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Electromechanical models of micro and nanoresonators

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Abstract. This work is devoted to numerical and partly experimental investigation of the electromechanical models of micro and nanoresonators. The main purpose of this resonators is detected the adherence of micro or nanoparticles and measurement its mass. The periodical oscillations of mono and many layers nanocapacitors, from which nanoresonator situated in electric field is constructed, are studied. The purpose of this work is creating of electromechanical models of MEMS and NEMS the aim of which is to identify the changes introduced into dynamic systems due to adhesion of micro or nanoparticles. It's significant to note that the time of overcharge of the nano-capacitor is much less, then the period of excited oscillations. This fact gives the possibility to apply asymptotic methods in numerical investigation. Physical experiments similar in model to the electromechanical nanoresonators were carried out. This work is an extended review article based on the results of previous our works.

Key words: graphene layer, condenser armature, electrical field, nonlinear oscillations

The new technologies and materials discovered in recent time enable to development the principal new micro and nanoelectromechanical systems (MEMS, NEMS), particular, micro and nanoresonators. One of possible application of the graphene nanoresonators is using of these devices as detectors of nanoparticle mass sticking to graphene layer. In result of adherence of the particle (biological cell or even molecules) to elastic graphene surface its resonance frequency is changed. This effect gives us the possibility to determine the mass of nanoparticle.

Now we have the great number of works devoted to study of nanoresonators based on graphene, for example [1-4]. The main problem in creating of nanoresonator is an increasing of the sensitivity to nanomass of this particle. This sensitivity determines the alteration of eigen flexural oscillations frequency at adhesion of nanoparticle on a nanolayer. At low Q-factor of resonators (order 100) may be occur that this alteration is comparable with width of resonance diapason. This fact does not give us the possibility to obtain the accurate data of particle mass in case if its mass represents the value, which is order 1% from mass of graphene layer of resonator, and compels to use new models and methods.

Now the certain articles consider resonators with a few graphene layers [2,4,8,9]. This article is a prolongation of works [7-9] and is devoted to investigation of possible increasing the precisions of measurement of resonators frequency with law Q-factor.

1. Monolayer nanoresonator.

The scheme of monolayer resonator is showed in Fig.1. The monolayer nanoresonator consists from graphene layer clamped on isolated rigid supports. Under graphene layer on some distance the plate conductive surface is situated. In a space between graphene layer and conductive surface with help of EMF the electric field is generated.

Fig.1. Monolayer nanoresonator

The characteristic sizes of graphene resonator: the length of layer span – $500 \div 1000$ nm, the width of layer – $10 \div 20$ nm, the gap between graphene layer and conductive surface is also $10 \div 20$ nm.

Let's consider the process of motion of this system taking into account the electrical forces. The different mechanical model of graphene layer may be considered – the stretched string, the beam with flexural stiffness and simultaneously under the influence of axial tension, elastic membrane, thin stretched plate etc. These models may be both linear and nonlinear, including geometrical and physical nonlinearity. In common case the equations of motions may be written in a view

$$L(w) + R\ddot{w} = F_e(w, t). \quad (1.1)$$

Here w – flexure of graphene layer, $L(w)$ – differential operator of elasticity, R – operator of inertia, $F_e(w, t)$ – the attracted force, induced by assistance of electrical field.

By the long-term exposure of constant EMF, the system becomes in equilibrium position, which can be found from the system of equation (1.1) with absence of inertial force

$$L(w_0) = F_e(w_0). \quad (1.2)$$

Equilibrium position does not coincide with zero flexure of graphene layer, because the load induced by assistance of electrical field doesn't equal zero. At increasing of flexure of layer the electrical force grows unlimited then the elastic force stays limited. Thus, the equilibrium positions, depending from EMF, may be either two or non-existent at all. In extreme case we have a single multiple equilibrium. From character of acting forced it's a clear that equilibrium may have two positions. Lesser flexure is stable, and other with large flexure – unstable (see Fig.4).

Let's consider the oscillations of graphene layer close to stable equilibrium at action of AC voltage. In this case the solution of equation (1.1) searches as deflection from equilibrium position $v = w - w_0$. Then the equation (1.1) taking in account inertial force has a form

$$L(w_0 + v) + R\ddot{v} = F_e(w_0 + v) \quad (1.3)$$

For obtaining the equations of graphene layer motions at regimes closed to resonance let's use Galerkin method. The solution represents as series of space coordinate function V_k with coefficients x_k depended from time

$$v = x_1(t)V_1 + x_2(t)V_2 + \dots \quad (1.4)$$

As coordinate functions the continuum of eigen function without taking into account their nonlinear elastic properties and action of electric field is considered. The eigen functions satisfy to equations

$$L_0 V_k - \lambda_k^2 R V_k = 0, \quad k = 1, 2, \dots \quad (1.5)$$

where $L_0(V)$ – linearized operator $L(V)$, λ_k^2 – eigen frequency of free oscillations increasing in series.

Near of resonance only one component from series (1.4) makes sense to keep, which corresponds to resonance frequency λ_i . As usual it may be λ_1

$$v = x(t) V_1 \quad (1.6)$$

The using of Galerkin procedure to equation (1.3) taking into account (1.6) gives the equation of oscillations closed to resonance regime:

$$m\ddot{x} + \beta\dot{x} + P(x) - \frac{1}{2} \frac{Q^2}{C^2} \frac{dC}{dx} = 0, \quad (1.7)$$

To the equation of motion (1.7) the equation of overcharge of capacitor can be added. This capacitor is development from graphene layer and plane conductive surface. The capacitance depends from the distance between armatures $d_0 - x$. Where d_0 – initial gap of condenser, x – the value of deformation of elastic graphene layer. The equation of electric circuit is

$$R\dot{Q} + \frac{1}{C(x)}Q = U(t). \quad (1.8)$$

In equations (1.7), (1.8) P – elastic forces, $Q(t)$ – the value of capacitor charge, $C(x)$ – capacity of capacitor, R – ohmic resistance of EMF source, $U(t)$ – set voltage in electric circuit.

Equation (1.7) and (1.8) are described the motion of electromechanical system, which consist of oscillation system with one degree of freedom and electric circuit consist of in-series EMF, resistor and capacitor. Interaction between mechanical system and electric circuit determines that electric field in capacitor, which creates the mechanical force affected on the oscillated mass. In the same time displacement of the mass determines the change of capacity.

