between the vibrating cable and the ideal cable for the end-effector variables

Variable	Maximum error ($ cl-cr $)	Maximum error ($ ce-cr $)	
	EA=1500 N	EA=1500 N	EA=15000 N
α	0.0034 rad	0.0032 rad	0.0011 rad
β	0.0032 rad	0.0028 rad	0.0013 rad
z_{BN}	1.76 cm	1.42 cm	0.14 cm

Figure 7 shows the dynamic coordinates of the vibrations related to the cables in three modes of linear vibration (q_{li}), nonlinear vibration (q_{ei}) with low and high Yang modulus.

Fig. 7 Dynamic coordinates of vibrations related to cables 1 to 6

As illustrated in Fig. 7, the dynamic coordinates of the cables vibration in the linear mode have a higher amplitude of oscillation and a lower frequency compared to the nonlinear one. The maximum oscillation amplitude of the cables are averagely 9.45 and 7.68 mm on in linear and nonlinear modes, respectively. Figures 4 and 5 demonstrate that the translational and rotational movements of the base and the end-effector are closer to the desired path in the non-linear (ce) mode compared to the linear (cl) one. Further, as shown in Fig. 6 and Table 2, it can be found that the maximum error of the load-carrying of the end-effector in the direction z_{BN} compared to ideal case is 1.76 cm for the linear vibrating mode and 1.42 cm for the non-linear one respectively. The maximum differences of α and β between the two vibrating and ideal cable modes are about 0.19 and 0.18 degrees in the linear mode, respectively, which decrease to 0.18 and 0.16 degrees in the nonlinear mode, respectively. The reason contributed to the ignorance of the mass elements of the cables and their related kinetic energy in the linear mode and that the vibration of each cable is modeled with the length change of a spring, as demonstrated in Fig. 3(a). Therefore, in the nonlinear state, a large portion of the cables energy is spent on the kinetic energy of the mass elements while a smaller part of the energy is assigned on the potential energy related to the vibration and strain of the cables. This fact increases the stiffness of the vibrating cables in the nonlinear state and reduces the amplitude of the vibrations. As a result, the nonlinear analysis of the cable vibration makes the modeling closer to reality, since the mass of the cables are considered.

The second scenario addresses the effect of increasing the Young's modulus of vibrating cables on the motion diagrams. In this case, the multiplication of the Young's modulus by the cross-sectional area of each cable (EA) is considered 15000 N, instead of 1500 N. Accordingly, an increase in the Young's modulus of the cables can significantly reduce the oscillation amplitude of the cables vibration dynamic coordinates so that it reaches to about 0.76 mm, as shown in Fig. 7. On the other hand, an increase in the Young's modulus of the cables makes the movement diagrams of the end-effector closer to the ideal cable state (cr) (Fig. 4). Further, as shown in Fig. 5 and Table 2, the maximum error of the end-effector in the direction z_{BN} of the nonlinear vibrating cable with high Young modulus reaches 0.14 cm compared to the ideal mode. Further, the maximum error of α and β of the end-effector in this case are about 0.06 and 0.07 degrees, respectively. Thus, these values highlight the accuracy of the robot's dynamic equations with vibrating cables.

In order to investigate the effect of cables' vibration on the kinetic response of the system, Eqs. (34)-(35) are used to be simulated. The path of the chassis and the end-effector in the estimated equation (34) is considered as the numerical solution of the robot with ideal cables. Also the values of q_{ei} is considered as Fig. 7. Then, by solving the inverse dynamics of Eq. (34) the motor's torque can be extracted for the robot with nonlinear flexible cables. Consequently, the related cable's tension with nonlinear flexibility can also be calculated through the Eq. (35). The cable's tension in its linear format can be also extracted using the same method. Figure 8 shows

the cable's tension and its comparison for four cases including solid, linear flexible and nonlinear flexible with two different modules.

