PU Adjacent Subcarriers Power Allocation for Controlling Interference of OFDM-CR Systems

Dipen Bepari
IIT (ISM): Indian Institute of Technology
SOUMEN MONDAL (soumen.durgapur@gmail.com)
National Institute of Technology Durgapur https://orcid.org/0000-0003-1632-4619
Ananya Chattopadhyay
Heritage Institute of Technology

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[1]Dipen Bepari · [2,*]soumen Mondal · [3]Ananya Chattopadhyay

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Abstract Interference is the primary limiting factor of Cognitive Radio (CR) systems. This paper proposes a new power allocation idea for orthogonal frequency division multiplexing (OFDM)-based CR systems to control the interference to the licensed primary users (PUs) caused for allowing cognitive users (CUs) to use a licensed band temporarily. The idea is *'redistribute the power among a minimum number of PU adjacent subcarriers'*. This idea has been implemented to modify two existing power loading schemes, and developed an evolutionary Genetic Algorithm (GA)-based power distribution method for maximizing CR system capacity of (i) a single PU, and (ii) multiple PUs CR paradigm under the constraint of total power budget, individual sub-channel power budget and, PU interference tolerance limit. Simulation results authenticated that the CU capacity improvement significantly depends on channel gain quality ('*good*' or '*bad*') of the PU adjacent subcarriers.

Keywords Cognitive Radio Systems · Capacity Maximization · Orthogonal Frequency Division Multiplexing (OFDM) · Subcarrier Power Allocation

1 Introduction

Now-a-days, growing demand for spectrum, and the problem of inefficient use of limited spectrum can be overcome efficiently and reliably by implementing cognitive radio (CR) systems recognised as a revolutionary dynamic spectrum access (DSA) technique [1] [2]. Unlicensed cognitive users (CUs) are allowed to share radio spectrum concurrently with licensed primary users (PUs) as long as interference from CU to PU is lower than the interference tolerance

[1]Department of Electronics Engineering, IIT (ISM), Dhanbad, Jharkhand, India
[2]Department of Electronics and Communication Engineering, National Institute of Technology, Durgapur, West Bengal, India.
[3]Department of Electronics and Communication Engineering, Heritage Institute of Technology, West Bengal, Kolkata
*Correspondence E-mail: soumen.durgapur@gmail.com

limit [3] [4]. Orthogonal Frequency Division Multiplexing (OFDM) is the most suitable technique that can allow impressively both type users (PU and CU) in side-by-side subcarriers of a same frequency band [5]. In OFDM-based CR system, CUs are flexibly allowed to use the spectral holes or unused subcarriers temporarily left by the licenced system. Maintaining a sufficient PU protection, CUs are also able to use the subcarriers with PU even when there are no unused subcarriers left by the licence system [6].

Though the CUs transmit through an opportunistic spectrum hole of OFDM-based CR systems, mutual interference between licensed systems and the cognitive systems still exists due to non-orthogonality when the PU and CU are operated in a side-by-side band [7]. Mutual interference is one of the fundamental restrictive issues of facilitating an excellent capacity performance of CR systems. The traditional water-filling algorithm is an optimal power allocation method that avoids power allocation to the subcarriers with poorer channel gain. However, the application of this algorithm is limited to the classical OFDM systems. Iteratively partitioned water-filling (IPWF) [6] algorithm is also incompetent to restrict the mutual interference between PU and CU. Lagrangian dual method is also reported for power allocation to enhance spectrum utilization [8] [9]. However, computational complexity of the Lagrangian multiplier based optimal methods is considerably high [10]. Therefore, an appropriate suboptimal power allocation method needs to be implemented not only to protect the PU from harmful interference but also to enhance the spectral utilization. Exponential power distribution [11], Full–Filling Algorithm [12], geometric water filling [13] and iterative Dinkelbach method (IDM) [14] are some of the identified methods reported to limit mutual interference and maximize system capacity. Though the PU adjacent cognitive subcarriers introduce a maximum amount of PU interference, most of the reported methods [11] [12] [13] maintain the PU interference constraint by reducing the power of all subcarriers.

We are motivated from [7] where authors illustrated the effect of non-orthogonality in the OFDM systems allowing two types users (PU and CU) to share a frequency spectrum. It clearly shows that the CU subcarriers that are adjacent to PU introduce maximum amount of interference, and CU subcarriers that are far (in term of spectral distance) from PU produce a negligible amount of PU interference. In various power loading schemes like scaling, suboptimal, stepladder [15] [16] [17], when PU interference constraint is not satisfied, power of all the CU subcarriers (PU adjacent and non-adjacent) are redistributed whereas, only PU adjacent CU subcarriers are mainly responsible for intolerable PU interference. The Nulling technique, in contrast, is supposed to be the simplest method of controlling PU interference, where zero power is allocated to the CU subcarriers adjacent to PU. However, this process loses frequency diversity as no power is allowed to allocate in the PU adjacent subcarriers even when channel gain quality is excellent.

Genetic Algorithm (GA), based on the Darwin's 'Theory of Evolution', is a class of evolutionary algorithms inspired by genetic evolution and natural selection of species in nature [18]. To the best of our knowledge, power allocation for capacity maximization in OFDM based CR system using GA has not been reported much. In [19], throughput is maximised by GA-based schedulers. However, power allocation was not their concerned. In [20], GA-based power allocation algorithm was proposed to maximize throughput of an underlay-based CR system. GA was found as an effective power distribution method for a sensing-free underlay spectrum sharing-based CR system [21]. However, capability of GA in power allocation for OFDM-CR system was not investigated in [20] [21]. Investigating the ability of GA-based power distribution in maximizing capacity of OFDM-CR system is one of the objectives of this paper. Based on the above discussions, the major contributions of this paper are summarised as below.