In case then EMF is a source with harmonic voltage $U(t) = U_0 \cos \omega t$ and at small time of capacitor discharging in compare with period of voltage, i.e. at fulfillment of condition $RC_0 \ll \frac{2\pi}{\omega}$, we may neglect the first summand of equation (1.8), when charge of capacitor in time will change by form:

$$Q = C(x) U_0 \cos \omega t \quad (1.9)$$

The electromechanical model of resonator, which is analogy of nano-resonator is showed in Fig.2

Fig.2. Electromechanical model of monolayer nano-resonator

Substituted the last expression (1.9) into equation (1.7), we obtain the equation of graphene layer oscillations in regime close to resonance

$$m\ddot{x} + \beta\dot{x} + P(x) - \frac{1}{2}C_0U_0^2 \frac{d_0 \cos^2(\omega t)}{(d_0 - x)^2} = 0. \quad (1.10)$$

For dynamic analysis let's transformable equation (1.10) in dimensionless form, included non-dimensional variables: time $\tau = \lambda t$, flexure $\xi = \frac{x}{d_0}$, frequency $\Omega = \frac{\omega}{\lambda}$, where λ – eigen frequency of linearized oscillation system without action of electric field. The equation (1.10) represents in form

$$\ddot{\xi} + 2n\dot{\xi} + P(\xi) - b^2 \frac{1 + \cos 2\Omega\tau}{(1 - \xi)^2} = 0, \quad (1.11)$$

where parameter $b^2 = \frac{C_0 U_0^2}{2m\lambda^2 d_0^2}$ is a ratio of amplitudes of electric energy and kinetic energy of oscillations with frequency λ and amplitude equals to the gap of capacitor. Evidently, we consider only the case, then $\xi < 1$, i.e. maximum of flexure deformation of the graphene layer don't exceed the initial value of the gap.

Equation (1.11) consists from two nonlinear components. Last of them connects with nonlinear influence of electrical force, acting at the presence of electric field and indefinitely increasing at approach of graphene layer flexure to the value of initial gap. Really this force plays the role of negative stiffness. First nonlinear component $P(\xi)$ connects with nonlinear elastic of graphene layer at its transversal deformation. Graphene layer by the method of its production fix so that its edges don't move either transversal or longitudinal directions. The stiffness of graphene layer on the expansion so large, that equivalent Young's modulus for graphene is 5 times more then by the constructed steels. The longitudinal tension at flexure of graphene layer will be increase owing to its stretching in longitudinal direction that stipulate the addition for its transversal stiffness. This is result of many physical experiments.

As the analog to the naonolayer, let's consider the static deformation of string previously stretched, the force of tension of which is T_0 . String clamped so, that displacements of its edges in transversal and longitudinal directions don't possible, and in middle is loaded by transversal force P (Fig.3).

Fig.3. Transversal deformation of string

Total value of tension is summed from initial and supplemental component owing to stretching of string at its transversal loading

$$T = T_0 + T_* \varepsilon, \varepsilon = \frac{ds-dx}{dx} = \sqrt{1 + (tg\alpha)^2} - 1 \quad (1.12)$$

Substituting the last expression into conditions of equilibrium and using demonstrable geometrical correlation between flexure w and angle α , we obtain connection between vertical force P and flexure

$$P = 4 \frac{w}{L} (T_0 + T(\sqrt{1 + (2 \frac{w}{L})^2} - 1)) / \sqrt{1 + (2 \frac{w}{L})^2} \quad (1.13)$$

The linearized approximation (1.13) gives the classical expression for transversal stiffness so called «inextensible» string in the same time more correctly believes that its name is infinitely stretched. The first approximation gives cubic relation

$$P = 4T_0 \frac{w}{L} + (T_* - T_0) \left(2 \frac{w}{L}\right)^3 = 4T_0 \left(\frac{w}{L} + \frac{2(T_* - T_0)}{T_0} \left(\frac{w}{L}\right)^3\right) = c(\xi + \gamma \xi^3) \quad (1.14)$$

The influence of cubic term is more strong than initial tension T_0 is smaller and larger than longitudinal stiffness of string T_* is great. Let's introduce dimensionless variable $\xi = \frac{w}{L}$ and time $\tau = \lambda t$, where λ –eigen frequency of small oscillation of string. Taking into account (1.14) let us overwrite equation (1.11) in dimensionless form

$$\ddot{\xi} + 2n\dot{\xi} + \xi + \gamma \xi^3 - b^2 \frac{1 + \cos 2\Omega\tau}{(1 - \xi)^2} = 0 \quad (1.15)$$

Static equilibrium positions at different sign of γ are shown in Fig.4

Fig.4 Static equilibriums $b = 0.3$, a) $\gamma = 0.5$, b) $\gamma = -0.5$

The stable oscillations at an excitation dimensionless frequency of the order of unity occur near stable equilibrium position as it can be seen in Fig.5

Fig.5 Oscillation near stable equilibrium position ($n = 0.05, \gamma = 0.5, b = 0.3, \omega = 1.1$)

For equation (1.15) it's interesting to obtain the resonance curve – the dependence of amplitude of oscillations in steady-state regime from exciting frequency. Amplitude-frequency characteristic for equation (1.15) may be obtained by harmonic balance method. However, the nonlinear algebraic equations, obtained by harmonic balance method, don't solve analytically. Therefore, for equation (1.15) numerical experiment with scanning of exciting frequency was fulfilled.

At first the Cauchy task is solved for exciting frequency close on resonance frequency at zero initial data. Then after the steady-state was came the Cauchy task is solved again for nearest value of frequency, and initial data was taken from solution for previous value of frequency in time of installation of steady-state regime. This procedure returns at many values of frequency. Responsible resonance curves are showed in Fig.6, Fig.7.

Fig.6 Resonance curve at small initial tension ($\gamma=10$)

Fig.7 Resonance curve at large initial tension ($\gamma=0.1$)

Demonstrated here resonance curves have characteristic breaking conditioned either elasticity nonlinear characteristic (at small initial tension), or nonlinear negative electric stiffness (at large initial tension and large amplitude of oscillations). Resonance curves with characteristic, which has breaking, observe in real physical experiments, in particular these results demonstrate in article [7].

In laboratory of «Mechanical and processes of control» department was produced the demonstrated device, which enables to observe «breaking» resonance curves. As oscillations system a segment of steel wire clamped by edge was used with possible of previous tension. The

oscillation of wire is excited by magnetic system, consisted from steel magnetic conductor (spring), permanent magnet and coil with alternating current. The coil feeds from generator of standard signals with possible adjustment of amplitude and frequency. The dependence of magnetic force, acted to wire, from values of wire flexure is analogy to the same dependence of electric force, acted in graphene. Namely this force rises monotonous at increasing of flexure. The equation, described the oscillation of wire in this experiment, is analogy to equation (1.11). In Fig.8 and Fig.9 the examples of oscillogram and resonance curves, obtained on the experimental device, are shown.

Fig.8.The oscillogram of oscillation of wire at exciting frequency 62 Hz

Fig.9.The resonance curve

The direction of scanning of frequency at execution of experiment is shown in Fig.9. On the resonance curve characteristic breaking at scanning of exciting frequency top-down is visible.

2. Differential resonator

The scheme of differential resonator, consisted from two graphene layers, was proposed in works [8,9]. Scheme of this resonator is shown in Fig.10. Differential resonator consists from two parallel graphene layers – basic and additional. Additional layer is situated under basic layer and represents

the plate conductive surface. The source of permanent EMF (basic) is aimed for creation of force interaction between basic and additional layers, the source EMF (additional) is proposed for assign the initial condition.

