Partial Relay Selection for Two-Way Mixed RF/FSO DF Networks in the Presence of I/Q Imbalance

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Abstract In this paper, the study of mixed radio frequency (RF)/ free space optical (FSO) communication decode and forward (DF) two way relaying (TWR) has been presented. In fact, it has been considered that multiple relays are present out of which the best operational relay is selected as per the partial relay selection (PRS) methodology in the presence of outdated channel state information (CSI). Importantly, the relay nodes are assumed to operate in the presence of in phase (I) quadrature phase (Q) imbalance (IQI). The atmospheric turbulence on the FSO link has been modeled using the Malaga distribution with pointing errors. In addition to this, the impact of type of optical demodulation has been considered in the analysis. For the system model, outage probability expression has been derived in terms of Meijer-G and Fox's H-functions. In addition to this, for the TWR system, the outage probability expressions have been modified to present asymptotic results in terms of elementary functions. The numerical analysis of the research work suggests that the overall mixed RF/FSO DF TWR system is impacted by the image rejection ratio (IRR) due to IQI, correlation between outdated CSI, atmospheric turbulence, pointing error and type of optical demodulation in addition to the amount of fading on the RF link.

Keywords RF/FSO relaying, in-(I) phase/quadrature-(Q) phase imbalance (IQI), outage probability, Meijer-G function, Fox's H-function.

The advent of free space optical (FSO) communication systems have been advocated to support the data deficiency in radio frequency (RF) systems. To accomplish this, mixed RF/FSO relaying systems have been envisioned [1]. Investigation of mixed RF/FSO relaying strategy has been performed in various research works then after where different models for FSO and RF links have been considered [2–4]. Specifically, RF and FSO links have been modeled using Nakagami- m and Gamma-Gamma distributions, respectively in [2], using which closed-form expression of performance metrics such as outage probability, bit error rate (BER) and ergodic capacity have been derived. In order to represent the turbulence with more accuracy, double generalized Gamma and Malaga distributions have been considered for modeling the second hop of relaying in [3, 4].

As the research in the pretext of RF/FSO relaying became progressive, researchers started imposing practical constraints in the system models. In fact, authors of [5–8] have considered presence of interference during the signal flow of dual hop transmission. While [5] is an attempt to model amplify-and-forward (AF) relaying, authors of [6] have presented multi-user scenario for decode-and-forward (DF) relayed mixed RF/FSO systems. Two way relaying (TWR) protocol has been investigated in [7] where source node incorporates multiuser diversity whereas the relay node operates in the presence of multiple co-channel interferers (CCIs). Another practical constraint called boresight pointing error along with multiple CCIs have been assumed to impact the performance of mixed RF/FSO relaying systems in [8].

However, a major practical performance delimit is introduced in the system when high switching speeds of the relay hardware restricts the available data rates in cooperative relaying systems. Such restrictions are generally analyzed in the context of hardware impairments. Specifically, aggregated hardware impairments have been considered for the case of mixed RF/FSO relaying in works [9–11]. Here, the impact of hardware impairments have been analyzed for partial relay selection (PRS) based mixed RF/FSO relaying systems. However, the impact of in-phase/quadrature phase imbalance (IQI) has not been considered in [9–11]. It is the fast switching speed of the relay hardware that prevents the overall system to achieve the theoretical limit of data rates [12, 13]. Of course, the in-phase (I) and quadrature-phase (Q) components of the carriers are ideally $\pi/2$ radians apart. However, due to practical constraints of the switching speeds of hardware along with involvement of analog components such as up and down converters, some mismatch remains between the two components. The impact of IQI on the performance of mixed RF/FSO relaying has been analyzed by the authors of [14] where closed-form expressions have been derived for the outage and ergodic capacity of an AF TWR system.

No research work has analyzed DF-TWR mixed RF/FSO systems where the relay node operates in the presence of transmit IQI. In fact, the effect of IQI has been considered in [14] for AF relaying strategy. In addition to this, study of the impact of outdated channel state information (CSI) on PRS assisted TWR remains unattended in the literature. Motivated by this, the authors of the present research work investigate PRS-aided DF TWR system with transmit IQI operation of the relay nodes. The best relay is selected based on the outdated CSI. The RF link of the mixed RF/FSO relaying has been modeled using the Rayleigh fading distribution, whereas the FSO link has been considered to be impacted by Malaga distributed atmospheric turbulence with pointing errors. For the given system model, exact and asymptotic expressions have been derived for the outage probability.

The organization of the remainder of paper is as follows: Section 2 presents the system and channel model whereas section 3 presents the analysis of exact outage probability. Section 4 presents the asymptotic analysis of the outage probability. In section 5, the numerical results of the research work has been provided followed by conclusion in section 6.

