Design and Analysis of Jittering Mitigator for Robot Arm-tip

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Title page

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Design and analysis of jittering mitigator for robot arm-tip

Shuyong Duan^{1†}, Chunlu Li¹, Pengfei Xu¹

Abstract

To tackle the jittering of a robot arm-tip, a novel method of the jittering mitigation is proposed. The key idea is to transmit the jittering to vibration of a mass block nested inside the jittering-mitigator. This method only requires the frequency of the jittering at the arm-tip. To verify the practicability of the method, a practical robot was applied through simulations and physical experiments. The jittering mitigator device can be directly attached to or detached from the arm-tip. The jittering at the arm-tip was first measured through experiments using accelerometers connected to the vibration and noise testing system. Then, the dominant frequency was identified through fast Fourier transform analysis. The theoretical parameters of the jittering mitigator were calculated accordingly. A model was established to simulate the jitter reduction

effect of the mitigator. The results revealed a 68% reduction in the average amplitude of the jittering vibration at the arm-tip, which is corroborated by the experiment results. The proposed method could be applicable to all types of robot because it only requires computing the frequency of jitter.

Keyword: Robot arm; arm-tip jittering; jittering mitigator

1. Introduction

Trajectory accuracy and kinematic stability are the most important indicators of precision for a robotic arm [1-6]. Previous studies found that the arm-tip jittering of a robot can seriously affect its trajectory accuracy and kinematic stability under typical working conditions. There are various factors influencing arm-tip jittering [7-17], including resonance of the robot arm [7,8], stiffness of the transmission gear in retarder and robot links [9,10], mismatch between the motor

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output torque and the actual demand of joints, installation accuracy, transmission gap [11], etc. A literature review shows that there are many complex and interconnected factors that are too difficult to be identified in a holistic manner. Therefore, to mitigate the arm-tip jittering while circumventing the complex causes, end-to-end techniques were developed.

Several methods have been developed for the vibration control of various mechanical systems. The first type of method involves redesigning the robot structures [18] or using smart materials [19-21] for vibration control. This type of method is not only uneconomical, but also ineffective in damping the vibrations. The second type of method optimises the operational trajectory [22-27] by establishing the relationship between motion parameters and robot position deviation. Vibration can be reduced substantially by optimising the mechanism to compensate for the position deviation. This type of method is limited by the region of designability. A substantial amount of vibrations often remain even after the parameters have been optimised [28]. This affects its applicability under various working conditions and environments. The third type of method actively controls the vibration source or external excitations [29-46], and includes the proportion integral differential control [29,30], optimal control [31], input

shaping [32-37], etc. All these methods require sophisticated real-time sensors and feedback systems, as well as control algorithms. This challenge has limited the practical industrial applications of robot arms [10]. In addition, the authors' earlier experimental studies have found that arm-tip jitter vibrations of robots exhibit high non-linearity. The design of an active control system can be challenging because it requires establishing an effective method and algorithm based on non-linear principles, which is difficult. For a non-linear vibrational system, internal resonance exhibits modal interactions. For example, if two natural frequencies in the linear portion of a set of non-linear equations of motion are commensurable, or nearly commensurable, internal resonances will occur [47]. Compared with active suppression methods, passive vibration absorption methods based on the dissipation of vibrational energy are more feasible and efficient. It is because passive vibration absorption methods ignore all the complicated and interrelated non-linearities to simply dissipate the vibration energy. However, such a simple damping system will require a substantial amount of additional mass, for the frequency range of a typical jittering vibration. Hence, it is not feasible to suppress jittering by a simple system. On top of that, in the damping-based method, the damper takes time to absorb

the vibrational energy, and industrial robots cannot delay operations. Thus, passive methods are generally not preferred in the robotics industry. Contrary to popular belief that the jittering vibration occurs due to resonance [7,8], our experimental studies demonstrated that this is often not true. In fact, the equations of jittering vibration at the arm-tip of a robot are much more complex than the simple resonance. For example, the step motors in the joints of a robot substantially may contribute to the jittering vibration, which complicates the equation of motion.

A thorough literature review revealed that the causes of the jittering vibration in a robot arm can be complicated. A control method based on the full understanding of the causes is far-fetched, or the resulted control rules could become too complex to be tractable. The control system alone could be too bulky to handle the entire load-carrying capacity of the robot. A passive control system that uses an energy damping system is also not feasible due to the increased time delays. This paper investigates the core factors that may influence the arm-tip jittering of a robot, to lay foundation for the active control of jitters by a control system. A new device called ‘jittering mitigator’ is proposed to alleviate jittering at the robot’s arm-tip. The jittering mitigator is a stand-alone modality and can thus be directly mounted

on and detached from the robot’s arm-tip. The key idea of the device is to transmit the jittering vibration of the arm-tip to the vibrations of a mass block nested inside our jittering mitigator. The proposed method only requires the frequency of jittering at the arm-tip for its computations, circumventing the complexity of jittering. Therefore, in theory, it is applicable to any type of robot. It integrates vibration transmission and damping into one system, which sets it apart from the existing active and passive control systems. According to current knowledge, the presence of any damper in a system is counterproductive. Based on the vibration transmission theory, in an ideal case, when the jittering vibration has a single frequency, no damping will occur; thus, the jittering vibration can be entirely transmitted to the vibration of the mass inside the mitigator box. No residual vibrations will remain on the mitigator body. Therefore, in theory, it is as good as a vibration eliminator. In practice, however, damping is unavoidable, and the frequency of jittering vibration will not be the only parameter. A small amount of residual vibration will always be present on the mitigator. Therefore, this device is termed ‘mitigator’ instead of ‘eliminator’. The underlying vibration mitigation theory for the jittering mitigator was formulated by installing a pendulum and originates from the earthquake reduction techniques used for tall buildings. This

work will make significant contributions to the mitigation of jittering vibrations at the robot arm-tip.