Fig.10 The scheme of differential resonator

The principal of work of differential resonator consists in next. The aim of one cycle of measuring is that voltage from additional source in a form of shot impulse excites combined free oscillations of graphene layers. Partial eigen oscillations of each layer (without connection) are very close and connection between them is soft in comparison the natural elastic of each layer. Thus, this system has two graphene layers closely approximated one from another and close to partial frequencies. Thus, free oscillations of this system will have the character of beating. The envelope of this process is determinate by detection. The characteristic envelope frequency is equal to the half of difference of partial frequencies and much less then partial frequency of every layer. At adherence of particle on top layer the partial frequency of this layer decreasing. And characteristic frequency also is changed, at that the small alteration of partial frequency gives the large alteration of envelope characteristic frequency. After some period, the free oscillations are damped and cycle of measuring may be repeated.

In laboratory «Mechanical and processes of control» department as well as at first case demonstrational device for observe the beating process was prepared. This device consists from two parallel steel cantilever beams.

3. Parametrical resonator.

The new scheme of graphene resonator, in which the oscillation is created only by parametric action, is proposed. The scheme of this resonator is shown in Fig.11.

Fig.11 Parametrical resonator

Parametrical resonator consists from one graphene layer, enclosed by two conductive surfaces. This combination is formed the two capacitors with one common armature – graphene layer. The sources of AC EMF with synchronous frequency connect to each of capacitor. Macro-model analogy to parametrical resonator, which was showed in Fig.11, is proposes in Fig.12

Fig.12 Electromechanical models of parametrical resonator

This model includes two capacitors so, that decreasing of distance between of first two armatures gives increasing this distance in second couple. Analogically equation (1.10) equation of motion for model of parametrical resonator may be written

$$m\ddot{x} + \beta\dot{x} + cx - \frac{Q_1^2}{2C_1} \frac{dC_1}{dx} - \frac{Q_2^2}{2C_2} \frac{dC_2}{dx} = 0. \quad (3.1)$$

The capacities of capacitors are changed at displacement of central mass x (that analogy flexure of graphene layer) and have a view

$$C_1 = C_0 \frac{d_0}{d_0 - x}, \quad C_2 = C_0 \frac{d_0}{d_0 + x},$$

where the same designations, that in section 1 are input.

The equation (3.1) with account relation between period of overcharging of capacitor circuit and period of external sources $RC_0 \ll \frac{2\pi}{\omega}$ may be written in dimensionless form:

$$\ddot{\xi} + 2n\dot{\xi} + \xi - \frac{b^2\xi}{(1-\xi^2)^2} (1 + \cos 2\Omega\tau) = 0. \quad (3.2)$$

The initial position of equilibrium of this resonator coincides with neutral unstrained position of graphene layer $\xi = 0$. At absent of external voltages this position is stable. However, at existence of external voltage this position began unstable. The linearized equation (3.2) has a form:

$$\ddot{\xi} + 2n\dot{\xi} + (1 - b^2) \left(1 + \frac{b^2}{1-b^2} \cos 2\Omega\tau \right) \xi = 0. \quad (3.3)$$

This is Mathieu equation, in which from parameter b^2 both coefficient of pulsation and eigen frequency are dependent. The border of unstable field is shown in Fig.13.

Fig.13. The borders of unstable field, $\Lambda = \sqrt{1 - b^2}$
1 – Q-factor 100, 2 – 70, 3 – 50.

For unstable equilibrium position the diapason of frequency between left and right branches is defined, that this diapason becomes narrow then Q-factor of oscillation system is less. That parametrical resonance is different from usual. This fact may be used for increasing the accuracy of measuring the resonance frequency at low Q-factor.

At parametric oscillations closed on equilibrium position the amplitude is limited by the nonlinear of the oscillations system. Amplitude of stationary regime may be found by harmonic balance method, i.e. to search solution of equation (3.2) in a view of harmonic oscillations with frequency Ω . In Fig.14 the dependence of amplitude of stationary regime from frequency of external voltage at different values of parameter b^2 is shown

Fig.14 Amplitude of stationary regime

4. Parametrical nanoresonator with magnetic exciting of oscillation

In this paragraph the equilibrium and oscillation nearly situated graphene layers with harmonic currents exciting parametrical resonance oscillations is considered. These parametrical oscillations are used for determine the mass of nanoparticle adherence. The task of this type is a prolongation of our works [7-9], in which the oscillations of the different types of nano-electromechanical system (NEMS) excite by alternative electric field, are considered. In these works the change of the spectrum of mechanical oscillations of different types NEMS, conditioned by adherence of nano-particle, is determined by the alteration of amplitude-frequency characteristic in a region of

parametrical or forced resonance. In this paragraph parametrical oscillations excite by alternative current conducted through two closely spaced graphene layers. The determination of nanoparticle mass, stuck to one of parallel graphene layers, consists in determination of shifting the zone of parametrical resonance.

The main problem of creating a resonator, as detector of particle adherence, is an increasing the sensitivity to nano-mass of this nanoparticle. The explanation of advantage the using of the parametrical resonance at adherent of nanoparticle on graphene layer at law Q-factor (order~100) was done in previous paragraph.

The scheme of installation of graphene layers is analogy that it was fulfilled in differential resonator [9]. In difference from differential resonator, the graphene layers are used as conductors with alternating current. In result around conductive layers the alternating magnetic field is excited. In every layer the alternating in time magnetic force is arise and mechanical oscillations are excited. It's shown that this action may get excitation of parametrical resonance. As it known the width of zone of parametrical resonance is narrowed at decreasing of Q-factor of electromechanical system. This condition may be useful for increasing the accuracy of measurement the frequency of resonator at its law Q-factor.

The scheme of this resonator is shown in Fig.15

Fig.15 The scheme of resonator 1 – graphene layers, 2 - supports

Resonator consists from two graphene layers situated on some distance one from another. At every layer the current is conducted as it shown in Fig.15.

The forces-distributed interaction between conductors with currents I_1, I_2 in case of its even directivity attraction is determined by relation

$$q = \frac{\mu_0 I_1 I_2}{2\pi} \frac{r}{r^2}, \quad (4.1)$$

where $\mathbf{r}$ –radius-vector, connecting the points of deformable conductors, initially lie on common normally [10].

At first let's fulfill the solution of task of static deformation initially parallel conductors at action of distributed attractive electromagnetic force. The boundary problem of symmetric deformation relatively dimensionless cumulative displacement of strings v has a form [10]

$$\frac{d^2 v}{ds^2} + \frac{\lambda^2}{1-v} = 0, \quad v(0) = v(1) = 0, \quad (4.2)$$

where $v = 2v_1/h$ is summary deflection of strings from non-deformable positions, h – initial gap, l –the length of string, $s = \frac{x}{l}$, $\lambda^2 = \frac{\mu_0 I^2 l^2}{\pi T h^2}$, T is the force of tension.