1 System and Channel Models

1.1 System Models

Consider a mixed RF/FSO TWR communication DF relaying network, where the two source nodes communicate with each other through a relay node. In this setup, it is assumed that source node S_1 is a RF mobile device whereas the source node S_2 is an FSO terminal equipped with single telescope. It is assumed that N relay nodes are present out of which the best relay is selected as per quality of link. In the first phase of communication, the signal received at the relay node can be presented as

$$y_{r_{RF}} = x_1 h_{RF} + N_{01} \quad (1)$$

where h_{RF} denotes the channel coefficient on the RF link from source node on the optical link, the average power is defined as $\mathbb{E}[|x_1|^2] = P_1$ where $\mathbb{E}[\cdot]$ denotes the expectation operator. Further to this, N_{01} denotes the additive white Gaussian noise (AWGN) on the $S_1 - R$ link with zero mean and variance σ_{n1}^2 . Similarly, on the FSO link, node S_2 sends the signal to the relay node which can be written as

$$y_{r_{FSO}} = (\eta I)^{r/2} x_2 + N_{02} \quad (2)$$

where η is the optical-to-electrical conversion ratio, N_{02} denotes the noise on the FSO link with variance σ_{n2}^2 , with irradiance fluctuation denoted by I to transmit information symbol x_2 . The average signal-to-noise ratios (SNRs) of the communication in the first time slot is defined as $\gamma_{RF} = \bar{\gamma}_{RF} |h_{RF}|^2$ and $\bar{\gamma}_{RF} = \frac{P_1}{\sigma_{n1}^2}$ and $\gamma_{FSO} = \frac{(\eta I)^r}{\sigma_{n2}^2}$. The constant r in the definition of γ_{FSO} is responsible for both kinds of optical demodulation techniques, where $r = 2$ corresponds to intensity modulation direct detection (IM/DD) demodulation whereas $r = 1$ is for coherent detection.

In the second phase of communication, the relay node decodes both the symbols, x_1 and x_2 and after processing again encodes them to transmit the symbols to the other end. It has been assumed that using the outdated CSI, the best relay is selected among the present N -relays is utilized for transmission. Suppose out of N relays, the j^{th} relay is selected for performing the communication. More specifically, the received signal at the source node S_1 from relay $r_{(j)}$ can be expressed as

$$y_{S1_{RF(j)}} = K_1^t \hat{h}_{RF} y_{r_{FSO(j)}} + K_2^{t*} \hat{h}_{RF}^* y_{r_{FSO(j)}} + N_{01} \quad (3)$$

where K_1^t represents transmit IQ imbalance from the relay node and $\hat{h}_{RF}$ denotes the outdated channel state of the RF link statistics. The coefficients K_1^t and K_2^t are further expressed as $K_1^t = \frac{1}{2} (1 + \epsilon^t e^{\pm j\phi^t})$ and $K_2^t = \frac{1}{2} (1 - \epsilon^t e^{\mp j\phi^t})$, where ϕ^t and ϵ^t quantify the phase and amplitude mismatch between the transmit hardwares at the node r_j relay node while the positive and negative signs in these relations account for up-conversion and down-conversion, respectively.

Similarly, the signal received from S_1 is forwarded by the relay node on the FSO link towards the node S_2 which can be modeled as

$$y_{S2_{FSO(j)}} = K_1^t y_{r_{RF(j)}} + K_2^{t*} y_{r_{RF(j)}} + N_{02} \quad (4)$$

After performing some mathematical manipulation, the signal-to-noise-plus-distortion ratio (SINR) can be expressed as

$$\gamma_{RS1} = \frac{|K_1^t|^2 |\hat{h}_{RF}|^2 (\eta I)^r}{|K_1^t|^2 |\hat{h}_{RF}|^2 \sigma_n^2 + |K_2^t|^2 |\hat{h}_{RF}|^2 \sigma_n^2 + |K_2^t|^2 |\hat{h}_{RF}|^2 (\eta I)^r + C} \quad (5)$$

where it is assumed that $\sigma_{n1}^2 = \sigma_{n2}^2 = \sigma_n^2$ and $C = \frac{1}{\sigma_n^2 |K_1^t|^2}$. The above fraction can be restated as

$$\gamma_{RS1} = \frac{\gamma_{RF(j)} \gamma_{FSO}}{(\text{IRR} + 1) \gamma_{RF(j)} + \text{IRR} \gamma_{RF(j)} \gamma_{FSO} + C} \quad (6)$$

where $\text{IRR} = \frac{|K_1^t|^2}{|K_2^t|^2}$ denotes the image rejection ratio (IRR) due to hardware impairment. Besides, the SNRs are defined as $\gamma_{RF(j)} = \bar{\gamma}_{RF(j)} |\hat{h}_{RF(j)}|^2$ and $\bar{\gamma}_{RF(j)} = \frac{P_1}{\sigma_n^2}$. The expression for SINR γ_{RS2} can be further expressed as

$$\gamma_{RS2} = \frac{\gamma_{RF(j)} \gamma_{FSO}}{(\text{IRR} + 1) \gamma_{FSO} + \text{IRR} \gamma_{RF(j)} \gamma_{FSO} + C} \quad (7)$$

