Forward Dynamic Solution and Optimization of a 1-DOF Parallel Mechanism for Vibrating Screen

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Abstract: In this paper, the ordered single-open-chain (SOC) method in combination with the principle of virtual work is adopted to model and solve the forward dynamics of a single degree-of-freedom (DOF) parallel mechanism (PM), featuring one translation and two rotations (1T2R), which is applied in spatial vibrating screen (i.e., parallel vibrating screen mechanism, PVSM). Afterwards, the dynamic performances of the PVSM is optimized using differential evolution algorithm. Based on the kinematics of the PM, the forward dynamic model is derived and the dynamic response equation is built, of which the coefficient matrix is determined by means of the generalized velocity equation. Moreover, the Euler method was adopted to solve the numerical solution of the differential equation of motion to characterize the motion law and dynamics of the screen surface of the PVSM, which is verified with ADAMS simulation. With the parametric model, the dynamic optimization of PVSM is carried out to maximize the energy transfer efficiency, subject to the constraints on the link mass. The comparison of the dynamic performances of the PVSM with and without optimization reveals the improvement of the PM.
Keywords: Ordered single-open-chain • Principle of virtual work • Dynamic response modeling • Parallel vibrating screen mechanism

1 Introduction

Modeling and solving the forward dynamics of the mechanism means to reveal the motion law of the mechanism with the prescribed external wrench (forces and/or moments) and resistance force, considering the mass distributions of the links in motion. Forward dynamics can help to study the movement stability of the mechanism for enhanced control [1], for which the crucial problem lies in the establishment of the dynamic response equation of the mechanism. Compared to the inverse dynamics, the forward dynamics solution and analysis of the mechanism are not so extensively reported in the literature, which can benefits to the design and control of the prototype.

The main approaches to model the mechanism dynamics include Roberson-Wittenburg method **Error! Reference source not found.**, Kane method [3], Lagrange equation [4], Lagrange multiplier method **Error! Reference source not found.**, Hamilton method **Error! Reference source not found.**, and Newton-Euler method [7], etc. The previous methods can make the dynamic equations concise and highly stylized, suitable for computer programming to ease the solving procedure. The principle of virtual work is efficient in solving the inverse dynamics problem, while, it not easy to calculate the reaction forces existing in the joints when solving the forward dynamics. From the perspective of energy, the Lagrange equation can be applied in modeling the dynamics through the computation of the instantaneous kinetic and potential energies of all the moving links, but with the increasing computational burden when the mechanism has multiple links. Similarly, the Newton-Euler method results in high computation cost with the increasing number of links, since it requires the force analysis of each separate link of the mechanism to build the dynamic equation by eliminating the internal forces.

Dynamic modeling and analysis of parallel mechanisms (PMs) have been extensively studied in the literature. For

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instance, based upon the generalized coordinates, Chen**Error! Reference source not found.** et al. applied the Newton-Euler method to model the dynamic response equation of the 6-UPS [1] spatial parallel mechanism. Zhang[9] et al. used the Lagrange method to model the dynamics of an eccentric cam mechanism. Liu[10] et al. adopted Kane's method to analyze the forward dynamics of a 3-RRRT PM. The foregoing modeling methods does not consider the orderly topological decomposition of the PM, leading to the complex derivation of the differential equation of motion and decreasing efficiency of solving dynamic equation, due to the highly nonlinear sets of equations and coupling of the motion parameters. Alternatively, Yang[1] proposed an ordered single-open-chain method based on the principle of virtual work for dynamic analysis of PMs, which has been applied to analyze the dynamics of a 3-DOF 3-RRR PM and a 4-DOF 2SPS-2RPS PM [11]. This paper will handle the dynamic problem in accordance with this method. With the equations of motion, dynamic performances of the mechanisms can be optimized for improvement. Regarding the optimization methods, particle swarm optimization (PSO), genetic algorithm (GA), ant colony algorithm (ACO), immune algorithm (IA), differential evolution algorithm (DE), and other intelligent algorithms [12] can be adopted to optimize the kinematic and dynamic performances of PMs, with the significant advantages in terms of its practicality and reliability [13]-[16].

