Protocol-Based Control for Cyber-Physical Systems with Environment-Dependent Energy Harvesting Sensors

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Abstract

In this paper, a protocol-based controller is designed for Cyber-Physical Systems (CPSs) with multiple sensors, which are powered by environment-dependent energy harvesting (EDEH) devices, respectively. The Round-Robin (RR) protocol is adopted to coordinate the data transmission of sensors. The protocol-based transmission can be realized only when the energy harvested by EDEH devices is sufficient. The aim of this paper is to design the protocol-based controller to ensure the stochastic finite-time boundedness (FTB) with EDEH and RR protocol. Firstly, modeling the EDEH by a switching sequence with varying sojourn probabilities, assuming a finite battery capacity constraint, and associating protocol-based transmission with a given energy cost, we propose a new recursive model to depict the dynamic of energy levels for each sensor. Then, combining with stochastic analysis and the dynamic of energy levels, the explicit expressions of the controller for each environment with average dwell time (ADT) are obtained. Finally, an example is provided to demonstrate the effectiveness of the designed controllers.

Keywords: Nonlinear systems, stochastic finite-time boundedness, environment-dependent energy harvesting, RR protocol.

1. Introduction

Due to the rapid development of computation, networking and communication, CPSs have attracted significant research interest [1, 2, 3], which combine the cyber and physical process quite tightly. So far, CPSs have been applied in many fields, i.e. autonomous vehicles [4], industrial automation[5]. In these applications, a hot research topic is to study the control performance under the resources constraint (energy, bandwidth, computation and so on) [6]. In [7] and [8], the event-triggering control scheme has been proposed for SCPs to decrease the data transmission burden with the limited communication resource. In [9], the resource allocation and control problem have been jointly studied for industrial CPSs under the power and subcarrier constraints.

As a vitally important branch of the resources constraint, energy problem often exists in CPSs [10, 11, 12, 13]. In some applications of CPSs, the communication devices are battery powered via a fixed power or a replaceable battery [14]. However, for some application scenarios, it is neither affordable nor feasible to always use a fixed supply or rechargeable battery, like Forest Fire

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Monitoring[15], oil spill remote sensing, Military environmental monitoring and so on. In the last decade, a new technology name Energy Harvest (EH) was presented in [16] and [17] to power the communication devices. EH technology can harvest energy from the environment, where the energy supply is potentially infinite but random and intermittent over time. In [18], EH process is assumed to be finite state Markov chains. In [19] and [20], the EH has been modeled by a two-state Markov chain ('Good' and 'Bad') and the Bernoulli distribution.

The EH devices is installed in fluctuating environments and environmental factors directly affect EH [21]. For the communication between the sensors and controllers under EDEH, the amount of energy harvested and stored is typically random and depends on the external environment. For example, solar panels only operate in sunlight and can collect energy with different probability distributions in different light conditions. As such, the energy analysis will be misleading if we use fixed statistical information, and our paper aims to provide a EDEH framework.

Moreover, to combat the scarcity of the communication resources, a series of communication protocol have been put forward. For example, as a periodic protocol, RR protocol has been proposed to dispatch the signals which are transmitted in a given circular function [22, 23]. RR protocol has got broad application in productions because of the structural simplicity and convenient implementation [24]. Consequently, an obvious problem is whether the stored energy under EDEH is insufficient to warrant the protocol-based transmission at certain time instants.

Based on the above discussions, we try to study protocol-based controllers to ensure the stochastic FTB of CPSs with EDEH sensors. The main contributions are: 1) Considering the phenomena of EH in environment-dependent modes, we make the first attempt to introduce the concept of EDEH modeled by environment-dependent Bernoulli distributions with known conditional probabilities. 2) A new recursive approach is proposed to calculate the probability of protocol-based energy storage level and information transmission under the EDDH. 3) Protocol-based controller is designed and the influence of the dwell time for each environment is analyzed under EDEH, where the technique is a recursive algorithm which is suitable for on-line computation.

The rest of this paper is organized as follows. Section II states preliminaries and problem formulation, followed by main results in Section III. In Section IV, an example is presented to demonstrate the effectiveness of the established design scheme. Finally, conclusions are drawn in Section V.

Notation: In this paper, $\lambda_{\max}(A)$ and $\lambda_{\min}(A)$ denote the maximum and minimum eigenvalue of A , respectively. $\mathbb{R}^n$ denotes the n dimensional Euclidean space. The superscript " T " stands for the transposition of considered matrices. The notation $P > 0$ shows that P is positive definite. I denotes the identity matrix of compatible dimension and $\text{diag}\{\cdot\}$ means a diagonal matrix.

