Adaptive Control for Quadrotors with Uncertain and Faulty Actuators^{*,**}

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ABSTRACT

Two adaptive control strategies are presented in this paper for two types of quadrotors to cope with potential uncertainties and faults in the actuator system. The four actuators of the considered quadrotors are distinct and suffered from loss of effectiveness (LOE) as well. To accommodate unknown actuator parameters, a filter-based immersion and invariance (I&I) adaptive controllers are designed for attitude and altitude subsystems, respectively, and an integral backstepping controller is developed for the horizontal subsystem to achieve null steady-state error. Both simulation and experiment results are carried out to illustrate the effectiveness of trajectory tracking performance and fault-tolerant accommodation ability of the proposed control schemes.

1. Introduction

Quadrotors have received much attention in both military and civilian fields due to mechanical simplicity, low development/maintenance cost and high hovering precision [1]. However, the characteristics of underactuation, non-linearity, strong coupling and uncertainty of quadrotors bring huge challenges to design the control laws.

The earlier results of quadrotor control are largely based on their physical parameters. In [2, 3, 4], backstepping controllers are designed to stabilize quadrotor system, and in [5, 6], controllers are designed based on the compensation of the Coriolis and gyroscopic torques. To handle the uncertainties in the quadrotor system such as mass, moment of inertia, etc., adaptive control methods are favored by many researchers. In [7], adaptive backstepping method is utilized to estimate the unknown mass, and in [8], an adaptive control scheme is presented to deal with external disturbances and varying center of mass (CoM). To improve transient behaviour, researchers in [9, 10, 11] utilize I&I adaptive method which provides a freedom via an additional proportion term to shape the estimation error manifolds, and I&I adaptive laws are constructed to estimate unknown parameters such as mass and aerodynamic damping coefficients.

With the development of fault-tolerant control (FTC) theory, many researchers are paying more attentions to FTC for quadrotors. It is reported that loss of effectiveness (LOE) fault of actuators occurs frequently in a quadrotor system [12, 13, 14]. In [12], an I&I observer is constructed to deal with partial LOE (PLOE) of a quadrotor, and it is combined with a sliding mode controller to stabilize the attitude system. In [13], a finite-time fault-tolerant trajectory tracking controller is designed for the quadrotor system against external disturbances, parametric uncertainties and actuator faults without fault diagnosis mechanism. In [14], a systematic design of fault detection, isolation and accommodation is presented for PLOE quadrotor actuator faults using nonlinear adaptive techniques; and a nonlinear robust adaptive fault-tolerant method is developed for altitude and attitude to accommodate actuator faults without the need of a fault diagnosis mechanism in [15]. In [16], fault-tolerant formation control problem is investigated for multi-quadrotor systems based on decentralised state observers and adaptive fault estimators. In [17], an adaptive compensation control scheme is developed via disturbance observer and quantum information technology for the quadrotor, and a reconfiguration control scheme is proposed in [18] for a quadrotor helicopter subject to LOE via simple adaptive control and quantum logic. In [19], a disturbance observer-based adaptive sliding mode control (SMC) strategy for a quadrotor helicopter subject to multiple actuator faults, parametric uncertainties and external disturbances is developed. Intelligent control methods are also investigated by researchers for quadrotor control problem, where in [20], an adaptive PID controller is proposed for a quadrotor system in the presence of actuator faults using the fuzzy inference scheme to tune the controller gains online; and in [21], an indirect neural network (NN)-based adaptive fault-tolerant control scheme is developed, which only involves one lumped parameter adaptation thus is more affordable for onboard implementation.

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An extreme situation is considered in [22], where an emergency fault-tolerant controller is developed for quadrotors suffering from a total loss of one rotor.

In this paper, we focus on dealing with the uncertainty in the quadrotor actuator system in controller design. In contrast with a number of existing literature, the parameters of four actuators of the quadrotor have different values. It is due to the fact that, on the one hand, inevitable errors often occur during the manufacturing process and, on the other hand, parameter variation are caused due to long-term operation of the devices. Furthermore, a sudden change in parameters caused by propeller and/or motor damage can also be considered as a special case of actuator uncertainties. To deal with those uncertainties, we take the advantage of I&I method to design adaptive schemes to compensate them. In the first control scheme, by defining intermediate control inputs, controllers based on I&I adaptive method with low-pass filters are designed for the attitude and altitude subsystems. However, it is shown that the designed scheme involves self-loops, i.e., algebraic loop, since each control input contains one another. To overcome this problem, we develop the second scheme using gain matrix decomposition technique since the input parameter matrix can be decomposed to several sub-matrices which contain a unity upper triangular matrix. For the horizontal subsystem, a non-adaptive controller is designed, which is combined with the integral term of the tracking error to improve the performance. Finally, we present numerical simulation and experiment to validate the proposed control law. The contribution of this paper are given as follows,

1. Two of the most common configurations of quadrotors are considered, i.e., “+”-type and “×”-type. Numerical simulations are carried out on the former type, while experiments are implemented on the latter one.
2. The designed adaptive controller for the corresponding quadrotor is robust to actuator parameter perturbations as well as LOE actuator faults.
3. Sufficient simulation and experiment results are given to show the superiority performance of the proposed controller compared to the classical adaptive controller.

This paper is organized as follows. Section 2 presents the quadrotor model and problem formulation. In Section 3, control designs for attitude, altitude and horizontal subsystems are presented respectively, and the stabilities of the closed-loop systems are analyzed. An improved design for attitude and altitude is given in 4 with several notable remarks. A simulated example is provided to show the effectiveness of the proposed approach in Section 5, and experimental results under various situations are carried out in Section 6. Finally, Section 7 draws a conclusion of this study.

2. Quadrotor Model and Problem Formulation

We consider two of the most common configurations of quadrotors, namely “+”-type and “×”-type, whose sketches are shown in the left and right side of Fig. 1, respectively. To describe the motion of quadrotor body, two reference frames are defined as earth inertial reference frame and body-fixed reference frame denoted by $\sum_e = \{x_e, y_e, z_e\}$ and $\sum_b = \{x_b, y_b, z_b\}$, respectively. The position and attitude of the quadrotor with respect to $\sum_e$ are represented by $[x, y, z]^T$ and $[\phi, \theta, \psi]^T$, where ϕ , θ and ψ denote roll angle, pitch angle and yaw angle, respectively.

For the two types of quadrotors, they have the same quadrotor body model given as,

$$\begin{cases} \ddot{\phi} = \frac{J_y - J_z}{J_x} \dot{\theta} \dot{\psi} - \frac{J_r}{J_x} \Omega_R \dot{\theta} + \frac{\tau_1}{J_x} \\ \ddot{\theta} = \frac{J_z - J_x}{J_y} \dot{\phi} \dot{\psi} + \frac{J_r}{J_y} \Omega_R \dot{\phi} + \frac{\tau_2}{J_y} \\ \ddot{\psi} = \frac{J_x - J_y}{J_z} \dot{\phi} \dot{\theta} + \frac{\tau_3}{J_z} \\ \ddot{z} = -g + (\cos \phi \cos \theta) \frac{f}{m} \\ \ddot{x} = (\cos \phi \sin \theta \cos \psi + \sin \phi \sin \psi) \frac{f}{m} \\ \ddot{y} = (\cos \phi \sin \theta \sin \psi - \sin \phi \cos \psi) \frac{f}{m} \end{cases} \quad (1)$$

Figure 1: The quadrotor sketches of “+” type and “x” type.

where J_x , J_y and J_z are the moments of inertia along the three body axes; J_r is the inertia of the rotor; $\Omega_R = \omega_1 - \omega_2 + \omega_3 - \omega_4$ with ω_i , $i = 1, \dots, 4$ being the rotor speed; g denotes the gravitational acceleration; m is the mass of quadrotor; f , τ_1 , τ_2 and τ_3 are control force and torques, respectively.