- Our proposed idea has been implemented to develop a GA based suboptimal power allocation scheme that maximizes CU sum capacity maintaining a probabilistic PU interference constraint with a minimal sacrifice of spectral efficiency. The algorithm selects a minimum number of PU adjacent subcarriers based on their interference factor, and efficiently reallocates the powers of those subcarriers keeping non-adjacent subcarriers powers unchanged.
- Furthermore, we have modified two existing algorithms namely scaling method [17], and suboptimal method [15]. Modified algorithms redistribute only the power of PU adjacent subcarriers, and ensure to maintain interference constraint with minimum sacrificing of spectral efficiency.
- Finally, we have investigated the impact of channel gain quality (good/bad) on the capacity maximization problem under (i) individual PU interference constraint of each PU band, and (ii) aggregate interference constraint of all PU bands. Good channel indicates the scenario when PU adjacent channel gain quality is better than the others, and bad channel indicates that the PU adjacent channel gain quality poorer than the others.

The rest of the paper is organized as follows: Section 2 presents an OFDM-CR system model in space and frequency domain. Section 3 discusses CR constraints, and formulates optimization problems. The proposed adjacent subcarriers selection process for power redistribution is illustrated in Section Section 4. Different power allocation algorithms schemes and comparative performance evaluation of proposed power allocation schemes, are analyzed in Section 5 and section 6 respectively. Finally, the conclusion of this paper is drawn in Section 7.

2 System Model in Space and Frequency Domain

A typical OFDM-based CR system model of two PU transmitter-receiver pairs, and one CU transmitter-receiver pair have been depicted in Fig. 1. The system model in special domain contains two types of PU: PU_1 and PU_2. Firstly, PU_1 geographically co-located in the same area as CU i.e., PU_1 located inside the CU reliable sensing region. Hence, CU can detect activities of PU_1, and acquire

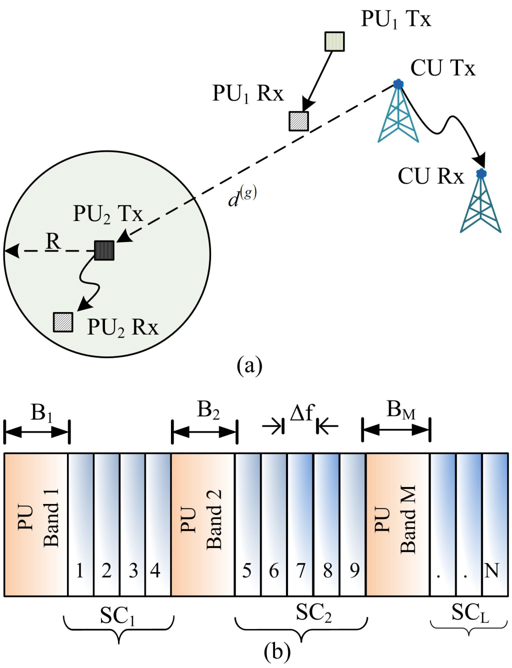

Fig. 1: OFDM-bsed cognitive radio system model in (a) special domain and (b) frequency domain

precise information of available spectral holes. Secondly, PU_2 is located outside the CU reliable sensing region, therefore, goes undetected. For first type PU, CU subcarriers are adjacent or/and in-between two PU occupied frequency bands. CU transmission demand highly flexibility in spectral shape of transmit signal for this interweave approach. A typical frequency domain spectrum arrangement has been shown in Fig. 1(b). For this scenario, N number of PU unoccupied subcarriers separated by Δf are grouped into a L number of sub-channels (SC) correspond to CU. On the other hand, M numbers of the frequency band of bandwidth $B_1, B_2, B_3, ..., B_M$ are occupied by PU. It is assumed that PU activities are un-correlated, and the information of the PU spectral occupancy in the sub-channel is available to CU.

For second type PU, in each sub-channel, there may be other undetected PUs geographically located outside $d^{(g)}$, the CU reliable sensing range. To protect undetected PU, interference from CU transmitter to the boundary of PU transmission region (of radius R) kept below P^{tx}, a certain predefined power level. The minimum distance from CU transmitter-to-PU receiver is given by $d^{(s)} = d^{(g)} - R$, and the received power at the boundary of PU transmission region expressed as $P^{sc} = P^{tx}(d^{(g)} - R)^{\eta}$, where η is the path loss exponent. For undetected PUs, exact location is not available to CU. Therefore, the CU assumes PU is located just outside of $d^{(g)}$, its reliable sensing region, and sets the distance from CU transmitter-to-PU receiver as $d^{(g)}$. Considering P_i as the allocated power to ith subcarrier, c_i the capacity of ith subcarrier is given by

$$c_i = \varDelta f \log_2 \left(1 + \frac{|{h_i}^{ss}|^2 P_i}{\sigma^2 + \tau}\right) \tag{1}$$

where ${h_i}^{ss}$ is the channel gain between CU transmitter to its corresponding receiver of ith subcarrier, σ^2 is additive white Gaussian noise (AWGN) variance, and τ is the aggregate interference to the ith CU subcarrier introduced by undetected PU of the same band and side by side co-located PU's band signal which is approximated as AWGN [16].

3 CR Constraints and Problem Formulation

This paper presents various types of constraints of the CR paradigm, and formulates an optimization problem for maximizing the sum capacity of the CUs.

3.1 Constraints of the CR Paradigm

There are mainly two types of constrains in the CR system; power constraint, and interference constraints, which are discussed in details.

Power constraint: The two type power constraints are (i) maximum allowable total power budget for N CU subcarriers, and (ii) maximum permissible sum power to each L sub-channels. P_T, the CU total power constraint expressed as $\sum_{i=1}^{N} P_i \leq P_T$. For the second type of PU, CUs are allowed to use a frequency band of undetected PU keeping sum power allocated to each sub-channel below P_j^{sc}, a threshold power of jth sub-channel. Zero power is allocated to the each sub-channel where a PU signal detected [6]. If S_j is the sum power allocated to jth sub-channel then individual CU sub-channel power constraint can be expressed as $S_j = \sum_{i \in j} P_i \leq P_j^{sc}, \quad \forall j \in \{1, 2, ..., L\}$.

