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Article

Keywords: quantum communication, quantum error correction, security proof

Posted Date: October 1st, 2021

DOI: <https://doi.org/10.21203/rs.3.rs-951115/v1>

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Stream privacy amplification for quantum cryptography

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Privacy amplification is the key step to guarantee the security of quantum communication. The existing security proofs require accumulating a large number of raw key bits for privacy amplification. This is similar to block ciphers in classical cryptography that would delay the final key generation since an entire block must be accumulated before privacy amplification. Moreover, any leftover errors after information reconciliation would corrupt the entire block. By modifying the security proof based on quantum error correction, we develop a stream privacy amplification scheme, which resembles the classical stream cipher, to solve the problems of final key generation delay and error spread. The stream scheme can also help to enhance the security of trusted-relay quantum networks. Inspired by the connection between stream ciphers and quantum error correction in our security analysis, we further develop a generic information-theoretic tool to study the security of classical encryption algorithms.

I. INTRODUCTION

As one of the first applications of quantum information science, quantum key distribution (QKD) aims at establishing an information-theoretic secure key between two distant parties, Alice and Bob [1, 2]. It applies fundamental laws of quantum physics to guarantee secure communication. The procedures of QKD can be divided into two parts: quantum operation and data postprocessing. Quantum operation includes the preparation, transmission, and measurement of quantum states for Alice and Bob to share raw key bits. The purpose of postprocessing is to extract an identical and private key from the raw data. This can be guaranteed by information reconciliation and privacy amplification, where the former guarantees the identity of key strings and the latter removes any potential information leakage to a potential eavesdropper, Eve [3, 4].

With the last three decades of development, QKD has experienced tremendous advancement from initial demonstrations in laboratories to practical implementation [5]. In fiber, the communication distance has been pushed over 500 km [6, 7]. Using quantum satellites, the communication distance has reached an intercontinental level [8]. Researchers have pushed QKD to a secret key rate of more than 10 Mbits/s as well [9]. In addition to point-to-point link, a number of field-test QKD networks have been conducted in many countries [10–15]. In particular, recently China has successfully completed the 2000-km-long fiber-optic backbone link between Beijing to Shanghai [16]. Therefore, QKD is already a mature technique for real-life applications [17].

With the exciting developments on the experimental side, privacy amplification might still be technically difficult to implement in some realistic conditions. Existing privacy amplification methods run as follows. After information reconciliation, Alice randomly chooses a hash function and sends it to Bob via a public classical channel. Both users hash their reconciled key strings with the hash function and obtain the final key. In practice, the family of Toeplitz-matrix hashing is widely adopted. To guarantee the efficiency of privacy amplification, the block size is normally big, typically in the order of megabits [18]. Alice and Bob cannot perform privacy amplification until they accumulate an entire block of a reconciled key. This block feature of privacy amplification would cause unpleasant delays in some practical scenarios. For instance, in the satellite case, since the quantum signals can be transferred only when the ground station can “see” the satellite with a clear atmosphere, it may take the satellite several orbits to accumulate enough data for one block of privacy amplification. Due to the unpredictable condition of the atmosphere, such a delay could take as long as days [19].

There are other cases where block privacy amplification could cause problems. If the ratio of the reconciled key to the final key is extremely large, the computational cost for the hash function can be very heavy. Such an issue has been encountered in randomness extraction of quantum random number generation as well, especially in device-independent case [20]. Due to the similarity between the definitions of randomness extractors and privacy amplification [21], the extractors can be constructed by universal hashing functions [22] and have the same block feature as existing privacy amplification schemes. The problem of heavy computational cost is more serious in quantum random number generation due to larger amount of data and it restricts further improvement of realtime generation speed.

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Moreover, with block privacy amplification, the leftover error in information reconciliation would spread out to the entire block. Information reconciliation is normally done by bilateral error correction. For some error correction schemes, there is a small probability to leave some errors uncorrected. If the two strings of Alice and Bob are not exactly the same, the output strings from block privacy amplification would be totally different due to the universal property of the hash family.

Furthermore, existing quantum network implementations rely on trusted relays for key distribution [16], due to the limited transmission distance of point-to-point QKD links. Trusted QKD network has been widely used in building intercity or backbone QKD communication links, such as the Hefei network [23] and the Beijing-Shanghai backbone link [16]. In the trusted network scenario, all intermediate relays must be trusted because each of the relays can produce the final key. If one relay becomes compromised, the security of the whole network will be seriously threatened. In practice, it is challenging and expensive to guarantee high-level secure relays, which hinders the further commercialization and application of QKD. There are some attempts to reduce the dependence on trusted relays. Unfortunately, these solutions either require duplicate resources [24] or still assume the intermediate relays do not attack the network intentionally [25].

To address these issues, we reexamine the Lo-Chau original security proof based on quantum error correction [26], where privacy amplification is reduced from phase error correction [27]. By rearranging the phase error correcting gates and error syndrome measurement, we divide privacy amplification into two steps: (a) generate pseudo-random bits from a pre-shared key seed and a hash function; (b) XOR the pseudo-random string from (a) and the reconciled key. We also prove that the hashing matrix in (a) can be reused. Then, Alice and Bob can generate pseudo-random bits offline. For realtime privacy amplification, they only need to perform the XOR operation in a stream output way.

A. Security analysis based on quantum error correction

In establishing the security analysis of QKD, important tools such as entanglement distillation [28] and quantum error correction have been proposed and developed. A profound discovery is the connection between key privacy and entanglement. The amount of generated keys can be elegantly linked with distillable entanglement under local operations and classical communication [26]. One can distill almost perfect entanglement via quantum bit and phase error correction before measuring the quantum state to get the final key. Though security can be proven via entanglement distillation, one does not need to carry out this procedure in reality. Quantum bit and phase error correction can be carried out with classical means once the parameters are well estimated in the virtual quantum scenario [27]. In this framework, quantum bit error correction corresponds to information reconciliation, and quantum phase error correction is transformed into privacy amplification. Below, we briefly introduce the security analysis based on quantum error correction [26, 27] and leave the details in Methods. In the following discussions, we call a state transmission and measurement generating a pair of raw key bits as a QKD *round*. After many rounds, Alice and Bob accumulate enough raw key bits as a *block* for postprocessing, which we call a QKD *session*.