1.2 Channel Models

It is assumed that the RF link follows the Rayleigh fading, the probability density of which is given as $f_{\gamma_{RF}}(x) = \frac{1}{\bar{\gamma}_{RF}} \exp\left(-\frac{x}{\bar{\gamma}_{RF}}\right)$, where $\gamma_{RF} = \bar{\gamma}_{RF} |h_{RF}|^2$. If based on outdated CSI, j^{th} -relay is selected for sending the symbols from the either ends during the second phase of communication, then the PDF of the $\gamma_{RF(j)}$ is given as [15]

$$f_{RF(j)}(\gamma) = j \binom{N}{j} \sum_{i=0}^{j-1} \binom{j-1}{i} \frac{(-1)^i}{(\bar{\gamma}_{RF})[(N-j+i)(1-\rho)+1]} \times \exp\left[-\frac{(N-j+i+1)\gamma}{((N-j+i)(1-\rho)+1)\bar{\gamma}_{RF}}\right] \quad (8)$$

where ρ represents the correlation coefficient quantifying the correlation between the Rayleigh channel state during selection and during execution. The corresponding CDF can be presented as [15]

$$F_{RF(j)}(\gamma) = 1 - j \binom{N}{j} \sum_{i=0}^{j-1} \frac{(-1)^i \binom{j-1}{i}}{[N-j+i+1]} \exp[-A_1 \gamma] \quad (9)$$

where $A_1 = \frac{(N-j+i+1)}{((N-j+i)(1-\rho)+1)\bar{\gamma}_{RF}}$. The irradiance fluctuation on the FSO link is modeled as $I = I_a I_p$, where I_a denotes the atmospheric turbulence, whereas I_p accounts for the pointing errors. The atmospheric turbulence fading I_a on the FSO link is modeled as Malaga distribution. The probability density function (PDF) of γ_{FSO} on the FSO link can be expressed as [4]

$$f_{\gamma_{\text{FSO}}}(\gamma) = \frac{\xi^2 A_3}{2^r \gamma} \sum_{m=1}^{\beta} x_2 G_{1,3}^{3,0} \left[A_4 \gamma^{1/r} \middle| \begin{matrix} \xi^2+1 \\ \xi^2, \alpha, m \end{matrix} \right] \quad (10)$$

where $G_{p,q}^{m,n} \left[Arg \middle| \begin{matrix} a \\ b \end{matrix} \right]$ is the Meijer-G function defined in [16, Eq. (9.301)], r represents the type of optical demodulation scheme, constants A_3, A_4 are defined as $A_3 = \frac{2\alpha^{\frac{\alpha}{2}}}{g^{1+\frac{\alpha}{2}} \Gamma(\alpha)} \left(\frac{g\beta}{g\beta+\Omega'} \right)^{\beta+\frac{\alpha}{2}}$, $A_4 = \frac{\xi^2 \alpha \beta (g+\Omega')}{[(\xi^2+1)(g\beta+\Omega')]^{\mu_r}}$ and $x_1 = \left(\frac{\beta-1}{m-1} \right) \frac{(g\beta+\Omega')^{1-\frac{m}{2}}}{\Gamma(m-1)} \left(\frac{\Omega'}{g} \right)^{m-1} \left(\frac{\alpha}{\beta} \right)^{\frac{m}{2}}$ [4]. The parameters involved in the Malaga distribution model are given as $g = 2b_0(1-\delta)$, $\Omega' = \Omega + 2b_0\delta + 2(2b_0\delta\Omega \cos(\phi_A - \phi_B))$. The pointing error parameters involved are $\xi = \frac{w_{zeq}}{2\sigma_s}$ [17], $w_{zeq}^2 = \frac{w_z^2 \sqrt{\pi} \text{erf}(v)}{2v \exp(-v^2)}$, $v = \frac{\sqrt{\pi}r}{2b_z}$ and $A_0 = [\text{erf}(v)]^2$ where w_z is the beam waist (calculated at e^{-2}) of the Gaussian spatial beam profile and w_{zeq} is equivalent beam waist at a distance of z .

$$F_{\text{FSO}}(\gamma) = A_5 \sum_{m=1}^{\beta} x_3 G_{r+1,3r+1}^{3r,1} \left[A_6 \gamma \middle| \begin{matrix} 1, \tau_1 \\ \tau_2, 0 \end{matrix} \right] \quad (11)$$

where $A_5 = \frac{\xi^2 A_3}{2^r (2\pi)^{r-1}}$, $x_3 = x_2 r^{\alpha+m+1}$, $A_6 = \frac{X_2^r}{r 2^r}$. with $\Delta(z : x)$ defined as $\left[\frac{x}{z}, \frac{x+1}{z}, \dots, \frac{x+z-1}{z} \right]$.