Interference Constraint: Aggregated PU interference constraint signifies that the sum of the interference from each CU subcarrier introduced to each PU band must be below I_{th}, the interference threshold. The individual PU band interference constraints signify that interference to each PU band must be below $I_{th}^{(m)}$, the interference threshold of mth $(m = 1, 2, 3, ..., M)$ PU band. The interference introduced by ith CU subcarrier to m^{th} PU sub-channel, $I_i^{(m)}$ expressed as [9] [15]

$$I_i^{(m)} = |h_m^{sp}|^2 P_i T_s \int_{d_{im} - B_m/2}^{d_{im} + B_m/2} \left(sinc\left(f T_s\right)\right)^2 df \tag{2}$$

where h_m^{sp} is the channel gain coefficient of the CU transmitter-to-mth PU receiver link, T_s is the symbol duration, d_{im} is the spectral distance from ith

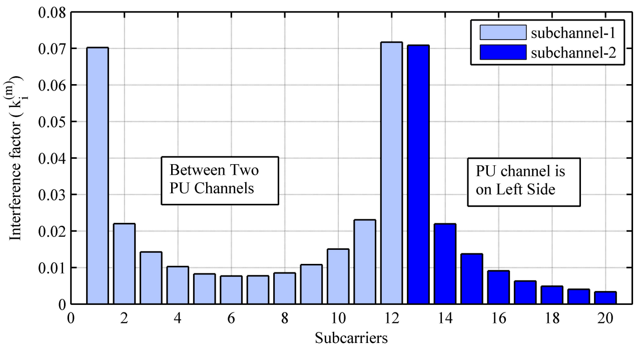

Fig. 2: Interference factors of the subcarriers, where $\Delta f = 0.3125$MHz, $B_1 =$ 2MHz, $B_2 = 3$ MHz, and $T_s = 4\mu s$, subchannel-1 is located inside two PU bands and subchannel-2 is located right side of PU band.

CU subcarrier to mth PU band, and B_m is the occupied bandwidth of mth PU band. The inference introduced by ith subcarrier is random. However, from (2), $k_i^{(m)}$, the deterministic part (interference factor) of (2) is expressed as

$$k_i^{(m)} = T_s \int_{d_{im}-B_m/2}^{d_{im}+B_m/2} (sinc(fT_s))^2 df \tag{3}$$

The deterministic interference factor depends on T_s, the symbol duration, d_{im}, the spectral distance from ith CU subcarrier to mth PU band and B_m, the occupied bandwidth of mth PU band. The value of interference factor, shown in Fig. 2, validates that the subcarriers adjacent to PU band introduce major amount of interference, and the subcarriers located far from PU band produces minor amount of interference.

Individual PU interference constraints: The interference constraint of mth PU band can be expressed as $|h_m^{sp}|^2 \sum_{i=1}^{N} P_i k_i^{(m)} \leq I_{th}^{(m)}$. The term $k_i^{(m)}$ is associated with spectral distance from PU frequency band to CU subcarrier. The channel gain qualities of CR transmitter to both the CR and PU receivers may not be available perfectly at the CR transmitter. Hence, the interference threshold of mth PU band is formulated as a probabilistic interference constraint, given as

$$\mathcal{P}_r \left(\sum_{i=1}^{N} |h_m^{sp}|^2 k_i^{(m)} P_i \leq I_{th}^{(m)} \right) \geq P_a, \qquad \forall m \in \{1, 2, ..., M\} \tag{4}$$

Here $\mathcal{P}_r$ is symbolised as a probability. The minimum probability of interference that a CU needs to maintain below $I_{th}^{(m)}$ is denoted by P_a. If the channel gains are Rayleigh distributed, then $|h_m^{sp}|^2$ is corresponding to an exponentially distributed with mean λ_m, expressed by

$$1 - \exp\left(\frac{-I_{th}^{(m)}}{2\lambda_m^2 \sum_{i=1}^{N} k_i^{(m)} P_i}\right) \geq P_a \tag{5}$$

Considering $\lambda_1 = \lambda_2 = ... = \lambda_M = \lambda$ and assuming

$$I_{eff}^{(m)} = \frac{I_{th}^{(m)}}{2\lambda^2\left(-\ln(1-P_a)\right)} \tag{6}$$

From (5), the interference constraint of mth PU band expressed as

$$\sum_{i=1}^{N} P_i k_i^{(m)} \leq I_{eff}^{(m)}, \quad \forall m \in M \tag{7}$$

Aggregated PU interference constraints: If I_{th} is the aggregated interference threshold, then probability-based interference constraint is given by

$$\mathcal{P}_r\left(\sum_{i=1}^{N}\sum_{m=1}^{M} |h_m^{sp}|^2 P_i k_i^{(m)} \leq I_{th}\right) \geq P_a \tag{8}$$

Comparing (8) with individual PU band interference constraint (4) and using (5), aggregated PU interference constraint can be expressed as [22]

$$\sum_{i=1}^{N} P_i\left(k_i^{(1)} + k_i^{(2)} + ... + k_i^{(M)}\right) \leq \left(\frac{I_{th}^{(1)}}{\lambda_1} + \frac{I_{th}^{(2)}}{\lambda_2} + ... + \frac{I_{th}^{(M)}}{\lambda_L}\right)\left(\frac{1}{-2\ln(1-P_a)}\right) \tag{9}$$

Assuming, $\sum_{m=1}^{M} k_i^{(m)} = K_i$, $\lambda_1 = \lambda_2 = ... = \lambda_M = \lambda$, $I_{th}^{(1)} + I_{th}^{(2)} + ... + I_{th}^{(M)} = I_{th}$, and $I_{eff} = \frac{I_{th}}{2\lambda^2(-\ln(1-P_a))}$ the aggregated PU interference constraint expressed as

$$\sum_{i=1}^{N} P_i K_i \leq I_{eff} \tag{10}$$

3.2 Problem Formulation for Capacity Maximization

Now we formulate a capacity maximization problem for two different scenarios of CR system. First, when all the PUs are inside the CU sensing region, and second, when some PUs are outside the CU sensing region.