Alice and Bob aim at establishing a perfect Einstein-Podolsky-Rosen (EPR) pair $|\Phi^+\rangle = (|00\rangle + |11\rangle)/\sqrt{2}$ for each round, where $|0\rangle$ and $|1\rangle$ are the eigenstates of Pauli operator σ_z . Due to channel disturbance or Eve's interference, the EPR pairs shared by Alice and Bob after n rounds of state transmission are usually imperfect. We denote the state of these data pairs as ρ_{AB} . The difference between ρ_{AB} and $|\Phi^+\rangle\langle\Phi^+|^{\otimes n}$ can be seen as disturbance and characterized by bit error rate e_b and phase error rate e_p ,

$$\begin{aligned} e_b &= \frac{1}{2}(1 - \text{Tr}[\rho_{AB}(\sigma_z \otimes \sigma_z)^{\otimes n}]), \\ e_p &= \frac{1}{2}(1 - \text{Tr}[\rho_{AB}(\sigma_x \otimes \sigma_x)^{\otimes n}]), \end{aligned} \tag{1}$$

where σ_x is another Pauli operation and we take the Z basis for key generation. The value of e_b and e_p can be obtained by parameter estimation. If $e_b = e_p = 0$, this means that for the pair of qubits ρ in each round,

$$\text{Tr}[\rho(\sigma_z \otimes \sigma_z)] = 1, \tag{2}$$

$$\text{Tr}[\rho(\sigma_x \otimes \sigma_x)] = 1, \tag{3}$$

which indicates that $\rho = |\Phi^+\rangle\langle\Phi^+|$. To achieve Eqs. (2) and (3), Alice and Bob can apply the quantum circuit shown in Figure 1 to do the quantum bit and phase error correction. Quantum bit error correction guarantees Eq. (2) and quantum phase error correction guarantees Eq. (3).

The cost of quantum error correction comes from the ancillary EPR pairs used during the procedure. As an analogue to the cost in classical error correction, the number of ancillary EPR pairs equals the number of parity check

FIG. 1. (a) Quantum bit and phase error correction. The measurements in all the figures are Z -basis measurements by default. “CC” is short for classical communication. The $\oplus$ operation means XOR operations on classical bit strings. H represents the Hadamard gate applied to each of the involved qubits. I/σ_x represents identity or σ_x operation on the qubits depending on the error syndrome. (b) In the linear case, the hash functions can be represented by matrices and realized by a series of CNOT gates between data (as control) and ancillary (as target) qubits. The measurement outcomes of ancillary qubits would give the parity information of the data qubits.

bits. Then, bit and phase error correction will cost $t_b = nh(e_b)$ and $t_p = nh(e_p)$ ancillary EPR pairs, respectively. Therefore, the net generation rate of EPR pairs is given by [27],

$$r = 1 - h(e_b) - h(e_p). \quad (4)$$

We can further reduce the procedure to a prepare-and-measure one by moving the final measurement to the front of quantum error correction. The reduction of quantum bit error correction is straightforward since all the operations can be done equivalently on classical data bits and it finally becomes information reconciliation. In contrast, the final measurement cannot be moved before the quantum phase error correction directly. Alice and Bob can replace the final measurements with a properly chosen joint measurement, which can be moved before quantum phase error correction [27]. Then the joint measurement becomes a classical operation called privacy amplification. Finally, quantum error correction is reduced to a “measurement + postprocessing” procedure. The reduction is in the spirit of Shor-Preskill’s security proof [27]. We leave the details in Methods.

Due to the joint measurement, each final key bit depends on the measurement results from all the n data qubits. Hence, for privacy amplification, Alice and Bob need to wait for all the quantum states to be transmitted and measured in a QKD session. We call it *block* privacy amplification.

II. RESULTS

A. Stream privacy amplification

In the aforementioned reduction, Alice and Bob cannot move the final key measurement ahead of phase error correction directly. The main obstacle is that the Hadamard gate does not commute with the dephasing operation caused by the final key measurement,

$$\Delta_{Z^{\otimes n}}[\rho_A] = |0\rangle\langle 0|^{\otimes n} \rho_A |0\rangle\langle 0|^{\otimes n} + |1\rangle\langle 1|^{\otimes n} \rho_A |1\rangle\langle 1|^{\otimes n}, \quad (5)$$

where ρ_A denotes the state Alice holds, and the dephasing operation is defined with respect to the key measurement basis $Z^{\otimes n}$. The operation on Bob’s side is similar. In the Shor-Preskill reduction, Alice and Bob essentially construct a joint Z -basis measurement commuting with hash operations to circumvent this problem. As a result, this makes privacy amplification operating in blocks. Here, we rearrange the reduction of the phase error correcting gates and keep the individual Z -basis measurements in the quantum phase error correction. Consequently, we can render stream privacy amplification.

The key idea of our new reduction is to cancel all the Hadamard gates in quantum phase error correction, shown in Figure 1. The CNOT gate in the circuit always appears in pairs on Alice’s and Bob’s sides. We focus on one

pair of CNOT gates in quantum phase error correction part, as depicted in Figure 2(a). First, noticing $H^2 = I$, we add two consecutive Hadamard gates after each output qubits of the CNOT gate. Then, the four Hadamard gates, before and after each CNOT gate, exchange the roles of control and target qubits, $H^{\otimes 2}C_{\alpha\beta}H^{\otimes 2} = C_{\beta\alpha}$, where $C_{\alpha\beta}$ denotes a CNOT gate with control qubit α and target qubit β and $C_{\beta\alpha}$ is the other way around. For Bob's data qubit, the phase-error correcting operator I/σ_x becomes I/σ_z since $\sigma_z = H\sigma_x H$. Hence, we prove that the circuits (a) and (b) are equivalent in Figure 2. Since the new phase-error correcting operator I/σ_z does not affect the Z-basis measurement, this step can be skipped along with the error syndrome measurements on the ancillary qubits. The rest operations commute with the dephasing operation, $\Delta_{Z^{\otimes n}}$. Alice and Bob can add Z-basis measurements on ancillary qubits after the CNOT gates, since they are irrelevant at that point. Finally, they can move the final measurement before quantum error correction, as shown in Figure 2(c).