2 Exact Outage Probability

The outage probability is an important performance indicator and is defined as probability of the event when the instantaneous SNR falls below a predefined threshold value γ_{th} . Due to DF operation of the cooperative system, the outage probability P_{out} can be defined as

$$P_{out} = P_{out}^{t1} + (1 - P_{out}^{t1}) P_{out}^{t2} \quad (12)$$

where $P_{out}^{t1} = \Pr[\min(\gamma_{s1R}, \gamma_{s2R}) < \gamma_{th}]$. Similarly, the outage for second phase of communication can be established as $P_{out}^{t2} = \Pr[\min(\gamma_{RS1}, \gamma_{RS2}) < \gamma_{th}]$. In the following, the derivation of the outage probability is provided in details.

For the first phase of communication, the outage probability can be written as

$$P_{out}^{t1} = F_{\gamma_{RF}}(\gamma_{th}) + F_{\gamma_{FSO}}(\gamma_{th}) - F_{\gamma_{RF}}(\gamma_{th}) F_{\gamma_{FSO}}(\gamma_{th}) \quad (13)$$

Using that the RF link fading is modeled as Rayleigh fading while the FSO link is modeled using the Malaga distribution, we have

$$P_{out}^{t1} = 1 - \exp\left(-\frac{\gamma_{th}}{\bar{\gamma}_{RF}}\right) \left(1 - A_5 \sum_{m=1}^{\beta} x_3 G_{r+1,3r+1}^{3r,1} \left[A_6 \gamma_{th} \middle| \begin{matrix} 1, \tau_1 \\ \tau_2, 0 \end{matrix} \right] \right) \quad (14)$$

The P_{out}^{t2} can be defined as $P_{out}^{t2} = F_{\gamma_{RS1}}(x) + F_{\gamma_{RS2}}(x) - F_{\gamma_{RS1}}(x) F_{\gamma_{RS2}}(x)$. Following the procedure given in Appendix A, expression for $F_{\gamma_{RS2}}(x)$ can be obtained as

$$\begin{aligned} F_{\gamma_{RS2}}(x) &= 1 - j \binom{N}{j} \sum_{i=0}^{j-1} \frac{(-1)^i \binom{j-1}{i}}{[N-j+i+1]} \exp\left(-\frac{A_1 x (1+x\text{IRR})}{(1-x\text{IRR})}\right) \\ &\times \frac{\xi^2 A_3}{2^r \gamma} \sum_{m=1}^{\beta} x_2 G_{r,3r+1}^{3r+1,0} \left[\frac{A_4 x C A_1}{r 2^r (1-x\text{IRR})} \middle| \begin{matrix} \tau_1 \\ \tau_2, 0 \end{matrix} \right] \end{aligned} \quad (15)$$

$$\begin{aligned}
P_{out} = & \left[1 - \exp\left(-\frac{\gamma_{th}}{\bar{\gamma}_{RF}}\right) \left(1 - A_5 \sum_{m=1}^{\beta} x_3 G_{r+1,3r+1}^{3r,1} \left[A_6 \gamma_{th} \mid \frac{1, \tau_1}{\tau_2, 0} \right] \right) \right] + \left[\exp\left(-\frac{\gamma_{th}}{\bar{\gamma}_{RF}}\right) \left(1 - A_5 \sum_{m=1}^{\beta} x_3 \right. \right. \\
& \times G_{r+1,3r+1}^{3r,1} \left[A_6 \gamma_{th} \mid \frac{1, \tau_1}{\tau_2, 0} \right] \left. \left. \left(1 - \left[j \binom{N}{j} \sum_{i=0}^{j-1} \frac{(-1)^i \binom{j-1}{i}}{[N-j+i+1]} \exp\left(-\frac{A_1 \gamma_{th} (1 + \gamma_{th} \text{IRR})}{(1 - \gamma_{th} \text{IRR})}\right) \right. \right. \right. \right. \\
& \times \frac{\xi^2 A_3}{2^r \gamma} \sum_{m=1}^{\beta} x_2 G_{r,3r+1}^{3r+1,0} \left[\frac{A_4^r \gamma_{th} C A_1}{r^{2r} (1 - \gamma_{th} \text{IRR})} \mid \frac{\tau_1}{\tau_2, 0} \right] \left[1 - j \binom{N}{j} \sum_{i=0}^{j-1} \frac{1}{[(N-j+i)(1-\rho)+1] \bar{\gamma}_{RF(i)}} \right. \\
& \left. \left. \left. \left. \times \mathcal{H}_1 \left(\frac{A_6 (1 - x \text{IRR}) x}{A_1}, \frac{(1 + \text{IRR})}{C A_1} \right) \right] \right] \right] \right] \quad (18)
\end{aligned}$$