To facilitate controller design, let $x_{11} = \phi$, $x_{21} = \theta$, $x_{31} = \psi$, $x_{41} = z$, $x_{51} = x$ and $x_{61} = y$. Each subsystem of the quadrotor model (1) can be expressed in state space form as the following second-order system,

$$\begin{aligned} \dot{x}_{i1} &= x_{i2} \\ \dot{x}_{i2} &= f_i + g_i U_i, \quad i = 1, \dots, 6 \end{aligned} \quad (2)$$

where f_i , g_i are given as

$$\begin{aligned} f_1 &= \frac{J_y - J_z}{J_x} x_4 x_6 - \frac{J_r}{J_x} \Omega_R x_4, & f_2 &= \frac{J_z - J_x}{J_y} x_2 x_6 + \frac{J_r}{J_y} \Omega_R x_2, & f_3 &= \frac{J_x - J_y}{J_z} x_2 x_4, & f_4 &= -g, \\ f_5 &= f_6 = 0, & g_1 &= \frac{1}{J_x}, & g_2 &= \frac{1}{J_y}, & g_3 &= \frac{1}{J_z}, & g_4 &= \frac{\cos x_1 \cos x_3}{m}, & g_5 &= g_6 = \frac{U_4}{m}. \end{aligned}$$

The six control inputs of subsystems (2) contain four actual inputs and two virtual inputs, i.e., $U_1 = \tau_1$, $U_2 = \tau_2$, $U_3 = \tau_3$ and $U_4 = f$ are the actual control inputs; U_5 and U_6 are the virtual inputs given as follows,

$$\begin{aligned} U_5 &= (\cos x_1 \sin x_3 \cos x_5 + \sin x_1 \sin x_5) \\ U_6 &= (\cos x_1 \sin x_3 \sin x_5 - \sin x_1 \cos x_5) \end{aligned} \quad (3)$$

which are used to obtain the desired roll and pitch angles.

The actual control inputs U_i , $i = 1, \dots, 4$ are generated by propellers, which have different formulations for “+”-type and “x”-type respectively. Let actual control inputs of the two types of quadrotor denote by U_i^+ and U_i^x , respectively. For U_i^+ , its expression with respect to the propeller speed ω_i is shown as,

$$\begin{aligned} U_1^+ &= -c_{i2} l \omega_2^2 + c_{i4} l \omega_4^2 \\ U_2^+ &= -c_{i1} l \omega_1^2 + c_{i3} l \omega_3^2 \\ U_3^+ &= c_{d1} \omega_1^2 - c_{d2} \omega_2^2 + c_{d3} \omega_3^2 - c_{d4} \omega_4^2 \\ U_4^+ &= c_{i1} \omega_1^2 + c_{i2} \omega_2^2 + c_{i3} \omega_3^2 + c_{i4} \omega_4^2. \end{aligned} \quad (4)$$

where $c_{ii} > 0$ and $c_{di} > 0$, $i = 1, \dots, 4$ are the thrust factor and the drag factor of each propeller, respectively, and l is

the quadrotor arm length. The expression of $U_i^\times, i = 1, \dots, 4$ is given as

$$\begin{aligned} U_1^\times &= \frac{\sqrt{2}}{2}(-c_{t1}l\omega_1^2 - c_{t2}l\omega_2^2 + c_{t3}l\omega_3^2 + c_{t4}l\omega_4^2) \\ U_2^\times &= \frac{\sqrt{2}}{2}(-c_{t1}l\omega_1^2 + c_{t2}l\omega_2^2 + c_{t3}l\omega_3^2 - c_{t4}l\omega_4^2) \\ U_3^\times &= c_{d1}\omega_1^2 - c_{d2}\omega_2^2 + c_{d3}\omega_3^2 - c_{d4}\omega_4^2 \\ U_4^\times &= c_{t1}\omega_1^2 + c_{t2}\omega_2^2 + c_{t3}\omega_3^2 + c_{t4}\omega_4^2. \end{aligned} \quad (5)$$

The four rotors of a quadrotor are all driven by corresponding direct current motors. The rotor speed ω_i is considered to be proportional to the motor voltage input u_i ,

$$\omega_i = K_{mi}u_i, \quad i = 1, \dots, 4 \quad (6)$$

with $K_{mi} > 0$ the gain coefficient.

Remark 1. Equations (4) and (5) present the relationship between control inputs and rotor speeds of considered two types subject to uncertain parameters. In fact, these equations can also represent the case when actuators are suffering from LOEs. Take “+”-type for example, for the nominal condition, i.e., the parameters of four rotors are identical, then (4) can be expressed as

$$\begin{aligned} U_1^+ &= -c_t l \omega_{2,a}^2 + c_t l \omega_{4,a}^2 \\ U_2^+ &= -c_t l \omega_{1,a}^2 + c_t l \omega_{3,a}^2 \\ U_3^+ &= c_d \omega_{1,a}^2 - c_d \omega_{2,a}^2 + c_d \omega_{3,a}^2 - c_d \omega_{4,a}^2 \\ U_4^+ &= c_t \omega_{1,a}^2 + c_t \omega_{2,a}^2 + c_t \omega_{3,a}^2 + c_t \omega_{4,a}^2. \end{aligned} \quad (7)$$

where $\omega_{i,a}$ is the rotor speed under the following actuator fault,

$$\omega_{i,a} = \varrho_i \omega_i \quad (8)$$

with $\varrho_i \in (\underline{\varrho}, 1]$ the LOE factor, and $\underline{\varrho} > 0$ is a lower bound needed to maintain the controllability of the system [14, 15]. Assume the LOE $\varrho_1 < 1$ occurs to the 1st rotor while other rotors stay healthy, i.e., $\varrho_2 = \varrho_3 = \varrho_4 = 1$, then substitute (8) into (7) yields

$$\begin{aligned} U_1^+ &= -c_t l \omega_2^2 + c_t l \omega_4^2 \\ U_2^+ &= -c_{t1} l \omega_1^2 + c_t l \omega_3^2 \\ U_3^+ &= c_{d1} \omega_1^2 - c_d \omega_2^2 + c_d \omega_3^2 - c_d \omega_4^2 \\ U_4^+ &= c_{t1} \omega_1^2 + c_t \omega_2^2 + c_t \omega_3^2 + c_t \omega_4^2. \end{aligned} \quad (9)$$

where $c_{t1} \triangleq c_t \varrho_1^2$ and $c_{d1} \triangleq c_d \varrho_1^2$. From this example we see that (4) and (5) can represent the case when all rotors suffer from LOEs.

To facilitate the control design, the following assumptions are made.

Assumption 1. The reference signals and their 2th-order derivatives are available and bounded for the subsystems in (2).

Assumption 2. All state variables are available for controller design.

Assumption 3. The roll angle and pitch angle operate within small ranges.

Remark 2. Assumption 3 is reasonable since it can be guaranteed by a successfully designed controller. Furthermore, it simplifies the model description and the controller design, e.g., the term g_4 in (2) becomes $g_4 \approx \frac{1}{m}$ which is constant.