FIG. 2. Circuit (a) is derived from the quantum phase error correction phase in Figure 1 by adding Hadamard gates in dashed boxes which form identity operations. We take one pair of CNOT operations for illustration. Circuit (b) is equivalent to Circuit (a) by considering the following facts: $H^{\otimes 2}C_{\alpha\beta}H^{\otimes 2} = C_{\beta\alpha}$; $H^2 = I$; $H\sigma_x H = \sigma_z$. Since neither the identity nor the σ_z gate affects the Z-basis measurement, the operations in the dashed box of Circuit (b) are redundant and can be removed. Then by moving the Z-basis measurement on ancillary qubits before hash operation and changing quantum-control-flips to classical-control-flips, Circuit (b) turns into Circuit (c), a “measurement + postprocessing” case. Circuit (d) shows the case of multiple CNOT pairs taking the hashing circuit in Figure 1 (b) as an example. In the end, both Alice and Bob employ Circuit (d) to get final key strings.

So far, we only consider one CNOT gate. The hash operation in phase error correction shown in Figure 1 is composed of many CNOT gates. This reduction also works for the general hash operation case. With this argument, by inserting consecutive Hadamard gates $H^2 = I$ after each CNOT gate of phase error correction part in Figure 1, we can reduce the whole quantum error correction circuit to the “measurement + postprocessing” case, as shown in Figure 2(d).

With the new reduction, the final key is determined by single-qubit measurements plus bit flips. The Z-basis measurement on the ancillary EPR pairs would provide Alice and Bob with a secure key *seed*. The bit flips are controlled by the seed and the hashing matrix. Then, the i th final key bit, extracted from the i th data qubit, is independent of the other data qubits. Hence, the new procedure can output the final key in a stream, i.e., the users can get a secure key bit once a pair of raw key bits is reconciled successfully between Alice and Bob. Following the name of stream cipher in classical cryptography, we call it *stream* privacy amplification, as presented in Box 1. The hashing matrix M in Step 1 is the transpose of the original hashing matrix used in the quantum phase error correction phase of Figure 1, because the original hashing matrix acts on X basis while M acts on Z basis. The cost of stream privacy amplification lies in Step 2, where $nh(e_p)$ pre-shared secure bits are consumed. The final key rate matches Eq. (4). Note that we consider the infinite data size limit here. The finite data-size effects due to statistical fluctuations have been well considered in literature and shown in Supplementary Information.

Box 1: Stream privacy amplification

After information reconciliation, denote Alice and Bob's reconciled key to be $\vec{a} \in \{0, 1\}^n$.

1. Alice and Bob randomly choose a hashing matrix M of size $nh(e_p) \times n$.
2. Alice and Bob use an $nh(e_p)$ -bit seed, $\vec{d} \in \{0, 1\}^{nh(e_p)}$, to generate a pseudo-random string, $\vec{d} \cdot M$, where the dot product between the row vector and the matrix need to take modulo 2 addition.
3. The final key is given by $\vec{k} = \vec{d} \cdot M \oplus \vec{a}$.

With the above deduction, the failure probability of privacy amplification ε is given by that of quantum phase error correction. Given a phase error pattern from a typical set determined by parameter estimation, a randomly chosen hash function can identify it with a high probability $1 - \varepsilon$. This is the reason why Alice and Bob need a random hashing matrix in Step 1 of Box 1. Details of error correction and its failure probability are presented in Supplementary Information.

Note that the phase error pattern must be set before choosing the random hash function. That is, Eve cannot know the hashing matrix before Alice and Bob obtain the raw key bits. Naively, Alice can randomly pick up a matrix and send it to Bob via an authenticated but non-encrypted channel, as in the conventional case in block privacy amplification. Since this public transmission of the hashing matrix must be done after quantum measurement, Alice and Bob have to wait for the whole block to be transmitted and measured. Then, they lose the “stream” property in privacy amplification.

To solve this problem, we apply a different approach, where Alice and Bob generate an identical random hashing matrix locally with a pre-shared key and never reveal it to public. Then, they can prepare this matrix (Step 1) and the pseudo-random string (Step 2) before quantum transmission. A naive implementation of this approach, in which Alice and Bob generate M and $\vec{d}$ in each run of the privacy amplification, could consume too many pre-shared secure bits, as for most of the universal hashing matrices, the number of random bits required to generate the matrix is larger than the data size n . Fortunately, with the following theorem, Alice and Bob can reuse the private hashing matrix in multiple QKD sessions with a failure probability increasing linearly, satisfying the composable security definition [29, 30]. Since the failure probability can be exponentially small, the same hashing matrix can be used for many QKD sessions. Therefore, the cost of generating this hashing matrix is shared with these sessions, making the average cost negligible.

Theorem 1 (Reuse of hashing matrix in privacy amplification). *Given a QKD session, the failure probability of a randomly chosen hashing matrix for privacy amplification is upper bounded by ε . Then, for m QKD sessions, if Alice and Bob applies the same randomly chosen matrix for each session, the probability that privacy amplification fails in at least one session is upper bounded by $m\varepsilon$.*

Proof. Following the aforementioned deduction of quantum error correction, the failure probability of privacy amplification is determined by phase error correction. Now, the question becomes that given m phase error patterns, if Alice and Bob randomly pick a hash function to correct all the errors, what is the failure probability that at least one phase error pattern is unsuccessfully corrected. The answer is $m\varepsilon$ as given by the lemma of reusing hash functions for error correction in Supplementary Information. $\square$

Before running QKD sessions, Alice and Bob can perform Steps 1 and 2 and prepare the pseudo-random string in advance. They only need to run Step 3 in privacy amplification during realtime QKD, which is essentially composed of simple XOR operations and much faster than hash operations. Moreover, the locations of leftover errors in information reconciliation will remain the same after stream privacy amplification since the final key bit is only decided by the pseudo-random bit and the raw key bit at the same location.