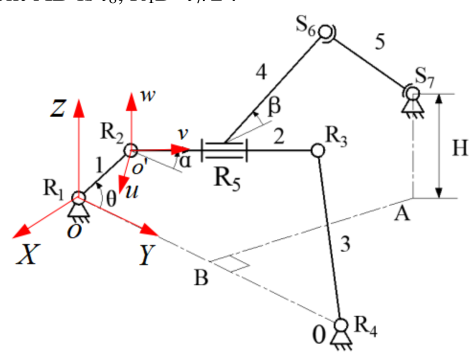
In this research, forward dynamic analysis of the one degree-of-freedom (DOF) parallel vibrating screen mechanism (PVSM) is carried out with the integrated principle of virtual work and ordered single-open-chain (SOC) method, wherein the coefficient matrices of the dynamic equation can be readily calculated based on the generalized coordinates. Numerical simulation is carried out to characterize the dynamics of the PVSM. Finally, with the differential evolution algorithm, taking the link mass as the constraints and energy transfer efficiency as the objective function, the dynamic performance of the PVSM is optimized for the structural improvement and further prototype design.

2 Parallel Vibrating Screen Mechanism

The 1-DOF PVSM with one translation and two rotations (1T2R) is shown in Fig.1[17][18]. It is composed of hybrid branches (R_1-R_2-R_3-R_4)-R_5 and spatial branches S_6-S_7 , both of which are respectively connected to the moving platform 4 and the base platform 0. Ref. [17] has proved that the PM DOF is equal to 1. The rotating joint R_1 is active, and the moving platform 4 can generate translation along the z-axis, rotation α around the x axis, and rotation β around the axis of rotation of joint R_5 (i.e., parallel to line R_2R_3), among which only one of the motion parameters (z, α, β) can be considered as an independent parameter with couple motions of the other two.

According to the SOC based mechanism composition principle, the PM can be decomposed into two sub-kinematic chains, i.e., SKC1: R_1-R_2-R_3-R_4, SKC2: R_5-S_6-S_7. Their coupling degrees are both equal to $\kappa=0$.

Let: $R_1R_2=l_1$, $R_2R_3=l_2$, $R_3R_4=l_3$, $R_5S_6=l_4$, $S_6S_7=l_5$, $AS=H$, $R_1R_4=l_7$, $AB=l_8$. The distance of the line from S_6 to line R_2R_3 is l_4, and the distance from the spherical joint S_7 to the plane of the base platform is H, and the length of segment AB is l_8, $R_1B=l_7/2$.



(a) The kinematic sketch of mechanism

(b) Experimental prototype

Figure 1 1-DOF parallel vibrating screen mechanism

3 Forward Dynamic Analysis

3.1 Determination of Velocity and Acceleration Coefficient Matrix

In accordance with Ref. [18], the angular/linear velocity and acceleration of the center of mass of each link can be expressed as a function of generalized velocity $\dot{q}$ and acceleration $\ddot{q}$, namely,

$$v_i = v(\dot{q}); \ \omega_i = \omega(\dot{q}); \ a_i = a(\ddot{q}) + a(\dot{q}); \ \varepsilon_i = \varepsilon(\ddot{q}) + \varepsilon(\dot{q}) \tag{1}$$

[1] Throughout this paper, P, R, U, S and T stand for the prismatic, revolute, universal, spherical and Hooke joints, respectively.

Thus, the velocities of the parallel vibrating screen is expressed as

$$\begin{cases}[\boldsymbol{v}] = \boldsymbol{B}\dot{q} \\ [\boldsymbol{\omega}] = \boldsymbol{C}\dot{q}\end{cases} \tag{2}$$

Where,

$$\boldsymbol{B} = \left[\frac{\partial r_1}{\partial q}, \frac{\partial r_2}{\partial q}, \frac{\partial r_3}{\partial q}, \frac{\partial r_4}{\partial q}, \frac{\partial r_5}{\partial q}\right]^T$$

$$\boldsymbol{C} = \left[\frac{\partial \varphi_1}{\partial q}, \frac{\partial \varphi_2}{\partial q}, \frac{\partial \varphi_3}{\partial q}, \frac{\partial \varphi_4}{\partial q}, \frac{\partial \varphi_5}{\partial q}\right]^T$$

r_i、φ_i ——the position vector and angular position vector of link i, $i = 1 \sim 5$.