Similarly, the expression for $F_{\gamma_{RS1}}(x)$ can be derived following the steps presented in Appendix B which is given below

$$\begin{aligned}
F_{\gamma_{RS1}}(x) = & j \binom{N}{j} \sum_{i=0}^{j-1} \frac{1}{[(N-j+i)(1-\rho)+1] \bar{\gamma}_{RF(i)}} \\
& \times \mathcal{H}_1 \left(\frac{A_6 (1 - x \text{IRR}) x}{A_1}, \frac{(1 + \text{IRR})}{C A_1} \right) \quad (16)
\end{aligned}$$

where

$$\mathcal{H}_1(x, y) = \mathbf{H} \left[\begin{array}{c} \begin{pmatrix} 0, 2 \\ 2, 0 \end{pmatrix} \\ \begin{pmatrix} 3r, 1 \\ 2+r, 3r+1 \end{pmatrix} \\ \begin{pmatrix} 1, 0 \\ 0, 1 \end{pmatrix} \end{array} \middle| \begin{array}{c} (0 : 1, 1), (1 : -1, 1) \\ (1, 1), \overline{(1, 1)}, (\tau_1, [1]_{\tau_1}) \\ (\tau_2, [1]_{\tau_2}), \overline{(0, 1)} \\ \overline{(0, 1)} \end{array} \middle| x, y \right] \quad (17)$$

is the bivariate Fox's H-function defined in [18, Eq. (1.1)]. The expression for the outage probability of the TWR DF relaying network has been presented in (18) on the top of next page. It can be noted that the overall outage probability depends on the average SNRs on both links, correlation coefficient on the PRS link, atmospheric turbulence and pointing error on the FSO link and the IQI at the relay node.

3 Asymptotic Outage Probability

Due to involvement of complex mathematical functions, it is generally of more importance to represent the derived expressions into simpler forms. From the theory of residual of contour integration, the Meijer-G and Fox's H-functions can be expressed in terms of dominant poles. Assuming $\min(\tau_2) = p$, we have $F_{RF}(\gamma_{th}) \simeq \frac{\gamma_{th}}{\bar{\gamma}_{RF}}$ and

$$F_{FSO}(\gamma_{th}) \simeq A_5 \sum_{m=1}^{\beta} x_3 (A_6 \gamma_{th})^p \frac{\prod_{j=1}^{3r} \Gamma(t_{21,j} - p)}{\prod_{j=1}^r \Gamma(t_{1,j} - p) \Gamma(1 + p)} \quad (19)$$

Similarly, assuming $\min(t_{21}, 0) = q$, we can express $F_{\gamma_{RS2}}(x)$ asymptotically as given below

$$\begin{aligned}
F_{\gamma_{RS2}}(\gamma_{th}) \simeq & 1 - j \binom{N}{j} \sum_{i=0}^{j-1} \frac{(-1)^i \binom{j-1}{i}}{[N-j+i+1]} \exp\left(-\frac{A_1 \gamma_{th} (1 + \gamma_{th} \text{IRR})}{(1 - \gamma_{th} \text{IRR})}\right) \\
& \times \frac{\xi^2 A_3}{2^r \gamma} \sum_{m=1}^{\beta} x_2 \left(\frac{A_4^r \gamma_{th} C A_1}{r^{2r} (1 - \gamma_{th} \text{IRR})} \right)^q \frac{\prod_{j=1}^{3r+1} \Gamma(t_{21,j} - q)}{\prod_{j=1}^r \Gamma(t_{1,j} - q)} \quad (20)
\end{aligned}$$

Fig. 1 Impact of correlated channels on IQI-TWR DF Relaying System

In addition to this, the asymptotic expression of P_{out}^{t1} for $\bar{\gamma}_{RF}, \bar{\gamma}_{FSO} \rightarrow \infty$ can be written as

$$P_{out}^{t1} \simeq \left(\frac{\gamma_{th}}{\bar{\gamma}_{RF}} \right) \left(1 - A_5 \sum_{m=1}^{\beta} x_3 \left(A_6 \gamma_{th} \right)^{\min(\tau_2)} \frac{\prod_{j=1}^{3r} \Gamma(\tau_{2,j} - \min(\tau_2)) \Gamma(\min(\tau_2))}{\Gamma(1 + \min(\tau_2))} \right) \quad (21)$$