The remainder of the paper is organized as follows. The phenomenon and underlying cause of robot arm-tip jittering are elaborated in Section 2. The proposed jittering mitigation method for robot arm is presented in Section 3. In Section 4, the effectiveness of the proposed method is verified by numerical simulation. Section 5 verifies the effectiveness of the proposed method and the correctness of the numerical simulation results by experiment. Section 6 draws some conclusions.

2 Jittering of robot arm-tip

2.1 Phenomenon of Robot Arm-tip Jittering

Any robotic operation will experience a certain degree of jittering. In this paper, a typical six-axes tandem robot arm (Fig. 1) was used to embody the proposed method and analyse the jittering phenomenon during operation. The main components of the robot arm were a base, six kinematic joints, an upper arm and a forearm. The driving system inside each joint was composed of a controller, a motor, and a speed reducer. The total mass of the robot was 23 kg, the maximum load-carrying capacity was 5 kg, and the maximum linear velocity of the arm-tip was 2,800 mm/s.

Fig. 1 Six-axes tandem robot arm (AUBO-i5)

In the authors' previous studies, when the speed of the arm-tip moving along the x-direction is different from points P_1 to P_2 without load (Fig. 2), the corresponding amplitude of the arm-tip jittering will be different (Fig. 3). As indicated in Fig. 2, the origin of the coordinate system is the centre of the lower surface of the robot-arm base.

The trajectory distance between P_1 and P_2 is 600 mm. The time taken by the robot arm-tip to reciprocate at a uniform speed of 100 mm/s was 12.35 s. Figure 3 also shows the arm-tip moving horizontally with a constant velocity of 100 mm/s. The arm experiences a strong jitter along its z-axis. This state of motion is a typical working

condition. Subsequent research in this study will

Fig. 2 Trajectory of robot arm

A three-directional accelerometer was placed on the top of Joint 6, as shown in Fig. 1, to collect the vibrational data for analysis in vibration and noise testing system (LMS Test Lab) [48,49]. To collect reliable vibration spectrum (frequency and amplitude) information, the following sampling theorems were used:

$$X(f)=0, \quad f > f_c \quad (1)$$

$$\Delta t \leq \frac{1}{2f_c} \quad \text{or} \quad f_s = \frac{1}{\Delta t} \geq 2f_c \quad (2)$$

where $X(f)$ indicates the frequency spectral function of the continuous signal $x(t)$; f stands for the frequency variable; f_c denotes the cut-off frequency of $x(t)$ signal; and f_s represents the sampling frequency of $x(t)$ signal.

This experiment requires a sampling frequency two times greater than the maximum cut-off frequency of the continuous time signal. The continuous signal $x(t)$ was sampled and recorded as discrete signals of $x(n\Delta t)$. To obtain

be based on this condition.

Fig. 3 Amplitude of jittering at different speeds

reliable data, the sampling frequency was set as 5–10 times of the signal cut-off frequency (2,048 Hz) to collect the acceleration signal. The recorded history of arm-tip jittering acceleration is shown in Fig. 4(a). Then, the fast Fourier transform (FFT) method was performed to derive the frequency spectrum:

$$X(k) = \sum_{n=0}^{N-1} x(n)W_N^{nk}, k = 0, 1, L, N-1 \quad (3)$$

where $x(n)$ expresses the measured time-domain signal; $X(k)$ represents the frequency spectral signal of $x(n)$; W_N^{nk} indicates the rotation factor matrix; and N stands for the number of sampling points.

In order to improve the proficiency of the spectrum analysis, zero-padding FFT or integral FFT transformation [50] were performed, as shown in Fig. 4(b). The recorded forward movement indicates a maximum arm-tip jitter

amplitude of 0.3 mm, as seen in Fig. 4(b). A distinct peak appears around 9.60 Hz, implying a high-energy vibration signal around this frequency. This peak frequency of 9.60 Hz coincides with the result of the rough estimation done by counting the number of peaks per unit time. Therefore, the vibration frequency of jittering in z-direction of the arm-tip was determined to be 9.60 Hz.

To further analyse the jittering of the robot arm-tip, the measured acceleration signal was integrated twice to obtain the displacement history, as shown in Fig. 4(c). As indicated in Fig. 4(c), the jitter displacement during the forward movement of the arm-tip from P₁ to P₂ is longer than that during the backward movement. This phenomenon can be explained by analysing the mechanisms during movement. Figure 5 shows the angles between Joint 1 and upper arm, upper arm and forearm, and forearm and Joint 5, which are denoted by θ_2 , θ_3 , and θ_4 , respectively. As the robot arm moves from P₁ to P₂ in the positive x-direction (+x) in the time interval of 0 to 6 s, θ_2 and θ_3 gradually increase, while θ_4 increases first and then decreases. When the robot arm moves from P₂ to P₁ in the negative x-direction (-x) in the time interval of 6 to 12 s, θ_2 and θ_3 gradually decrease, while θ_4 increases first and then decreases.