The integral of this equation has a form

$$\frac{1}{2}(v') - \lambda^2 \ln(1 - v) = h, \quad (4.3)$$

here $h = -\lambda^2 \ln(1 - v_m)$, $v_m = v(1/2)$ – maximum of displacement in a middle of the string. In result we obtain relation

$$s = \frac{1}{\sqrt{2}\lambda} \int_0^v \frac{dv}{\sqrt{\ln(1 - v) - \ln(1 - v_m)}} = \frac{\sqrt{2}}{\lambda} (1 - v_m) \int_{\varphi_1(v)}^{\varphi_1(0)} e^{z^2} dz, \quad (4.4)$$

$$\varphi_1(v) = \sqrt{\ln(1 - v) - \ln(1 - v_m)}, \quad \varphi_1(0) = \sqrt{-\ln(1 - v_m)}$$

From condition $v_m = v(1/2)$ we obtain the equation determining $v_m(\lambda)$

$$\lambda = 2\sqrt{2}(1 - v_m) \int_0^{\sqrt{-\ln(1 - v_m)}} e^{z^2} dz \quad (4.5)$$

Another way for obtaining this equation is using Galerkin method, supposed approximation $v = z \sin \pi s$. The projection conditions have a form

$$-z\pi^2 \int_0^1 \sin^2 \pi s ds + \lambda^2 \int_0^1 \frac{\sin \pi s}{(1 - z \sin \pi s)} ds = 0 \quad (4.6)$$

In result we obtain nonlinear transcendental equation for coefficient $z = v_m$

$$-\frac{\pi^2 z}{2} + \frac{\lambda^2}{\pi} \left\{ -\frac{\pi}{z} + \frac{2}{z\sqrt{1 - z^2}} \left[\frac{\pi}{2} + \arcsin z \right] \right\} = -\frac{\pi^2 z}{2} + \frac{\lambda^2}{z\pi} \left[\pi \left(\frac{1}{\sqrt{1 - z^2}} - 1 \right) + \frac{2 \arcsin z}{\sqrt{1 - z^2}} \right] = 0 \quad (4.7)$$

From this equation we also can find the dependence $z = v_m(\lambda)$, which comparison with analytic solution (4.5) is shown in Fig.16

Fig.16. Dependence $v_m(\lambda)$ 1-Galerkin solution, 2-analytical solution

The small free oscillations of tension string (filament) relatively non-deformable position are described by equation

$$\frac{T}{l^2} v'' - \rho \ddot{v} = 0 \quad (4.8)$$

with boundary conditions $v(0, t) = v(1, t) = 0$. In this equation the next designations are input: $\rho = \rho_V S$ – linear density of string, ρ_V – volume density of nano-string material, S – square of transversal profile, stroke $()'$ designates the derivative on dimensionless coordinate. The frequency of transverse oscillations on first eigen form is equal $\bar{\omega} = \pi \sqrt{\frac{T}{\rho l^2}}$.

Let's found the value of current, at which the static deformation by attraction of graphene layers can be estimate, used parameters of graphene nanoresonator: length $l = 1000nm$, width $\delta = 10nm$, thickness $b = 0.3nm$, resonance frequency $\bar{\omega} = 30MHz$ and Q-factor~100. From formula for frequency of free oscillation we find the tension force

$$T = \rho l^2 \bar{\omega}^2 / \pi^2 = 1,5 \cdot 10^{-12} N$$

Used the expression for parameter $\lambda^2 \approx 1$ (Fig.16) for gap $h \sim 10nm$ we obtain value of current strength $I_0 = \frac{h \lambda}{l} \sqrt{\frac{\pi T}{\mu_0}} = 40 mA$.

Introduced non-dimensional time $\tau = \frac{\bar{\omega}}{\pi} t$, we transform the equation of free oscillation (4.8) into the form:

$$v'' - \ddot{v} = 0 \quad (4.9)$$

with boundary condition $v(0, \tau) = v(1, \tau) = 0$, $()'$ – differencing by non-dimensional time. The eigenforms and frequency of free oscillations for tension string:

$$V_k(s) = C_k \sin \pi k s, \quad \Omega_k = \pi k, \quad k = 1, 2, \dots \quad (4.10)$$

At normalization $\frac{1}{\pi} \int_0^1 V_k V_j ds = \delta_{kj}$ we choose $C_k = \sqrt{2}$.

Let's consider the dynamic task, introduced into equation (4.2) inertial component and believed that currents are harmonic with amplitude I_0 ($I(\tau) = I_0 \sin \omega \tau$, $\omega = \frac{\pi \tilde{\omega}}{\bar{\omega}}$, $\tilde{\omega}$ – frequency of alternating current). Boundary problem takes a form:

$$\begin{aligned} v'' + \frac{\lambda^2 (1 - \cos 2\omega \tau)}{2(1 - v)} - \ddot{v} &= 0, \\ v(0, \tau) = v(1, \tau) &= 0 \end{aligned} \quad (4.11)$$

For simplification in future let's consider the small oscillation relatively static equilibrium position, which was founded taking into account the nonlinearity of deformation (for example as at (1.14)). The complete dynamical flexure we search as sum of static and alternating component: $v(s, \tau) = v_0(s) + \tilde{v}(s, \tau)$. In result the equation of oscillations relatively dynamical flexure takes a form:

$$\tilde{v}'' + \frac{\lambda^2 \tilde{v}}{(1 - v_0)(1 - v_0 - \tilde{v})} - \frac{\lambda^2 \cos 2\omega \tau}{(1 - v_0 - \tilde{v})} - \ddot{\tilde{v}} = 0. \quad (4.12)$$

The boundary conditions are homogenous $\tilde{v}(0, \tau) = \tilde{v}(1, \tau) = 0$. The solution is fulfilled in a diapason of parameter value $\lambda < \lambda_*$, where value λ_* conforms to parameter, responsible to static point of branching.

Let's investigated small free oscillations (without consideration the component in equation (4.12), clearly depended from time), relatively stable static equilibrium form taking into account

negative stiffness, excited by electrical force at deformable equilibrium form of graphene layer. The motion of this type without dissipation is described by linearized equation (4.12):

$$\zeta'' + \frac{\lambda^2}{(1-v_0(s,\lambda))^2} \zeta - \ddot{\zeta} = 0. \quad (4.13)$$

This is a linear equation with alternative coordinate positive coefficient

$$a(s, \lambda) = \frac{\lambda^2}{(1 - v_0(s, \lambda))^2}$$

Thus, we obtain the boundary problem: the equation

$$\zeta'' + a(s, \lambda)\zeta - \ddot{\zeta} = 0 \quad (4.14)$$

and border condition: $\zeta(0, \tau) = \zeta(1, \tau) = 0$.