Omitting the product terms in the definition of P_{out}^{t1} and P_{out}^{t2} due to high SNR approximations, overall outage probability can be given as

$$\begin{aligned} P_{out} &\simeq P_{out}^{t1} + (1 - P_{out}^{t1}) \left[1 - A_5 \sum_{m=1}^{\beta} x_3 \left(A_6 \gamma_{th} \right)^p \frac{\prod_{j=1}^{3r} \Gamma(t_{21,j} - p)}{\prod_{j=1}^r \Gamma(t_{1,j} - p) \Gamma(1 + p)} \right. \\ &\quad - j \binom{N}{j} \sum_{i=0}^{j-1} \frac{(-1)^i \binom{j-1}{i}}{[N - j + i + 1]} \exp \left(- \frac{A_1 \gamma_{th} (1 + \gamma_{th} \text{IRR})}{(1 - \gamma_{th} \text{IRR})} \right) \frac{\xi^2 A_3}{2^r \gamma} \sum_{m=1}^{\beta} x_2 \\ &\quad \times \left(\frac{A_4^r \gamma_{th} C A_1}{r^{2r} (1 - \gamma_{th} \text{IRR})} \right)^q \frac{\prod_{j=1}^{3r+1} \Gamma(t_{21,j} - q)}{\prod_{j=1}^r \Gamma(t_{1,j} - q)} \left. \right] \quad (22) \end{aligned}$$

The results obtained in the above expression can be very useful in quick evaluation when compared to the exact results which require the implementation of Meijer-G and bivariate Fox's function. The asymptotic result for the outage probability depends on the correlation coefficient on the PRS link along with the atmospheric turbulence and pointing error on the FSO link.

4 Result

In this section, the numerical results obtained in the research work have been presented. The numerical results have been verified using the Monte-Carlo simulations. The model parameters on the FSO link has been obtained from [4]. In Fig. 1, the impact of PRS strategy on the outage probability has been presented where the optical link parameters are varied. The analysis has been demonstrated for both the types of optical demodulation schemes. In fact, the parameter $r = 1$, which corresponds to coherent demodulation scheme provides the best outage performance. However, as the pointing error increases from $\xi = 2.06$ to $\xi = 1.14$, the outage probability of the network increases, suggesting degradation in performance. Interestingly, IM/DD demodulation presents a more challenging condition to the TWR network. Importantly, in all the circumstances, when the correlation between $M = 5$ relays is increased, the outage probability decreases significantly. The analysis has been done for the case when $j = 4$ relays have been selected for communication. During the analysis, it was notable that when the number of

Fig. 2 Effect of atmospheric turbulence and the number of available links IQI-TWR DF Relaying System

Fig. 3 Effect of IQI and correlated relays for the TWR DF Relaying System

available relays are less, the betterment due to correlation is subtle. The Monte-Carlo simulation has been used to validate the results.

Fig. 2 demonstrates the impact of varying turbulence and number of relays selected through the PRS scheme. Moderate atmospheric turbulence has been modeled using parameter settings $\alpha = 4.2$ and $\beta = 3$ and strong turbulence has been considered using $\alpha = 2.296$ and $\beta = 2$. For analysis, the number of relays have been assumed to be $M = 5$ and out of which it has been considered that $j = 1$ and $j = 5$ relays are available for transmission. The performance in Fig. 2 is studied for IM/DD optical detection for a fixed threshold SNR of $\gamma_{th} = 1$ dB. The correlation among the channels of the multiple relays is assumed to be $\rho = 0.8$. Clearly, when all the relays are available for transmission, the reliability increases. On the other hand, the increase in atmospheric turbulence presents a challenging situation, deteriorating the outage experienced by the TWR-DF system. Importantly, the asymptotic analysis derived in the paper matches with the exact results when the average SNR per hop increases. The numerical results have been verified using the Monte-Carlo simulations.

The impact of IQI on the performance of mixed RF/FSO DF TWR has been demonstrated in Fig. 3. The correlation between the $M = 5$ relays has been varied as $\rho = [0.3, 0.8]$ and it has been assumed that IM/DD demodulation scheme is utilized to demodulate the optical signal. The IQI has been opted as $IRR = [-10, -20]$ dB. Even though the DF protocol improves the outage performance of the system, as IRR increases from -20 dB to -10 dB, the outage probability of the overall system increases. For

Fig. 4 Effect of type of optical demodulation and strength of pointing error on DF TWR system.

representing the characteristics, the atmospheric turbulence on the FSO link has been modeled for strong fading conditions.

The FSO link parameters play significant part in the end-to-end reliability of the TWR DF system. Fig. 4 illustrates the impact of optical demodulation and pointing error on the outage performance. As per the system model, $r = 1$ represents the coherent demodulation scheme whereas $r = 2$ is corresponding to IM/DD detection. For the case when $M = 5$ relays are present and out of which $j = 2$ are selected for communication, coherent demodulation scheme provides better outage experience. In addition to this, $\xi = 1.14$ corresponds to severe pointing error and induces larger outage probability as compared to when the pointing error is small with $\xi = 2.06$. The analysis has been conducted for moderate turbulence on the FSO link, when the threshold SNR is assumed to be $\gamma_{th} = -3$ dB. Interestingly, the high SNR approximate expressions of the outage probability have been plotted on the same axis as exact results, and as the average SNR per hop is increased, the asymptotic and exact values match in the figure.