The directions of the driving and resisting

torques of Joints 2, 3 and 4 as the arm-tip reciprocates from P₁ to P₂ to P₁ again are listed in Table 1. From 0 to 3 s, the directions of driving and resisting torques of Joints 2, 3 and 4 are different. From 3 to 6 s, the directions of the driving and resisting torques of Joints 2 and 4 are the same, whereas Joint 3 exhibits different driving torque and anti-torque directions. From 6 to 9 s, the directions of the driving and resisting torques of Joints 2 and 4 are different, while Joint 3 displays a similar driving torque and anti-torque directions. From 9 to 12 s, the directions of driving and resisting torques of Joints 2, 3 and 4 are the same. For backward movement, from 0 s to 6 s, only the driving torque and anti-torque directions of Joint 3 are different. In contrast, from 6 to 12 s, only Joint 3 exhibits similar driving torque and anti-torque directions. Comparing the results in Table 1 with the time-domain signals in Fig. 4(c), when the driving torque and anti-torque directions of Joint 3 are different, the displacement of the arm-tip jittering is larger than that of Joint 3, exhibiting similar driving torque and anti-torque directions. Therefore, we speculate that Joint 3 is the most influential factor for the vibration.

We also surmise that the drive system is less susceptible to interference, to obtain a stable driving torque when the drag torques and driving torques of the joint are in the same direction.

Contrarily, the drive system will be substantially affected by interference and will negate the output stability when the directions of the drag torque and driving torque are different. Hence, it can be

inferred that this is another potential cause for the difference in jittering amplitude at the different stages of movement, as shown in Fig. 4(c).

Fig. 4 Jittering signal of robot arm-tip: (a) acceleration; (b) frequency spectrum; (c) time-domain signal

Fig. 5 Analyses of movement: (a) posture at 0 and 12 s; (b) posture at 3 and 9 s; (c) posture at 6 s

Table 1 Direction of joint torque during different time periods of movement

Time	Torque direction of joint 2		Torque direction of joint 3		Torque direction of joint 4	
	Driving	Impedance	Driving	Impedance	Driving	Impedance
	torque	torque	torque	torque	torque	torque
0–3 s
3–6 s
6–9 s
9–12 s

2.2 Root Cause of Robot Arm-tip Jitter

The tandem robot arm exhibits a low overall stiffness which may incur coupling resonance of the drive system on the robot joint and low-order natural frequencies on the robot system. It may be the major contributor to the arm-tip jitter. Another potential source of jitter during the arm-tip movement is the unstable angular displacement generated by the drive system of each joint. Based on these two inferences, and by referring to the analysis of the jitter phenomenon from Section 2.1, three tests were designed to explore the key factors responsible for the arm-tip jittering. The flow chart for the tests is shown in Fig. 6.

Test 1: Constrained modal experiments were performed under inert conditions to obtain the natural frequencies of the robotic arm. The natural frequencies were compared with the vibration frequencies measured in Section 2.1. If the measured natural frequencies are equal or

close to the corresponding frequencies measured in Section 2.1, it may be preliminarily inferred that the jittering is caused by the resonance of the robotic arm's structure. To confirm this inference, a further test (Test 2) will be performed. Conversely, if the measured natural frequencies are not equal or close to the corresponding frequencies measured in Section 2.1, it may be deduced that the jittering is not caused by the resonance of the robotic arm's structure, and another test (Test 3) will be conducted to explore the root cause of jittering in a different direction.

Test 2: Based on the test conditions in Section 2.1, different load blocks were applied to the arm-tip. The jittering signals under different load conditions were measured [51] for compared with the vibration frequency measured in Section 2.1. If the jittering frequencies measured under different loading conditions are not equal to the vibration frequency in Section 2.1, it can be

confirmed that the jittering is caused by the resonance of the robot body, which will conclude

Fig. 6 Flow chart for the exploration of the underlying cause of robot arm jitter

the research. Otherwise, if the natural frequency under a specific loading condition coincides with that in Section 2.1, the jittering is not caused by the resonance of the robot body. Test 3 will be performed with different settings.

Test 3: Based on the robot operating system (ROS), the output signals of the main kinematic joints and the jittering signals at the robot arm-tip were compared and analysed, to assess the relevance of the jittering and output signals.

2.2.1 Test 1: Modal Test

The constrained modal test was conducted based on the impact testing module in LMS Test Lab. The arm-tip was struck by a hammer with a force sensor, and the output signal of the robot arm system was recorded by a three-directional

sensor. The mode of vibration of the robotic arm obtained from the constrained modal experiment is shown in Fig. 7. The procedure of the constrained modal experiments is as follows:

1) Calibration of sampling points: Before the test, the location and quantity of the sampling sensors on the robotic arm should be deployed appropriately. The base of the robot, the joints, the upper arm and the forearm are divided into basic units. Each unit has sufficient sampling points to profile its morphological features. According to the shape and size of each unit, 52 sampling points are specified on the surface of the robot. Then, the relative distances and locations of the specified sampling points are calibrated and measured. The measurement location coordinates

of each sensor are input to the LMS Test Lab.

2) Sensor attachment: The three-directional acceleration sensors are glued at the 52 sampling locations, as shown in Fig. 7, to collect the acceleration signals of the sampling points subject to impact excitation.