2. Problem Formulation

As Figure 1 shows, the control process is equipped with EDEH sensors under the communication protocol, where the sensors harvest energy from different environments. The energy status and the communication protocols coordinatively determine the transmission of measurements to the remote controller. The system in Figure 1 is given as

$$x(k+1) = A(k)x(k) + B(k)u(k) + D(k)w(k) + f(k, x(k), \psi(k)) \quad (1)$$

in a finite time horizon $k \in [1, N]$ with the i -th sensor's measurement output

$$y_i(k) = C_i(k)x(k) + E_i(k)v(k)$$

Figure 1: The control with EDEH sensors

where $x(k) \in \mathbb{R}^{n_x}$ represents the state variable, $u(k) \in \mathbb{R}^{n_u}$ and $y(k) \in \mathbb{R}^{n_y}$ denote the input and the output, $w(k) \in \mathbb{R}^{n_x}$ and $v(k) \in \mathbb{R}^{n_y}$ are the noise, which are unknown but satisfy

$$\begin{aligned}\mathcal{W}_k &= \{w^T(k)\bar{Q}(k)w(k) < \delta_1^2\}, \\ \mathcal{V}_k &= \{v^T(k)\tilde{Q}(k)v(k) < \delta_2^2\}\end{aligned}$$

where matrices $\bar{Q}(k) > 0$ and $\tilde{Q}(k) > 0$. $A(k)$, $B(k)$, $C_i(k)$, $D(k)$ and $E_i(k)$ are known time-varying matrices. $\psi(k)$ is a zero mean Gaussian noise sequence. The nonlinear item $f(k, x(k), \psi(k))$ satisfies

$$\begin{aligned}\mathbb{E}\{f(k, x(k), \psi(k))|x(k)\} &= 0, \\ \mathbb{E}\{f(k, x(k), \psi(k))f^T(j, x(j), \psi(j))|x(k)\} &= 0, \quad k \neq j \\ \mathbb{E}\{f(k, x(k), \psi(k))f^T(k, x(k), \psi(k))|x(k)\} &= \sum_{l=1}^s \pi_{l,k} \pi_{l,k}^T \mathbb{E}\{x^T(k) \Gamma_{l,k} x(k)\}\end{aligned}$$

where $s \in \mathbb{Z}^+$, $\pi_{l,k}$ and $\Gamma_{l,k}$ are known matrices.

The information exchange between sensors and controllers takes place over a communication network with limited resources. In the following, we will establish the measurement model under RR protocol and the environment-dependent energy harvesting.

2.1. Round Robin Protocol

In this paper, RR protocol is used to schedule the transmission, under which only one sensor can transmit information in turns. By defining $r(k) = \text{mod}(k-1, n_y) + 1$ as the label of the active sensor, the measurement can be denoted as

$$\tilde{y}(k) = \Phi_{r(k)} y(k)$$

where $\Phi_j = \text{diag}\{\delta(1-j), \delta(2-j), \dots, \delta(n_y-j)\}$, $j = 1, 2, \dots, n_y$, $\delta(\cdot) \in \{0, 1\}$ is the Kronecker delta function.

2.2. Environment-dependent energy harvesting

We denote the energy level for i -th sensor as $z_k^i \in \{0, 1, 2, \dots, S\}$ where S is the maximum energy level. The harvested energy at the i -th environment is represented by h_k^i which is an environment-dependent independently identically distributed stochastic process with the probability distribution

$$\Pr(h_k^i = \gamma | \sigma_k = \beta) = p_{\gamma, \beta}^i,$$

$$\gamma = 0, 1, 2, 3, \dots, \beta = 1, 2, \dots, M$$

where $\sigma_k \in \{1, 2, \dots, M\}$ is the environment factor, $p_{\gamma, \beta}^i$ satisfies $0 \leq p_{\gamma, \beta}^i \leq 1$ and $\sum_{\gamma=1}^{+\infty} p_{\gamma, \beta}^i = 1$.

When $z_k \neq 0$, the sensor can transmit the measurement to the controller consuming 1 unit of energy. Thus, the indicator function and the measurement are further expressed by

$$\eta_{k, \beta}^i = \begin{cases} 1, & r(k) = i, z_k^i \geq 1, \sigma_k = \beta \\ 0, & \text{else} \end{cases}$$

and $\hat{y}(k) = [\hat{y}_1(k), \hat{y}_2(k), \dots, \hat{y}_{n_y}(k)]^T$ where $\hat{y}_i(k) = \eta_k^i y_i(k) + (1 - \eta_k^i) \hat{y}_i(k-1)$. Moreover, the dynamics of the energy level for i -th sensor is

$$z_{k+1}^i = \min\{z_k^i + h_{k+1}^i - \eta_k^i, S\} \quad (2)$$

with $z_0^i \leq S$. Then, the controller can be described as

$$u(k) = K_{\sigma_k}(k) \hat{y}(k)$$

where $K_\beta(k)$ is the controller gain to be designed.

Consequently, denoting $\bar{x}(k) = [x^T(k), \hat{y}^T(k)]^T$ and $\bar{w}(k) = [w(k), v(k)]^T$, we have

$$\begin{aligned} \bar{x}(k+1) = \sum_{\beta=1}^M \sigma_\beta(k) & \left[\theta_{k, \beta} (\mathcal{A}_{1, \beta}(k) \bar{x}(k) + \mathcal{D}_{1, \beta}(k) \bar{w}(k)) + (I - \theta_{k, \beta}) (\mathcal{A}_{2, \beta}(k) \bar{x}(k) \right. \\ & \left. + \mathcal{D}_{2, \beta}(k) \bar{w}(k)) + \bar{f}(k, x(k), \psi(k)) \right] \end{aligned} \quad (3)$$