The control objective is to design adaptive controllers for the two types of quadrotors subject to uncertain and faulty actuators, and guarantees that the plant can track the reference given by users. Through out this paper, for simplicity concern, we only make a distinction between the variable names related to the actuator system, since the body dynamics of the two types of quadrotors are the same.

3. Adaptive Control Design

3.1. Extract actuator parameters

Observing (4) and (5), where the control inputs U^+ and $U^\times$ contain unknown and distinct factors. In addition to that, from (6) the gain parameter of each rotor speed to the voltage input is also unknown. Hence, we need to extract actuator parameters from the original control inputs U^+ and $U^\times$ to facilitate the control design.

For “+”-type quadrotor, substituting (6) into (4),

$$\begin{aligned} U_1^+ &= -\bar{\vartheta}_{11}^+ u_2^2 + \bar{\vartheta}_{12}^+ u_4^2 \\ U_2^+ &= -\bar{\vartheta}_{21}^+ u_1^2 + \bar{\vartheta}_{22}^+ u_3^2 \\ U_3^+ &= \bar{\vartheta}_{31}^+ u_1^2 - \bar{\vartheta}_{32}^+ u_2^2 + \bar{\vartheta}_{33}^+ u_3^2 - \bar{\vartheta}_{34}^+ u_4^2 \\ U_4^+ &= \bar{\vartheta}_{41}^+ u_1^2 + \bar{\vartheta}_{42}^+ u_2^2 + \bar{\vartheta}_{43}^+ u_3^2 + \bar{\vartheta}_{44}^+ u_4^2 \end{aligned} \quad (10)$$

where $\bar{\vartheta}_{11}^+ = c_{l2} l K_{m2}^2$, $\bar{\vartheta}_{12}^+ = c_{l4} l K_{m4}^2$, $\bar{\vartheta}_{21}^+ = c_{l1} l K_{m1}^2$, $\bar{\vartheta}_{22}^+ = c_{l3} l K_{m3}^2$, $\bar{\vartheta}_{3i}^+ = c_{di} K_{mi}^2$ and $\bar{\vartheta}_{4i}^+ = c_{ti} K_{mi}^2$.

Define the intermediate variables v_i^+ , $i = 1, \dots, 4$ as

$$\begin{aligned} v_1^+ &= -u_2^2 + u_4^2 \\ v_2^+ &= -u_1^2 + u_3^2 \\ v_3^+ &= u_1^2 - u_2^2 + u_3^2 - u_4^2 \\ v_4^+ &= u_1^2 + u_2^2 + u_3^2 + u_4^2 \end{aligned} \quad (11)$$

Using (8), then (7) can be rewritten as,

$$\begin{aligned} U_1^+ &= \vartheta_{11}^+ v_1^+ + \vartheta_{12}^+ (v_3^+ - v_4^+) \\ U_2^+ &= \vartheta_{21}^+ (v_3^+ + v_4^+) + \vartheta_{22}^+ v_2^+ \\ U_3^+ &= \vartheta_{31}^+ v_1^+ + \vartheta_{32}^+ v_2^+ + \vartheta_{33}^+ v_3^+ + \vartheta_{34}^+ v_4^+ \\ U_4^+ &= \vartheta_{41}^+ v_1^+ + \vartheta_{42}^+ v_2^+ + \vartheta_{43}^+ v_3^+ + \vartheta_{44}^+ v_4^+ \end{aligned} \quad (12)$$

where

$$\begin{aligned} \vartheta_{11}^+ &= \frac{\bar{\vartheta}_{11}^+ + \bar{\vartheta}_{12}^+}{2}, & \vartheta_{12}^+ &= \frac{\bar{\vartheta}_{11}^+ - \bar{\vartheta}_{12}^+}{4}, & \vartheta_{21}^+ &= \frac{-\bar{\vartheta}_{21}^+ + \bar{\vartheta}_{22}^+}{4}, & \vartheta_{22}^+ &= \frac{\bar{\vartheta}_{21}^+ + \bar{\vartheta}_{22}^+}{2}, \\ \vartheta_{31}^+ &= \frac{\bar{\vartheta}_{32}^+ - \bar{\vartheta}_{34}^+}{2}, & \vartheta_{32}^+ &= \frac{-\bar{\vartheta}_{31}^+ + \bar{\vartheta}_{33}^+}{2}, & \vartheta_{33}^+ &= \frac{\sum_{i=1}^4 \bar{\vartheta}_{3i}^+}{4}, & \vartheta_{34}^+ &= \frac{\sum_{i=1}^4 (-1)^{i+1} \bar{\vartheta}_{3i}^+}{4}, \\ \vartheta_{41}^+ &= \frac{-\bar{\vartheta}_{42}^+ + \bar{\vartheta}_{44}^+}{2}, & \vartheta_{42}^+ &= \frac{-\bar{\vartheta}_{41}^+ + \bar{\vartheta}_{43}^+}{2}, & \vartheta_{43}^+ &= \frac{\sum_{i=1}^4 (-1)^{i+1} \bar{\vartheta}_{4i}^+}{4}, & \vartheta_{44}^+ &= \frac{\sum_{i=1}^4 \bar{\vartheta}_{4i}^+}{4} \end{aligned}$$

are unknown parameters.

For “ $\times$ ”-type quadrotor, substituting (6) into (5),

$$\begin{aligned} U_1^\times &= -\bar{\vartheta}_{11}^\times u_1^2 - \bar{\vartheta}_{12}^\times u_2^2 + \bar{\vartheta}_{13}^\times u_3^2 + \bar{\vartheta}_{14}^\times u_4^2 \\ U_2^\times &= -\bar{\vartheta}_{21}^\times u_1^2 + \bar{\vartheta}_{22}^\times u_2^2 + \bar{\vartheta}_{23}^\times u_3^2 - \bar{\vartheta}_{24}^\times u_4^2 \\ U_3^\times &= \bar{\vartheta}_{31}^\times u_1^2 - \bar{\vartheta}_{32}^\times u_2^2 + \bar{\vartheta}_{33}^\times u_3^2 - \bar{\vartheta}_{34}^\times u_4^2 \\ U_4^\times &= \bar{\vartheta}_{41}^\times u_1^2 + \bar{\vartheta}_{42}^\times u_2^2 + \bar{\vartheta}_{43}^\times u_3^2 + \bar{\vartheta}_{44}^\times u_4^2 \end{aligned} \quad (13)$$

where $\bar{\vartheta}_{1i}^\times = \bar{\vartheta}_{2i}^\times = \sqrt{2} c_{ti} l K_{mi}^2 / 2$, $\bar{\vartheta}_{3i}^\times = c_{di} K_{mi}^2$ and $\bar{\vartheta}_{4i}^\times = c_{ti} K_{mi}^2$.