3) Sampling parameters: The low natural frequency of the robot arm, due to its flexible linkage structure, causes structural resonance. Therefore, the current experimental tests analyse the first to third natural frequencies in the z -direction of the robot arm. To ensure that the sampled data are effective, the sampling bandwidth is set between 0 and 512 Hz, and the frequency resolution is set as 0.25 Hz.

4) Test system calibration: Nineteen data channels—18 for the acceleration sensor data and 1 for the impact force of the hammer—are employed to transmit data. The sensor is struck by the hammer repeatedly, to excite the system for calibration.

5) Data collection for excitations: After calibrating the sampling system, the arm-tip is impacted with a soft rubber-headed hammer equipped with a force sensor. The acceleration signals in the x -axis, y -axis and z -axis directions triggered by the impact of the hammer are collected by the sensors attached to the sampling points.

6) Data post-processing and result analysis:

After the calculations and analysis, the frequency response function of the test input and output can be derived. The overall stabilisation diagram of the robot arm can be deduced by summing up the spectrogram for the 52 sampling points, as shown in Fig. 8. The abscissa and ordinate in Fig. 8 are frequency and acceleration, respectively. The frequency corresponding to a distinct peak in Fig. 8 may be a natural frequency of the robot arm. The identified natural frequency can be confirmed with reference to the vibration mode at the peak. As the vibration of the arm-tip is mainly concentrated in the z -axis direction, the internal excitation of the robot tends to couple with the low natural vibration modes in the z -direction. The first three bending natural frequencies in the z -direction from the modal analyses are listed in Table 2. The first natural frequency in the z -direction is 11.55 Hz, the second is 62.21 Hz, and the third is 125.10 Hz. Compared with the arm-tip jittering frequency of 9.60 Hz measured in Section 2, the tested first natural frequency (11.55 Hz) exhibits a percentage difference of 20.92%. Therefore, this marginal discrepancy cannot confirm resonance to be the root cause of jittering. Thus, to further assess whether resonance of the robot arm structure is the cause of jitters in the arm, Test 2 was performed.

Fig. 7 Wireframe diagram of the robotic arm

Fig. 8 Stabilisation diagram from modal analysis

Table 2 Modal shapes and corresponding natural frequencies

Natural frequency in z-direction	Modal shape	Natural frequency (Hz)
First z-direction bending		11.50
Second z-direction bending		62.21
Third z-direction bending		125.10

2.2.2 Test 2: Jittering Tests under Different Loads

The kinematic path, speed, and posture of the robot arm in this test were the same as those of the jittering tests in Section 2. The governing factors for the natural frequencies of the robot arm are mass and stiffness [52].

$$f_n = \frac{A_n}{2\pi} \sqrt{\frac{K_s}{m_s}} \quad (4)$$

where f_n represents the natural frequencies of the cantilever beam; K_s represents the overall structural stiffness of robot arm; m_s denotes the equivalent mass; and A_n stands for the vibration

mode coefficients ($A_1 = 3.52, A_2 = 22.4, A_3 = 61.7, A_4 = 121.0...$).

Fig. 9 Robotic arm under different loads

If the jittering is caused by resonance, the change in the mass of the robot arm will shift the jittering frequencies. If not, then the vibration

frequencies of arm-tip jittering under different load conditions will remain constant. According to the practical experimental conditions, the change of mass by the addition of load on the arm-tip does not change the overall stiffness of the robot arm. The arm-tip jitter was measured under additional loads of 1, 3 and 5 kg, respectively, as shown in Fig. 9. The frequency spectra obtained through the FFT of the measured accelerations are shown in Fig. 10 with the corresponding jittering signals of the arm-tip with

1, 3 and 5 kg surplus loads in Figs. 10(a), (b) and (c), respectively. As observed in the figures, all the maximum amplitudes in Figs. 10(a), (b) and (c) fall at 9.59 Hz. The test results are summarised for comparison with the measured jittering frequency in Section 2.1. Since the vibration frequencies of jittering under different loads of 0, 1, 3 and 5 kg are identical, the jittering was not caused by the resonance of the robot arm structure.

Fig. 10 Frequency spectra of jittering signals with different additional loads: (a) 1 kg; (b) 3 kg; (c) 5 kg

2.2.3 Test 3: Vibration Analysis of Drive

System

Based on the test results in Section 2.2.2, the vibration frequency does not change with the addition of masses to the robot arm. We speculate that the arm-tip jittering is due to the unstable outputs of the main kinematic joint drive systems. Subsequently, the motor output and jittering data of each main joint during the operation of the robotic arm were measured. The FFT was performed on these data to derive the corresponding spectrogram by Eq. (3), as shown in Fig. 11. As shown in Fig. 11, the vibration amplitude of the output torque of Joint 3 is the largest among the driving output torques of the joints. Therefore, Joint 3 was identified as the main source of the arm-tip jitter. The kinematic frequency spectra of other joints were also compared with the jittering frequency spectra. The robot arm was subject to three different angular velocities, 0.35 rad/s, 0.50 rad/s, and 0.75 rad/s, respectively, in ROS. The kinematic trajectory and load conditions were the same as the test in Section 2. The ROS was used to directly monitor the output displacement signal of each joint drive system of the arm [53].

Meanwhile, the acceleration signal of arm-tip jittering was measured using an external acceleration sensor and the LMS Test Lab test system.