Problem Formulation for individual PU interference: It is assumed that only first type PU exists in the CR system i.e., all PU are geographically located inside the CU reliable sensing region. The presence/absence information of PU is accurately available to CU. Therefore, for individual PU band interference scenario power constraint of each sub-channel does not need to consider in capacity optimization problem. The optimization problem is expressed as

$$C = \max_{\mathbf{P}} \sum_{i=1}^{N} \Delta f \log_2\left(1 + \frac{|h_i^{ss}|^2 P_i}{\sigma^2 + \tau}\right) \tag{11}$$

subject to

$$P_i \geq 0, \quad \forall i \tag{12}$$

$$\sum_{i=1}^{N} P_i \leq P_T \tag{13}$$

$$\sum_{i=1}^{N} P_i k_i^{(m)} \leq I_{th}^{(m)}, \quad \forall m \tag{14}$$

Theorem 1: The optimal power allocation vector $\mathbf{P}$ ($\mathbf{P} = P_i,\ i = 1, 2, ..., N$) for capacity maximization of problem (11) with constraints (12, 13, 14) can be expressed as

$$P_i = \left[\frac{1}{\alpha + \sum_{m=1}^{M} \delta_m k_i^{(m)}} - \frac{\sigma^2 + \tau}{|h_i^{ss}|^2} \right]^+, \quad \forall i \in N \tag{15}$$

where, $[x]^+ = \max(0, x)$, α, and δ_m are the Lagrange parameters.
Proof The proof is presented in **Appendix A**.
Problem formulation for aggregated PU interference: The interference constraint (8) denotes the PU aggregated interference constraint to be incorporated in the optimization problem. The capacity maximization problem can be expressed in totality as

$$C = \max_{\mathbf{P}} \sum_{i=1}^{N} \Delta f \log_2 \left(1 + \frac{|h_i^{ss}|^2 P_i}{\sigma^2 + \tau} \right) \tag{16}$$

subject to

$$P_i \geq 0 \tag{17}$$

$$\sum_{i=1}^{N} P_i \leq P_T \tag{18}$$

$$S_j \leq P_j^{sc} \quad \forall j \in \{1, 2, ..., L\} \tag{19}$$

$$\sum_{i=1}^{N} P_i K_i \leq I_{eff} \tag{20}$$

Theorem 1: The power allocation vector $\mathbf{P}$ ($\mathbf{P} = P_i$, $i = 1, 2, ..., N$) for optimal solution of capacity maximization problem (16) with constraints (17)-(20) can be expressed as

$$P_i = \left[\frac{1}{\alpha + \beta_{j,\phi(i)} + \delta K_i} - \frac{\sigma^2 + \tau}{|h_i^{ss}|^2} \right]^+, \quad \forall i \tag{21}$$

where, $[x]^+ = \max(0, x)$, α, β_j, $(j = 1, 2, ..., M)$ and δ are the Lagrange parameters.
Proof : The proof is reproduced from standard literature in **Appendix B**.

4 Adjacent Subcarriers Selection and Interference Constraint Modification

In this section, the processes of selecting '*n-adjacent*' subcarriers for both the individual PU interference and aggregated PU interference constraint are explained. The '*n-adjacent*' denotes n number of subcarriers of each side of PU band.

4.1 Adjacent Subcarriers for Individual PU Band interference Constraint

The CU sub-channels may be located (i) either side of a PU band or (ii) only right side a PU band or (iii) only left side of a PU band. If CU sub-channels are existing each side of mth PU band, then $A_m^{(b)}$, the '*n-adjacent*' subcarrier set with respect to mth PU band selected for power modification is given by

$$A_m^{(b)} = \{ \big(X_{(m-1)} - (n-1)\big), \big(X_{(m-1)} - (n-2)\big), ..., X_{(m-1)}, \big(X_{(m-1)} + 1\big), \\ \big(X_{(m-1)} + 2\big), ..., \big(X_{(m-1)} + n\big) \} \tag{22}$$

where $X_m = \sum_{k=1}^{m} N_k$, the number of subcarriers of kth ($k = 1, 2, ..., m$) SC is N_k and $X_0 = 0$. For example, in Fig.1(b), with respect to PU band 2 when for $m = 2$, $A_2^{(b)} = \{3, 4, 5, 6\}$ subcarriers are selected for power modification. When SC situated only left side of mth PU band then $A_m^{(l)}$, the '*n-adjacent*' subcarrier set selected for power modification is given by

$$A_m^{(l)} = \Big\{ \big(X_{(m-1)} - (n-1)\big), \big(X_{(m-1)} - (n-2)\big), ..., X_{(m-1)} \Big\} \tag{23}$$

When SC is existing only right side of mth PU band, $A_m^{(r)}$, the '*n-adjacent*' subcarrier set selected for power modification is expressed as

$$A_m^{(r)} = \Big\{ \big(X_{(m-1)} + 1\big), \big(X_{(m-1)} + 2\big), ..., \big(X_{(m-1)} + n\big) \Big\} \tag{24}$$

Based on the location of PU and CU band in frequency domain, only the power of $A_m^{(b)}$, $A_m^{(l)}$, and $A_m^{(r)}$ subcarriers need to modify for maintaining the individual PU band interference constraint as given below

$$S_n^{(m)} = \begin{cases} A_m^{(b)}, & \text{SC located both side of PU band} \\ A_m^{(l)}, & \text{SC located left side of PU band} \qquad \forall m \in M \\ A_m^{(r)}, & \text{SC located right side of PU band} \end{cases} \tag{25}$$

4.2 Adjacent Subcarriers for Aggregated PU Band Interference Constraint

According to the system model, first SC is located on the right side of first PU band and others are located in between PU bands. S_n , the '*n-adjacent*' subcarrier set selected from SC for power modification to meet (20) is expressed as

$$S_n = \left\{A_1^{(r)}, A_2^{(b)}, A_3^{(b)}, ..., A_L^{(b)}\right\} \tag{26}$$

4.3 Interference Constraint Modification

In the previous section, we have discussed the procedure for selecting '*n-adjacent*' subcarriers. Here, we will find minimum value of n, and maximum allowable interference by '*n-adjacent*' subcarriers.