B. Enhancing the security of trusted-relay QKD network

The major issue of a trusted-relay QKD network lies in the trustworthiness of intermediate nodes. There are some attempts to reduce the requirement on trusted relays, one of which is delayed privacy amplification [31, 32]. In the normal case, all the QKD links between two end-users Alice and Bob would generate secure keys between neighboring nodes. Then, the intermediate relays swap the keys by announcing the XOR results of two keys generated with two neighbors. In the delayed privacy amplification case, the relays swap the key right after information reconciliation. Then, Alice and Bob perform privacy amplification without the relays. In this case, they can eliminate the relays

from the final key generation process. That is, the relays do not get the final key directly. Of course, if the relays listen to classical communication to get privacy amplification matrices, they can still get the whole final key string. Therefore, the delayed privacy amplification scheme only works for *honest but curious* relays.

Now, we can combine stream privacy amplification with delayed privacy amplification to further reduce the trustworthiness of intermediate relays. After information reconciliation, all relays swap the keys by announcing the XOR results of two neighboring keys. Then, Alice and Bob would share a reconciled key string $\vec{a}$, which is also known to relays. Notice that Alice and Bob perform the steps in Box 1 locally. In particular, in Step 2, they generate the pseudo-random string $\vec{d} \cdot M$ privately. Hence, the relays cannot know the final key without $\vec{d} \cdot M$. If the relays want to learn the final key, they need to figure out $\vec{d}$ and M . The seed $\vec{d}$ is private and changes in every QKD session. The hashing matrix M , on the other hand, is reused for many sessions in stream privacy amplification, so the relays might figure it out from final and reconciled key strings in past sessions by methods like differential cryptanalysis. These analysis methods often consume a lot of computational resources. For an even higher security level with fewer assumptions on the relays, we can add another layer of security based on the computational complexity on intermediate nodes. In practice, this combined scheme further reduces the requirement of the trustworthiness of relays and enhances the security of trusted-relay QKD networks.

C. New tool for information-theoretic analysis of classical encryption

There is an interesting property of the pseudo-random string $\vec{d} \cdot M$ generated in Step 2 of Box 1. On the one hand, parameter estimation provides an upper bound on the information leakage of $nh(e_p)$ bits about the raw key string $\vec{a}$. On the other hand, the security proof guarantees that the information leakage is removed via the simple XOR operation $\vec{k} = \vec{d} \cdot M \oplus \vec{a}$. This implies that $\vec{d} \cdot M$ has at least $nh(e_p)$ -bit uncertainty to Eve. In the security analysis, we do not pre-assume which part of $nh(e_p)$ -bit information on $\vec{a}$ is known by Eve. Then, for any $nh(e_p)$ bits from $\vec{d} \cdot M$, the corresponding $nh(e_p)$ -bit substring will be uniformly distributed from Eve's point of view, as shown in Figure 3. This property is called k -wise independence in classical cryptography, an essential requirement of many stream ciphers [33].

FIG. 3. Illustration of the pseudo-randomness property of $\vec{d} \cdot M$. The red shaded area in reconciled key denotes the $nh(e_p)$ -bit information leakage to Eve. This leaked information can be the bit values of the key or the parity information about the key. After the XOR of the reconciled key and the pseudo-random string $\vec{d} \cdot M$, the reconciled key becomes the final secure key and is uniformly distributed from Eve's point of view. That is, Eve's knowledge about the key is removed. As a corollary, the corresponding yellow shaded area in the pseudo-random string should be uniform to Eve as well.

The above observation inspires a new tool for the security analysis of classical encryption based on quantum phase error correction. First, we notice that the hash function for phase error correction is not necessarily linear. Alice and Bob can employ an arbitrarily nonlinear code to extend the seed to a pseudo-random string in Step 2 of Box 1, as shown in Figure 4(a). This extension operation can be treated as a pseudo-random number generator. Second, we can further generalize this stream privacy amplification as a joint function of the reconciled key and the seed, as shown in Figure 4(b). The joint operation can be treated as a classical encryption box. Alice and Bob can employ sophisticated schemes here, like Advanced Encryption Standard (AES) and lattice-based encryption algorithms. Third, we can express the classical operation in the quantum form, as a joint operation Λ acting on data and ancillary qubits, as shown in Figure 4(c). Since the operation is classical, Λ commutes with the dephasing operation in Eq. (5), $\Lambda[\Delta_{Z^{\otimes n}}(\rho)] = \Delta_{Z^{\otimes n}}[\Lambda(\rho)]$. By definition, Λ is a dephasing incoherent operation (DIO) [34]. At last, we can move Λ ahead of measurement. Since phase error correction is virtual in our security analysis, Alice and Bob can add an extra operation Ω on the ancillary qubits if necessary, as shown in Figure 4(d).

Similar to stream privacy amplification, we can analyze the security of the final output of Figure 4(d) by going

FIG. 4. (a) Schematic diagram for stream privacy amplification. The process of extending the seed can be treated as a pseudo-random number generator in Step 2 of Box 1. (b) Generalization of the extension and XOR operations as a joint operation, which can be seen as a classical encryption box. (c) Quantum form of (b). The classical joint operation becomes a DIO Λ . (d) Change the order of Λ and measurement. The final measurement on ancillary qubits can be arbitrary with an extra operation Ω .

back to the Lo-Chau security proof based on quantum error correction. The DIO Λ is determined by the classical encryption algorithm, which is viewed as the encoding for a quantum error-correcting code. With this code, Alice and Bob exchange the measurement results of ancillary qubits as a phase error syndrome which should give information about the phase errors of data qubits. According to the security analysis, Alice and Bob do not need to actually correct phase errors since it would not affect the final key. Then, Alice and Bob can add an extra virtual operation Ω on the ancillary qubits between Λ and measurement. They can optimize Ω to maximize the error correction capability led by Λ . The security of the final output of the classical encryption would be determined by the phase error correction capability of the corresponding code. Since we consider the most general attacks in QKD, our method provides a generic information-theoretic analysis tool for classical encryption algorithms.