The acceleration history was integrated twice with respect to the time history of displacement. The spectrogram of the displacement signal of jittering and that of the displacement output by each joint drive system were compared. To ensure the objectivity of experimental data, the vibration frequencies of each joint and arm-tip were taken as the average of results from five identical experiments. The vibration frequency corresponding to the maximum amplitude is shown in Table 3. The frequency differences corresponding to the maximum amplitudes of Joints 2, 3, 4 and those for the arm-tip were in the ranges of 6.75–10.81%, 20.06–26.44% and 21.89–26.33%, respectively. From this, it can be inferred that the major factor causing jittering is the superposition and transmission of output vibration signals of the main kinematic joint drive systems, i.e., the output excitation signals of the joints of the robot arm are the cause of arm-tip jittering.

Fig. 11 Frequency spectrum of joint driving system with different angular velocities: (a) 0.35 rad/s; (b) 0.5 rad/s; (c) 0.75 rad/s

Table 3. Frequency corresponding to maximum vibration amplitude with different joint angular velocities

Angular velocity (rad/s)	Vibration frequency of Joint 2 (Hz)	Vibration frequency of Joint 3 (Hz)	Vibration frequency of Joint 4 (Hz)	Vibration frequency of Arm- tip (Hz)
0.35	2.67	2.64	2.76	2.96
0.50	2.42	2.53	2.63	3.29
0.75	2.52	2.49	2.64	3.38

3 Jittering mitigation method

To mitigate the arm-tip jittering, a jittering mitigator was devised. The schematic of the

mitigator considering a robot arm with jittering vibration in the vertical direction is shown in Fig. 12(a). We assumed a no-damping condition for

this system. However, since damping cannot be neglected in practice, Fig. 12(b) illustrates the mitigator with damping. The jittering mitigator constitutes of three basic components: a box with mass m_B , an oscillator with mass m_O mounted within the box through a spring k_O . The box is connected to the arm-tip via a spring k_B . The operational tools for the robot were mounted on the jittering mitigator box. It should be noted that the proposed mitigator does not rely on damping for the mitigation. Instead, the presence of damping negates the mitigation effects. Figure 12(b) illustrates the mitigator with dampers c_B and c_O . c_B connects the box to the arm-tip, and c_O to the inner oscillator. When c_B or/and c_O are set to zero as in Fig. 12(a), the system becomes undamped.

In the mechanism of the jittering mitigator, the vertical jittering in the arm-tip triggers the mitigator box to vibrate. The mass m_O installed

within the box responds to the vibration of the host box. If the parameters (k_B , k_O , m_B , m_O , etc.) are designed properly, the vibration at the arm-tip (base movements) can be transmitted to vibrations of the mass m_O installed inside the box, while the box remains still. Because the operational tool of the robot arm is mounted on the box, the jittering vibration is practically removed. This conjecture will be proved in theory for both the undamped and damped cases as follows.

3.1 Undamped Jittering Mitigator

The undamped mitigator in Fig. 12(a) can be expressed as an equivalent simple spring–mass–spring–mass system excited by the base of the arm-tip undergoing jittering vibration in the vertical direction. Denoting the arm-tip transient displacement as y , we express the dynamic equations of motions for m_B and m_O as

Fig. 12 Schematic for the proposed jittering mitigator: (a) Undamped; (b) Damped

$$m_B \ddot{x}_B + k_B(x_B - y) + k_O(x_B - x_O) = 0 \quad (5)$$

$$m_O \ddot{x}_B + k_O(x_O - x_B) = 0 \quad (6)$$

where m_B and m_O are the mass of the mitigator box and the oscillator inside the box, respectively; k_B is the stiffness of the spring connecting the mitigator box to the robot arm-tip; k_O represents the stiffness of the spring connecting the inner oscillator to the mitigator box; and, x_B and x_O are the displacements of m_B and m_O , respectively.

Considering the steady state of the system undergoing harmonic motion y with a known frequency, the displacement of x_B takes the form of $x_B = X_B e^{i\omega t}$, where X_B is the amplitude of the steady state solution of x_B . If the amplitude of y is denoted by Y , then the amplitude ratio $|X_B/Y|$ can be derived from Eqs. (5) and (6):

$$\left| \frac{X_B}{Y} \right| = \left| \frac{k_B(-m_O\omega^2 + k_O)}{(-m_B\omega^2 + k_O + k_B)(-m_O\omega^2 + k_O) - k_O^2} \right| \quad (7)$$

where ω is the circular frequency of the base excitation at the arm-tip.

The purpose of the mitigator is to minimize X_B by tuning parameters k_B , k_O , m_B and m_O . According to Eq. (7), when the natural frequency ω_O ($\omega_O = \sqrt{k_O/m_O}$) of the stand-alone k_O - m_O system is equal to ω , $|X_B/Y|$ approaches zero. The natural frequency ω_B ($\omega_B = \sqrt{k_B/m_B}$) of the

stand-alone k_B - m_B system, however, does not affect $|X_B/Y|$.

Fig.13 Relationships between ω/ω_O and $|X_B/Y|$ with different m_O/m_B

To further explore the correlations among m_O/m_B , ω/ω_O and $|X_B/Y|$, ω_O is set to be equal to ω_B for simplicity. The relationships between ω/ω_O and $|X_B/Y|$ with respect to different m_O/m_B of 0.05, 0.5, 1.0 and 2.0 are shown in Fig. 13. As shown in the figure, zero amplitude registers at $\omega/\omega_O = 1$ despite the different m_O/m_B . The flat region approximating zero amplitude expands as m_O/m_B increases. It indicates that m_O/m_B should be as large as possible for effective jitter mitigation.