And the accelerations are derived as

$$\begin{cases}[\boldsymbol{a}] = \boldsymbol{B}\ddot{q} + \dot{\boldsymbol{B}}\dot{q} = [\boldsymbol{a}(\ddot{q})] + [\boldsymbol{a}(\dot{q})] \\ [\varepsilon] = \boldsymbol{C}\ddot{q} + \dot{\boldsymbol{C}}\dot{q} = [\varepsilon(\ddot{q})] + [\varepsilon(\dot{q})]\end{cases} \tag{3}$$

Where,

$$\dot{\boldsymbol{B}} = \left[\frac{d}{dt}\frac{\partial \boldsymbol{r}_1}{\partial q}, \frac{d}{dt}\frac{\partial \boldsymbol{r}_2}{\partial q}, \frac{d}{dt}\frac{\partial \boldsymbol{r}_3}{\partial q}, \frac{d}{dt}\frac{\partial \boldsymbol{r}_4}{\partial q}, \frac{d}{dt}\frac{\partial \boldsymbol{r}_5}{\partial q}\right]^T$$

$$\dot{\boldsymbol{C}} = \left[\frac{d}{dt}\frac{\partial \boldsymbol{\varphi}_1}{\partial q}, \frac{d}{dt}\frac{\partial \boldsymbol{\varphi}_2}{\partial q}, \frac{d}{dt}\frac{\partial \boldsymbol{\varphi}_3}{\partial q}, \frac{d}{dt}\frac{\partial \boldsymbol{\varphi}_4}{\partial q}, \frac{d}{dt}\frac{\partial \boldsymbol{\varphi}_5}{\partial q}\right]^T$$

$$[\boldsymbol{a}(\ddot{q})] = \boldsymbol{B}\ddot{q} = [\boldsymbol{a}_1(\ddot{q}), \boldsymbol{a}_2(\ddot{q}) \ldots \boldsymbol{a}_5(\ddot{q})]^T$$

$$[\boldsymbol{a}(\dot{q})] = \dot{\boldsymbol{B}}\dot{q} = [\boldsymbol{a}_1(\dot{q}), \boldsymbol{a}_2(\dot{q}) \ldots \boldsymbol{a}_5(\dot{q})]^T$$

$$[\varepsilon(\ddot{q})] = \boldsymbol{C}\ddot{q} = [\varepsilon_1(\ddot{q}), \varepsilon_2(\ddot{q}) \ldots \varepsilon_5(\ddot{q})]^T$$

$$[\varepsilon(\dot{q})] = \dot{\boldsymbol{C}}\dot{q} = [\varepsilon_1(\dot{q}), \varepsilon_2(\dot{q}) \ldots \varepsilon_5(\dot{q})]^T$$

3.2 Establishment of the Forward Dynamic Equation

With the principle of virtual work [13]-[16], the sum of the external force/moment) and inertial force/moment of the system on any virtual displacement of the mechanical system should be equal to zero, which yields

$$\boldsymbol{\delta}_{r_i}^T \tilde{\boldsymbol{F}} + \boldsymbol{\delta}_{\varphi}^T \tilde{\boldsymbol{M}} + \boldsymbol{\delta}_q \boldsymbol{M}_d = 0 \tag{4}$$

Where,

$\tilde{\boldsymbol{F}} = \left[\tilde{\boldsymbol{F}}_1, \tilde{\boldsymbol{F}}_i, \ldots, \tilde{\boldsymbol{F}}_5\right]^T$,

$\tilde{\boldsymbol{M}} = \left[\tilde{\boldsymbol{M}}_1, \tilde{\boldsymbol{M}}_i, \ldots, \tilde{\boldsymbol{M}}_5\right]^T$,

$\boldsymbol{M}_d = \left[\boldsymbol{M}_1, \ldots, \boldsymbol{M}_f\right]^T$,

$\tilde{\boldsymbol{F}}_i = \boldsymbol{F}_i - \boldsymbol{m}_i \boldsymbol{a}_i$,

$\tilde{\boldsymbol{M}}_i = \boldsymbol{M}_i - \boldsymbol{J}_i \boldsymbol{\varepsilon}_i$.

$\tilde{\boldsymbol{F}}$、$\tilde{\boldsymbol{M}}$——The principal vector and principal moment of the external force on each link of the PM.

$\boldsymbol{F}_i$、$\boldsymbol{M}_i$——The principal vector and principal moment of the external force of the i-th link;

$\boldsymbol{J}_i$ ——The inertia tensor of link i to the center of mass.

$\boldsymbol{M}_d$ ——The driving torque matrix of the driving rod.