III. CONCLUSION AND DISCUSSION

In this work, we propose a stream privacy amplification scheme, where Alice and Bob locally generate a pseudo-random bit string and XOR it with the reconciled key to get the final key. The new scheme has a stream output and hence can prevent unpleasant delay and error spreading in practical QKD. In addition, stream privacy amplification can enhance the security of a trusted-relay QKD network.

We need to emphasize that although we reduce the stream privacy amplification from the Lo-Chau security proof, the technique is independent of security proofs. Other security proof methods, such as Koashi's complementarity approach [35], can also be easily extended to the stream privacy amplification case. Moreover, the concept is rather generic and can be applied to other QKD schemes, such as six-state, continuous-variable, measurement-device-independent, two-way communication postprocessing, and decoy-state [5]. Furthermore, the stream technique can be directly applied to quantum random number generation and obtain *stream* randomness extraction. This would help solve the current bottleneck of realtime random number generation — slow extraction. Whether this property has further applications in cryptography is also worth studying.

Our security analysis provides a new perspective to examine classical encryption algorithms information theoretically through quantum information theories. Rigorous assessment of classical encryption algorithms, such as AES and lattice-based encryption, is often a formidable challenge. To our best knowledge, little consideration has been put forward in the context of the information-theoretic study of these encryption algorithms.

IV. METHODS

A. QKD protocols and security definition

Here, we shall introduce the first and probably the most well-known QKD protocol, Bennett-Brassard-1984 (BB84) [1], and its entanglement version, Bennett-Brassard-Mermin-1992 (BBM92) [36]. Then, we will show how their security can be established.

The procedures of the BB84 protocol are listed in Box 2. Alice and Bob have a quantum channel for state transmission and an authenticated classical channel for data postprocessing. In practice, photons are widely used as the information carrier in quantum communication. Various degrees of freedom of a photon can be used for qubit encoding. For example, the four BB84 states $\{|0\rangle, |1\rangle, |+\rangle, |-\rangle\}$ can be encoded into four polarization states of a photon, namely, vertical, horizontal, 45° and 135° , respectively.

Box 2: BB84 protocol [1], a prepare-and-measure protocol.

- (1) *State Preparation:* Alice randomly prepares qubits in one of the four states, $|0\rangle$, $|1\rangle$, $|+\rangle$, and $|-\rangle$, where $|0\rangle$ and $|1\rangle$ form the Z basis and $|\pm\rangle = (|0\rangle \pm |1\rangle)/\sqrt{2}$ form the X basis.
- (2) *State Transmission and Measurement:* Alice sends the encoded qubits to Bob through a quantum channel. Bob measures each received qubit in the Z or X basis randomly.
- (3) *Key Sifting:* Alice and Bob announce their bases choices publicly through an authenticated classical channel. They keep only the bits where they use the same bases and discard the rest. They accumulate n sifted key bits.
- (4) *Key Distillation:* Alice and Bob perform classical postprocessing, including information reconciliation and privacy amplification, to generate a secret key from the n -bit sifted key.

When Alice prepares a qubit in the Z basis and Bob measures in the same basis, an error occurs when their bit values are different. The errors in the Z basis are called *bit errors*. Similarly, when they operate in the X basis, a *phase error* occurs when their bits are different. Denote the bit and phase error rates to be e_b and e_p , respectively,

$$\begin{aligned} e_b &= \frac{\# \text{ of bit errors}}{n}, \\ e_p &= \frac{\# \text{ of phase errors}}{n}. \end{aligned} \tag{6}$$

Measurements on the two bases are non-commuting. Thanks to quantum mechanics, with any attempt to extract nontrivial information from the qubits, Eve would inevitably introduce disturbance, such as errors, making $e_b, e_p \neq 0$. Intuitively, Alice and Bob shall share a private key if $e_b = e_p = 0$.

Before the security analysis, we first introduce an entanglement-based protocol with EPR pairs, BBM92, in Box 3. The BBM92 protocol can be reduced to the BB84 protocol if Alice measures her half of the state in the Z or X basis. Based on her measurement result, Alice equivalently sends a qubit in $|0\rangle$, $|1\rangle$, $|+\rangle$, or $|-\rangle$ to Bob.

Box 3: BBM92 protocol [36], an entanglement version of BB84.

- (1) *State Preparation:* Alice prepares EPR pairs, $|\Phi^+\rangle = (|00\rangle + |11\rangle)/\sqrt{2}$. She stores one half of the pairs locally and sends the other half to Bob.
- (2) *State Storage:* Upon receiving a qubit, Bob stores the state in quantum memories. If the qubit has been lost in the channel or the quantum storage fails, they discard the pair.
- (3) *Quantum Error Correction:* Alice and Bob measure a random sample of the stored qubit pairs to estimate the quantum bit and phase error rates, e_b and e_p . They apply quantum error correction to the rest stored qubit pairs. They share n (almost) perfect EPR pairs.
- (4) *Key Measurement:* Both Alice and Bob measure the EPR pairs in the local Z basis to obtain the final key.