Define intermediate variables $v_i^\times, i = 1, \dots, 4$, as

$$\begin{aligned} v_1^\times &= -u_1^2 - u_2^2 + u_3^2 + u_4^2 \\ v_2^\times &= -u_1^2 + u_2^2 + u_3^2 - u_4^2 \\ v_3^\times &= u_1^2 - u_2^2 + u_3^2 - u_4^2 \\ v_4^\times &= u_1^2 + u_2^2 + u_3^2 + u_4^2 \end{aligned} \quad (14)$$

then (10) can be rewritten using (11),

$$\begin{aligned} U_1^\times &= \vartheta_{11}^\times v_1^\times + \vartheta_{12}^\times v_2^\times + \vartheta_{13}^\times v_3^\times + \vartheta_{14}^\times v_4^\times \\ U_2^\times &= \vartheta_{21}^\times v_1^\times + \vartheta_{22}^\times v_2^\times + \vartheta_{23}^\times v_3^\times + \vartheta_{24}^\times v_4^\times \\ U_3^\times &= \vartheta_{31}^\times v_1^\times + \vartheta_{32}^\times v_2^\times + \vartheta_{33}^\times v_3^\times + \vartheta_{34}^\times v_4^\times \\ U_4^\times &= \vartheta_{41}^\times v_1^\times + \vartheta_{42}^\times v_2^\times + \vartheta_{43}^\times v_3^\times + \vartheta_{44}^\times v_4^\times \end{aligned} \quad (15)$$

where

$$\begin{aligned} \vartheta_{11}^\times &= \frac{\sum_{i=1}^4 \bar{\vartheta}_{1i}^\times}{4}, \quad \vartheta_{12}^\times = \frac{\sum_{i=1}^4 (-1)^{i+1} \bar{\vartheta}_{1i}^\times}{4}, \quad \vartheta_{13}^\times = \frac{-\bar{\vartheta}_{11}^\times + \bar{\vartheta}_{12}^\times + \bar{\vartheta}_{13}^\times - \bar{\vartheta}_{14}^\times}{4}, \quad \vartheta_{14}^\times = \frac{-\bar{\vartheta}_{11}^\times - \bar{\vartheta}_{12}^\times + \bar{\vartheta}_{13}^\times + \bar{\vartheta}_{14}^\times}{4}, \\ \vartheta_{21}^\times &= \frac{\sum_{i=1}^4 (-1)^{i+1} \bar{\vartheta}_{2i}^\times}{4}, \quad \vartheta_{22}^\times = \frac{\sum_{i=1}^4 \bar{\vartheta}_{2i}^\times}{4}, \quad \vartheta_{23}^\times = \frac{-\bar{\vartheta}_{21}^\times - \bar{\vartheta}_{22}^\times + \bar{\vartheta}_{23}^\times + \bar{\vartheta}_{24}^\times}{4}, \quad \vartheta_{24}^\times = \frac{-\bar{\vartheta}_{21}^\times + \bar{\vartheta}_{22}^\times + \bar{\vartheta}_{23}^\times - \bar{\vartheta}_{24}^\times}{4}, \\ \vartheta_{31}^\times &= \frac{-\bar{\vartheta}_{31}^\times + \bar{\vartheta}_{32}^\times + \bar{\vartheta}_{33}^\times - \bar{\vartheta}_{34}^\times}{4}, \quad \vartheta_{32}^\times = \frac{-\bar{\vartheta}_{31}^\times - \bar{\vartheta}_{32}^\times + \bar{\vartheta}_{33}^\times + \bar{\vartheta}_{34}^\times}{4}, \quad \vartheta_{33}^\times = \frac{\sum_{i=1}^4 \bar{\vartheta}_{3i}^\times}{4}, \quad \vartheta_{34}^\times = \frac{\sum_{i=1}^4 (-1)^{i+1} \bar{\vartheta}_{3i}^\times}{4}, \\ \vartheta_{41}^\times &= \frac{-\bar{\vartheta}_{41}^\times - \bar{\vartheta}_{42}^\times + \bar{\vartheta}_{43}^\times + \bar{\vartheta}_{44}^\times}{4}, \quad \vartheta_{42}^\times = \frac{-\bar{\vartheta}_{41}^\times + \bar{\vartheta}_{42}^\times + \bar{\vartheta}_{43}^\times - \bar{\vartheta}_{44}^\times}{4}, \quad \vartheta_{43}^\times = \frac{\sum_{i=1}^4 (-1)^{i+1} \bar{\vartheta}_{4i}^\times}{4}, \quad \vartheta_{44}^\times = \frac{\sum_{i=1}^4 \bar{\vartheta}_{4i}^\times}{4} \end{aligned}$$

are unknown parameters.

To unify the expressions of “+”-type and “×”-type, the attitude and altitude subsystems in system (2) is rewritten as

$$\begin{aligned} \dot{x}_{i1} &= x_{i2} \\ \dot{x}_{i2} &= f_i + g_i(\vartheta_{ii}^* v_i^* + \varphi_i^{*T} \vartheta_i^*), \quad i = 1, \dots, 4 \end{aligned} \quad (16)$$

where the superscript * denotes + or ×, and

$$\begin{aligned} \varphi_1^+ &= v_3^+ - v_4^+, \quad \varphi_2^+ = v_3^+ + v_4^+, \quad \varphi_3^+ = [v_1^+, v_2^+, v_4^+]^T, \quad \varphi_4^+ = [v_1^+, v_2^+, v_3^+]^T, \\ \vartheta_1^+ &= \vartheta_{12}^+, \quad \vartheta_2^+ = \vartheta_{21}^+, \quad \vartheta_3^+ = [\vartheta_{31}^+, \vartheta_{32}^+, \vartheta_{34}^+]^T, \quad \vartheta_4^+ = [\vartheta_{41}^+, \vartheta_{42}^+, \vartheta_{43}^+]^T, \\ \varphi_1^\times &= [v_2^\times, v_3^\times, v_4^\times]^T, \quad \varphi_2^\times = [v_1^\times, v_3^\times, v_4^\times]^T, \quad \varphi_3^\times = [v_1^\times, v_2^\times, v_4^\times]^T, \quad \varphi_4^\times = [v_1^\times, v_2^\times, v_3^\times]^T, \\ \vartheta_1^\times &= [\vartheta_{12}^\times, \vartheta_{13}^\times, \vartheta_{14}^\times]^T, \quad \vartheta_2^\times = [\vartheta_{21}^\times, \vartheta_{23}^\times, \vartheta_{24}^\times]^T, \quad \vartheta_3^\times = [\vartheta_{31}^\times, \vartheta_{32}^\times, \vartheta_{34}^\times]^T, \quad \vartheta_4^\times = [\vartheta_{41}^\times, \vartheta_{42}^\times, \vartheta_{43}^\times]^T. \end{aligned}$$

In what follows, v_i^* is the control input to be designed. Obviously, after determining v_i^* , one can obtain the desired voltage inputs u_i according to (11) and (14), respectively.

3.2. Adaptive Control Design for Attitude and Altitude Subsystems

Compared with the classical adaptive method which depends on cancellation of the terms of the estimation error related in the derivative of Lyapunov function, I&I adaptive approach provides an effective means of shaping the dynamic response of the estimation error via domination [23]. In view of this, we design an adaptive controller based on I&I adaptive method for attitude and altitude subsystems of the quadrotor. Since solving the partial differential equation (PDE) is needed in I&I adaptive method, hence, the control design will be presented via the introduction of linear low-pass filters to bypass the solving of PDEs. The filter-based I&I adaptive control method has been applied in various kinds of systems (see [23, 24, 25] for example).