Substituting Eqs. (2) and (3) into Eq. (4), one obtains

$$\boldsymbol{B}^{\mathrm{T}} \tilde{\boldsymbol{F}} + \boldsymbol{C}^T \tilde{\boldsymbol{M}} + \boldsymbol{M}_d = 0 \tag{5}$$

Rearranging Eq. (5), the general equation of the dynamic response equation can be obtained as

$$\boldsymbol{H}_{f\times f}\ddot{q}_{f\times 1} + \boldsymbol{P}(\dot{q})_{f\times 1} + \boldsymbol{Q}_{f\times 1} = 0 \tag{6}$$

Where,

$$\boldsymbol{H} = \boldsymbol{B}^T \boldsymbol{diag}[\boldsymbol{m}]\boldsymbol{B} + \boldsymbol{C}^T \boldsymbol{diag}[\boldsymbol{J}]\boldsymbol{C} \tag{7}$$

$$\boldsymbol{P}(\dot{q}) = \boldsymbol{B}^T \boldsymbol{diag}[\boldsymbol{m}]\boldsymbol{a}(\dot{q}) + \boldsymbol{C}^T \boldsymbol{diag}[\boldsymbol{J}]\varepsilon(\dot{q}) \tag{8}$$

$$\boldsymbol{Q} = -(\boldsymbol{B}^T \tilde{\boldsymbol{F}} + \boldsymbol{C}^T \tilde{\boldsymbol{M}} + \boldsymbol{M}_d) \tag{9}$$

And the elements in the matrices H, $P(\dot{q})$, and Q are given by

$$h_{ij} = \sum_{i=1}^{5}\left\{m_i \frac{\partial r_{si}}{\partial q}\frac{\partial r_{si}}{\partial q} + (\boldsymbol{J}_i \times \frac{\partial \varphi_i}{\partial q})\frac{\partial \varphi_i}{\partial q}\right\} \tag{10}$$

$$p_i\dot{q} = \sum_{i=1}^{5}\left\{m_i \boldsymbol{a}_i \frac{\partial r_i}{\partial q} + (\boldsymbol{J}_i \times \boldsymbol{\varepsilon}_i(\dot{q}))\frac{\partial \varphi_i}{\partial q}\right\} \tag{11}$$

$$Q_i(\dot{q}_i) = -(\sum_{i=1}^{5}\left\{F_i \frac{\partial r_i}{\partial q} + M_i \frac{\partial \varphi_i}{\partial q}\right\} + M_{d_i}) \tag{12}$$

3.3 Determination of the Coefficients of the Dynamic Response Equation

The key issue for Eq. (6) is to determine the coefficients $\frac{\partial \boldsymbol{r}_i}{\partial q}$, $\frac{\partial \boldsymbol{\varphi}_i}{\partial q}$, $\boldsymbol{a}_i(\dot{q})$, and $\boldsymbol{\varepsilon}_i(\dot{q})$ in Eqs. (10)~(12).

1）Determination of $\frac{\partial \boldsymbol{r}_i}{\partial q}$ and $\frac{\partial \boldsymbol{\varphi}_i}{\partial q}$

Expanding Eq. (2), the velocities of mass center of each link are derived as

$$\begin{cases}\boldsymbol{v}_i = \dfrac{\partial \boldsymbol{r}_i}{\partial q}\dot{q} \\ \boldsymbol{\omega}_i = \dfrac{\partial \boldsymbol{\varphi}_i}{\partial q}\dot{q}\end{cases} \tag{13}$$

Let $\dot{q}$ =1, the velocities $\boldsymbol{v}_i(\dot{q}=1) = \frac{\partial \boldsymbol{r}_i}{\partial q}$ and $\boldsymbol{\omega}_i(\dot{q}=1) = \frac{\partial \boldsymbol{\varphi}_i}{\partial q}$ can be determined by means of velocity analysis.

2）Determination of $\boldsymbol{a}_i(\dot{q})$ and $\boldsymbol{\varepsilon}_i(\dot{q})$

Expanding the Eq. (3) leads to

$$\begin{cases} \boldsymbol{a}_i = \sum_{i=1}^{f} \frac{\partial r_i}{\partial r_q} \ddot{q}_t + \boldsymbol{a}_i(\dot{q}) \\ \boldsymbol{\varepsilon}_i = \sum_{i=1}^{f} \frac{\partial r_i}{\partial q} \ddot{q}_t + \boldsymbol{\varepsilon}_i(\dot{q}) \end{cases} \tag{14}$$

Let the generalized acceleration to be $\ddot{q} = 0$, with the input speed $\dot{q}$, the accelerations of the mass center of each link $\boldsymbol{a}_i(\dot{q})$ and $\boldsymbol{\varepsilon}_i(\dot{q})$ can be derived from acceleration analysis.