Here, the bit and phase error rates defined in Eq. (6) are consistent with the ones in Eq. (1). Note that Eq. (6) defines error frequencies, while Eq. (1) defines error probabilities. Strictly speaking, these two definitions are only the

same in the infinite data size limit, $n \rightarrow \infty$. In the finite data size regime, there is a deviation between them caused by statistical fluctuations. For simplicity, we ignore the difference for the moment, which we shall take into account in the evaluation of the failure probability of quantum error correction. According to Eqs. (2) and (3), if $e_b = e_p = 0$, Alice and Bob share perfect EPR pairs $|\Phi^+\rangle$ in each round. The following security definition then ensures that Alice and Bob can generate a secure key.

For the key distribution task, Alice and Bob need to make sure their key bit strings are *identical* and *uniformly random* from Eve's point of view. To satisfy these two requirements, the ideal key state shared by Alice and Bob, and any outside adversary Eve, is defined to be the following classical-classical-quantum state,

$$\rho_{ideal} = 2^{-n} \sum_{k \in \{0,1\}^n} |k, k\rangle_{A,B} \langle k, k| \otimes \rho_E, \quad (7)$$

where systems A, B are keys held by Alice and Bob and system E is held by Eve. In the ideal key state, Alice and Bob's key bit strings are identical. Eve's system ρ_E is independent of the key k , which brings her no more information about the key string than a random guess. This definition follows the works of Ben-Or *et al.* [29] and Renner and König [30].

In practice, however, Alice and Bob cannot generate an ideal key. It is reasonable to allow for a small failure probability. That is, Alice and Bob can generate a key state very close to an ideal one. To put the idea into a rigorous manner, if the realistic key state

$$\rho_{key} = \sum_{k_A, k_B \in \{0,1\}^n} \Pr(k_A, k_B) |k_A\rangle_A \langle k_A| \otimes |k_B\rangle_B \langle k_B| \otimes \rho_E(k_A, k_B), \quad (8)$$

satisfies

$$\frac{1}{2} \min_{\rho_E} \|\rho_{key} - \rho_{ideal}\|_1 \leq \varepsilon, \quad (9)$$

the protocol shall be ε -secure. Here, the distance measure is the trace distance. For two density matrices ρ, σ , the measure is defined as

$$\frac{1}{2} \|\rho - \sigma\|_1 = \frac{1}{2} \sum_i |\lambda_i|, \quad (10)$$

where λ_i are eigenvalues of the operator $\rho - \sigma$. The choice of the trace-distance measure is that it satisfies the requirements of a composable security framework [29, 30].

Definition 1 (QKD ε -security). *A QKD protocol is ε -secure if the generated state ρ_{key} given in Eq. (8) is ε -close to the ideal key state ρ_{ideal} given in Eq. (7) with respect to the trace distance definition.*

This definition, usually referred to as *soundness*, guarantees that once the protocol is not aborted, the generated key is private with a high probability. In the composable security framework, there is another security parameter, *completeness*, to phrase the protocol success probability, which guarantees one protocol is not trivial and it does not always abort. For most protocols, completeness can be easily established, so we shall not discuss the completeness parameter in this work. To connect the security definition and the EPR pairs, we employ the following lemma.

Lemma 1 (Lemma 1 in [18]). *If Alice and Bob share a quantum state ρ_{AB} which is ε_f -close to the ideal key state before projective measurements onto $|\Phi^+\rangle^{\otimes n}$, i.e.,*

$$\langle \Phi^+ |^{\otimes n} \rho_{AB} | \Phi^+ \rangle^{\otimes n} \geq 1 - \varepsilon_f, \quad (11)$$

then the QKD protocol is ε -secure with $\varepsilon = \sqrt{\varepsilon_f(2 - \varepsilon_f)}$.

Combining Lemma 1 and the security definition, we can conclude that if the state ρ_{AB} shared by Alice and Bob before the measurement of a QKD protocol satisfies Eq. (11) with a small ε_f , this protocol is ε -secure. Therefore, the security of QKD is closely related to entanglement and the purpose of security proof is to achieve Eq. (11).

B. Quantum error correction with hashing

As mentioned in the main text, we can use Eq. (11) to further derive Eqs. (2) and (3), which are related to bit and phase error, respectively, and these two equations can be achieved by quantum error correction. We divide the procedure of quantum error correction into two steps, bit error correction to guarantee Eq. (2) and phase error correction to guarantee Eq. (3).

Quantum error correction can be seen as an extension of classical error correction and accomplished by using universal hashing [37–39]. In the classical case, Alice and Bob each possess a bit string. The differences between these two strings are called “errors”. The main job of classical bilateral error correction is to figure out the error locations. Alice hashes her string and sends the parity information to Bob. With the same hash function, Bob hashes his string and compares it to Alice’s. After enough times of this procedure, Bob can figure out the error locations and flip his corresponding bits to correct the errors. In the end, Bob’s bit string is reconciled with Alice’s. The number of parity-check bits is given by $nh(e)$ in the Shannon limit, where n is the bit string length and e is the error rate. For the case of a finite data size, there is the possibility that error correction fails. Details of error correction along with analysis of finite size effect and failure probability are presented in Supplementary Information.

In the following discussions, we consider linear hash functions, which can be represented by hashing matrices, for simplicity. One of the most widely used linear hash families is the family of Toeplitz matrices. The elements in a Toeplitz matrix M satisfies $\forall i - j = i' - j', M_{ij} = M_{i'j'}$ and there are $m + n - 1$ free bits in a Toeplitz matrix of size $m \times n$. We give an example of the Toeplitz hashing matrix as follows,

$$M = \begin{pmatrix} 1 & 0 & 1 & 1 & 0 & \cdots \\ 0 & 1 & 0 & 1 & 1 & \cdots \\ 0 & 0 & 1 & 0 & 1 & \cdots \\ \vdots & & & \ddots & & \cdots \end{pmatrix}_{nh(e) \times n}. \quad (12)$$

As will be shown in Supplementary Information, if Alice and Bob randomly choose a Toeplitz matrix of size $nh(e) \times n$ for error correction, the efficiency converges to the Shannon limit very fast when n is large.