The frequency and forms of small string oscillations as eigen functions taking in account attraction we search in form of the time harmonic function of boundary problem (4.13): $\zeta = Z(s)\sin(\Lambda\tau + \alpha)$, from where we receive the space boundary problem:

$$\begin{aligned} Z'' + a(s, \lambda)Z + \Lambda^2 Z &= 0, \\ Z(0) = Z(1) &= 0. \end{aligned} \quad (4.15)$$

The symmetric forms of oscillations taking into account attraction represent in form of series of odd forms of string oscillations without external action, determined by relations (4.10)

$$Z(s) = \sum_{i=1,3,\dots}^n \beta_i V_i(s). \quad (4.16)$$

Projective conditions for coefficients of mode expansion β_i have a form:

$$\int_0^1 (Z'' + a(s, \lambda)Z + \Lambda^2 Z) V_i(s) ds = 0, i = 1, 3, \dots \quad (4.17)$$

After transformation of system (4.17) taking in account (4.16) we receive a system of linear homogenous algebraic equations relatively of coefficients β_i

$$\beta_i (\Lambda^2 - \Omega_i^2) + \sum_{k=1,3,\dots}^n \beta_k a_{ki} = 0, i = 1, \dots, \quad (4.18)$$

where coefficient $a_{ki} = \int_0^1 a(s, \lambda) V_i(s) V_k(s) ds$. Equated to zero the determinant of system (4.18), we find the frequencies Λ_i , $i = 1, 3, \dots$ every value of which includes a set of coefficients $\beta_k^{(i)}$, $k = 1, 3, \dots$, and, accordingly, their modes of oscillations $V_i(s)$.

Keeping in expansion (4.16) only first form, we may find approximate alteration of first eigen frequency, corresponded to static attraction of strings:

$$\Lambda_1^2 = \pi^2 - \int_0^1 a(s, \lambda) V_1^2(s) ds \quad (4.19)$$

As the function $a(s, \lambda) = \frac{\lambda^2}{(1-v_0(s,\lambda))^2}$ is positive on the segment $[0, 1]$ on coordinate s , so integrand also is positive – integral nonnegative, and, accordingly, the frequency of free oscillations taking into account attraction decreases. This decreasing is droningly to come up to zero at value of parameter $\lambda = \lambda_*$, responsible to point of branching of static boundary problem. Namely in this

case variational equation for nonlinear boundary problem (4.13) has non-zero eigen-function, coincide with solution of boundary problem (4.15) at $\Lambda = 0$. The dependence of first eigen-frequency taking in account attraction at increasing of parameter λ , i.e. considering of increasing of current amplitude, is shown in Fig.17.

Fig.17.

Here Λ_1 – relative frequency, i.e. divided on the first eigen frequency of string without taking into account their attraction, which in dimensionless time is equal π .

Let's transform equation (4.12), leaving only linear relatively $\tilde{v}$ component,

$$\tilde{v}'' + \frac{\lambda^2(1-\cos 2\omega\tau)\tilde{v}}{(1-v_0)^2} - \frac{\lambda^2 \cos 2\omega\tau}{(1-v_0)} - \ddot{\tilde{v}} = 0, \quad (4.20)$$

The boundary conditions are homogenous $\tilde{v}(0, \tau) = \tilde{v}(1, \tau) = 0$. Initial condition for deformed string conforms to stable static form at given value of parameter λ . Thus, we may suppose that $\tilde{v}(s, 0) = 0, \dot{\tilde{v}}(s, 0) = 0$.

In case of small oscillations, we consider approximate solution of boundary problem (4.20) in form $\tilde{v} = Z_1(s)u(\tau)$, where $Z_1(s)$ – first mode taking into account attraction of strings. Projected equation on this form, we have differential equation in time:

$$u \int_0^1 Z_1'' Z_1 ds + u(1 - \cos 2\omega\tau) \int_0^1 \frac{\lambda^2 Z_1^2}{(1-v_0)^2} ds - \ddot{u} \int_0^1 Z_1^2 ds = \cos 2\omega\tau \int_0^1 \frac{\lambda^2 Z_1}{(1-v_0)} ds \quad (4.21)$$

Let's designate the integrals, including into (4.21), as $\int_0^1 \frac{\lambda^2 Z_1^2}{(1-v_0)^2} ds = p(\lambda), \int_0^1 \frac{\lambda^2 Z_1}{(1-v_0)} ds = f(\lambda)$.

Used equation for mode Z_1 (4.15), we obtain:

$$\ddot{u} + (\Lambda_1^2 + p(\lambda) \cos 2\omega\tau)u = -f(\lambda) \cos 2\omega\tau \quad (4.22)$$

Equation (4.22) – this is a linear non-autonomous differential equation with periodic coefficient and periodic right part. Homogenous part of equation (4.22) taking in consideration dissipation component presents in form

$$\ddot{u} + 2n\dot{u} + (a(\lambda) + p(\lambda) \cos 2\omega\tau)u = 0, \quad (4.23)$$

where $a(\lambda) = \Lambda_1^2$. Equation (4.23) has a form of Mathieu equation with constant excitation coefficient. At that both the frequency of free oscillations and coefficient of exciting depends from parameter λ , determined by current strength in conductors. Parametrical resonance takes place at

next relations of frequencies: $\omega = \omega_0, \omega_0/2, \omega_0/3, \dots$, where $\omega_0 = \Lambda_1$. The right part of equation (4.22) has harmonic component with frequency 2ω therefore at $2\omega = \omega_0$ the resonance of forced oscillation, the frequency of which coincides with second parametric resonance, take place. At $\lambda \rightarrow 0, v_0 \rightarrow 0, a(\lambda) = \Lambda_1 \rightarrow \pi, p(\lambda) \rightarrow 0$ and, consequently, dimensionless frequency of parametrical oscillation tends to π .

In first approximation periodic solution coincides to border of first (main) parametric resonance can found by harmonic balance method with frequency equal to half of frequency of external action

$$u = A \cos \omega \tau + B \sin \omega \tau. \quad (4.24)$$

In result we receive two linear algebraic equations relatively coefficients A and B , that gives the equation for frequencies, corresponded for borders of main parametrical resonance:

$$\begin{aligned} (a(\lambda) - \omega^2)A + 0.5 p(\lambda)A + 2n\omega B &= 0, \\ (a(\lambda) - \omega^2)B - 0.5 p(\lambda)B - 2n\omega A &= 0 \end{aligned} \quad (4.25)$$

The dependence of the zone of parametrical resonance from λ in mentioned approximation is determined by equal to zero the determinant of system (4.25) (Fig.18).

Fig.18 The zone of parametric resonance

The main idea of determination of nanoparticle mass, adherence to graphene layer, consists in alteration of parametrical resonance zone taking into account the change of eigen frequency and mode of free flexure oscillations. For simplification in further we assume, that significant alteration has place only with first symmetrical form and, thereafter, with first eigen frequency, what may be calculated, for example, at adherence of nanoparticle near string middle. At adhesion of nanoparticle both coefficients $p(\lambda), f(\lambda)$ in equations (4.22), (4.23) and eigen frequency Λ_1 are changed. The aim of analytical represent of approximated zone of parametrical resonance consists in estimation of conformity spectral properties of nano-string alteration at adherence of the nanoparticle, with alteration of excitement frequency (frequency of current).