where

$$\begin{aligned} \mathcal{A}_{1, \beta}(k) &= \begin{bmatrix} A(k) + B(k)K_\beta(k)C(k) & 0 \\ C(k) & 0 \end{bmatrix}, \mathcal{A}_{2, \beta}(k) = \begin{bmatrix} A(k) & B(k)K_\beta(k) \\ 0 & I \end{bmatrix}, \\ \mathcal{D}_{1, \beta}(k) &= \begin{bmatrix} D(k) & B(k)K_\beta(k)E(k) \\ 0 & E(k) \end{bmatrix}, \mathcal{D}_{2, \beta}(k) = \begin{bmatrix} D(k) & 0 \\ 0 & 0 \end{bmatrix}, \\ \theta_{k, \beta} &= \text{diag}\{\eta_{k, \beta}^1, \eta_{k, \beta}^2, \dots, \eta_{k, \beta}^m, \eta_{k, \beta}^1, \eta_{k, \beta}^2, \dots, \eta_{k, \beta}^m\}, C(k) = [C_1(k), C_2(k), \dots, C_m(k)]^T, \\ E(k) &= [E_1(k), E_2(k), \dots, E_m(k)]^T, \bar{f}(k, x(k), \psi(k)) = [f(k, x(k), \psi(k)), 0]^T, \end{aligned}$$

$\sigma_\beta(k) = 1$ means that $\sigma_k = \beta$ and $\sum_{\beta=1}^M \sigma_\beta(k) = 1$.

Given the stochastic variables $\alpha_1(k, \beta) = \eta_{k, \beta}^{r(k)}$, $\alpha_2(k, \beta) = 1 - \eta_{k, \beta}^{r(k)}$ and $\sigma_\beta(k) = \delta(\beta - \sigma_k)$ where $\delta(\cdot) \in \{0, 1\}$ is the Kronecker delta function, we can rewrite the closed-loop nonlinear system as

$$x(k+1) = \sum_{\beta=1}^M \sigma_\beta(k) \sum_{\mu=1}^2 \alpha_\mu(k, \beta) \Theta_{k, \beta, \mu} \left[\mathcal{A}_{\mu, \beta}(k) \bar{x}(k) + \mathcal{D}_{\mu, \beta}(k) \bar{w}(k) + \bar{f}(k, x(k), \psi(k)) \right] \quad (4)$$

where

$$\Theta_{k, \beta, \mu} = \begin{cases} \theta_{k, \beta}, & \mu = 1 \\ I - \theta_{k, \beta}, & \mu = 2 \end{cases}.$$

Definition 1. [25] The discrete system with $w^T(k)\bar{Q}(k)w(k) < \delta_1^2$, $v^T(k)\tilde{Q}_i(k)v(k) < \delta_2^2$ is said to be stochastic FTB with $(c_0, c_1, \tau, R, \delta_1, \delta_2)$, if

$$x^T(0)Rx(0) \leq c_0 \Rightarrow E\{x^T(k)Rx(k)\} < c_1, k = 1, 2, \dots, \tau$$

where $c_1 > c_0 > 0$ are real scalars and R is a weight coefficient matrix.

The purpose is to design the controller such that system (4) stochastic FTB under RR protocol and EDEH.

3. Main Results

Firstly, we denote $\rho_{k,\beta}^i = [\Pr\{z_k^i = 0 | \sigma_k = \beta\}, \Pr\{z_k^i = 1 | \sigma_k = \beta\}, \dots, \Pr\{z_k^i = S | \sigma_k = \beta\}]^T$ and

$$\tilde{Q}_\beta^i = \begin{bmatrix} p_{0,\beta}^i & 0 & 0 & \cdots & 0 & 0 \\ p_{1,\beta}^i & p_{0,\beta}^i & 0 & \cdots & 0 & 0 \\ p_{2,\beta}^i & p_{1,\beta}^i & p_{0,\beta}^i & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ p_{S-1,\beta}^i & p_{S-2,\beta}^i & p_{S-3,\beta}^i & \cdots & p_{0,\beta}^i & 0 \\ -\sum_{\mu=0}^{S-1} p_{\mu,\beta}^i & -\sum_{\mu=0}^{S-2} p_{\mu,\beta}^i & -\sum_{\mu=0}^{S-3} p_{\mu,\beta}^i & \cdots & -p_{0,\beta}^i & 0 \end{bmatrix},$$

$$\bar{Q}_\beta^i = \begin{bmatrix} p_{0,\beta}^i & p_{0,\beta}^i & 0 & 0 & \cdots & 0 \\ p_{1,\beta}^i & p_{1,\beta}^i & p_{0,\beta}^i & 0 & \cdots & 0 \\ p_{2,\beta}^i & p_{2,\beta}^i & p_{1,\beta}^i & p_{0,\beta}^i & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ p_{S-1,\beta}^i & p_{S-1,\beta}^i & p_{S-2,\beta}^i & p_{S-3,\beta}^i & \cdots & p_{0,\beta}^i \\ -\sum_{\mu=0}^{S-1} p_{\mu,\beta}^i & -\sum_{\mu=0}^{S-1} p_{\mu,\beta}^i & -\sum_{\mu=0}^{S-2} p_{\mu,\beta}^i & -\sum_{\mu=0}^{S-3} p_{\mu,\beta}^i & \cdots & -p_{0,\beta}^i \end{bmatrix}.$$

Lemma 1. For $\{z_k^i\}_{k \geq 0}$, the recursion of the probability $\rho_{k,\beta}^i$ under the β -th environment is given by

$$\rho_{k+1,\beta}^i = [0, 0, \dots, 0, 1]_{S+1}^T + [\eta_{k,\beta}^i \bar{Q}_\beta^i + (1 - \eta_{k,\beta}^i) \tilde{Q}_\beta^i] \rho_{k,\beta}^i, \quad (5)$$

where $\rho_{0,\beta}^i = [\underbrace{0, \dots, 0}_{z_0^i}, 1, \underbrace{0, \dots, 0}_{S-z_0^i}]^T$ and $\beta = 1, 2, \dots, M$.