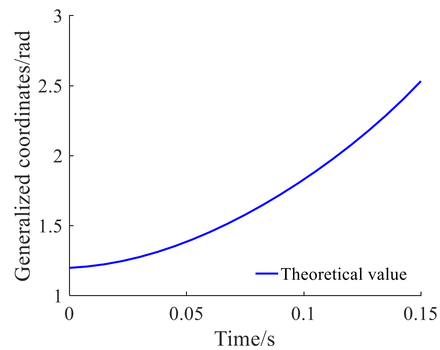
4 Numerical Solution and Verification of Forward Dynamic Equation

By setting the motion parameters to be special values, the elements in the coefficient matrix of the differential equation of motion (6) can be determined. In order to figure out the motion law of each link of the PM, numerical analysis can be adopted to solve the differential equation of motion. The Common methods to solve differential equations include Euler method and RUNGE-KUTTA method [24]. The Euler method features smaller computation burden, which can provide acceptable computation accuracy with smaller increments, which is adopted in this work. The basic principle is depicted as: at the initial condition $t=t_0$, the initial values of the generalized coordinates and generalized speed of the vibrating screen are $q = q_0$ and $\dot{q} = \dot{q}_{t0}$, and the generalized acceleration can be solved as $\ddot{q} = \ddot{q}_{t0}$ when $t=t_0$. With the step size Δt, applying Euler's method leads to

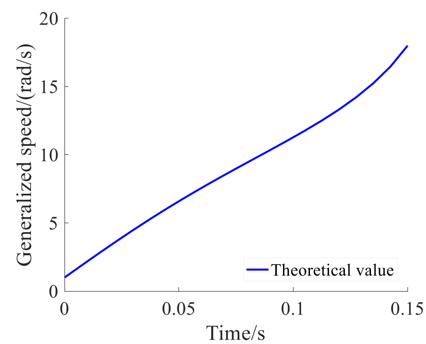
$$\begin{cases} \dot{q}_{t(i+1)} = \dot{q}_{ti} + \ddot{q}_{ti}\Delta t \\ q_{t(i+1)} = q_{ti} + \dot{q}_{ti}\Delta t + \frac{1}{2}\ddot{q}_{ti}\Delta t^2 \end{cases} \quad (t = 0,1,2,\ldots) \tag{15}$$

Sequentially, the generalized speed $\dot{q}_{t_1}$ and generalized coordinates q_{t_1} at the moment $t = t_1 = t_0 + \Delta t$ can be deduced, which can be treated as the initial conditions for the next moment. Repeating the previous procedures results in the motion law of each link of the PM.

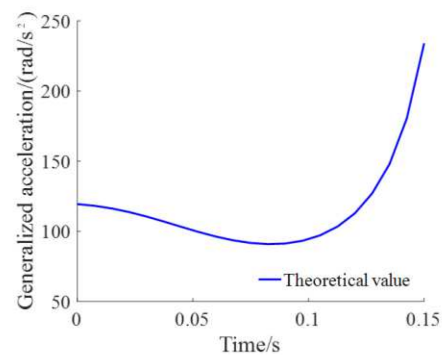
In order to calculate Eq. (6), the parameters of the PM are set in accordance with Ref. [18]. The constant torque acting on the drive joint of the PM in the counterclockwise is T=50(N·mm). The initial angular input position is q_{t0}=1.2 (rad), and the initial velocity is $\dot{q}_{t0} = 1$ (rad/s). Based on Eq.(6), with the step size 10^{-4}, the motion profiles of parameters q, $\dot{q}$ and $\ddot{q}$ during the period t=0~0.15s (60°~146°) are calculated, as shown in Fig. 2, whlile the simulation results by ADAMS are shown in Fig. 3.



(a)Generalized coordinates

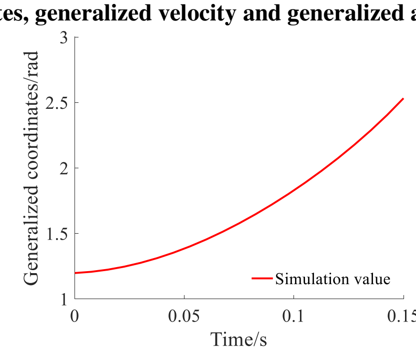


(b)Generalized speed

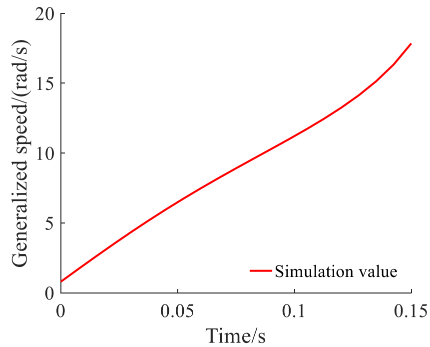


(c) Generalized acceleration

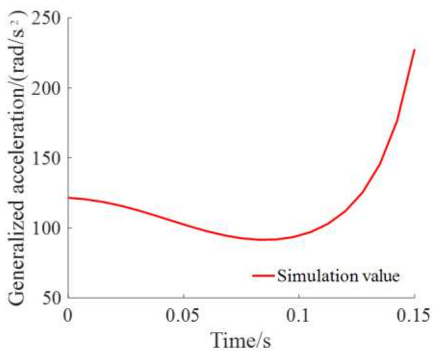
Figure 2 Theoretical calculation curves of generalized coordinates, generalized velocity and generalized acceleration