Let's consider forced and parametrical nonlinear oscillations of string taking into account damping, described by equation (4.12) in the assumption of viscous damping for considerable nano-scales. We will look the solution in a form $\tilde{v} = Z_1(s)u(\tau)$, taken only first mode of free oscillations, which was founded only keeping the first mode of free oscillation taking into account attraction of the strings. Projective equation on the first mode of free oscillations, considered nonlinear components, can be written in a view:

$$\varphi(u)(\ddot{u} + 2n\dot{u}) + (b(\lambda)\Lambda_1^2 + \lambda^2 e(\lambda))u^2 - \Lambda_1^2 c(\lambda)u = \lambda^2 d(\lambda) \cos(2\omega\tau) \quad (4.26)$$

Here next designations are input:

$$\begin{aligned} \varphi(u, \lambda) &= b(\lambda)u - c(\lambda), \\ b(\lambda) &= \int_0^1 (1 - v_0(s, \lambda)) Z_1^3(s) ds, \\ c(\lambda) &= \int_0^1 (1 - v_0(s, \lambda))^2 Z_1^2(s) ds, \\ d(\lambda) &= \int_0^1 (1 - v_0(s, \lambda)) Z_1(s) ds, \\ e(\lambda) &= \int_0^1 \frac{Z_1^3(s)}{(1 - v_0(s, \lambda))} ds. \end{aligned} \quad (4.27)$$

Coefficient $\varphi(u)$ at highest derivations in equations (4.26) depends from unknown u and changing in time may have the return point, where $\varphi(u)$ equal 0.

The parameters, corresponded to contact of strings at their oscillations are determined the equality to zero the function $1 - v$. Substitution $v = v_0 + \tilde{v}$, where $\tilde{v} = Z_1(s)u(\tau)$, gives possibility to estimate the critical value of coefficient $u(\tau)$, at which strings may touch. Clearly, that $u_{crit} = \min\left(\frac{1-v_0}{Z_1}\right) = \frac{1-\max(v_0)}{\sqrt{2}} = 0$, where $\max(v_0)$ corresponds to flexure in a middle, coefficient $\sqrt{2}$ at Z_1 inputs from condition of normalization. Thus, it's needed to check, how the critical value of amplitude of oscillation, corresponded of string touch, and critical point in dependence $\varphi(u, \lambda)$ from parameter λ correlate:

Fig.19 The dependence of critical value of maximal flexure v_{cr} from λ

From this graphic we can see that at every values of parameters λ the level of return point is situated over then level of critical value, i.e. addition condition has a form $u < u_{crit}$. Indicated value it is necessary to check at calculation the motion of dynamical deformation at solution of the equation (4.26).

By the same meaning, the main interest represents the behavior of system at the frequencies near to main parametrical resonance and, consequently, at value of current frequency near to free oscillations frequency. Let us stay on the determination of amplitude-frequency characteristic of parametrical resonance, believed that $\omega \sim \Lambda_1$.

Attempts to receive the stationary regimen in zone of parametrical resonance by harmonic balance method give the system algebraic nonlinear equations, the analytic solution of which looks impossible. Therefore, the numerical experiment with nonlinear equation (4.26) was fulfilled as near so inside zone of parametrical resonance. In time of realization of numerical experiment, the Cauchy task solved firstly nearly of right border of parametrical resonance zone at zero initial condition. At achievement of steady-state regime the Cauchy task solved again for slightly smaller value of frequency, but at initial condition, taken from solution in some moment of time. This procedure repeated at many values of frequencies until the left border of parametric zone is reached. In Fig. 20, 21, 22, 23 are shown the examples of oscillograms, obtained in result of fulfillment the numerical experiments

Fig.20.Oscillogram of oscillations at $\omega = 3.1190$

Fig.21.Oscillogram of oscillation at $\omega = 3.1160$

Fig.22.Oscillogram of oscillation at $\omega = 3.1130$

Fig.23.Oscillogram of oscillation at $\omega = 3.1129$

From these oscillograms it's seen, that out of zone of parametrical resonance amplitude of oscillation extremely small and oscillations have the second harmonic of excitement frequency. Inside of parametrical resonance zone the amplitude receives the value is order 0.7 of initial distance between graphene layers and consist only first harmonic of excitement frequency.

Thereafter results of numerical experiment the resonance curve was built, which was shown in Fig.24.

Fig.24 The resonance curve in a zone of parametric resonance

In Fig.24 the points designate the values of steady-state amplitudes of oscillations at appropriated values of frequency. Arrowheads are shown the direction of frequency alteration at transition from one regime to another.

Obtained zone of parametrical resonance is the enough narrow. The resonance curve turns out nonsymmetrical relatively to medium-sized of frequency band (zone of resonance). Moreover, we can observe the breaking on the resonance curve at scanner of exciting frequency top-down. To the small (relatively with zone of resonance) alteration of frequency corresponds abrupt decreasing of amplitude of steady-state regimes.

Thus, it was proposed the new scheme of graphene nano-resonator. Investigation of possible regimes of this resonator work is shown that in this device excitement of parametric oscillation with large amplitude is possible. The distribution of amplitudes on frequency diapason has resonance character and resonance curve has characteristic breaking.

Obtained results may be useful at using of this construction resonator as detector of mass of nanoparticle adherent on the one of its layers. For determination of size of adherent particle, it may be used the value of frequency at amplitude breaking, which more accuracy to measuring then the frequency of maximum on the resonance curve.

5. Self-oscillation regime of nanoresonator

In overwhelming majority of experimental works on study the graphene resonator [1-6], so as in works, where the various mechanical models of nano-resonators are considered [7-9], it's proposed to determine the eigen frequency of resonator by the method of building the amplitude-frequency characteristic (resonance curve). It means, in essence, the next. On the resonator the forced periodical action with some frequency is realized. Future the amplitude of steady-state oscillations of resonator is measured. After that the frequency of action is changed and again the amplitude of steady-state oscillation is measured. This procedure is realized in frequency diapason inside of which hypothetically the eigen frequency of resonator is situated. After that the dependence of amplitude of steady-state oscillations from frequency of action is built. The frequency corresponded to maximum value of amplitude assumes as required eigen frequency. The preference of this method is simplicity of experiment fulfillment.

Let's note the disadvantage of this method. At-first, the fulfillment of experiment demands the significant expenditures of time. The measuring must be carried out for large values of frequencies. At second, the determination of maximum values of amplitude may have the large inaccuracy. Namely nearly of resonance the amplitude deeply changes at small alteration of frequency, that make difficult the searching of this extremum. Using nonlinear resonance curves, which have abrupt breaking, significantly simplify the task of searching resonance frequency. At that we have another unpleasant factor – the frequency of breaking strongly depends from amplitude of action.

Oscillating system, in which self-oscillations possible hasn't these enumerated imperfections. Appearance of self-oscillations regime is possible at availability of positive feedback between oscillations system and the source of excitement of oscillations.

The important degree of self-oscillations regime is self-regulation on the resonance frequency at slow (in comparison with period of oscillation) alteration of parameters of oscillations system. For example, at adherent of nanoparticle on the graphene layer, the mass, taken part in oscillation process, is changed that stipulate to alteration of self-oscillation frequency.

The proposed scheme of self-oscillation resonator represents in Fig.25

Fig.25 The scheme of self-oscillation resonator

Self-oscillation resonator consists from amplifier, which must have positive feed-back connection. Feed-back circuit properly has graphene resonator and vibration transducer of graphene layer.