Now, we can apply classical error correction to the quantum case. Let us start with quantum bit error correction. The parity hashing on the raw key bit strings can be implemented by a series of CNOT gates between the data qubits and ancillary EPR pairs, as shown in Figure 5, which is a concrete example of the quantum bit error correction part in Figure 1(a). Alice and Bob can get the parity information by measuring the ancillary qubits. The measurement result of one ancillary qubit will reflect one bit parity information of data qubits. The measurement results on the ancillary pair of Alice and Bob will be different if there is an odd number of errors in these control qubits, and the results will be the same if there is no error or an even number of errors. Alice sends the parity information to Bob, who then figures out the error syndromes and corrects the errors. The property of the universal hash family guarantees that Bob can correct all the errors with a small failure probability. A similar approach can be employed for quantum phase error correction, with additional Hadamard gates before hash operations and measurements.

In entanglement distillation, both bit and phase error correction should be successfully implemented. The two quantum error correction procedures are carried out sequentially. Hence, we need to make sure that these two error correction procedures do not interfere with each other. Fortunately, by using ancillary EPR pairs, we can decouple these two steps with the following lemma.

Lemma 2 (Bit and phase error correction decoupling [40]). *By using EPR pairs as ancillary qubits, bit error correction has no effect on phase errors and vice versa.*

Proof. Let us first show that the phase-error measurement results are the same with or without the bit error correcting operations. The phase error is evaluated when both Alice and Bob perform the X -basis measurement on the data qubits, denoted by the measurement of the joint observable $\sigma_x \otimes \sigma_x$. From Figure 1(a), we can see that the bit error correcting operations on data qubits are essentially I , σ_x , and σ_z . The operations I and σ_x would not change the X -basis measurement outcomes. The operation σ_z comes from the CNOT gate between the data and ancillary qubits. Since Alice’s and Bob’s hash functions are the same, the CNOT gates always appear in pairs. That is, if there is a CNOT between Alice’s share of a data qubit pair and an ancillary EPR pair, there will be a CNOT between Bob’s share of the data qubit pair and the ancillary EPR pair. From the circuit equivalency shown in Figure 6, we can see that the σ_z operation always appear in pairs on Alice’s and Bob’s data qubits. That is, the σ_z operation will simultaneously flip Alice’s and Bob’s X -basis measurement results, which leaves the measurement results of $\sigma_x \otimes \sigma_x$ on data qubit pairs unchanged. Therefore, quantum bit error correction does not affect phase errors.

With the duality between the X and Z bases, we can also prove that the phase error correction would not affect bit errors with the same arguments. □

FIG. 5. Illustration of quantum bit error correction. As an example, the circuit corresponds to the hashing matrix M in Eq. (12). The i th row of M corresponds to the CNOT control data qubits targeting on the i th ancillary qubit, and the j th column of M corresponds to the CNOT target ancillary qubits controlled by the j th data qubit. The measurement result of one ancillary qubit equals to the XOR sum of the Z -basis measurement results of the ancillary qubit and its controlling data qubits. By comparing the measurement results, Bob can learn one bit parity information of the data qubits. After knowing enough parity information on the data qubits, Bob can locate the bit errors and correct them with the quantum gate σ_x . A similar circuit can be applied for quantum phase error correction with additional Hadamard gates according to Figure 1(a).

FIG. 6. A diagram showing the reason why bit error correction does not change phase errors. In the showcase, we take a pair of CNOT gates as an example. In the figures we only depict the circuit on Alice's side and the circuit is the same on Bob's side. Here, the control qubit is one of the data qubits and the target qubit is one of the ancillary qubits, as shown in Figure 1(a). The equivalence of (a), (b) and (c) comes from the facts that $\sigma_x = H\sigma_z H$ and $H^{\otimes 2}|\Phi^+\rangle = |\Phi^+\rangle$.

C. Reduction from quantum operations to classical ones

Now, we want to reduce the quantum operations to classical ones, mainly following the Shor-Preskill security proof [27]. By classical, we mean that Alice and Bob can equivalently perform the operations after key measurement, which become classical data processing on key bits. The key idea is to move the final measurement before quantum error correction. Then, bit error correction becomes classical bilateral error correction and phase error correction becomes privacy amplification. To do so, we need to make sure that the dephasing operation $\Delta_{Z^{\otimes n}}$ in Eq. (5) caused by the final Z -basis measurement on each side commutes with all the quantum error correction operations.

Looking back at Figure 1(a), it is not difficult to see that bit error correction only consists of I , σ_x , and CNOT gates. These quantum operations commute with the dephasing operation $\Delta_{Z^{\otimes n}}$. Therefore, Alice and Bob can change the order of the Z -basis measurement and bit error correction. By doing so, quantum operations are replaced with corresponding classical operations on the measurement results, as shown in Figure 7. For example, the σ_x gate becomes flip on classical bits and the CNOT gate becomes classical control-flip operation. Finally, quantum bit error correction becomes classical bilateral error correction, a typical information reconciliation method.

Quantum phase error correction is more complicated because it contains one more operation, Hadamard gate, which does not commute with the dephasing operation, $\Delta_{Z^{\otimes n}}$. Here, we follow the idea of the Shor-Preskill security proof to reduce phase error correction into classical privacy amplification [27].

FIG. 7. Transformation from quantum bit error correction to information reconciliation. The left circuit is the quantum bit error correction part in Figure 1. The “Flip” gate means Alice and Bob flip the corresponding classical bits. Bob calculates the error syndrome using his parity information and the one sent by Alice, and then flips his erroneous bits to reconcile the key.

First, in the quantum circuit shown in Figure 1(a), we combine the Hadamard gate with the adjacent Z -basis key generation measurement together, which becomes the X -basis measurement. The phase error correcting operations on Bob’s side is essentially σ_x , which does not affect the result of the X -basis measurement. Therefore, neither the correction operation nor the phase-error-syndrome measurement is necessary and the circuit of quantum phase error correction becomes the one shown in Figure 8(a).

