Origin of Probability in Quantum Mechanics and the Physical Interpretation of the Wave Function

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Research Article

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Posted Date: November 16th, 2020

DOI: <https://doi.org/10.21203/rs.3.rs-95171/v2>

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Origin of Probability in Quantum Mechanics and the Physical Interpretation of the Wave Function

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Abstract

The theoretical calculation of quantum mechanics has been accurately verified by experiments, but Copenhagen interpretation with probability is still controversial. To find the source of the probability, we revised the definition of the energy quantum and reconstructed the wave function of the physical particle. Here, we found that the energy quantum $\hat{e}$ is $6.62606896 \times 10^{-34} \text{J}$ instead of $h\nu$ as proposed by Planck. Additionally, the value of the quality quantum $\hat{o}$ is $7.372496 \times 10^{-51} \text{kg}$. This discontinuity of energy leads to a periodic non-uniform spatial distribution of the particles that transmit energy. A quantum objective system (QOS) consists of many physical particles whose wave function is the superposition of the wave functions of all physical particles. The probability of quantum mechanics originates from the distribution rate of the particles in the QOS per unit volume at time t and near position r . Based on the revision of the energy quantum assumption and the origin of the probability, we proposed new certainty and uncertainty relationships, explained the physical mechanism of wave-function collapse and the quantum tunnelling effect, derived the quantum theoretical expression of double-slit and single-slit experiments.

Keywords: probability origin, wave-function collapse, uncertainty principle, quantum tunnelling, double-slit and single-slit experiments.

Introduction

As one of the pillars of modern physics, quantum mechanics has many notable achievements, with its theoretical calculation verified by experiments. However, its physical nature is still controversial. Studies on the counterintuitive phenomena of quantum mechanics, such as the quantum superposition state, wave-function collapse, quantum tunnelling effect, uncertainty principle, double-slit interference, and

single-slit diffraction, have not reached a consensus. From a practical point of view, if the calculation results are correct, then the physical mechanism does not need to be debated. However, to understand nature, revealing the essence of matter and its interactions is the fundamental task of physics. Therefore, quantum mechanics must be studied to explore the essence of the material world.

The quantum superposition state, wave-function collapse, the quantum tunnelling effect, the uncertainty principle, double-slit interference, single-slit diffraction, and other counterintuitive phenomena are seemingly related to the probability of quantum mechanics, which has been physically interpreted by the Copenhagen school. Although Planck, Einstein, Schrödinger, and other physicists strongly opposed it, they did not propose a better interpretation. In quantum mechanics, the probability is based on Born's hypothesis, which lacks physical sources; thus, Steven Weinberg has repeatedly questioned that the integration of probability into physics has confused physicists, but the difficulty of quantum mechanics related to its source rather than probability since 2016. Therefore, to understand the counterintuitive phenomenon of quantum mechanics, one must trace the origin of probability in quantum mechanics.

Results and discussion

Revision of the energy quantum assumption

Smallest unit of energy - the energy quantum

Planck¹ proposed an energy density Formula (1) for black-body radiation in 1900, consistent with experimental data. Based on an assumption that the resonator of the black-body radiation source discontinuously radiates energy according to the smallest unit of $\varepsilon = h\nu$, he theoretically derived the following formula.

$$\varepsilon_\nu = \frac{8\pi h\nu^3}{c^3} \frac{1}{\exp(h\nu/kT)-1} \quad (1)$$

where ε_ν is the energy density of black-body radiation; ν is the frequency of the resonator; c is the speed of light; k is the Boltzmann constant; T is the thermodynamic temperature; and h is the Planck constant.

In the process of deriving Formula (1), Planck^{2,3} obtained Formula (2), which shows the energy radiated by the resonator of the black-body radiation source.

$$E_n = nh\nu, n=1, 2, 3... (2)$$

Based on Formula (2), Planck believed that the energy radiated by a black body is discontinuous and thus can be taken only as an integral multiple of $h\nu$. If $n=1$, the resulting energy $h\nu$ is the smallest energy unit possible (i.e., it cannot be subdivided), defined as the energy quantum. According to the definition of this energy quantum, de Broglie assumption that the energy E of a physical particle is equal to the energy of an energy quantum $h\nu$ is clearly illogical, and it is also difficult to logically establish the wave function of a physical particle.

In the assumption of $\varepsilon = h\nu$, Planck sets ε as the minimum energy released by the resonator of the black-body radiation source. Moreover, the dimension of the coefficient h is Joule·sec (J.s) to maintain the dimension of energy of ε when the dimension of the vibration frequency of the resonator is s^{-1} . However, the assumption of $\varepsilon = h\nu$ shows that the resonator, which radiates energy externally, is actually doing work outside. The minimum amount of power of a single resonator of a black-body radiation source is $\varepsilon = h\nu$, where the dimension of ε is $J.s^{-1}$ and the dimension of the coefficient h is Joule (J). Based on the above analysis, if both sides of Formula (2) are divided by unit time T_0 (1 s), then Formula (2) becomes Formula (3).

$$E_n/T_0 = nh\nu/T_0 (3)$$

Let $E = E_n/T_0$ and $\hat{e} = h/T_0$; then

$$E = n\nu\hat{e} (4)$$

where E is the energy radiated externally by a single resonator of the black-body radiation source per unit time, which is the power with dimension $J.s^{-1}$. If $n = 1$ in Formula (4), then Formula (5) holds.

$$\hat{e} = E/\nu = h/T_0 = 6.62606896 \times 10^{-34} J (5)$$

Formula (5) shows that $\hat{e}$ is the smallest unit of energy radiated by a single resonator of a black-body radiation source. We define the minimum unit of energy $\hat{e}$ as an energy quantum with the unit of Joule. Clearly, the energy of the material world is discontinuous, and the energy quantum is $\hat{e}$ instead of $h\nu$, with a value of 6.62606896

$\times 10^{-34}$ J. The energy of the energy quantum is a physical constant that cannot be subdivided and changed and is not related to time or any other factor. However, the period of the corresponding vibration can vary.