PROOF. According to (2), the probability $z_k^i = t$ ($0 \leq t < S$) under the β -th environment can be denoted by

$$\begin{aligned} & \text{Prob}\{z_{k+1}^i = t | \sigma_{k+1} = \beta\} \\ &= \text{Prob}\{\min\{z_k^i + h_{k+1}^i - \eta_k^i, S\} = t | \sigma_{k+1} = \beta\} \\ &= \eta_k^i \text{Prob}\{z_k^i + h_{k+1}^i - 1 = t | \sigma_{k+1} = \beta\} + (1 - \eta_k^i) \text{Prob}\{z_k^i + h_{k+1}^i = t | \sigma_{k+1} = \beta\} \\ &= \text{Prob}\{z_k^i = 0, h_{k+1}^i = t | \sigma_{k+1} = \beta\} + \eta_k^i \sum_{\mu=1}^{t+1} \text{Prob}\{z_k^i = \mu, h_{k+1}^i = t + 1 - \mu | \sigma_{k+1} = \beta\} \end{aligned}$$

$$\begin{aligned}
& + (1 - \eta_k^i) \sum_{\mu=1}^t \text{Prob}\{z_k^i = \mu, h_{k+1}^i = t - \mu | \sigma_{k+1} = \beta\} \\
& = \text{Prob}\{z_k^i = 0\} \text{Prob}\{h_{k+1}^i = t | \sigma_{k+1} = \beta\} + \eta_k^i \sum_{\mu=1}^{t+1} \text{Prob}\{z_k^i = \mu\} \text{Prob}\{h_{k+1}^i = t + 1 - \mu | \sigma_{k+1} = \beta\} \\
& + (1 - \eta_k^i) \sum_{\mu=1}^t \text{Prob}\{z_k^i = \mu\} \text{Prob}\{h_{k+1}^i = t - \mu | \sigma_{k+1} = \beta\} \\
& = p_{t,\beta}^i \text{Prob}\{z_k^i = 0\} + \eta_k^i \sum_{\mu=1}^{t+1} p_{t+1-\mu,\beta}^i \text{Prob}\{z_k^i = \mu\} + (1 - \eta_k^i) \sum_{\mu=1}^t p_{t-\mu,\beta}^i \text{Prob}\{z_k^i = \mu\}. \tag{6}
\end{aligned}$$

Afterwards, the probability of $z_k^i = S$ can be expressed as

$$\text{Prob}\{z_{k+1}^i = S | \sigma_k = \beta\} = 1 - \sum_{\mu=0}^{S-1} \text{Prob}\{z_{k+1}^i = \mu | \sigma_k = \beta\}. \tag{7}$$

Combining (6) and (9), we can obtain the recursion (8). The proof is complete.

According to $\rho_{k,\beta}^i$ under the β -th environment, the transmission probability for i -th sensor can be obtained as $\varrho_\beta^i(k) = [0, 1, \dots, 1]_{S+1} \rho_{k,\beta}^i$. Then, the expectation of $\alpha_1(k, \beta) = \eta_{k,\beta}^{r(k)} = 1$ under the β -th environment is $\bar{\alpha}_{1,k,\beta} = \varrho_\beta^{r(k)}(k)$ and the expectation of $\alpha_2(k, \beta) = 1 - \eta_{k,\beta}^{r(k)} = 1$ is $\bar{\alpha}_{2,k,\beta} = 1 - \varrho_\beta^{r(k)}(k)$.

Now, we will study the corresponding control problems.

Theorem 1. Consider the closed-loop nonlinear system (4). Let the controller parameters $K_\beta(k)$, the parameters $c_1 > 0$, $c_0 > 0$, $\delta_1 > 0$, $\delta_2 > 0$ and matrices $P_0 > 0$ and $R > 0$ be given. The expectation $\bar{\alpha}_{\mu,k,\beta}(\mu = 1, 2; \beta = 1, 2, \dots, M)$ can be obtained by Lemma 1. System (4) is stochastic FTB with $(c_0, c_1, N, R, \delta_1, \delta_2)$ if there exist matrices $P_k > 0$, $S > 0$, nonsingular matrices $\Lambda(k)$ and scalars $\psi > 1$, $\eta \geq 1$, $\lambda_1 > 0$ such that

- 1). for $k \in \mathbb{N}^{[1,N]}$

$$\begin{bmatrix} \Omega_{t,k}^{11} & * \\ \Omega_{t,k}^{21} & \Omega_{t,k}^{22} \end{bmatrix} < 0, \tag{8}$$

$$\begin{bmatrix} -P_{k+1} - I & \lambda_1 P_{k+1}^{-1} R \\ * & -\lambda_1 R \end{bmatrix} < 0; \tag{9}$$