3.2 Damped Jittering Mitigator

Considering the damped mitigator in Fig. 12(b), the equivalent spring-mass-damper system is also excited by the base of the arm-tip undergoing harmonic motion in the vertical direction.

3.2.1 Damped Jittering Mitigator with c_B only

The governing equations of motions of m_B and

m_O can be expressed as

$$m_B \ddot{x}_B + k_B(x_B - y) + k_O(x_B - x_O) + c_B(\dot{y} - \dot{x}_B) = 0 \quad (8)$$

$$m_O \ddot{x}_O + k_O(x_O - x_B) = 0 \quad (9)$$

where c_B expresses the damping coefficient of

$$\left| \frac{X_B}{Y} \right| = \left| \frac{(-m_O \omega^2 + k_O)(k_B - i\omega c_B)}{(-m_B \omega^2 + k_O + k_B - i\omega c_B)(-m_O \omega^2 + k_O) - k_O^2} \right| \quad (10)$$

As inferred from Eq. (10), when ω_O is equal to ω , $|X_B/Y|$ approaches zero and the variation of ω_B still does not affect $|X_B/Y|$.

Again, for simplicity, ω_O is set to be equal to ω_B and $m_O/m_B = 1$, the correlation between ω/ω_O and $|X_B/Y|$ with respect to different damping ratios (ζ) of 0, 0.1, 0.5 and 1 is explored, where $\zeta = c_B/c_c$ and c_c is the critical damping coefficient ($c_c = 2m_B\omega_B$). The calculated results for the damped mitigator are plotted in Fig. 14. In the plot, the zero amplitude registers at $\omega/\omega_O = 1$ despite the different ζ . The flat region approximating the zero amplitude shrinks as ζ increases. This shows that damping does not mitigate jittering against the conventional understanding of the effect of the damper. The maximum amplitude ratio $|X_B/Y|$ significantly decreases with increasing ζ . Therefore, for effective jitter mitigation, ζ should be increased as much as possible.

the damper connecting the mitigator to the arm-tip. From Eqs. (8) and (9), the amplitude ratio $|X_B/Y|$ can be derived as

Fig. 14 Relationships between ω/ω_O and $|X_B/Y|$ with different ζ

3.2.2 Damped Jittering Mitigator with both c_B and c_O

The governing equations of motions for m_B and m_O can be expressed as

$$m_B \ddot{x}_B + k_B(x_B - y) + k_O(x_B - x_O) + c_B(\dot{x}_B - \dot{y}) + c_O(\dot{x}_O - \dot{x}_B) = 0 \quad (11)$$

$$m_O \ddot{x}_O + k_O(x_O - x_B) + c_O(\dot{x}_O - \dot{x}_B) = 0 \quad (12)$$

where c_O is the damping coefficient of the damper connecting the box to the inner oscillator. Similarly, from Eqs. (11) and (12), the amplitude ratio $|X_B/Y|$ can be derived as

$$\frac{X_B}{Y} = \frac{(k_B k_O - c_B c_O \omega^2 - k_B m_O \omega^2)^2 + \omega^2 (c_B k_O - c_B m_O \omega^2 + k_B c_O)^2}{\sqrt{\{\omega^2 [-c_B c_O - m_B (k_O - m_O \omega^2) - m_O (k_B + k_O)] + k_B k_O\}^2 + \omega^2 [c_B k_O + c_O k_B - \omega^2 (c_B m_O + c_O m_O + c_O m_B)]^2}} \quad (13)$$

According to Eq. (13), when ω_O equals ω , $|X_B/Y|$ approaches zero. The variation of k_B affects the relationship between the minimum amplitude $|X_B/Y|$ and the ratio ω/ω_O . The relationship between k_B and $|X_B/Y|$ is shown in Fig. 15. The maximum amplitude ratio $|X_B/Y|$ is almost unaffected by the increase of k_B .

According to the analyses of results in Section 3.1, the mass ratio m_O/m_B should be as large as possible. Therefore, it is still specified that m_O/m_B is equal to 2 and ω_O is equal to ω_B . The relationships between ω/ω_O and $|X_B/Y|$

Fig. 15 Relationships between ω/ω_O and X_B/Y with different k_B with c_O is equal to zero and different damping c_B from 3 to 17 were derived as shown in Fig. 16. Analyses of the results indicated that the minimum amplitude registered at $\omega/\omega_O = 1$

was almost unchanged with increasing c_B . This implies that the presence of damping did not mitigate jittering. The flat region in the vicinity of the minimum amplitude remained almost unchanged as c_B increases, indicating that the presence of damper c_B did not mitigate the jitter. The maximum amplitude ratio $|X_B/Y|$ decreases significantly with increasing c_B . Therefore, the highest possible c_B should be specified.

According to the analyses of results in Section 3.1, the mass ratio m_O/m_B should be as large as possible. Therefore, we specify an m_O/m_B of 2 and a ω_O that is equal to ω_B . The relationships between ω/ω_O and $|X_B/Y|$ with $c_B = 40$ and different damping c_O from 0 to 8 were derived as shown in Fig. 17. The analyses of results indicated that the minimum amplitude registering at ω/ω_O of 1 increased with increasing c_O . The maximum amplitude ratio $|X_B/Y|$ decreases with increasing c_O . Therefore, the highest possible value of c_O should be specified.