To begin with, for attitude and altitude subsystems (16), define the following error variables as ,

$$z_{i1} = x_{i1} - x_{id}, \quad z_{i2} = x_{i2} - \alpha_i, \quad i = 1, \dots, 4 \quad (17)$$

where x_{id} is the reference signal of each subsystem, α_i is the virtual control to be designed. The derivative of z_{i1} is calculated as

$$\dot{z}_{i1} = \dot{x}_{i1} + \alpha_i - \dot{x}_{id} \quad (18)$$

α_i is designed as

$$\alpha_i = -k_{i1}z_{i1} + \dot{x}_{id} \quad (19)$$

with design parameter $k_{i1} > 0$, then (18) is written as

$$\dot{z}_{i1} = -k_{i1}z_{i1} + z_{i2} \quad (20)$$

The derivative of z_{i2} is calculated as

$$\begin{aligned} \dot{z}_{i2} &= \dot{x}_{i2} - \dot{\alpha}_i \\ &= f_i + g_i(\vartheta_{ii}^* v_i^* + \varphi_i^{*T} \vartheta_i^*) - \dot{\alpha}_i \\ &= g_i \vartheta_{ii}^* v_i^* + \bar{\varphi}_i^{*T} \bar{\vartheta}_i^* \end{aligned} \quad (21)$$

where $\bar{\varphi}_i^{*T} \bar{\vartheta}_i^* = g_i \varphi_i^{*T} \vartheta_i^* + f_i - \dot{\alpha}_i$ with $\bar{\varphi}_i^*$ a known function vector and $\bar{\vartheta}_i^*$ an unknown constant vector.

Based on the above analysis, the adaptive control law v_i^* will be specified as the following theorem.

Theorem 1. Define the following filters

$$\dot{z}_{i1,f} = -z_{i1,f} + z_{i1} \quad (22)$$

$$\dot{z}_{i2,f} = -z_{i2,f} + z_{i2} \quad (23)$$

$$\dot{\bar{\varphi}}_{i,f}^* = -\bar{\varphi}_{i,f}^* + \bar{\varphi}_i^* \quad (24)$$

$$\dot{v}_{i,f}^* = -v_{i,f}^* + v_i^* \quad (25)$$

which yield the following error dynamics,

$$\dot{z}_{i1,f} = -k_{i1}z_{i1,f} + z_{i2,f} + \varepsilon_1 \quad (26)$$

$$\dot{z}_{i2,f} = g_i \vartheta_{ii}^* v_{i,f}^* + \bar{\varphi}_{i,f}^{*T} \bar{\vartheta}_i^* + \varepsilon_2 \quad (27)$$

where ε_1 and ε_2 are exponentially decaying terms to be specified later. For system (2), the I&I adaptive controller is designed as,

$$v_{i,f}^* = -z_{i1,f} - k_{i2}z_{i2,f} - \bar{\varphi}_{i,f}^{*T}(\hat{\vartheta}_i^* + \beta_i^*) \quad (28)$$

$$\dot{\hat{\vartheta}}_i^* = \gamma_i \bar{\varphi}_{i,f}^* (z_{i1,f} + k_{i2}z_{i2,f}) - \gamma_i z_{i2,f} \dot{\bar{\varphi}}_{i,f}^* \quad (29)$$

$$\dot{\beta}_i^* = \gamma_i \bar{\varphi}_{i,f}^* z_{i2,f} \quad (30)$$

where $k_{i2} > 0$, $\gamma_i > 0$ are the design parameters and $(\hat{\vartheta}_i^* + \beta_i^*)$ are the estimate of $\frac{\bar{\vartheta}_i^*}{g_i \vartheta_{ii}^*}$. Then, all the signals in the closed-loop system are ultimately bounded and attitude and altitude tracking errors z_{i1} converge to zero.

PROOF. Define the following variables

$$\eta_{i1} = k_{i1}z_{i1,f} - z_{i2,f} - z_{i1,f} + z_{i1} \quad (31)$$

$$\eta_{i2} = -g_i(\vartheta_{ii}^* v_{i,f}^* + \bar{\varphi}_{i,f}^{*T} \bar{\vartheta}_i^*) - z_{i2,f} + z_{i2}$$

It has $\dot{\eta}_{i1} = -\eta_{i1}$ and $\dot{\eta}_{i2} = -\eta_{i2}$ from (22)-(27), and ε_1 and ε_2 are dropped hereafter since they do not affect the validity of the stability analysis[23, 24, 25]. The filtered error system is now represented as

$$\dot{z}_{i1,f} = -k_{i1}z_{i1,f} + z_{i2,f} \quad (32)$$

$$\dot{z}_{i2,f} = g_i \vartheta_{ii}^* v_{i,f}^* + \bar{\varphi}_{i,f}^{*T} \bar{\zeta}_i^* \quad (33)$$

Substituting control law (28) into (33) yields

$$\dot{z}_{i2,f} = g_i \vartheta_{ii}^* (-z_{i1,f} - k_{i2}z_{i2,f} - \bar{\varphi}_{i,f}^{*T} \zeta_i^*) \quad (34)$$

with

$$\zeta_i^* = \hat{\vartheta}_i^* - \frac{\bar{\vartheta}_i^*}{g_i \vartheta_{ii}^*} + \beta_i^* \quad (35)$$

defined as the off-the-manifold, whose derivative can be obtained by invoking (29) and (30),

$$\dot{\zeta}_i^* = -\gamma_i \bar{\varphi}_{i,f}^* \bar{\varphi}_{i,f}^{*T} \zeta_i^* \quad (36)$$

Finally, the stability of whole closed-loop system can be verified by choosing the Lyapunov function

$$V_1^* = \frac{1}{2} \sum_{i=1}^4 \left(z_{i1,f}^2 + \frac{1}{g_i \vartheta_{ii}^*} z_{i2,f}^2 + \frac{1}{k_{i2}} \zeta_i^{*T} \gamma_i^{-1} \zeta_i^* \right) \quad (37)$$

whose time derivative along the solutions of (32), (34) and (36) is given by

$$\begin{aligned} \dot{V}_1^* &= \sum_{i=1}^4 \left(z_{i1,f} \dot{z}_{i1,f} + \frac{1}{g_i \vartheta_{ii}^*} z_{i2,f} \dot{z}_{i2,f} + \frac{1}{k_{i2}} \zeta_i^{*T} \gamma_i^{-1} \dot{\zeta}_i^* \right) \\ &= \sum_{i=1}^4 \left(-k_{i1} z_{i1,f}^2 - k_{i2} z_{i2,f}^2 - \bar{\varphi}_{i,f}^{*T} \zeta_i^* z_{i2,f} - \frac{1}{k_{i2}} (\bar{\varphi}_{i,f}^{*T} \zeta_i^*)^2 \right) \\ &\leq \sum_{i=1}^4 \left(-k_{i1} z_{i1,f}^2 - \frac{k_{i2}}{2} z_{i2,f}^2 - \frac{1}{2k_{i2}} (\bar{\varphi}_{i,f}^{*T} \zeta_i^*)^2 \right) \\ &\leq 0. \end{aligned} \quad (38)$$

Then it can be concluded that $V_1^*(t) \in \ell_\infty$, and thus $z_{i1,f} \in \ell_\infty$, $z_{i2,f} \in \ell_\infty$ and $\zeta_i^* \in \ell_\infty$. From the definition of ζ_i^* , the boundedness of $\hat{\vartheta}_i^* + \beta_i^*$ is guaranteed. Then $v_{i,f}^*$ is bounded and consequently all the closed-loop signals are bounded. On the other hand, we know $z_{i1,f}, z_{i2,f} \in \ell_2 \cap \ell_\infty$, then it can be concluded that $\lim_{t \rightarrow \infty} z_{i1,f} = 0$ and $\lim_{t \rightarrow \infty} z_{i2,f} = 0$ by invoking Barbalat's lemma. Finally, from (22) and (23) we know $\lim_{t \rightarrow \infty} z_{i1} = 0$ and $\lim_{t \rightarrow \infty} z_{i2} = 0$.