- 2). if $\sigma_k = \sigma_{k+1}$, then $P_{k+1} = P_k$ and $\eta_k = 1$ hold, otherwise $P_{k+1} < \eta_k P_k$ and $\eta_k = \eta$ hold;
- 3). the inequality $\psi^N (\lambda_2 c_0 + \delta^2 \lambda_{\max}(S)) < \lambda_1 c_1$;
- 4). the ADT satisfies

$$N_a > N_a^* = \frac{N \ln \eta}{\ln \kappa_1 - \ln \kappa_2} \tag{10}$$

where

$$\Omega_{t,k}^{22} = \text{diag}\{P_{k+1} - \Lambda(k) - \Lambda^T(k), P_{k+1} - \Lambda(k) - \Lambda^T(k)\},$$

$$\Omega_{t,k}^{11} = \text{diag}\{\eta_k \sum_{l=1}^s \Gamma_{l,k} \text{tr}(\pi_{l,k}^T P_k \pi_{l,k}) - \psi P_k, -\psi S\}, \delta^2 = N(\delta_1^2 + \delta_2^2) / \min\{\lambda_{\min}(\bar{Q}), \lambda_{\min}(\tilde{Q})\},$$

$$\Omega_{t,k}^{21} = \left[\sqrt{\bar{\alpha}_{1,k,\beta}} [\Theta_{k,\beta,\mu} \Lambda(k) \mathcal{A}_{1,\beta}(k), \Theta_{k,\beta,\mu} \Lambda(k) \mathcal{D}_{1,\beta}(k)]^T, \right. \\ \left. \sqrt{\bar{\alpha}_{2,k,\beta}} [\Theta_{k,\beta,\mu} \Lambda(k) \mathcal{A}_{2,\beta}(k), \Theta_{k,\beta,\mu} \Lambda(k) \mathcal{D}_{2,\beta}(k)]^T \right]^T,$$

$$\lambda_2 = \lambda_{\max}(R^{-\frac{1}{2}} P_0 R^{-\frac{1}{2}}), \kappa_1 = (\eta - 1) c_1 \lambda_1 \psi^N + \delta^2 \lambda_{\max}(S) \text{ and } \kappa_2 = (\eta - 1) c_0 \lambda_2 + \delta^2 \lambda_{\max}(S).$$

PROOF. For any time step k ($0 < k \leq N$) and the switching signal σ_k , let $0 = k_0 < k_1 < \dots < k_j < \dots < k$ ($j = 0, 1, 2, \dots$) denote the switching instants.

For $V(x(k), k) = x^T(k) P_k x(k)$ and the system (4) at $k \in \mathbb{N}^{[k_j, k_{j+1})}$, we have

$$\begin{aligned} & \mathbb{E}\{x^T(k+1) P_{k+1} x(k+1)\} \\ &= \sum_{\beta=1}^M \sigma_{\beta}(k) \mathbb{E}\left\{ \sigma_{\beta}(k) \sum_{\mu=1}^2 \alpha_{\mu}(k, \beta) \Theta_{k,\beta,\mu} \{ \mathcal{A}_{\mu,\beta}(k) \bar{x}(k) + \mathcal{D}_{\mu,\beta}(k) \bar{w}(k) + \bar{f}(k, x(k), \psi(k)) \}^T \right. \\ & \quad \times P_{k+1} \{ \mathcal{A}_{\mu,\beta}(k) \bar{x}(k) + \mathcal{D}_{\mu,\beta}(k) \bar{w}(k) + \bar{f}(k, x(k), \psi(k)) \} \} \\ &= \sum_{\beta=1}^M \sigma_{\beta}(k) [\bar{x}^T(k), \bar{w}^T(k)] \Pi_{t,k} [\bar{x}^T(k), \bar{w}^T(k)]^T \end{aligned}$$

where

$$\begin{aligned} \Pi_{t,k} &= \sum_{\mu=1}^2 \bar{\alpha}_{\mu,k,\beta} \begin{bmatrix} \Theta_{k,\beta,\mu} \Pi_{t,k}^1 & \\ \Theta_{k,\beta,\mu} \mathcal{A}_{\mu,\beta}^T(k) P_{k+1} \mathcal{D}_{\mu,\beta}(k) & \Theta_{k,\beta,\mu} \mathcal{D}_{\mu,\beta}^T(k) P_{k+1} \mathcal{D}_{\mu,\beta}(k) \end{bmatrix}^* \\ \Pi_{t,k}^1 &= \mathcal{A}_{\mu,\beta}^T(k) P_{k+1} \mathcal{A}_{\mu,\beta}(k) + \sum_{l=1}^s \Gamma_{l,k} \text{tr}(\pi_{l,k}^T P_k \pi_{l,k}). \end{aligned}$$

Hence, one has

$$\begin{aligned} & \mathbb{E}\{V(x(k+1), k+1)\} - \psi \mathbb{E}\{V(x(k), k)\} - \psi \bar{w}^T(k) S \bar{w}(k) \\ &= \sum_{\beta=1}^M \sigma_{\beta}(k) [x^T(k), \bar{w}^T(k)] \Omega_{t,k} [x^T(k), \bar{w}^T(k)]^T \end{aligned}$$

where

$$\Omega_{t,k} = \sum_{\mu=1}^2 \bar{\alpha}_{\mu,k,\beta} \begin{bmatrix} \Pi_{t,k}^1 - \psi P_k & \\ \Theta_{k,\beta,\mu} \mathcal{A}_{\mu,\beta}^T(k) P_{k+1} \mathcal{D}_{\mu,\beta}(k) & \Theta_{k,\beta,\mu} \mathcal{D}_{\mu,\beta}^T(k) P_{k+1} \mathcal{D}_{\mu,\beta}(k) - \psi S \end{bmatrix}^*.$$