(a)Generalized coordinates



(b)Generalized speed



(c) Generalized acceleration

Figure 3 Simulation check curves of generalized coordinates, generalized velocity and generalized acceleration

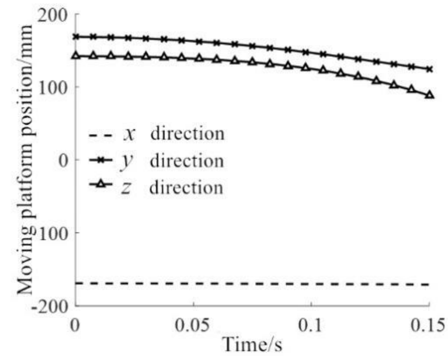
It can be seen from Figs.2~3,

(1) The theoretically calculated values of the input angle, angular velocity, and angular acceleration of the drive joint are consistent with the simulated values, which verifies the correctness of the dynamic model based on the principle of virtual work.

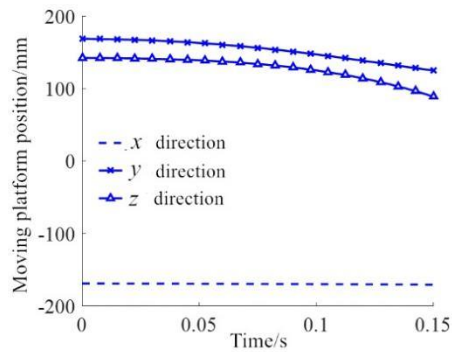
(2) The input angle and speed gradually increase with time, while the input acceleration changes suddenly and sharply after a decrease.

Because the word mainly investigates the motion law of the vibrating screen surface (moving platform is 4), the results obtained are substituted into the displacement, velocity, and acceleration equations of the center of mass of the moving platform 4, and calculated with MATLAB programming, and the results are shown in Fig.4(a)~Fig.8(a).

Then use the ADAMS software for simulation analysis, and get the corresponding curve of displacement, velocity and acceleration of the center of mass of the moving platform 4, as shown in Fig. 4 (b) ~ Fig. 8 (b).