Output current of amplifier conducts by graphene layer. Itself graphene layer is situated in magnetic field. Induction vector is normal to current direction and direction of layer flexure. In result the magnetic force acts on graphene layer and excites its flexure.

Graphene layer and conductive surface under layer are like, which produces capacitor. To armature of capacitor the source of constant EMF U_0 is applied. At alteration of flexure of graphene layer the capacity is changed, that excite overcharge of capacitor. The current of overcharge depends from velocity of flexure alteration. Voltage applied to resistor R is proportionately to current of overcharge and input on entry of amplifier, thereby feedback of amplifier close. This feedback may be positive or negative, that is determined of magnetic field direction.

Let us use mechanical model of graphene layer at its transversal oscillation. This model represents itself electromechanical system, consisted from oscillation mechanical system with one degree of freedom and electrical circuit from in-series source EMF, resistor and capacitor, presented in Fig.26. Interference of mechanical system and electric circuit stipulates due to exciting of mechanical force acted on the oscillation mass and displacement of mass excites the change of capacitance of capacitor. In a case of current transmission on graphene layer and at the present of magnetic field the magnetic force will be done on the mass. The value of this force is proportional to value of current q_2 , magnetic induction B and length of conductor (graphene layer) L .

Fig.26 Electromechanical model of self-oscillating resonator

The equation of motion with addition force connected with availability of magnetic field (last term in equation (5.1))

$$m\ddot{x} + b\dot{x} + P(x) - \frac{1}{2} \frac{q_1^2}{C} + BL\dot{q}_2 = 0. \quad (5.1)$$

For two electric contours adequacy with scheme showed in Fig.26 let's write Kirchhoff equations

$$U_0 - R\dot{q}_1 - \frac{q_1}{C} = 0, \quad F(u_{BX}) - R_i\dot{q}_2 = 0, \quad (5.2)$$

where $F(u_{BX})$ - characteristic of amplifier. To this equation it is next to add relation

$$u_{BX} = R\dot{q}_1. \quad (5.3)$$

For convenience instead of charges in contours let us input the new unknowns. In particular it is more simply to input the voltage on condenser v , which is expressed from the charge

$$v = \frac{q_1}{C}. \quad (5.4)$$

Then the first from equation of system (5.2) takes a form

$$R(C\dot{v} + \dot{C}v) + v = U_0. \quad (5.5)$$

And output current of amplifier also may be expressed through voltage on the condenser v capacitance C and their derivatives

$$\dot{q}_2 = F\left(\frac{R}{R_i}(C\dot{v} + \dot{C}v)\right). \quad (5.6)$$

At last capacitance of capacitor may be expressed through flexure of graphene layer

$$C = C_0 \frac{d_0}{(d_0 - x)}, \quad (5.7)$$

where C_0 – capacitance of condenser at non deformable graphene layer.

Finally, we obtain the system of equations, described the oscillation process in considerable electromechanical system, which is written in form

$$m\ddot{x} + b\dot{x} + P(x) - \frac{1}{2} C_0 v_0^2 \frac{d_0}{(d_0 - x)^2} -$$

$$-BLF \left(\frac{R}{R_i} C_0 \left(\frac{d_0}{d_0 - x} \dot{v} + \frac{d_0}{(d_0 - x)^2} v \dot{x} \right) \right) = 0,$$

$$RC_0 \left(\frac{d_0}{d_0 - x} \dot{v} + \frac{d_0}{(d_0 - x)^2} v \dot{x} \right) + v = U_0. \quad (5.8)$$

The characteristic of amplifier may be difference. Let's believe that in common case the characteristic of amplifier has a form

$$F(u) = V_0 \frac{2}{\pi} \arctg \left(\frac{\pi}{2} K \frac{u}{V_0} \right), \quad (5.9)$$

here K – coefficient of increasing at small input signals, V_0 – voltage of limited signals on the exit of amplifier. This characteristic allows to taking into account almost linear sector at small input signals, smooth nonlinear at large input signals and limitation of output signal under some level, depended from feeding of amplifier.

The voltage V_0 must to choose from next consideration. At appearance of self-oscillating regime, the amplitude of output signal from amplifier will be closed to V_0 . In steady-state self-oscillation regime it's necessary that amplitude of graphene layer will be enough large, but not be exceed the value of initial gap. From this at first it is necessary to solve the task about resonance forced oscillations of graphene layer under action of magnetic interference and to choose the value of output current of amplifier so, that amplitude of graphene layer in resonance regime will be specified value in order 0,3 – 0,5 from the value from initial gap. Without losing generality it's may to choose that $V_0 = U_0$, and necessary of output current provide by the value of resistor R_i .

Equations (5.8) may be written in non-dimensional form

$$\xi'' + 2\nu\xi' + \xi + \gamma\xi^3 - \beta^2 \frac{\eta}{(1-\xi)^2} - \alpha F \left(\delta \left(\frac{1}{1-\xi} \eta' + \frac{1}{(1-\xi)^2} \eta \xi' \right) \right) = 0,$$

$$\delta \left(\frac{1}{1-\xi} \eta' + \frac{1}{(1-\xi)^2} \eta \xi' \right) + \eta = 1, \quad (5.10)$$

where dimensionless variables: $\xi = \frac{x}{d_0}$ – dimensionless flexure, $\eta = \frac{v}{U_0}$ – dimensionless voltage on the capacitor, $\tau = \lambda t$ – non-dimensional time, λ – eigen frequency of graphene resonator, calculated in case, then in elastic characteristic $P(\xi)$ taking into account only linear part, $()'$ – derivative on non-dimension time.

In equation (5.10) three dimensionless multipliers are inputted, which have concrete physical sense. First from theirs $\frac{1}{2} \frac{C_0 U_0^2}{m d_0^2 \lambda^2} = \beta^2$ – relation of energy of electrical field to maximum of kinetic energy of oscillation at amplitude equal to full gap. At that estimations it corresponds to value is order 0,01~0,1.

The second multiplier $\frac{\frac{U_0 BL}{R_i}}{m \lambda^2 d_0} = \frac{\frac{U_0 BL}{R_i}}{c d_0} = \alpha$ – relation of magnetic force at current, created by source with output voltage U_0 and internal resistor R_i , and elastic force, corresponded to flexure of graphene layer on the value of initial gap. About estimation of magnetic force has been said before. Multipliers α – value, inversed Q-factor of oscillations system.

The third multiplier $RC_0 \lambda = 2\pi \frac{RC_0}{T} = \delta$ – relation of time constant of capacitor charging and the period of eigen frequency. At the large value of this multiplier the condenser dos not have time

to overcharge by one period of graphene oscillations. At too small value of this multiplier – the voltage on the resistor R will be also small. Estimation of this parameter also is order $0,01 \sim 0,1$.

At last in equations (5.10) we have more one dimensionless multiplier – coefficients of increasing K . Choosing of this coefficient gives the possibility of excitation of oscillations.