- Obtain S_n and $S_n^{(m)}$ using (26) and (25) respectively for $n = 1$. To meet the aggregated PU band interference constrain, set $I_n = I_{th}$, $A_n = S_n$, and for individual PU band interference constraint of mth PU band set $I_n = I_{th}^{(m)}$ and $A_n = S_n^{(m)}$.
- I_{far}, the aggregated interference of the subcarriers that are not '*n- adjacent*' to the PU band is given as

$$I_{far} = \begin{cases} \sum_{i=1, i \neq A_n}^{N} P_i k_i^{(m)} & \text{individual PU interference} \\ \sum_{i=1, i \neq A_n}^{N} P_i K_i & \text{aggregated PU interference} \end{cases} \tag{27}$$

- Save I_d and A_n when $I_d = (I_n - I_{far}) > 0$, otherwise, increase n by one and repeat the above two steps till $I_d > 0$.

Interference constraint is reformulated for individual PU interference case as

$$\sum_{i \in A_n}^{N} P_i k_i^{(m)} \leq I_d \tag{28}$$

and for aggregated PU interference case as

$$\sum_{i \in A_n}^{N} P_i K_i \leq I_d \tag{29}$$

Satisfying of (28) and (29) guarantee fulfilment of (14) and (20) respectively. The simulation results reported here is to *3-adjacent* cases having considered a small number of subcarriers.

5 Power Allocation Algorithms

In this section, GA based power allocation method, modified suboptimal method and modified scaling techniques are discussed.

5.1 Power Allocation Method for Meeting Individual PU Band Interference Constraint:

5.1.1 Genetic algorithm-based power allocation method

GA is recognised as a potential robust search engine for optimization problems. It is capable of solving problems in complex spaces without performing any training, even in unknown environments. In power allocation, GA considers subcarriers powers as chromosomes, and based on the fitness value of the objective function, evaluates good or bad chromosomes. Next, it generates a new solution set by recombining the 'good' chromosomes, and if the solutions pass a fitness test survives in the next generation. The rest of the chromosomes are discarded from the solution set. The solutions which pass the fitness test meet all the constraints of the optimization problem. This procedure is followed at each generation unless the optimized solution is found or stopping criteria is satisfied. GA-based interference control method for individual PU bands has been given below.

Step-I: In order to meet (13), the total power is distributed to N CR subcarriers using traditional water-filling algorithm (TWF) as adopted in [23,24]. $P_i^{(t)}$, allocated power to ith subcarrier by TWF given as

$$P_i^{(t)} = \left\{0, \frac{1}{\gamma} - H_i^{-1}\right\} \qquad \forall i \in \{1, 2, ..., N\} \tag{30}$$

where H_i represents carrier to noise ratio (CRN) defines as $H_i = |h_i^{ss}|^2/(\sigma^2 + \delta)$, $1/\gamma$ represent the water level, and γ is the Lagrange constant calculated from

$$\sum_{i=1}^{N} max\left\{0, \frac{1}{\gamma} - H_i^{-1}\right\} = P_T \tag{31}$$

Step-II: Based on power allocation vector $\mathbf{P}^{(t)}$, M PU bands are grouped in two sets X and Y. The mth PU band belongs to in group X if $\sum_{i=1}^{N} {P_i}^{(t)} {k_i}^{(t)} \leq I_{th}^{(m)}$ otherwise in set Y. For $|Y| = 0$, $\mathbf{P}^{(t)}$ is the final allocated power. Otherwise go to Step-III.

Step-III: Find $S_n^{(m)}$ and I_d by applying adjacent subcarrier selection process as discussed in section 4 for mth, $m \in Y$ PU where $\mathbf{P} = \mathbf{P}^{(t)}$. Apply GA to modify the power of A_n group subcarriers of mth, $m \in Y$ PU that satisfies the interference constraint. The objective function and constraint of GA are given by (32), (33) and (34). Update the power $\mathbf{P}^{(t)}$ and go to step-II. GA objective function and constraints as expressed below

Objective function

$$C = min - \sum_{i \in A_n} log_2\left(1 + \frac{|h_i^{ss}|^2}{\sigma^2 + \delta}\right) \tag{32}$$

Linear constraint:

$$\sum_{i\in A_n} k_i^{(m)} P_i^{(t)} \le I_d \tag{33}$$

Bound Constraint:

$$0 \le P_i \le P_i^{(t)} \qquad \forall i \in A_n \tag{34}$$

5.1.2 Modified Suboptimal Method

Now we modify the suboptimal method proposed in [15], where the authors reduce the power of all the subcarriers equally until interference constraint is met. The Step-I and Step-II of this method are similar to the first two steps of GA based power allocation method. To meet the interference constraint of mth PU, we select A_n, and redistribute power of only A_n group subcarriers keeping powers of others subcarriers unchanged.
Step-III: Redistribute powers of A_n group subcarriers by

$$P_i^{(m)} = \frac{I_d}{N_a k_i^{(m)}}, \qquad \forall i \in A_n \tag{35}$$

Step-IV: update power $P_i^{(t)}, \forall i$, and go to Step-II.

Unlike the suboptimal method proposed in [15], in modified suboptimal method, the interference constraint is satisfied by modifying the power of only PU adjacent subcarriers. It is also noticeable that this approach guarantees at least one of the $(M+1)$ constraints (M interference constraints of (14), and total power constraint) meet strictly whereas in suboptimal method of [15], the power was scaled to do this.

5.2 Power Allocation Method for Aggregated PU Interference Constraint

In aggregated PU interference case, (18) and (19) are satisfied by implementing the IPWF method as presented in most of the works [6, 9, 17]. Let, $P_i^{(t)}$, $\forall i \in N$ is the power allocated to the subcarriers satisfying the constraints (18) and (19). The interference constraint (20) is met by GA and modified scaling method discussed below.