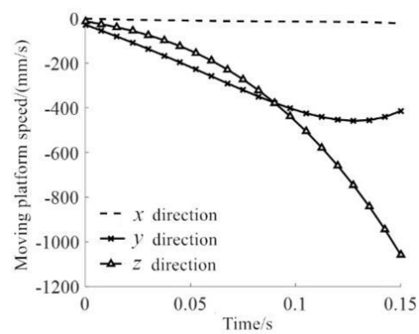


(a) Theoretical calculation value

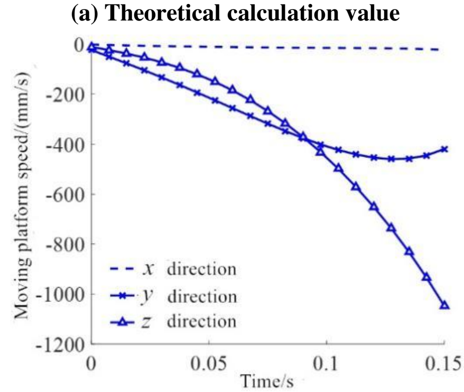


(b) Simulation check value

Figure 4 Displacement change of moving platform 4

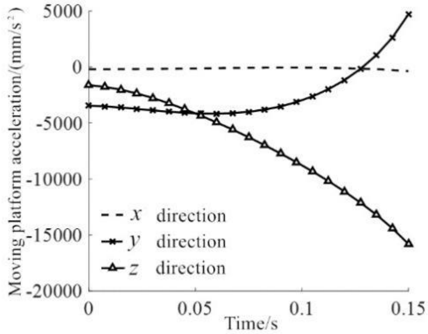


(a) Theoretical calculation value

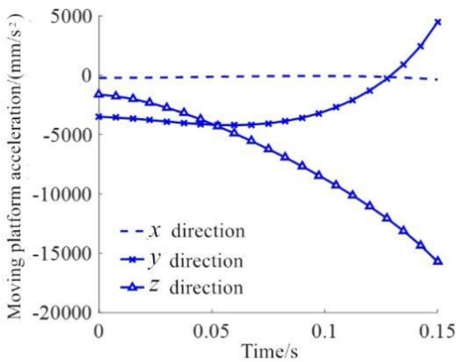


(b) Simulation check value

Figure 5 Velocity change of moving platform 4

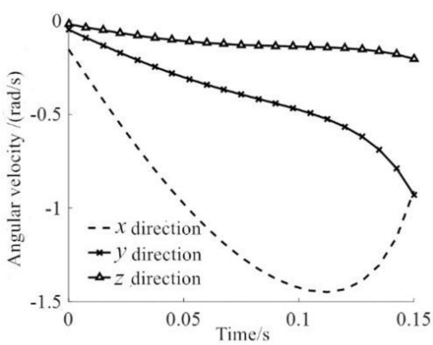


(a) Theoretical calculation value

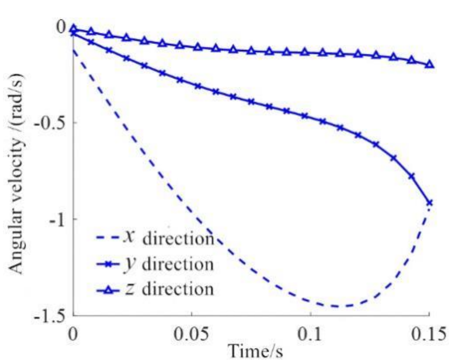


(b) Simulation check value

Figure 6 Acceleration change of moving platform 4

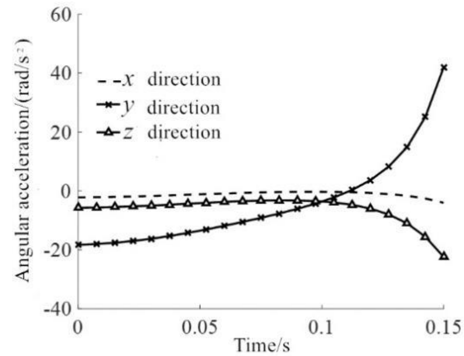


(a) Theoretical calculation value

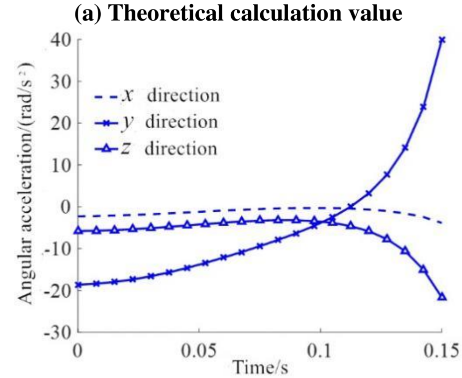


(b) Simulation check value

Figure 7 Angular velocity variation of moving platform 4



(a) Theoretical calculation value



(b) Simulation check value

Figure 8 Angular acceleration variation of moving platform

It can be seen from Fig. 4~Fig. 8:

(1) The position change of the moving platform 4 is relatively stable.

(2) Velocity/angular velocity, acceleration/angular acceleration fluctuate greatly in a short period of time, which provides protection for the screening efficiency of the vibrating screen. However the drastic fluctuation of velocity/acceleration in a short period of time can easily cause vibrations in each joint, which results in increased mechanical vibration of the PVSM, which requires optimization of the driving auxiliary input to improve the working stability of the PVSM.

5 Dynamic Performance Optimization with Differential Evolution Algorithm

5.1 Determine Optimization Parameters and Objective Function

(1) Design variables

The previous work Ref. [17] has presented the kinematic optimization of the 1-DOF parallel vibrating screen mechanism (PVSM), such as the dimensions, the trajectory of the screen surface, and the equilibrium of the four-bar linkage. On the other hand, optimization of dynamic performances such as mass parameters and energy transfer efficiency can be considered for further improvement.

In this section, the mass parameters of the links will be optimized based on previously determined link lengths.

Suppose m_1, m_2, m_3, m_4, and m_5 are the masses of the five links of the PVSM, therefore, the design variables are written as

$$\boldsymbol{m} = \begin{bmatrix} m_1 & m_2 & m_3 & m_4 & m_5 \end{bmatrix} \tag{16}$$

(2) Optimization constraints

According to the original masses of each link, the upper and lower bounds of the design variables are given in Table 1.