Excitation of self-oscillations is possible then natural damping may be compensated by arrival of energy from external source. In a first from equation (5.10) we have two components, connected with velocity ξ' . First from its – viscous friction, corresponded energy absorption, and second is a force, creating by interference of current and magnetic field, corresponded for feeding of energy. At conform choosing of amplifier coefficients it's may to obtain the positive balance of energy and as result - excitation of oscillations.

The fulfilled numerical experiment with equations (5.10) is shown that at corresponded of choosing the amplifier coefficient K and limited voltage V_0 it is may to obtain excitation of oscillation with output on the steady-state regime. In Fig.27 one from oscillogram obtained by numerical integration of the system (5.10) is shown.

Fig.27 Oscillogram of self-oscillating process

At this oscillogram we see the process of excitation and next limitation of amplitude. At adherent of a nanoparticle on graphene layer the change of effective mass is occurred. In equation (5.10) it may taking into account the alteration of coefficient at highest derivative

$$(1 + \mu)\xi'' + 2\nu\xi' + \xi + \gamma\xi^3 - \beta^2 \frac{\eta}{(1-\xi)^2} - \alpha F \left(\delta \left(\frac{1}{1-\xi} \eta' + \frac{1}{(1-\xi)^2} \eta \xi' \right) \right) = 0,$$

$$\delta \left(\frac{1}{1-\xi} \eta' + \frac{1}{(1-\xi)^2} \eta \xi' \right) + \eta = 1, \quad (5.11)$$

where μ – relative increasing of effected mass.

In Fig.28 the oscillograms of steady-state self-oscillation at various values of additional mass are shown.

Fig.28 Steady-state self-oscillations at different values of adherent particle

$$1 - \mu = 0, 2 - \mu = 0.1, 3 - \mu = 0.2$$

The increasing of the mass of adherent particle is extended the period of steady-state self-oscillation and, consequently, the frequency decrease.

Separately let us consider the case of small time constant of time of capacitor charging. Then in second from equation (5.10) we may to neglect the speed of alteration the capacitor voltage and in an explicit form to obtain the voltage on the capacitor.

$$\eta = \frac{1}{1 + \frac{\delta \xi'}{(1-\xi)^2}} \quad (5.11)$$

Then at substitution (5.11) to the first equation from (5.10) we obtain the equation

$$\xi'' + 2\nu\xi' + \xi + \gamma\xi^3 - \beta^2 \frac{1}{(1-\xi)^2} - \alpha F\left(\delta \left(\frac{1}{(1-\xi)^2 + \delta \xi'} \xi' \right)\right) = 0. \quad (5.12)$$

For obtaining the representation about possibility of limit cycle from equations (5.12) let's consider phase portrait of free oscillation of graphene layer without magnetic field. Corresponding equation of free oscillation has a form

$$\xi'' + \xi + \gamma\xi^3 - \beta^2 \frac{1}{(1-\xi)^2} = 0 \quad (5.13)$$

In Fig.29 phase portrait, described by equation (5.13), is shown

Fig.29. Phase portrait of free oscillation

Evidently, that this system has two equilibrium position stable (center) and unstable (saddle). Equation (5.13) has energy integral

$$\frac{1}{2}(\xi')^2 + \frac{1}{2}\xi^2 + \frac{\gamma}{4}\xi^4 - \frac{\beta^2}{1-\xi} = H, \quad (5.14)$$

which allows to find H , corresponded to homoclinic separatrix. For its determination it is necessary to substitute phase coordinate of critical point (saddle). After that it is possible to find intersection of separatrix with abscissa axe 0 ξ , solved the equation

$$\frac{1}{2}\xi^2 + \frac{\gamma}{4}\xi^4 - \frac{\beta^2}{1-\xi} = H. \quad (5.15)$$

Let's consider the motion nearly stable position at small amplitudes, i.e. believing, that flexure ξ is small comparable with 1 (gap in resonator). Then equation (5.12) may to linearize relatively by ξ

$$\xi'' + 2\nu\xi' + \xi - \beta^2 - \alpha F(\delta\xi') = 0 \quad (5.16)$$

Keeping in amplifier characteristic only linear and cubic component, we obtain the equation

$$\xi'' + (2\nu - \alpha\delta K + \varepsilon(\xi')^2)\xi' + \xi - \beta^2 - \alpha F(\delta\xi') = 0 \quad (5.17)$$

It is known, that this equation has single limit cycle. Approximate estimation of its amplitude a

$$a = \sqrt{\frac{3}{4} \frac{\varepsilon}{\alpha\delta K - 2\nu}} \quad (5.18)$$

Existence of limit cycle is possible only at fulfillment the condition $\alpha\delta K > 2\nu$, i.e. at condition that inputting energy from source at small amplitudes more, then loosing energy for the account damping, and at large amplitude inversely smaller. Estimation of its amplitude gives the possible to show that limit cycle belongs to the area, which lies inside separatrix, determined by relation (5.15). The stable of limit cycle is determined by the sign of its characteristic ratio [11]

$$h = \frac{1}{T} \int_0^T (-2\nu + \alpha\delta K - 3\varepsilon y^2 + \alpha\delta \frac{\partial F}{\partial y}) d\tau. \quad (5.19)$$

Here integral takes at closed phase trajectory, corresponded limit cycle for periodic function $y(\tau) = \xi'(\tau)$ with period T . In result we obtain its expression

$$h = (\alpha\delta K - 2\nu) - \frac{1}{T} \int_0^T (3\varepsilon y^2 - \alpha\delta \frac{\partial F}{\partial y}) d\tau, \quad (5.20)$$

which determines its stability in case then $h < 0$.

Thus, the scheme of self-oscillation nanoresonator on graphene nanolayer is proposed.

Conclusions.

Obtained results may be useful at construction of the graphene nanoresonator as detector of mass of nanoparticle adherence on the one of its layers. At adherence of nanoparticles to graphene layer its spectral characteristics are changed. The using of forced resonance for determination the eigen frequency of nanolayer has some disadvantages. The determination of maximum value of amplitude may have a large inaccuracy and requires a dense set of excitation frequencies. At second maximum of amplitude-frequency characteristic is difficult to determine at law Q factor. In our work, we propose other ways to determine the changes in spectral characteristics which

have greater accuracy. For determination of size of stuck particle, it may be used the value of amplitude breaking, which take place in parametric resonance. In addition, with a low Q-factor, the parametric resonance zone narrows, which also gives an advantage in determination the spectral characteristics. In differential resonator free oscillations have character of beating. The characteristic envelope frequency is equal to the half of difference of partial frequencies and much less then partial frequency of every layer. At adherence of particle on graphene layer the partial frequency of this layer decreasing and the small alteration of partial frequency gives the large alteration of envelope characteristic frequency. The important advantage of self-oscillations regime is self-regulation on the resonance frequency at slow (in comparison with period of oscillation) alteration of parameters of oscillations system. For example, at adherence of nanoparticle on the graphene layer, the mass, taken part in oscillation process, is changed that stipulate to alteration of self-oscillation frequency.

Data availability Statement

The dataset, which was used and analyzed during the current study, at introducing physical parameters are available from the corresponding author Dr. Dmitry Skubov on request.

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