5.2.1 GA based aggregated PU interference control

Step-I: Find A_n, the group of '*n-adjacent*' subcarriers, and I_d by applying adjacent subcarrier selection process as discussed in section 4 where $\mathbf{P} = \mathbf{P}^{(t)}$.
Step-II: Apply GA to modify the power of group subcarriers. The objective function and constraint of GA are given by (32), (34) and (36).

$$\sum_{\forall i\in A_n} K_i P_i^{(t)} \le I_d \tag{36}$$

5.2.2 Modified Scaling Method

If $\sum_{i=1}^{N} P_i^{(t)} > I_{eff}$, unlike scaling method [17], powers of only A_n group subcarriers are scale down to satisfy interference constraint strictly. I_d and A_n are determined by adjacent subcarrier selection process. Modified scaling powers of A_n group subcarriers obtained as

$$P^s(t) = \frac{I_d \mathbf{P}(t-1)}{\sum_{\forall i \in A_n} K_i P_i^{(0)}} \tag{37}$$

where $\mathbf{P}(t)$ and $\mathbf{P}(t-1)$ are the subcarrier power of tth and $(t-1)$th iteration respectively.

5.3 Nulling Method

The nulling mechanism avoids power allocation (i.e., zero power) to the PU adjacent subcarriers to reduce the interference since the adjacent subcarriers introduce a maximum amount of interference. It is one of the simplest power allocation techniques for OFDM-CR systems with computational complexity $O(1)$ [9]. However, nulling mechanism losses frequency diversity and affects CU achievable sum-rate capacity due to assigning zero power to the PU adjacent subcarriers even when the channel gain condition is excellent. We have compared the performance of the proposed methods with a one-nulling mechanism that allocates zero power to the first subcarrier on either side of the PU frequency band.

6 Performance Evaluation

The simulation results for aggregated PU band interference have been carried out based on the following system parameters. We consider a simple OFDM-based CR system for simulation with two opportunistic sub-channels accessed by CU. Subcarrier 1 to 7 and subcarrier 8 to 15 are comprised with sub-channel-1 and sub-channel-2 respectively. All the channels are Rayleigh faded with unity average channel power gain. The numerical simulation has been performed considering the following parameters: $N = 17$, $P_T = 4 \times 10^{-3}$ watt, $P_1^{sc} = 1 \times 10^{-3}$ watt, $P_2^{sc} = 3 \times 10^{-3}$ watt, $I_{th} = 5 \times 10^{-6}$ watt, $P_a = 95\%$, $\lambda = 0.5$, $\sigma^2 = 3.437 \times 10^{-4}$, $\Delta f = 0.3125$ MHz, $B_1 = 2$ MHz, $B_2 = 3$ MHz, and $\tau = 1.65 \times 10^{-6}$ watt. Interference factors K_i are calculated from $k_i^{(m)}$ of (2).

The simulation result of individual PU band interference case is presented assuming a OFDM based CR system model with $M = 3$ primary bands (bandwidth $B_1 = 3$ MHz, $B_2 = 5$ MHz, and $B_3 = 4$ MHz) and $N = 19$ available subcarriers grouped into $L = 3$ CU sub-channels. First sub-channel comprises

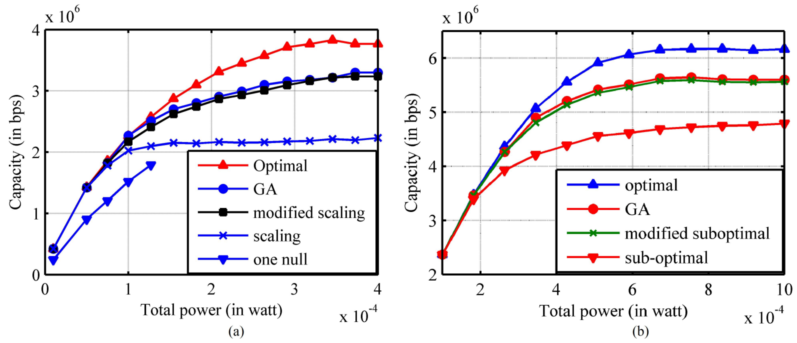

Fig. 3: Achievable CR system capacity as a function of total transmission power budget for a) individual, and b) aggregate PU interference constraint.

subcarrier 1 to 5 and located between B_1 and B_2. Second sub-channel is consisting subcarrier 6 to 13 and located between B_2 and B_3. Subcarriers 14 to 19 belong to third sub-channel located right side of B_3. The individual PU band interference constraints are $I_{th}^{(1)} = 3 \times 10^{-6}$ watt, $I_{th}^{(2)} = 5 \times 10^{-6}$ watt, and $I_{th}^{(3)} = 1 \times 10^{-6}$ watt. The other parameters are used for numerical simulation are same as used in the simulation of aggregated PU interference case. For both the aggregate and individual PU band interference scenario GA is performed based on following parameters: population size = 30, number of generations = 150, crossover fraction = 0.85, mutation probability = 1%, elitism= 4, function tolerance = 1×10^{-9} , Roulette wheel selection process and, a hybrid minimization function 'fmincon' has been used for simulation. The simulation results are obtained after averaging 10^5 independent simulations run.

Figure 3 depicts the relation between average achievable CR system capacity and total power budget for a) individual and b) aggregate PU band interference. The suboptimal method referred here is the one proposed in [15]. It is observed in both the Fig.3 (a) and (b) that all the methods are almost equally efficient for lower power budget (below 2×10^{-4} for Fig.3 (a) and 1×10^{-4} for Fig.3 (b), except one null) as satisfying the interference constraint is not much challenging. The sum capacity tends to saturate beyond a certain power level as interference constraint restricts higher power allocation to the CU subcarriers. It is also noticeable that the modified scaling and modified suboptimal methods outperform the scaling and suboptimal methods respectively, and one nulling performance is the poorest among all processes. The reason can be explained from Fig.2, which shows the typical variation of interference factor values of subcarriers. It is certainly noticeable that the significant amount of PU interference is introduced from PU adjacent subcarriers. Our proposed modified methods, unlike scaling and suboptimal, only redistribute the power of PU adjacent subcarriers that are the leading source of PU interference. GA based power allocation is much better than scaling and suboptimal. However, the capacity improvement using GA is not that much