Fig. 8 Cable's tension with flexibility

According to the above figure, it can be concluded that the vibration of the cable's tension with nonlinear flexibility has lower amplitude and higher frequency. Also it is observed that by increasing the Young module, the profiles converge to solid case.

3.2. Experimental verification

As shown in Fig. 9, the rear wheels are driver ones and the front one is caster wheel. In this paper, no slipping is assumed in the motion of the robot wheels. The front wheel, which is used for balancing, is considered spherical and without friction. This robot consists of eight DC motors, six of which are connected to the pulleys and related to the cable parts, and the other two motors are related to the robot mobile base.

Fig. 9 Mobile base cable robot built in the laboratory

Figure 10 shows the motion diagrams of the base and end-effector, obtained from the robot laboratory test for the desired path of Eq. (36) case. Comparison between the experimental test and simulation one is conducted here.

Fig. 10 a) Two-dimensional path of the base b) Three-dimensional path of the end-effector

As observed, the motion paths of the base and end-effector resulting from the numerical simulation follow the path obtained from the laboratory test with a small difference. Based on the result, the accuracy of the dynamic equations of the mobile base robot with vibrating cables is confirmed. Furthermore, the base and end-effector's path in the nonlinear vibrating cable mode are closer to the laboratory and actual path compared to the linear mode, which shows the advantage of nonlinear vibration analysis of cables over the linear mode which can be used for exact controlling of the system with less parametric uncertainties.

4. Conclusions and discussion

The modeling of a cable suspended robot with moving platform was modified here considering the elasticity of the cables with nonlinear vibrations. The effect of this cables' elasticity was studied on the robot's kinematic and kinetic performance. It was stated that the longitudinal vibration of the cable is just important here since the mass of the end-effector is high enough to provide the required tensile force in the cables and avoid the transversal vibrations. Thus the kinematics of the robot with longitudinal elasticity of the cables was extracted using Rayleigh-Ritz method and the dynamic equation of this flexible system was calculated by the aid of Lagrange multiplier approach. The correctness of the modeling was verified using two approaches. Firstly, the elasticity of the cables was converged to solid state and it was shown that the response of the system is converged to the response of a cable robot with rigid cables. At the second approach, the response of the flexible system was compared with the experimental tests conducted on the manufactured cable robot at IUST and it was shown that the results of the flexible system is more close to the experimental ones compared to the simulation of the cable robot with rigid cables. Also it was investigated that, modeling the elasticity of the cables using nonlinear vibrating equation provides more accurate results compared to the system in which the elastic cables are modeled using linear equations. This important result was achieved by comparing the simulation results of these two models with experimental tests. It was shown that the response of the nonlinear model is significantly more compatible with the experimental results compared to the linear one. It was explained that this phenomenon is contributed to the fact that in linear modeling, the mass elements of the cables are ignored and thus the cables have less stiffness and higher amplitude. Moreover analyzing the effect of the cables' elasticity on the robot behavior showed that, the moving chassis DOFs together with x_{BN} , y_{BN} , and γ DOFs of the end-effector are not affected from the cables vibration while diagrams z_{BN} , α , and β of the end-effector are the DOFs with the most affectability from these vibrations. It was seen that the maximum error of the end-effector along the gravitation direction for the flexible system respect to the experimental results is about 1.95% for the case in which the cables are modeled linear while this value is about 1.57% when the cables are modeled nonlinear. Thus it can be concluded that modeling the cables in a nonlinear way increases the accuracy of modeling. Also it was shown that increasing the elasticity module of the cables can decrease the maximum error up to 0.15. Considering the fact that according to the achieved observations of this paper, this modeling can provide a more accurate prediction of the robot behavior, it can be concluded that employing this model as the feedforward section of the designed controller can increase the compensating ability of the parametric uncertainties related to the engaged cables.

Conflict of Interest

The authors have no conflicts of interest to declare that are relevant to the content of this article.

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Data Availability

The datasets generated during and/or analyzed during the current study are available from the corresponding author on reasonable request.

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