Performing the congruence transformation $\text{diag}\{I, I, \Lambda^{-T}(k), \Lambda^{-T}(k)\}$ to (8), then utilizing the inequality

$$-P_{k+1}^{-1} \leq \Lambda^{-1}(k) P_{k+1} \Lambda^{-T}(k) - \Lambda^{-1}(k) - \Lambda^{-T}(k),$$

the Schur complement lemma and condition (2), we get $\Omega_{t,k} < 0$ and

$$\mathbb{E}\{V(x(k+1), k+1)\} < \psi \mathbb{E}\{V(x(k), k)\} + \psi \bar{w}^T(k) S \bar{w}(k)$$

for $k \in \mathbb{N}^{[k_j, k_{j+1})}$.

Thus, for $\forall k \in \mathbb{N}^{[k_j, k_{j+1})}$, we can obtain that

$$\mathbb{E}\{V(x(k), k)\} < \psi^{k-k_j} \mathbb{E}\{V(x(k_j), k_j)\} + \sum_{s=k_j}^{k-1} \psi^{k-s} \bar{w}^T(s) S \bar{w}(s).$$

Combining $P_{k+1} < \eta P_k (\Pi_{k+1}^* \neq \Pi_k^*)$ and denoting $\tilde{P}_k = R^{-\frac{1}{2}} P_k R^{-\frac{1}{2}}$, we have

$$\mathbb{E}\{V(x(k), k_{j+1})\} < \eta \mathbb{E}\{V(x(k), k_j)\}$$

and

$$\begin{aligned} \mathbb{E}\{V(x(k), k)\} &< \psi^{k-k_0} (\eta^{\frac{k-k_0}{N_a}} c_0 (\lambda_{\max}(P_0)) x^T(k_0) x(k_0) + \frac{\eta^{\frac{k}{N_a}} - 1}{\eta - 1} \delta^2 \lambda_{\max}(S)) \\ &= \psi^{k-k_0} (\eta^{\frac{k-k_0}{N_a}} (\lambda_{\max}(\tilde{P}_0)) x^T(k_0) R x(k_0) + \frac{\eta^{\frac{k}{N_a}} - 1}{\eta - 1} \delta^2 \lambda_{\max}(S)). \end{aligned}$$

Moreover, because of

$$V(x(k), k) = x^T(k) R^{\frac{1}{2}} \tilde{P}_k R^{\frac{1}{2}} x(k) \geq \inf_{k \in [1, N]} (\lambda_{\min}(\tilde{P}_k)) x^T(k) R x(k),$$

we have

$$\mathbb{E}\{x^T(k) R x(k)\} \leq \frac{\psi^{k-k_0}}{\inf_{k \in [1, N]} (\lambda_{\min}(\tilde{P}_k))} \left(\eta^{\frac{k-k_0}{N_a}} c_0 (\lambda_{\max}(\tilde{P}_0)) + \frac{\eta^{\frac{k}{N_a}} - 1}{\eta - 1} \delta^2 \lambda_{\max}(S) \right).$$

According to (9), we have $\tilde{P}_k > \lambda_1 I$. Then, according to $\psi^N (\lambda_2 c_0 + \delta^2 \lambda_{\max}(S)) < \lambda_1 c_1$ and $N_a > N_a^*$, we obtain

$$\mathbb{E}\{x^T(k) R x(k)\} \leq \frac{\psi^{k-k_0}}{\lambda_1} (\eta^{\frac{k-k_0}{N_a}} c_0 \lambda_2 + \frac{\eta^{\frac{k}{N_a}} - 1}{\eta - 1} \delta^2 \lambda_{\max}(S)) = c_1.$$

The proof is complete.

Theorem 2. For known parameters $c_1 > 0$, $c_0 > 0$, $\delta_1 > 0$, $\delta_2 > 0$, matrices $P_0 > 0$, $R > 0$ and the expectation $\bar{\alpha}_{\mu, k, \beta} (\mu = 1, 2; \beta = 1, 2, \dots, M)$ obtained by Lemma 1, system (4) is stochastic FTB with $(c_0, c_1, \tau, R, \delta_1, \delta_2)$ if exist $P_k > 0$, $S > 0$, $\Lambda(k)$, $\bar{K}_i(k)$ ($i = 1, 2, \dots, N$), $\psi > 1$, $\eta \geq 1$, $\mu_k \geq 1$ and $\lambda_1 > 0$ such that

$$\begin{bmatrix} \Omega_{t,k}^{11} & * \\ \bar{\Omega}_{t,k}^{21} & \Omega_k^{22} \end{bmatrix} < 0 \quad (11)$$