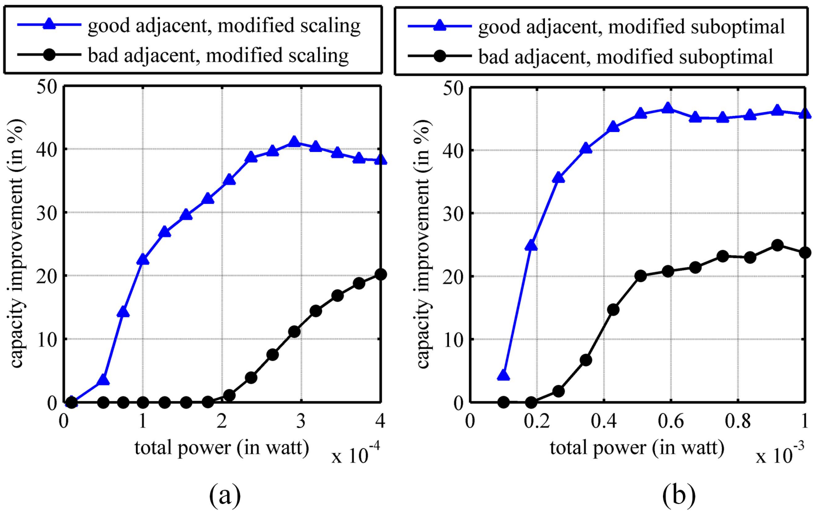

Fig. 4: Capacity of (a) modified suboptimal and (b) modified scaling methods for good and bad adjacent subcarriers.

significant while compare with modified method for both the aggregate and individual PU interference. The idea of redistributing power among only PU adjacent subcarriers group for controlling interference remarkably improves the CR system capacity.

Now, the capacity performance of modified methods is analysed considering the 'channel gain quality', i.e., 'good adjacent' and 'bad adjacent' subcarriers. The good adjacent denotes the subcarriers of '*n-adjacent*' group have better channel gain as compared to non-'*n-adjacent*' subcarriers, and bad adjacent signifies the '*n-adjacent*' subcarriers have relatively poorer channel gain as compared to non-'*n-adjacent*' group subcarriers. The effectiveness of the proposed modified methods is observed clearly in Fig.4 and Fig.5 when we consider the channel gain quality of the CU subcarriers adjacent to PU frequency band in the evaluation of capacity. It is observed in Fig.4 that channel gains quality of non-adjacent subcarriers mainly influence the capacity. Interestingly, the achievable capacity is more for bad adjacent subcarrier. During the bad adjacent case, most of the power is allocated to non-adjacent subcarriers. Unlike scaling and suboptimal methods, our proposed modified scaling and modified suboptimal methods redistribute the power of PU adjacent subcarriers, and maintain interference constraint by withdrawing very less amount of power from PU adjacent subcarriers. However, more amount of power needs to be withdrawn from PU adjacent subcarriers as most of the power is allocated to these subcarriers.

Figure 5 signifies magnitude of capacity improvement of (a) modified scaling over scaling methods for aggregated PU interference, and (b) modified

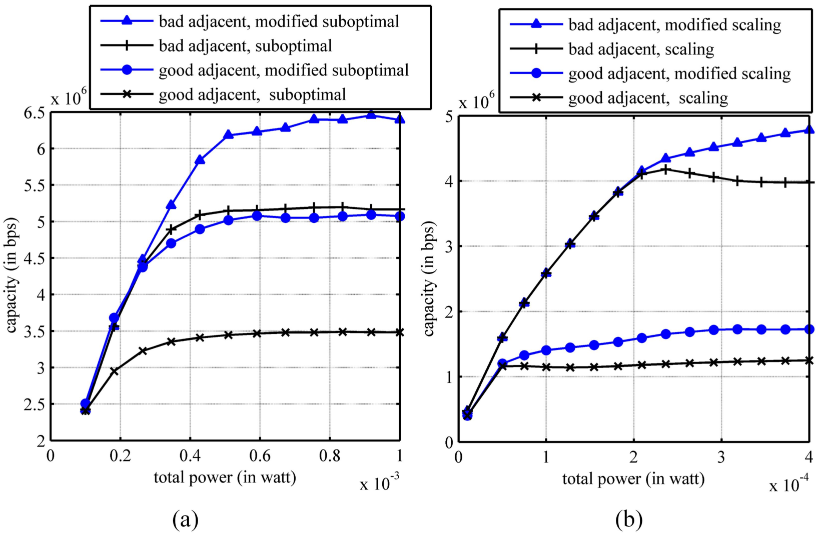

Fig. 5: Capacity improvement of (a) modified scaling and (b) modified suboptimal methods for good and bad adjacent subcarriers.

suboptimal over suboptimal methods for aggregated PU interference. The improvement of capacity has been investigated for both the good and bad adjacent subcarriers. The capacity improvement presented here by taking respective differences of capacity. We observe that for a given power budget, the capacity improvement is remarkably high for both the modified methods. In the good adjacent subcarriers, the improvement reaches 40% level which is much higher than the bad adjacent case for both the aggregate and individual PU interference constraint. This is because, in good adjacent case, major amount of power is assigned to the subcarriers that are adjacent to the PU frequency and mainly responsible for interference. The modified methods, to fulfil interference constraint, redistribute only the power of PU adjacent subcarriers (which are primary source of interference) keeping the power of non-adjacent subcarriers unchanged. In contrast, scaling and suboptimal methods redistribute the power of all subcarriers which reduces the achievable capacity.

7 Conclusion

The effectiveness of the PU adjacent subcarriers power modification for controlling the PU interference of OFDM-based CR system is analyzed. The strategy - *redistribute the power of a minimum number of PU adjacent subcarriers group keeping the power of non-adjacent subcarriers unchanged* is implemented to develop three suboptimal power allocation techniques named as modified suboptimal, modified scaling and GA based power allocation method. Numerical simulation results authenticate that the proposed methods significantly improve CR system capacity, and efficiently maintain interference constraint for both the aggregate and individual PU band interference constraint. GA based power allocation in term of capacity is equally efficient as the modified scaling and modified suboptimal algorithms in controlling the PU interference of OFDM-based CR systems. Benefits of modified algorithms are sufficiently acknowledged when channel gain qualities of PU adjacent subcarriers are considered. Performance of proposed modified scaling and modified suboptimal algorithms is more than 40% better scaling and suboptimal algorithms respectively for *good* adjacent subcarriers. The improvement is also remarkable for the *bad* adjacent scenario.