Table 1 Design variables and their ranges

m_1/g	m_2/g	m_3/g	m_4/g	m_5/g
7~25	44~176	33~124	21~122	22~66

Then the design variables in the optimization process should meet the overall constraint as follows.

$$m_1 + m_2 + m_3 + m_4 + m_5 \geq m_{\min} \tag{17}$$

(3) Objective function

From Ref. [25], the energy transfer efficiency ζ_p can be used as an index to solve the problem of dimensionally inconsistent inertia matrix, which is expressed by

$$\zeta_P = \frac{E_P}{T} \tag{18}$$

Where,

E_p——Kinetic energy of moving platform 4,

T——Overall kinetic energy of PVSM.

Thus, $T = \sum_{i=1}^{5} T_i$, $E_p = \frac{1}{2} m_4 \boldsymbol{v}_4^T \boldsymbol{v}_4 + \frac{1}{2} \boldsymbol{\omega}_4^T \, {}^0\boldsymbol{I}_4 \boldsymbol{\omega}_4$

For the whole PM, the kinetic energy of the moving platform can be considered as the effective energy of the PM, and the higher the proportion of kinetic energy occupied by the moving platform, the better the energy transmission effect of the PM. In order to evaluate the overall energy transfer performance of the organization, the global energy transfer efficiency index f_p is defined as

$$f_{\mathrm{P}} = \frac{\int_w \zeta_{\mathrm{P}} \mathrm{d}w}{\int_w \mathrm{d}w} \tag{19}$$

It is calculated that the energy transfer efficiency of the mechanism is f_{p_0} under the original mass parameters. From this, the objective function f_d for dynamic performance optimization is calculated as

$$f_d = \frac{f_P - f_{P_0}}{f_{P_0}} \to \max \tag{20}$$

5.2 Optimization Analysis and Comparison

After defining the optimization problem, the differential evolution algorithm compiled in MATLAB can be used to optimize the dynamic performance of the PM, where the algorithm parameters are selected according to the Refs. [26] and [27].

(1) Algorithm parameter initialization

To reduce the computational cost, the population size and iteration times of the algorithm are reduced The parameter settings shown in Table 2 are obtained from the preliminary estimation.

Table 2 Initialization of differential evolution algorithm

	Population size	The maximum number of iterations	Algorithm parameters	
			Scaling factor	Crossover factor
DE	30	200	0~1	0.8~1

(2) Comparison and analysis of optimization results

Figure 9 shows the iterative optimization, from which it can be seen that the objective function begin to converge after 50 iterations, and the final solution is obtained, in comparison with the evaluation index of the PM without optimization, as listed in Table 3.

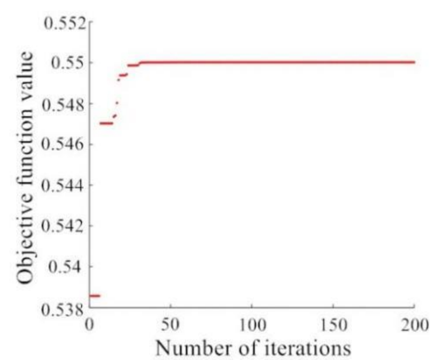


Figure 9 Calculation curve of differential evolution algorithm

Table 3. Comparison of mass and performance indexes before and after optimization

	m_1/g	m_2/g	m_3/g	m_4/g	m_5/g	f_p
Before optimization	13	83	65	82	44	0.35
After optimization	8	46	35	122	25	0.55
Increase（%）	-38.5	-44.6	-46.2	48.8	-43.2	57.1

It can be seen from Table 3 that after optimization, the mass of the driving link and the driven link are significantly reduced, the mass of the moving platform is significantly increased. It is noteworthy that the energy transfer efficiency increases by 57.1%.

6 Conclusions

(1) This paper presents dynamic analysis of a 1-DOF parallel vibrating screen mechanism using the ordered SOC method and the principle of virtual work, for which the concise equation of motion is derived, wherein the coefficient matrices of the dynamic equation can be readily calculated based on the generalized coordinates.
(2) Numerical simulation is carried out to characterize the motion laws of the moving platform of the PVSM by means of the Euler method.
(3) With the parametric model, the dynamic optimization of PVSM is carried out to maximize the energy transfer efficiency, subject to the constraints on the link mass. The results verify the feasibility of the optimization.

7 Declaration

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Availability of data and materials

The datasets supporting the conclusions of this article are included within the article.

Authors' contributions

The author' contributions are as follows: **Ju Li** and **Hui-Ping Shen** was in charge of the whole trial; **Si-Jin Xiao** wrote the manuscript; **Guang-Lei Wu** assisted with the revision of the paper. **Ting-Li Yang** participated in the discussion of the key issues of the paper.

Competing interests

The authors declare no competing financial interests.

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