5 Conclusion

In this research work, the impact of IQI has been studied for mixed RF/FSO TWR DF network. The PRS link has been modeled using the Rayleigh distribution while the FSO link has been assumed to be affected by atmospheric turbulence following Malaga distribution, and pointing error. In addition to this, the analysis takes into account for the total number of relays available, the number of selected relays, the fading on the RF link, the type of demodulation on the FSO link. For the considered system model, closed-form expression for the outage probability has been derived in terms of Meijer-G and bivariate Fox's H-function. The numerical results suggest that the correlation among the channels on the RF link plays an important role in determining the reliability of the network along with FSO link factors like turbulence, pointing errors, type of optical demodulation.

6 Declarations

Availability of data and material Data sharing not applicable to this article as no datasets were generated or analyzed during the current study

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A Derivation of $F_{\gamma_{RS2}}(x)$

In this section, the derivation of $F_{\gamma_{RS2}}(x)$ has been presented. From the definition of $\gamma_{RS2}(x)$ in (7), the expression for $F_{\gamma_{RS2}}(x)$ can be defined using the relationship

$$F_{\gamma_{RS2}}(x) = \Pr[\gamma_{RS2} < x] = \int_0^\infty F_{\gamma_{RF}}\left(\frac{x((1+\text{IRR})\gamma_{FSO} + c)}{(1-x\text{IRR})\gamma_{FSO}}\right) f_{\gamma_{FSO}}(\gamma_{FSO}) d\gamma_{FSO} \quad (23)$$

Substituting the requisites in the above integral, the expression for $F_{\gamma_{RS2}}(x)$ can be deduced as

$$\begin{aligned} F_{\gamma_{RS2}}(x) &= 1 - j \binom{N}{j} \sum_{i=0}^{j-1} \frac{(-1)^i \binom{j-1}{i}}{[N-j+i+1]} \exp\left(-\frac{A_1 x(1+x\text{IRR})}{(1-x\text{IRR})}\right) \frac{\xi^2 A_3}{2^r \gamma} \sum_{m=1}^{\beta} x_2 \\ &\times \int_0^\infty \gamma_{FSO}^{-1} \exp\left(-\frac{x C A_1}{(1-\text{IRR}\gamma_{FSO})}\right) G_{1,3}^{3,0} \left[A_4 \gamma_{FSO}^{1/r} \left| \frac{\xi^2 + 1}{\xi^2, \alpha, m} \right| d\gamma_{FSO} \right] \end{aligned} \quad (24)$$

The above integral can be further formulated as

$$\begin{aligned} F_{\gamma_{RS2}}(x) &= 1 - j \binom{N}{j} \sum_{i=0}^{j-1} \frac{(-1)^i \binom{j-1}{i}}{[N-j+i+1]} \exp\left(-\frac{A_1 x(1+x\text{IRR})}{(1-x\text{IRR})}\right) \frac{\xi^2 A_3}{2^r \gamma} \sum_{m=1}^{\beta} x_2 \\ &\times \int_0^\infty \gamma_{FSO}^{-1} G_{1,0}^{0,1} \left[\frac{(1-\text{IRR}\gamma_{FSO})}{x C A_1} \left| \frac{1}{-} \right| G_{1,3}^{3,0} \left[A_4 \gamma_{FSO}^{1/r} \left| \frac{\xi^2 + 1}{\xi^2, \alpha, m} \right| d\gamma_{FSO} \right] \end{aligned} \quad (25)$$

Applying [19, Eq. (07.34.21.0013.01)], a closed-form solution to the integral can be obtained which yields the expression for $F_{\gamma_{RS2}}(x)$.

B Derivation of $F_{\gamma_{RS1}}(x)$

The overall CDF for γ_{RS1} can be evaluated as

$$F_{\gamma_{RS1}}(x) = \int_0^\infty F_{\gamma_{FSO}}\left(x \left\{ \frac{(1+\text{IRR})y + C}{(1-x\text{IRR})y} \right\}\right) f_{RF(j)}(y) dy \quad (26)$$

Substituting the requites in the above expression, the above integral can be re-written as

$$\begin{aligned} F_{\gamma_{RS1}}(x) &= j \binom{N}{j} \sum_{i=0}^{j-1} \binom{j-1}{i} \frac{(-1)^i}{(\bar{\gamma}_{RF})[(N-j+i)(1-\rho)+1]} A_5 \sum_{m=1}^{\beta} x_3 \\ &\times \int_0^\infty \exp\left[-\frac{(N-j+i+1)}{((N-j+i)(1-\rho)+1)\bar{\gamma}_{RF}} y\right] G_{r+1,3r+1}^{3r,1} \left[A_6 \left(\frac{(1+\text{IRR})y + C}{(1-x\text{IRR})y}\right) \left| \frac{1, \tau_1}{\tau_2, 0} \right| \right] \end{aligned} \quad (27)$$