Moreover, the actual control v_i^* should be recovered according to (25) as,

$$v_i^* = \dot{v}_{i,f}^* + v_{i,f}^* \quad (39)$$

from which we know that the analytical expression of $\dot{v}_{i,f}^*$ is available from (22)-(24). However, it can be seen that $\dot{v}_{i,f}^*$ contains $\dot{\bar{\varphi}}_{i,f}^*$ and thus contains the unfiltered regressors $\bar{\varphi}_i^*$ by recalling (24). From the definition of $\bar{\varphi}_i^*$, we can observe that it involves φ_i^* , which consists of v_i^* . Thus, it causes a self-loop problem where each control input v_i^* contains one another. In Section 4, an alternative adaptive design will be presented to address the problem. The design for horizontal control will be presented here first.

3.3. Control Design for Horizontal Subsystem

Rewrite the horizontal subsystem from (2) as

$$\begin{aligned}\dot{x}_{j1} &= x_{j2} \\ \dot{x}_{j2} &= \rho U_j, j = 5, 6\end{aligned}\quad (40)$$

where $\rho = \frac{U_4^*}{m}$ is unknown from (4) or (5) and is always positive for dominating the gravity. Define the following error variables

$$z_{j1} = e_j + k_0 \int_0^t e_j dt, \quad z_{j2} = x_{j2} - \alpha_j \quad (41)$$

where $e_j = x_{j1} - x_{jd}$ is the tracking error, x_{jd} is the reference signal, $k_0 > 0$ is a positive constant, and α_j is the virtual control law. Differentiating z_{j1} gives

$$\begin{aligned}\dot{z}_{j1} &= \dot{e}_j + k_0 e_j \\ &= z_{j2} + \alpha_j - \dot{x}_{jd} + k_0 e_j\end{aligned}\quad (42)$$

Design α_j in (42) as

$$\alpha_j = -k_{j1} z_{j1} + \dot{x}_{jd} - k_0 e_j \quad (43)$$

with $k_{j1} > 0$, (42) is rewritten as

$$\dot{z}_{j1} = -k_{j1} z_{j1} + z_{j2}. \quad (44)$$

Differentiating z_{j2} gives

$$\dot{z}_{j2} = \rho U_j - \dot{\alpha}_j \quad (45)$$

The horizontal control law U_j is designed as

$$U_j = -(k_{j2} + \dot{\alpha}_j^2) z_{j2} \quad (46)$$

with the control parameter $k_{j2} > 0$. The stability of under control (46) can be checked by Lyapunov function $V_h = \frac{z_{51}^2}{2} + \frac{z_{61}^2}{2k_{51}} + \frac{z_{52}^2}{2} + \frac{z_{62}^2}{2k_{61}}$, whose derivative along the solutions of (44) and (45) is

$$\dot{V}_h = \sum_{j=5}^6 \left[-k_{j1} z_{j1}^2 + z_{j1} z_{j2} - k_{j1}^{-1} \rho (k_{j2} + \dot{\alpha}_j^2) z_{j2}^2 - k_{j1}^{-1} \dot{\alpha}_j z_{j2} \right] \quad (47)$$

Using Young's inequality on the cross terms in (47),

$$\begin{aligned}z_{j1} z_{j2} &\leq \frac{k_{j1}}{2} z_{j1}^2 + \frac{1}{2k_{j1}} z_{j2}^2 \\ \dot{\alpha}_j z_{j2} &\leq \frac{\rho}{k_{j1}} \dot{\alpha}_j^2 z_{j2}^2 + \lambda_j\end{aligned}$$

where $\underline{\rho} = \inf\{\rho\} > 0$ and $\lambda_j = \frac{k_{j1}}{4\rho}$, (47) becomes

$$\begin{aligned}\dot{V}_h &\leq \sum_{j=5}^6 \left(-\frac{1}{2} k_{j1} z_{j1}^2 - \frac{\rho k_{j2}}{k_{j1}} z_{j2}^2 + \lambda_j \right) \\ &\leq -k_h V_h + \lambda_h\end{aligned}\quad (48)$$

where $k_h = \min\{k_{jh}\}$ with $k_{jh} = \min\{k_{j1}, 2\rho k_{j2}\}$ and $\lambda_h = \lambda_5 + \lambda_6$. Integrating both side of (48), one has

$$V_h(t) \leq (V_h(0) - \frac{\lambda_h}{k_h}) \exp(-k_h t) + \frac{\lambda_h}{k_h}. \quad (49)$$

Therefore, $V_h(t) \in \ell_\infty$, and thus $z_{j1}, z_{j2} \in \ell_\infty$. The bound of horizontal tracking error z_{j1} can be found as $\sqrt{2\lambda_h/k_h}$.

Remark 3. It can be observed that the above horizontal control design includes the integral of the tracking error. The existence of integral action provides an additional design of freedom, and the horizontal tracking accuracy could be improved even when the performance of roll and pitch angle are not good enough.

4. Adaptive Control Design via Gain Matrix Decomposition

As illustrated in Section 3, the above adaptive control design for attitude and altitude subsystems involves algebraic loops. In this section, inspired by [26, 27, 28], an adaptive control design is designed to solve self-loop problem using gain matrix decomposition method. After presenting this control scheme, we will also present an alternative design with less filters applied and is easier to implement in practical.

Lemma 1. [26, 27, 28] Define the leading principal minors of a real matrix $G \in \mathbb{R}^{q \times q}$ as $\Delta_1, \dots, \Delta_q$. If $\Delta_1, \dots, \Delta_q$ are nonzero, then G can be decomposed as

$$G = SDQ \quad (50)$$

where S is symmetric positive definite matrix, $D = D_1 D_2$ is diagonal with D_1 arbitrary positive diagonal and $D_2 = \text{diag}\{\text{sgn}(\Delta_1), \text{sgn}(\frac{\Delta_2}{\Delta_1}), \dots, \text{sgn}(\frac{\Delta_q}{\Delta_{q-1}})\}$, and Q is unity upper triangular.

Define $U^* = [U_1^*, U_2^*, U_3^*, U_4^*]^T$, $v^* = [v_1^*, v_2^*, v_3^*, v_4^*]^T$, and rewrite (12) and (15) in the matrix form as

$$\begin{aligned} U^+ &= G^+ v^+ = \begin{bmatrix} \vartheta_{11}^+ & 0 & \vartheta_{12}^+ & -\vartheta_{12}^+ \\ 0 & \vartheta_{22}^+ & \vartheta_{21}^+ & \vartheta_{21}^+ \\ \vartheta_{31}^+ & \vartheta_{32}^+ & \vartheta_{33}^+ & \vartheta_{34}^+ \\ \vartheta_{41}^+ & \vartheta_{42}^+ & \vartheta_{43}^+ & \vartheta_{44}^+ \end{bmatrix} v^+ \\ U^\times &= G^\times v^\times = \begin{bmatrix} \vartheta_{11}^\times & \vartheta_{12}^\times & \vartheta_{13}^\times & \vartheta_{14}^\times \\ \vartheta_{21}^\times & \vartheta_{22}^\times & \vartheta_{23}^\times & \vartheta_{24}^\times \\ \vartheta_{31}^\times & \vartheta_{32}^\times & \vartheta_{33}^\times & \vartheta_{34}^\times \\ \vartheta_{41}^\times & \vartheta_{42}^\times & \vartheta_{43}^\times & \vartheta_{44}^\times \end{bmatrix} v^\times \end{aligned} \quad (51)$$

To utilize Lemma 1, the following assumption is made in addition.

Assumption 4. The leading principal minors of matrix G^* are nonzero, and their signs are known.