Fig. 16 Relationship between ω/ω_0 and X_B/Y with different c_B

Fig. 17 Relationship between ω/ω_0 and X_B/Y with different c_0

4 Numerical simulations

Based on the above analyses, as shown in Fig. 18, a numerical model [55] was constructed using ADAMS®, to simulate the jittering vibration in the robot used in the present experiments with the damped or undamped jittering mitigator.

4.1 Calculation of Parameters for Jittering

Mitigator

As indicated in Fig. 10, the base vibration frequency f ranges from 9.32–10.31 Hz, with the maximum amplitude of vibration at 9.6 Hz. Thus, the corresponding circular frequency $\omega = 2\pi f$ falls in the range of 58.56–64.78 rad/s, with the maximum amplitude of vibration at 60.3186 rad/s. To mitigate the jittering vibration, the parameters of the mitigator must be designed appropriately. The design conditions are as

follows.

- 1) For the undamped absorber, the target is $|X_B/Y| \leq 50\%$, as shown in Eq. (14).
- 2) For the mitigator with damping, Eq. (12) shall be used.
- 3) In Section 3, when $\omega/\omega_0 = 1$, $|X_B/Y|$ = zero, and the change of ω_B does not influence $|X_B/Y|$; thus, $\omega_0/\omega_B = 1$.
- 4) For the maximum frequency $\omega_0 = 60.3186$ rad/s, the change range of ω_0/ω_B is from 0.97 to 1.073. m_0/m_B as ω changes from 58.560 to 64.780 rad/s is calculated thereafter.

Based on the above conditions, $m_0/m_B \geq$

$$0.34, \quad k_B = k_0 = 3638.3 \text{ N/m}, \quad \left| \frac{X_B}{Y} \right|$$

can be derived as

$$\left| \frac{X_B}{Y} \right| = \left| \frac{(-m_0\omega^2 + k_0)(k_B - i\omega c_B)}{(-m_B\omega^2 + k_0 + k_B - i\omega c_B)(-m_0\omega^2 + k_0) - k_0^2} \right| \leq 0.5 \quad (14)$$

4.2 Numerical Simulations of Jittering

Vibration

Multi-rigid-body dynamic simulations using ADAMS were implemented to evaluate the jittering mitigator system. The simulation step is as follows.

1) Establishing robot arm model: According to the connections, fitting relations and relative positions of the robot arm parts in Fig. 1, a three-dimensional model of the robot arm was developed on the ADAMS-view module, as shown in Fig. 18. To improve the efficiency of the model, the major mechanical aspects were captured, while irrelevant details such as small screws, surface texture, non-bearing boss, and other complex and small features were eliminated. The internal structure of each joint was simplified as a solid block with an equivalent stiffness. The equivalent mass for each part was specified by the respective equivalent material densities. The total mass of the robot arm in the model is similar with the practical value of 23 kg. Both the masses m_B and m_O of the jittering mitigator were set to be 1 kg.

Fig. 18 Multi-rigid dynamic simulation model

2) Defining the connection relationships among different parts: When the robot arm moves in the trajectory, as in Section 2, only Joints 2, 3 and 4 rotate. Therefore, Joints 2, 3 and 4 are defined as revolute joints, while the rest, the upper arm, forearm and base are defined as rigid bodies. Masses m_B and m_O were set 100 and 200 mm away from the arm-tip along the z-axis. Springs k_B connect the arm-tip and the mass m_B , and spring k_O connects masses m_B and m_O . The stiffnesses of springs k_B and k_O were specified as 3,638.33 N/m.

3) Defining the boundary and loading conditions: The base is fixed, and the arm-tip, m_B and m_O move along the z-axis to ensure system stability. The time-history displacement in Fig. 10 is applied on the arm-tip as the vibration source.

Fig.19 Jittering vibration with $m_O/m_B = 1$

Fig. 20 Jittering vibration with $m_O/m_B = 2$

4) Result analysis: Based on the conditions that $\omega_B = \omega_O = 2\pi f = 60.3186$ rad/s, $m_B = m_O = 1$ kg or $m_O/m_B = 1$, $k_B/m_B = k_O/m_O$, $k_B = k_O = 3,638.33$ N/m and $\zeta = 0.08$, the time-history-displacement of m_B is derived, as shown in Fig. 19. The average vibration amplitude of the jittering signal at the arm-tip is 0.2 mm, and that of m_B is less than 0.0625 mm, according to Fig. 19. Compared with the amplitude of the original jittering vibration at the arm-tip, the average vibration amplitude of m_B significantly decreased by 68.75%, so that $|X_B/Y| < 31.25\%$. For $m_B = 0.275$ kg and

$m_O = 0.55$ kg, i.e., $m_O/m_B = 2$, the time-history displacement of m_B is shown in Fig. 20. The average vibration amplitude of m_B is 0.06 mm. Compared with the amplitude of the original jittering vibration at the arm-tip, the average amplitude m_B significantly decreased by 68.8%, which indicates $|X_B/Y| < 31.1\%$. Comparing the results in Figs. 19 and 20, the presence of different m_O/m_B does not alter the damping effect. Therefore, in the following experimental study, a constant ratio of 2 will be specified for m_O/m_B .