For finding the solution to the integral, the Meijer-G function can be expressed in contour integral form as per [19, Eq. (07.34.02.0001.01)] as shown below

$$\begin{aligned} G_{r+1,3r+1}^{3r,1} \left[A_6 \left(\frac{(1+\text{IRR})y + C}{(1-x\text{IRR})y}\right) \left| \frac{1, \tau_1}{\tau_2, 0} \right| \right] &= \frac{1}{2\pi i} \int_{\mathcal{L}_1} \Theta_1(s_1) A_6^{s_1} ((1+\text{IRR})y + C)^{-s_1} \\ &\times (1-x\text{IRR})y^{s_1} ds_1 \end{aligned} \quad (28)$$

Placing from (28) to (27), the integral required to be attended can be framed as

$$\mathcal{I}_1 = \int_0^\infty ((1+\text{IRR})y + C)^{-s_1} y^{s_1} \exp\left[-\frac{(N-j+i+1)}{((N-j+i)(1-\rho)+1)\bar{\gamma}_{RF}} y\right] dy \quad (29)$$

Invoking identity [19, Eq. (07.34.03.0271.01)], the integral can be again presented as follows

$$\mathcal{I}_1 = \frac{C^{-s_1}}{\Gamma(s_1)} \int_0^\infty y^{-s_1} G_{1,1}^{1,1} \left[\frac{(1+\text{IRR})y}{C} \left| \frac{1-s_1}{0} \right| \right] \exp\left[-\frac{(N-j+i+1)}{((N-j+i)(1-\rho)+1)\bar{\gamma}_{RF}} y\right] dy \quad (30)$$

With the help of [19, Eq. (07.34.21.0088.01)], the solution to the above integral can be given as

$$\begin{aligned} \mathcal{I}_1 &= \frac{C^{-s_1}}{\Gamma(s_1)} \left(\frac{(N-j+i+1)}{((N-j+i)(1-\rho)+1)\bar{\gamma}_{RF}}\right)^{s_1-1} \\ &\times G_{2,1}^{1,2} \left[\frac{(1+\text{IRR})}{C} \frac{((N-j+i)(1-\rho)+1)\bar{\gamma}_{RF}}{(N-j+i+1)} \left| \frac{s_1, 1-s_1}{0} \right| \right] \end{aligned} \quad (31)$$

In order to establish a closed-form solution, the relationship [19, Eq. (07.34.02.0001.01)] can be used to express the Meijer-G function into complex integral form, and therefore, the above integral can be further represented as

$$\begin{aligned} \mathcal{I}_1 = & \frac{C^{-s_1}}{\Gamma(s_1)} \left(\frac{(N-j+i+1)}{((N-j+i)(1-\rho)+1)\bar{\gamma}_{RF}} \right)^{s_1-1} \frac{1}{(2\pi i)} \int_{\mathcal{L}_2} \Gamma(s_2) \Gamma(s_1-s_2) \\ & \times \Gamma(1-s_1-s_2) \left(\frac{(1+IRR)}{C} \frac{((N-j+i)(1-\rho)+1)\bar{\gamma}_{RF}}{(N-j+i+1)} \right)^{-s_2} ds_2 \end{aligned} \quad (32)$$

Combining together, the results from (B.2) and (B.7), the overall expression for $F_{\gamma_{RS1}}(x)$ can be expressed in double contour integral form as given below

$$\begin{aligned} F_{\gamma_{RS1}}(x) = & j \binom{N}{j} \sum_{i=0}^{j-1} \frac{1}{[(N-j+i)(1-\rho)+1]\bar{\gamma}_{RF}} \frac{1}{(2\pi i)^2} \int_{\mathcal{L}_1} \int_{\mathcal{L}_2} \Gamma(s_1-s_2) \Gamma(1-s_1-s_2) \\ & \times \frac{C^{-s_1}}{\Gamma(s_1)} \Theta_1(s_1) A_6^{s_1} \left(\frac{(N-j+i+1)}{((N-j+i)(1-\rho)+1)\bar{\gamma}_{RF}} \right)^{s_1-1} ((1-xIRR))^{s_1} \Gamma(s_2) \\ & \times \left(\frac{(1+IRR)}{C} \frac{((N-j+i)(1-\rho)+1)\bar{\gamma}_{RF}}{(N-j+i+1)} \right)^{-s_2} ds_1 ds_2 \end{aligned} \quad (33)$$

Comparing the obtained contour integral with the [18, Eq. (2.1)], the resulting CDF for $F_{\gamma_{RS1}}(x)$ can be expressed as bivariate Fox's H-function as given in (16).

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