Using Lemma 1 under Assumption 4, matrix G^* can be decomposed as

$$G^* = S^* D^* Q^* \quad (52)$$

then (12) can be rewritten as

$$\begin{aligned} (\bar{g}S^*)^{-1} \dot{z}_2 &= D^* Q^* v^* + (\bar{g}S^*)^{-1} (f - \dot{\alpha}) \\ &= D^* [v^* - (1 - Q^*) v^*] + (\bar{g}S^*)^{-1} (f - \dot{\alpha}) \\ &= D^* v^* + \mu \Theta^* \end{aligned} \quad (53)$$

where $\bar{g} = \text{diag}\{g_1, g_2, g_3, g_4\}$, $f = [f_1, f_2, f_3, f_4]^T$, $\dot{\alpha} = [\dot{\alpha}_1, \dot{\alpha}_2, \dot{\alpha}_3, \dot{\alpha}_4]^T$, $z_2 = [z_{12}, z_{22}, z_{32}, z_{42}]^T$, $v^* = [v_1^*, v_2^*, v_3^*, v_4^*]^T$ and $\mu \Theta^*$ denotes

$$\mu \Theta^* = -D^* (1 - Q^*) v^* + (\bar{g}S^*)^{-1} (f - \dot{\alpha})$$

with $\mu^* = \text{diag}\{\mu_1^{*T}, \mu_2^{*T}, \mu_3^{*T}, \mu_4^{*T}\} \in \mathbb{R}^{4 \times p_i}$, $p_i > 0$ being a known function matrix and $\Theta^* \in \mathbb{R}^{p_i}$ an unknown parameter vector.

Introducing the following filter

$$\dot{\mu}_f^* = -\mu_f^* + \mu^* \quad (54)$$

with filters given in (22), (23) and (25), the filtered version of (53) is

$$(\bar{g}S^*)^{-1}\dot{z}_{2,f} = D^*v_f^* + \mu_f\Theta^* \quad (55)$$

where $z_{2,f} = [z_{12,f}, z_{22,f}, z_{32,f}, z_{42,f}]^T$ and $v_f^* = [v_{1,f}^*, v_{2,f}^*, v_{3,f}^*, v_{4,f}^*]^T$. The control will be given in the following theorem.

Theorem 2. *For system (2), the I&I adaptive controller is designed as*

$$v_f^* = D^{*-1}[-z_{1,f} - k_2 z_{2,f} - \mu_f(\hat{\Theta}^* + \beta_\Theta^*)] \quad (56)$$

$$\dot{\hat{\Theta}}^* = \bar{\gamma}\mu_f^{*T}(z_{1,f} + k_2 z_{2,f}) - \bar{\gamma}\dot{\mu}_f^{*T} z_{2,f} \quad (57)$$

$$\beta_\Theta^* = \bar{\gamma}\mu_f^{*T} z_{2,f} \quad (58)$$

where $z_{1,f} = [z_{11,f}, z_{21,f}, z_{31,f}, z_{41,f}]^T$, $k_2 = \text{diag}\{k_{12}, k_{22}, k_{32}, k_{42}\}$, and $\bar{\gamma} > 0$ is the adaptation gain with appropriate dimensions. Then, the boundedness of all the closed-loop signals is ensured and the tracking errors converge to zero asymptotically.

PROOF. Substituting (56) into (55) yields

$$(\bar{g}S^*)^{-1}\dot{z}_{2,f} = -z_{1,f} - k_2 z_{2,f} - \mu_f \zeta_\Theta^* \quad (59)$$

where $\zeta_\Theta^* = \hat{\Theta}^* - \Theta^* + \beta_\Theta^*$ is the off-the-manifold. Then, note that (32),(59) and the calculation of the derivative of ζ_Θ^* by using (57) and (58), it can be obtained that the conclusion of Theorem 2 holds by following a similar procedure of the proof of Theorem 1. The details are thus omitted here.

Remark 4. As in Theorem 2, algebraic loops are avoided since matrix $(1 - Q^*)$ is strictly upper triangular, which implies that v_1^* depends on v_2^* , v_3^* and v_4^* ; v_2^* depends on v_3^* and v_4^* ; and v_3^* only depends on v_4^* .

Remark 5. Except for actuator parameters, all other physical parameters such as mass, moment of inertia, etc. are not used in the controller design. Hence the proposed scheme is also robust to the uncertainties in the quadrotor body dynamics.

Remark 6. The controller (56)-(58) could be designed in an alternative way. Define the following filter,

$$\dot{\mu}_f^* = -k_f(\mu_f^* - \mu^*) \quad (60)$$

with $k_f > 0$ the filter constant, then a modified version of the proposed control law is given by

$$v^* = D^{*-1}[-z_1 - k_2 z_2 - \mu_f(\hat{\Theta}^* + \beta_\Theta^*)] \quad (61)$$

$$\dot{\hat{\Theta}}^* = \bar{\gamma}\mu_f^{*T}(z_1 + k_2 z_2) - \bar{\gamma}\dot{\mu}_f^{*T} z_2 \quad (62)$$

$$\beta_\Theta^* = \bar{\gamma}\mu_f^{*T} z_2 \quad (63)$$

From the above design, it is straightforward to conclude that if the filter parameter k_f are chosen large enough such that $\mu_f^* \approx \mu^*$, one can ensure the stability of the closed-loop system by following a similar procedure mentioned above. Obviously, compared with the previous controller design, the modified one is easier to implement in practical and, the dynamical order of the whole system is reduced as well, at the cost of tracking accuracy (which depends on the value of the filter constant).

5. Simulation Results

Simulation results are carried out in this section to verify the proposed controller. The (nominal) physical parameters of a “+”-type quadrotor used in simulation are borrowed from [5] as: $m = 0.468\text{kg}$, $g = 9.81\text{m/s}^2$, $l = 0.225\text{m}$, $J_x = J_y = 4.856 \times 10^{-3}\text{kg} \cdot \text{m}^2$, $J_z = 8.801 \times 10^{-3}\text{kg} \cdot \text{m}^2$, $J_r = 3.357 \times 10^{-5}\text{kg} \cdot \text{m}^2$, $c_t = 2.98 \times 10^{-6}\text{N} \cdot \text{s}^2/\text{rad}^2$ and $c_d = 1.14 \times 10^{-7}\text{Nm} \cdot \text{s}^2/\text{rad}^2$.

To verify the robustness of the designed controller, we consider the following different values of the time-varying actuator parameters: 1) at the initial stage ($t = 0\text{s}$), the thrust factors for the four rotors are set as $c_{t1} = c_t$, $c_{t2} = 0.97c_t$, $c_{t3} = 1.02c_t$ and $c_{t4} = 0.98c_t$; the drag factors are $c_{d1} = c_d$, $c_{d2} = 1.02c_d$, $c_{d3} = 0.96c_d$ and $c_{d4} = 0.97c_d$, and the motor-voltage gains are chosen as $K_{m1} = 100$, $K_{m2} = 97$, $K_{m3} = 95$ and $K_{m4} = 98$. 2) at time instant $t = 60\text{s}$, the motor of the 3rd rotor suffers from 30% LOE, i.e., $K_{m3} = 95$ is changed to $K_{m3} = 66.5$. 3) at time instant $t = 90\text{s}$, LOE is occurred to the propeller system of the 4th rotor, where c_{t4} and c_{d4} are lowered by 30%. The controller parameters are chosen as: $k_{11} = k_{21} = k_{31} = 20$, $k_{12} = k_{22} = k_{32} = 5$, $k_{41} = 0.5$, $k_{42} = 2$, $k_{51} = k_{61} = 1$, $k_{52} = k_{62} = 0.6$ and $k_0 = 0.5$; the adaptation gain for each adaptation law for the attitude system is fixed at 1×10^{-4} , while for the altitude system is set to 1×10^{-2} .