FIG. 8. (a) Reduced circuit from the phase error correction part in Figure 1 by considering the following two facts: Hadamard+ Z -basis measurement = X -basis measurement; neither the identity nor the σ_x gate affects the X -basis measurement. Now, Alice and Bob’s circuits become the same. (b) Reduction of Alice’s circuit: remove the measurement on ancillary qubits since the results do not affect the measurement on data qubits; replace individual X -basis measurements with joint X -basis measurements on data qubits; and explicitly express the hash operation with CNOT gates shown in Figure 1(b) as an example. If the joint X -basis measurements commute with the hash operation, the circuit in the red dashed box will not affect the measurements and hence can be removed. (c) Further reduction of Alice’s circuit: remove the redundant circuit in the red dashed box and combine joint X -basis measurements with the Hadamard gates which become joint Z -basis measurements. Finally, Alice can get an $n[1 - h(e_p)]$ -bit secure key from the joint Z -basis measurements in a QKD session.

Second, since Alice and Bob’s positions are symmetric now, we focus on Alice’s side. We shall show that she can replace the individual key measurement with a series of joint X -basis measurements and still get an $[n - nh(e_p)]$ -bit secure final key, and as shown in Figure 8(b). The joint measurement can be represented by an n -bit vector $\vec{v}$, which measures the observable

$$O_{\vec{v},x} = \bigotimes_{i=1}^n \sigma_x^{v_i}, \quad (13)$$

where v_i is the element values of $\vec{v}$. Here, we require $\vec{v}$ s to be linearly independent so that the final key is still secure after phase error correction. Note that since the key measurements differ from the ones in Figure 8(a), the obtained key bits may be different.

The effect of hash operations on the n data qubits is either identity when the measurement outcome of the ancillary qubit is 0 or a series of n -qubit operations consisting of σ_z and I when the outcome is 1. For example, the hash operation corresponding to the first row vector $\vec{m}_1$ of the matrix in Eq. (12) is an identity or

$$O_{\vec{m}_1,z} = \sigma_z \otimes I \otimes \sigma_z \otimes \sigma_z \otimes I \cdots. \quad (14)$$

In general, we represent the operation corresponding to the i th row vector $\vec{m}_i$ of the matrix M as follows,

$$O_{\vec{m}_i,z} = \bigotimes_{j=1}^n \sigma_z^{M_{ij}}, \quad (15)$$

where M_{ij} is the value of the element in the i th row and the j th column of the matrix.

Normally, the operators in Eqs. (13) and (15) do not commute because $[\sigma_x, \sigma_z] \neq 0$. Fortunately, we have $[\sigma_x \otimes \sigma_x, \sigma_z \otimes \sigma_z] = 0$. To make sure that a joint X -basis measurement commutes with a series of Z operations, we only need to design the joint X -basis measurement such that the number of qubits having been applied with the σ_z operation and measured in the X basis is even. That is, $\vec{m}_i \cdot \vec{v} = 0$ holds, $\forall i \in \{1, 2, 3, \dots, nh(e_p)\}$. This is equivalent to finding the kernel of M . The kernel of the matrix M is defined as the set of vectors such that

$$\ker M = \{\vec{v} : M\vec{v} = \vec{0}\}. \quad (16)$$

In phase error correction, the hashing matrix we use has a full rank. Therefore, the rank of $\ker M$ is $n - nh(e_p)$, or, $\ker M$ can be constructed from $[n - nh(e_p)]$ linearly independent vectors. We arrange these vectors in columns and form a matrix, V , which is of the size $n \times [n - nh(e_p)]$. We call the matrix V the dual matrix of M . Then, we can design joint X -basis measurements according to this dual matrix, where each joint measurement corresponds to a column vector of V according to the correspondence in Eq. (13). By construction, these joint measurements all commute with the hash operation.

Finally, the commuting property shows that the hash operation would not affect the results of the joint X -basis measurements. Then, Alice and Bob can remove the hash operation along with the ancillary qubits. The joint X -basis measurements can be further combined with the Hadamard gate and become the joint Z -basis measurements, as shown in Figure 8(c). The measurement results of the joint Z -basis measurements give an $[n - nh(e_p)]$ -bit secret key.

With the reduction, one can see that the ancillary EPR pairs are unnecessary for phase error correction, as shown in Figure 8(c). Does that mean, there is no cost of shared private randomness in phase error correction? Unfortunately, the answer is no. The cost is reflected in the joint Z -basis measurement. The number of final key bits is determined by the number of joint Z -basis measurements, which is limited by the kernel of the hashing matrix for phase error correction.

Meanwhile, the joint Z -basis measurement is compatible with bit error correction reduction. After considering the cost of shared private randomness in bit error correction, the net gain of secret key is $n[1 - h(e_b) - h(e_p)]$ bits, which matches the key rate in Eq. (4).

There are a few notes on information reconciliation and privacy amplification.

1. We take the family of Toeplitz matrices as an example here. The decoding for such random-Toeplitz-matrix hashing is computationally hard in practice. To bypass the hardness in information reconciliation, people use more practical error correcting codes, such as the low-density parity-check code [41, 42]. For privacy amplification, Alice and Bob can still adopt random-Toeplitz-matrix hashing, as the decoding is unnecessary for phase error correction.
2. In general, the failure probability of error correction ε is a property of the hash family. But if Alice and Bob can perform error verification, the failure probability of error correction is determined by the verification process instead [18]. In this case, the hash function can be pre-fixed and known to Eve in the beginning. This is the case for information reconciliation. Unfortunately, there is no way to verify the result of privacy amplification because the virtual quantum phase error correction is not implemented in real life.
3. The reduction from quantum phase error correction to privacy amplification is different from Shor-Preskill's original argument, where the Calderbank-Shor-Steane quantum error-correcting code is adopted.

ACKNOWLEDGMENTS

This work was supported by the National Natural Science Foundation of China Grants No. 11875173 and No. 1217040781, the National Key Research and Development Program of China Grant No. 2019QY0702 and No. 2017YFA0303903.

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