and b), c), d), e) in Theorem 1 hold where

$$\begin{aligned} \bar{\Omega}_{t,k}^{21} &= \left[\sqrt{\bar{\alpha}_{1,k,\beta}} \Theta_{k,\beta,1} \hat{\Omega}_{t,k}^{1T}, \sqrt{\bar{\alpha}_{2,k,\beta}} \Theta_{k,\beta,2} \hat{\Omega}_{t,k}^{2T} \right]^T, \quad \hat{\Omega}_{t,k}^j = [\bar{\Omega}_1^j, \bar{\Omega}_2^j], j = 1, 2, \\ \bar{\Omega}_1^1 &= \begin{bmatrix} \Lambda_{11}(k) W(k) A(k) + \bar{K}_\beta(k) C(k) + \Lambda_{12}(k) C(k) & 0 \\ \Lambda_{22}(k) C(k) & 0 \end{bmatrix}, \end{aligned}$$

$$\begin{aligned}
\bar{\Omega}_2^1 &= \begin{bmatrix} \Lambda_{11}(k)W(k)D(k) & \bar{K}_\beta(k)E(k) + \Lambda_{12}(k)E(k) \\ 0 & \Lambda_{22}(k)E(k) \end{bmatrix}, \\
\bar{\Omega}_1^2 &= \begin{bmatrix} \Lambda_{11}(k)W(k)A(k) & \bar{K}_\beta(k) + \Lambda_{12} \\ 0 & \Lambda_{22} \end{bmatrix}, \\
\bar{\Omega}_2^2 &= \begin{bmatrix} \Lambda_{11}(k)W(k)D(k) & 0 \\ 0 & 0 \end{bmatrix}, \Lambda_{11}(k) = \begin{bmatrix} \tilde{\Lambda}_{11}(k) & \tilde{\Lambda}_{12}(k) \\ 0 & \tilde{\Lambda}_{22}(k) \end{bmatrix}, \\
\bar{K}_\beta(k) &= [\hat{K}_\beta^T(k), 0_{n_u \times (n_y - n_x)}]^T = \Lambda_{11}(k)W(k)B(k)K_\beta(k) \\
\Lambda(k) &= \begin{bmatrix} \Lambda_{11}(k)W(k) & \Lambda_{12}(k) \\ 0 & \Lambda_{22}(k) \end{bmatrix}, W(k) = [B(k)(B^T(k)B(k))^{-1} (B^T(k))^\perp]^T
\end{aligned}$$

$\tilde{\Lambda}_{11}(k) \in \mathbb{R}^{n_u \times n_u}$, $\tilde{\Lambda}_{12}(k) \in \mathbb{R}^{n_u \times (n_x - n_u)}$ and $\tilde{\Lambda}_{22}(k) \in \mathbb{R}^{(n_x - n_u) \times (n_x - n_u)}$. Then, the controller gain is $K_\beta(k) = \tilde{\Lambda}_{11}^{-1}(k)\hat{K}_\beta(k)$.

PROOF. Substituting $\mathcal{A}_{1,\beta}(k)$, $\mathcal{A}_{2,\beta}(k)$, $\mathcal{D}_{1,\beta}(k)$, $\mathcal{D}_{2,\beta}(k)$ and $\bar{K}_\beta(k) = \Lambda_{11}(k)W(k)B(k)K_\beta(k)$ to **Condition a** in Theorem 1, we have **Condition a** is equivalent to (11).

Besides, it can be found from the structure of the matrix

$$\begin{aligned}
\bar{K}_\beta(k) &= \Lambda_{11}(k)W(k)B(k)K_\beta(k) = \Lambda_{11}(k)[I, 0]^T K_\beta(k) \\
&= \begin{bmatrix} \tilde{\Lambda}_{11}(k) & \tilde{\Lambda}_{12}(k) \\ 0 & \tilde{\Lambda}_{22}(k) \end{bmatrix} \begin{bmatrix} K_\beta(k) \\ 0 \end{bmatrix} \\
&= \left[(\tilde{\Lambda}_{11}(k)K_\beta(k))^T, 0 \right]^T
\end{aligned}$$

by plugging $\hat{K}_\beta(k) = \tilde{\Lambda}_{11}(k)K_\beta(k)$. Based on Theorem 1, the desired controller can be obtained as $K_\beta(k) = \Lambda_{11}^{-1}(k)\hat{K}_\beta(k)$ by solving (11), which completes the proof.

According to Lemma 1 and the conditions in Theorem 2, the energy harvesting-dependent control algorithm can be concluded as follows.

Algorithm

Require: Set the recursive time N and $k = 0$. Initialize the environment factor $\sigma_0 \in \{1, 2, \dots, M\}$ and the energy level for i -th sensor $z_0^i (i = 1, 2, \dots, n_y)$. Given the initial condition x_0 , the transmit sequence $\{r(k)\}_{k=1,2,\dots,N}$ under the Round-Robin protocol, the parameters $c_1 > 0$, $c_0 > 0$, $\delta_1 > 0$, $\delta_2 > 0$, matrices $P_0 > 0$ and $R > 0$. Calculate the average dwell time N_a according to the actual environment changes.

Step 1. Under the σ_k -th environment, obtain the expectations $\bar{\alpha}_{1,k,\sigma_k}$ and $\bar{\alpha}_{2,k,\sigma_k}$ according to Lemma 1. If the conditions in Theorem 2 is satisfied, calculate the control gain $K_{\sigma_k}(k)$ and the constraint N_a^* of average dwell time, set $k = k + 1$, and go to Step 2, else go to step 3.

Step 2. If $k < N$, then go to Step 1, else go to Step 3.

Step 3. Stop.