At the beginning stage of this simulated example, the horizontal reference of the quadrotor is set as $x_d = y_d = 1 - \exp(-t)$, and then it is changed to $x_d = \sin(0.05\pi(t - 30)) + 1$ and $y_d = \cos(0.05\pi(t - 30))$ at $t = 30\text{s}$. The altitude and yaw angle reference are fixed at $0.05t$ and $0.1 \sin(0.1\pi t)$, respectively.

For the sake of comparison, we also present a control scheme based on classical adaptive control method. For attitude and altitude subsystems, the adaptive controller is designed as

$$\begin{aligned} v_{CA}^* &= D^{*-1}(-z_1 - k_2 z_2 - \mu^* \hat{\Theta}_{CA}^*) \\ \dot{\hat{\Theta}}_{CA}^* &= \gamma_{CA} \mu^{*T} z_2 \end{aligned} \quad (64)$$

and the horizontal controller is the same with (46). All the control parameters of the classical adaptive controller are the same as the proposed one. In addition, the baseline controller is also considered in this simulation, where all the adaptation gains are set as zero. All the three controllers are tested under same conditions.

Simulation results are shown in Fig.2 and Fig.3, where the tracking errors of position and attitude are depicted respectively. From both figures we can see that, in the first 60s, the baseline controller exposes steady-state errors in the attitude and the height response, due to the effect of small uncertainties in the actuator parameters c_t , c_d and K_m . However, the uncertainties are overcome by both adaptive controllers. In contrast, from Fig. 2, the horizontal response of the baseline controller performs similar to that of the adaptive controllers, because of the existence of the integral action in the control law. At 60s, where the first LOE occurs, it is shown that the baseline controller cannot adapt to the sudden change of the actuator parameter and become unstable. In the mean time, the system response of the two adaptive controllers are affected temporarily and then converge very fast, so is the case when another LOE occurring at 90s. Although both adaptive schemes can stay stable subject to the change of parameters, the I&I adaptive one performs much better than that of the classical one, in terms of the transient behaviour, as shown in these figures.

6. Experimental Results

In this section, experimental results are carried out to illustrate the effectiveness of the proposed control scheme applied to an “X”-type quadrotor setup, as shown in Fig.4. In this setup, the remote sends only the take off or landing commands to the quadrotor system, and is connected with the ground station via USB. The position and attitude data are sent to the remote through the wireless communication, and then these data can be transmitted to the ground station. The reference trajectories are already programmed in the system environment. In the experiments, we consider three cases: the first case exams the performance of the classical adaptive controller under nominal situation (i.e., no hardware or software faults occur); in the second case, the quadrotor is equipped with an intentionally damaged propeller in one of the rotors, and a comparison between the classical adaptive controller and the I&I adaptive controller is given; the third case is implemented with the I&I adaptive controller under the damaged propeller as the same as case 2, and LOE factors are injected to all of the four actuators.

6.1. Case 1: Nominal Situation

In this case, the experiment is carried out using the classical adaptive scheme (64). The purpose of this case is, to verify that the parameters of four actuators are actually distinct which are needed to be considered in the control

Figure 2: Simulation: the evolution of position errors of the three control schemes.

Figure 3: Simulation: the evolution of attitude errors of the three control schemes.

design, even no obvious changes or damages of the system are observed. To make a comparison, we also present experimental results with the baseline controller, whose adaption gains are all set as zero. The references in this case are $x_d = y_d = \psi_d = 0$ and $z_d = 0.6 / (1 + \exp(-t + 4))$.

Results of this case are shown in Fig.5 and Fig.6, where the curves of position and attitude errors are depicted respectively. As expected, the performances are improved by the adaptive controller, especially in the attitude responses.

Figure 4: “X”-type quadrotor platform.

Figure 5: Case 1: the evolution of position error.

6.2. Case 2: Performance Comparison

In this case, to test the robustness of the system, the quadrotor platform is equipped with a partially damaged propeller of the 4th rotor, which makes the parameter of this rotor significantly different from all other ones. We are also interested in a further comparison of the classical and I&I adaptive controllers experimentally. To implement the controller more easily, the adaptation gains of attitude and altitude subsystems are set as the same value. Different adaptation gains are set for the two controllers respectively. For the classical adaptive controller, the adaptation gains are chosen for three different values, 0.01, 0.1 and 1, respectively, where $\gamma_{CA} = 1$ is the max value can be chosen in the tuning process; and for the I&I adaptive controller, the adaptation gains are all fixed at 0.01, which is also the extreme one can be chosen in the experiment. The references in this case are: $x_d = 0.7 \sin(0.05\pi t)$, $y_d = -0.7 \cos(0.05\pi t) + 0.7$, $z_d = 0.05t$ and $\psi_d = 0$.

Experiment results are carried out using the above four controllers, namely, controller 1-4, as shown in Fig. 7 and Fig.8. Fig. 7 shows the trajectories in x-y plane and Fig.8 presents the time profiles of the norms of the position error vector $e_\xi = [z_{41}, z_{51}, z_{61}]^T$ and the attitude error vector $e_\theta = [z_{11}, z_{21}, z_{31}]^T$ respectively. In both figures, the improving performance are observed of the classical adaptive controller (controller 1-3) with the increasing of the adaptation gain. However, without larger adaptation gain, the I&I adaptive one (controller 4) outperforms all other controllers. These figures illustrate the superiority of I&I adaptive method in the practical situation.

Figure 6: Case 1: the evolution of attitude error.

Figure 7: Case 2: the evolution of trajectory tracking in x-y plane.

6.3. Case 3: Software Fault Injection

In this case, the LOE faults will be injected artificially in the system programming environment to test the robustness of the I&I adaptive controller. This case is implemented on the basis of case 2, where the damaged propeller is still equipped. Initially, no LOEs occur in the first 20s; at 20s, the 1st rotor suffers from a 30% LOE; at 40s, the 2nd and the 3rd rotors are injected with 20% and 10% LOE, respectively; at 60s, the 2nd, 3rd and 4th rotors all suffer from 10% LOE; finally, at 80s, all faulty rotors return to healthy states, and remain unchanged in the rest of the flight. The quadrotor in this case is commanded to follow a circular trajectory in the horizontal direction, with the period of about 20s and the magnitude of 0.7m. The height command is 1m with a smooth transient stage and the yaw command is still zero. Results are shown in Fig. 9–11, where the system can track the reference constantly and the output errors

Figure 8: Case 2: the evolution of norms of position and attitude errors.

Figure 9: Case 3: the evolution of trajectory tracking in x-y plane.

of the subsystems are kept within small ranges.

7. Conclusion

Two adaptive control schemes are proposed in this paper to cope with uncertain and faulty actuators for two of common configurations of quadrotors, namely, “+”-type and “x”-type. By extracting unknown parameters from the original control inputs, actuator uncertainties are compensated via the constructed I&I adaptive controller. To avoid the self-loop problem, we propose an enhanced control design which utilizes gain matrix decomposition technique. Simulation results show that the steady-state error and the PLOE of actuators can be efficiently handled. Finally, three

Figure 10: Case 3: the evolution of position error.

Figure 11: Case 3: the evolution of attitude error.

different scenarios of experimental results are carried out to show the superiority of the proposed controller.

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