5 Experimental verification

Experimental tests as shown Fig. 21 were carried out to evaluate the performance of the jittering mitigator. In Fig. 21, $m_B = 0.275$ kg and $m_O = 0.55$ kg, and $m_O/m_B = 2$. Because the change of k_B does not affect $|X_B/Y|$ as shown in Fig. 17, k_B and k_O were specified as 1,000 N/m and 2,000 N/m, respectively, to ensure $k_B/m_B = k_O/m_O$. The mitigator was installed at the robot arm-tip. The acceleration sensors were mounted on the robot arm tip, the box with mass m_B and the oscillator with mass m_O . The LMS

Test Lab was used to record the vibration signals in the z-direction of the arm-tip, box, and oscillator, under the same operating conditions as

Fig. 21 Experimental test apparatus

Fig. 22 Original displacement histories of robot arm-tip and m_B

Fig. 23 Displacement histories by experiment and simulation for m_B

in Section 4. Through FFT analyses, the experimental vibration signals of m_B were compared with that of the arm-tip, as shown in Fig. 22. The average amplitude of m_B was 0.58 mm. Compared with the original vibration amplitude of jittering at the arm-tip, the average amplitude of m_B significantly decreased by 69.0%, i.e., $|X_B/Y| < 31\%$. Figure 23 compares the simulated and experimental vibration signals in the z-direction for m_B . As indicated in Fig. 23, variations from both the experiments and numerical simulations exhibited similar patterns with equal average amplitudes. Therefore, the simulation results are reasonable. Both experiments and numerical simulations demonstrated the effectiveness of the proposed jitter mitigation method.

6 Conclusions

This paper investigates the key factors influencing the arm-tip jittering of a robot under typical working conditions. A new method for mitigating the jittering vibration in the robot arm-tip was proposed. A jittering mitigator was designed according to the mitigation target and damping ratio. The effectiveness of the mitigator was demonstrated through numerical multi-rigid body dynamics simulations and experiments. The major findings of this study are summarised and concluded as follows.

- 1) A load-sweeping technique was used to apply different loads at the arm-tip and find out if jittering was indeed caused by resonance, while retaining the overall stiffness of the arm. If the jittering frequencies do not change, the frequency of jittering and the fundamental natural frequency of the robot arm are not related.
- 2) The jittering of the robot arm does not arise from the resonance of the robotic arm. Instead, it is highly probable that the jittering is due to the output excitation of the internal joint drivers. Joint 3 had the most influence on the jittering, followed by Joints 2 and 4.
- 3) Using the proposed mitigator, the average amplitude of the robot arm-tip jittering was reduced by 68%. The proposed jitter mitigation method is straightforward without any time delay, and it is effective in reducing the jittering vibration of robot arms.
- 4) The change in the mass ratio m_O/m_B had a significant impact on the damping effect [56]. The damping effect becomes more significant with the increase in mass ratio.
- 5) The natural frequency of the second mass system m_O affects the damping effect more than that of the main mass system m_B . The influence of damping coefficient c_O on the amplitude of the robot arm-tip jitter is

greater than that of the damping coefficient c_B .

- 6) The principle and mechanism of the proposed method are based on transmission, rather than damping or active suppression for the traditional devices. The method for mitigating jitters vibration paves the way for improving the trajectory accuracy of the robot.

7 Declaration

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Availability of data and materials

The datasets supporting the conclusions of this article are included within the article.

Authors' contributions

The author's contributions are as follows:

Shuyong Duan was in charge of the whole trial; Shuyong Duan wrote the manuscript; Chunlu Li and Pengfei Xu assisted with sampling and laboratory analyses.

Competing interests

The authors declare no competing financial interests.

Consent for publication

Not applicable

Ethics approval and consent to participate

Not applicable

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Figures

Figure 1

Six-axes tandem robot arm (AUBO-i5)

Figure 2

Trajectory of robot arm

Figure 3

Amplitude of jittering at different speeds

(a)

(b)

(c)

Figure 4

Jittering signal of robot arm-tip: (a) acceleration; (b) frequency spectrum; (c) time-domain signal

Figure 5

Analyses of movement: (a) posture at 0 and 12 s; (b) posture at 3 and 9 s; (c) posture at 6 s

Figure 6

Flow chart for the exploration of the underlying cause of robot arm jitter

Figure 7

Wireframe diagram of the robotic arm

Figure 8

Stabilisation diagram from modal analysis

Figure 9

Robotic arm under different loads

(a)

(b)

(c)

Figure 10

Frequency spectra of jittering signals with different additional loads: (a) 1 kg; (b) 3 kg; (c) 5 kg

(a)

(b)

(c)

Figure 11

Frequency spectrum of joint driving system with different angular velocities: (a) 0.35 rad/s; (b) 0.5 rad/s; (c) 0.75 rad/s

Figure 12

Schematic for the proposed jittering mitigator: (a) Undamped; (b) Damped

Figure 13

Relationships between ω/ω_0 and $|X_B/Y|$ with different m_O/m_B

Figure 14

Relationships between ω/ω_0 and $|X_B/Y|$ with different ζ

Figure 15

Relationships between ω/ω_0 and X_B/Y with different k_B

Figure 16

Relationship between ω/ω_0 and X_B/Y with different c_B

Figure 17

Relationship between ω/ω_0 and X_B/Y with different c_o

Figure 18

Multi-rigid dynamic simulation model

Figure 19

Jittering vibration with $m_O/m_B=1$

Figure 20

Jittering vibration with $m_O/m_B=2$

Figure 21

Experimental test apparatus

Figure 22

Original displacement histories of robot arm-tip and m_B

Figure 23

Displacement histories by experiment and simulation for m_B