A Appendix

PROOF OF PROPOSITION 1: Maximizing a concave function is nothing but a minimizing the concave function. Introducing Lagrange parameters μ_i, α, and δ_m for (12), (13), and (14) inequality constraints of (11) respectively, the KKT conditions are written as

$$\mu_i \geq 0, \forall i \in \{1, 2, ..., N\} \tag{A.1}$$

$$\mu_i P_i = 0, \forall i \in \{1, 2, ..., N\} \tag{A.2}$$

$$\alpha \geq 0 \tag{A.3}$$

$$\alpha \left(\sum_{i=1}^{N} P_i - P_{total} \right) = 0 \tag{A.4}$$

$$\delta_m \geq 0 \tag{A.5}$$

$$\delta_m \left(\sum_{i=1}^{N} P_i k_i^{(m)} - I_{eff}^{(m)} \right) = 0, \forall m \tag{A.6}$$

$$-\frac{1}{h_i^{-1} + P_i} - \mu_i + \alpha + \sum_{m=1}^{M} \delta_m k_i^{(m)} = 0 \tag{A.7}$$

where, $h_i = \frac{|h_i^{ss}|^2}{\sigma^2 + \tau}$. Now, obtain μ_i from (A.7) and substitute into (A.2)

$$\mu_i = \alpha + \sum_{m=1}^{M} \delta_m k_i^{(m)} - \frac{1}{h_i^{-1} + P_i} \tag{A.8}$$

$$P_i \left(\alpha + \sum_{m=1}^{M} \delta_m k_i^{(m)} - \frac{1}{h_i^{-1} + P_i} \right) = 0 \tag{A.9}$$

Substituting (A.8) into (A.1)

$$P_i > \frac{1}{\alpha + \sum_{m=1}^{M} \delta_m k_i^{(m)}} - h_i^{-1} \tag{A.10}$$

If $\alpha + \sum_{m=1}^{M} \delta_m k_i^{(m)} < h_i$, then (A.9) only holds $P_i > 0$ and solving (A.9) P_i can be obtained which is $P_i = \frac{1}{\alpha + \sum_{m=1}^{M} \delta_m k_i^{(m)}} - h_i^{-1}$. On the other hand, if

$\alpha+\sum_{m=1}^{M}\delta_m k_i^{(m)} \geq h_i$, then $P_i > 0$ is not viable as it will violate (A.9) and the only solution is $P_i = 0$. Combining these two results, the solution is expressed as

$$P_i = \left[\frac{1}{\alpha+\beta_{j,\phi(i)}+\delta K_i} - \frac{\sigma^2+\tau}{\left|h_i^{ss}\right|^2}\right]^+, \forall i \tag{A.11}$$

The theorem 1 is proved.

B Appendix

PROOF OF PROPOSITION 2: Considering the Lagrange parameters α, β_j, δ, and μ_i for (17), (18), (19) and (20) inequality constraints of (16) respectively, the KKT conditions are written as

$$\mu_i \geq 0, \forall i \in \{1,2,...,N\} \tag{B.1}$$

$$\mu_i P_i = 0, \forall i \in \{1,2,...,N\} \tag{B.2}$$

$$\alpha \geq 0 \tag{B.3}$$

$$\alpha\left(\sum_{i=1}^{N} P_i - P_{total}\right) = 0 \tag{B.4}$$

$$\beta_j \geq 0, \forall j \in \{1,2,...,L\} \tag{B.5}$$

$$\beta_j\left(P_j^T - P_j^{sc}\right) = 0, \forall j \in \{1,2,...,L\} \tag{B.6}$$

$$\delta_m \geq 0 \tag{B.7}$$

$$\delta\left(\sum_{i=1}^{N} K_i P_i - I_{eff}\right) = 0 \tag{B.8}$$

$$-\frac{1}{h_i^{-1}+P_i} - \mu_i + \alpha + \beta_{j,\phi(i)} + \delta K_i = 0, \forall i \tag{B.9}$$

where, $h_i = \frac{|h_i^{ss}|^2}{\sigma^2+\tau}$. From (B.9) it can be written that

$$\mu_i = \alpha + \beta_{j,\phi(i)} + \delta K_i - \frac{1}{h_i^{-1}+P_i} \tag{B.10}$$

Now, substituting (B.10) into (B.2)

$$P_i\left(\alpha + \beta_{j,\phi(i)} + \delta K_i - \frac{1}{h_i^{-1}+P_i}\right) = 0, \forall i \in N \tag{B.11}$$

and substituting (B.10) into (B.1)

$$P_i > \left(\frac{1}{\alpha+\beta_{j,\phi(i)}+\delta K_i} - h_i^{-1}\right), \forall i \tag{B.12}$$

From (B.12), it is observed that $\alpha+\beta_{j,\phi(i)}+\delta K_i < h_i^{-1}$ when $P_i > 0$ and from (B.11) P_i expressed as $P_i = \frac{1}{\alpha+\beta_{j,\phi(i)}+\delta K_i} - h_i^{-1}$. On the other hand, if $\alpha+\beta_{j,\phi(i)}+\delta K_i \geq h_i^{-1}$ then due to violation of (B.11), $P_i > 0$ is impossible and the only solution is $P_i = 0$.
Combining these two results, the solution is expressed as

$$P_i = \left[\frac{1}{\alpha+\beta_{j,\phi(i)}+\delta K_i} - \frac{\sigma^2+\tau}{\left|h_i^{ss}\right|^2}\right]^+, \forall i \tag{B.13}$$

The theorem 2 is proved.

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Conflict of interest: There is no conflict of interest.
Code Availability: There is no specific code in this work.

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