4. Numerical simulation

A numerical example is proposed in this section. For a nonlinear system with three sensors and the following parameters

$$A(k) = \begin{bmatrix} 0.8 + \cos(k) & 0.45 \\ 0.3 & 0.6 - \sin(k) \end{bmatrix}, B_1(k) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, C(k) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}^T, \\ E(k) = [0.1, 0.1, 0.5]^T, D(k) = [0.4, 0.2]^T, \pi = [0.2, 0.3]^T, \Gamma = \text{diag}\{0.01, 0.16\}$$

$c_0 = 0.1$ and $c_1 = 1.2$. The probabilities of energy harvesting at different environments are given in Table 1. The disturbance is $w_i(k) = v_i(k) = 0.2 \sin(2k)$, the initial state value is $x_1(0) = [0.2, -0.2]^T$, $x_2(0) = [0.2, -0.1]^T$, $x_3(0) = [0.1, -0.2]^T$ and $\tau = 50$. The nonlinear function is

$$f(k, x(k), \psi(k)) = (0.1 \text{sign}(x_1(k))x_1(k)\psi_1(k) + 0.4 \text{sign}(x_2(k))x_2(k)\psi_2(k)) \times [0.2, 0.3]^T (i = 1, 2, 3).$$

According to the given recursive algorithm, the controller gain can be obtained recursively by Matlab. Table 2 lists the desired parameters of controller $K_1(k)$ or $K_2(k)$ from the time $k = 1$ to $k = 12$. The bound is shown in Fig. 2. The switched environments are shown in Fig. 3. The energy level and measurement for i -th sensor are shown in Fig. 4-5. The simulation has confirmed that the designed controllers perform very well.

Table 1: the probabilities of energy harvesting

Sensor	1				2				3			
<i>Energy levels</i>	0	1	2	3	0	1	2	3	0	1	2	3
<i>Environment 1</i>	0.2	0.5	0.3	0	0.4	0.5	0.1	0	0.5	0.4	0.1	0
<i>Environment 2</i>	0.5	0.1	0.2	0.2	0.4	0.4	0.2	0	0.3	0.5	0.2	0

Figure 2: The open-loop system and the closed-loop system

Table 2: the controller gain

Time(mode)	1(1)	2(1)
Control gain	$\begin{bmatrix} -0.8353 & -0.1801 & -0.1854 \\ -0.2151 & 0.1043 & 0.1034 \end{bmatrix}$	$\begin{bmatrix} -0.3569 & -0.1414 & -0.1286 \\ -0.2424 & 0.0750 & 0.0743 \end{bmatrix}$
Time(mode)	3(1)	4(1)
Control gain	$\begin{bmatrix} 0.1714 & -0.2110 & -0.1918 \\ -0.2819 & -0.2362 & -0.1696 \end{bmatrix}$	$\begin{bmatrix} -0.0783 & -0.1233 & -0.0980 \\ -0.1666 & -0.3636 & -0.2949 \end{bmatrix}$
Time(mode)	5(1)	6(1)
Control gain	$\begin{bmatrix} -0.3024 & -0.2885 & -0.2377 \\ -0.0352 & -0.6550 & -0.5914 \end{bmatrix}$	$\begin{bmatrix} -1.7098 & -0.3404 & -0.1379 \\ -0.2921 & -0.4847 & -0.3967 \end{bmatrix}$
Time(mode)	7(1)	8(1)
Control gain	$\begin{bmatrix} -0.5797 & -0.1267 & -0.0913 \\ -0.2337 & 0.1244 & 0.1333 \end{bmatrix}$	$\begin{bmatrix} 0.1110 & -0.2344 & -0.2146 \\ -0.2997 & -0.1356 & -0.0517 \end{bmatrix}$
Time(mode)	9(2)	10(2)
Control gain	$\begin{bmatrix} -1.0773 & -0.3393 & -0.2351 \\ -0.2097 & -0.0003 & 0.0150 \end{bmatrix}$	$\begin{bmatrix} 0.0329 & -0.1716 & -0.1563 \\ -0.2720 & -0.4253 & -0.3529 \end{bmatrix}$
Time(mode)	11(2)	12(2)
Control gain	$\begin{bmatrix} -0.4618 & -0.1626 & -0.1284 \\ -0.1495 & -0.5166 & -0.4833 \end{bmatrix}$	$\begin{bmatrix} -1.6429 & -0.2381 & -0.2124 \\ -0.2999 & -0.5673 & -0.5692 \end{bmatrix}$

Figure 3: The modes of the environment.

(a) The first sensor.

(b) The second sensor.

Figure 4: The energy.

(a) The first sensor.

(b) The second sensor.

Figure 5: The measurement.

5. Conclusions

In this paper, the protocol-based control problem has been investigated for CPSs with EDEH devices. Firstly, an iterative algorithm of varying probabilities under EDEH and the RR protocol has been employed to describe the dynamics of the energy level for each sensor. Then, by utilizing ADT method and the varying probability of energy harvesting, the environment-dependent control problem case has been equivalently reduced to a general switched system with time-varying probability. A sufficient condition has been derived to guarantee the SFTB of the closed-loop system.

Subsequently, the desired controller has been designed in terms of the proposed algorithm that can be adopted effectively by using available software. At last, an example has been given to proof the availability of the proposed design method.

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Compliance with ethical standards

Conflict of interest

The authors declare that they have no conflict of interest.

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