Based on Planck's hypothesis, A. Einstein^{4,5} believed that light wave energy is discontinuous and that the smallest unit of each discontinuous and indivisible packet of energy is $\varepsilon=hf$, named the photon.

Everyone knows that the radiation of a black-body radiation source takes the form of electromagnetic waves, namely, light waves. According to the definition of the energy quantum, the minimum energy of a period of an electromagnetic wave radiated by a vibration of a resonator in a black-body radiation source is equivalent to the energy of an energy quantum, with a value of $6.62606896 \times 10^{-34}$ J, which is transmitted outward at the speed of light. Therefore, we defined the minimum energy transmitted by a periodic electromagnetic wave as a photon, then, the energy of a photon is $\hat{e}$ instead of hf of Einstein's hypothesis. As a physical constant that cannot be divided and changed, it is not related to time or any other factor. However, the period and wavelength of the corresponding electromagnetic wave can vary.

Smallest unit of quality - the quality quantum

Einstein proposed that the energy and quality of matter are equivalent; Formula (6) shows their relationship.

$$E = mc^2 \quad (6)$$

where E is the energy; m is the quality; and c is the speed of light in a vacuum.

When the energy takes the energy quantum $\hat{e}$, an indivisible minimum unit of quality exists, namely, the quality quantum $\hat{o}$. Formula (7) shows the relationship between $\hat{e}$ and $\hat{o}$.

$$\hat{o} = \hat{e}/c^2 \quad (7)$$

Substituting the values of the energy quantum $\hat{e}$ and the speed of light c into Formula (7), the quality quantum $\hat{o}$ is 7.372496×10^{-51} kg. In addition, the photon rest mass is zero, with a total mass of 7.372496×10^{-51} kg. In the material world, the minimum value of the rest mass of physical particles other than photons is $\hat{o}$. Therefore, the

mass of photons and physical particles can increase or decrease only following the integral multiples of the quality quantum. Mass and energy discontinuities are the essences of the material world. If the energy quantum, quality quantum, and light speed are regarded as natural constants of the material world, then Formula (8) shows their relationship.

$$\hat{e} = \hat{o}c^2 \quad (8)$$

Reconstruction of the physical particle wave function

Wave function of photons

According to electromagnetic theory, the electromagnetic wave function is as shown in Formula (9).

$$\psi = \psi_0 e^{2\pi i(r/\lambda - \nu t)} \quad (9)$$

where ψ and ψ_0 are the amplitude and maximum amplitude of the electromagnetic wave, respectively; ν and λ are the frequency and wavelength of the wave, respectively; and r and t are the spatial position and time, respectively.

According to its definition, a photon's energy is $\hat{e}$, which corresponds to one period and one wavelength of electromagnetic wave energy in a single electromagnetic wave. $n(n = \nu)$ photons with a frequency ν are superimposed to form a superimposed photon, the energy E and momentum p of which are defined in Formulas (10) and (11).

$$E = n\hat{e} = h\nu \quad (10)$$

$$p = mc = n\hat{e}/c = h\nu/c = h/\lambda \quad (11)$$

Formula (10) shows that the superimposed photon is a photon as defined by Einstein. In addition, Formulas (12) and (13) indicate that each photon's phase, frequency, and wavelength in the superimposed photon are the same.

$$\nu = E/h \quad (12)$$

$$\lambda = h/p \quad (13)$$

Inserting Formulas (12) and (13) into Formula (9) with $\hbar = h/2\pi$ yields Formula (14).

$$\psi = \psi_0 e^{i(p \cdot r - Et)/\hbar} \quad (14)$$

Formula (14) is a function relation that characterizes the spatial propagation of

superimposed photon streams after the electromagnetic wave function is quantized. In the wave function (14) of photon streams, ψ and ψ_0 represent the intensity amplitude and the maximum intensity amplitude of energy transmitted by the photon streams, respectively. The modular square of ψ is the energy density of the photon streams.

Since Formula (9) is the wave function of a monochromatic electromagnetic wave with frequency ν , Formula (14) is the wave function of a single energy superimposed photon stream with frequency ν . More complex electromagnetic waves are mixed with multiple frequencies in the objective world, the wave function of which is formed by the linear superposition of the monochromatic electromagnetic wave functions, expressed as

$$\psi = \sum \psi_{0\nu_j} e^{2\pi i(r/\lambda_j - \nu_j t)}, \quad j=1, 2, 3\cdots \quad (15)$$

where $\psi_{0\nu_j}$ represents the maximum intensity amplitude of electromagnetic waves with frequencies ν_j in the composite electromagnetic waves and j is a positive integer from 1 to ∞ .

Formula (16) is the photon stream wave function at the composite frequency quantized by the composite electromagnetic wave function (15).

$$\psi = \sum \psi_{0p_j} e^{i(p_j \cdot r - E_j t)/\hbar}, \quad j=1, 2, 3\cdots \quad (16)$$

where ψ_{0p_j} represents the maximum intensity amplitude of energy transmitted by the photon stream with momentum p_j in the composite photon stream and ψ represents the energy amplitude delivered by the composite photon stream. In addition, the modular square of the wave function ψ of the composite photon stream is the energy density transmitted by the composite photon stream.

The quantization of the electromagnetic wave function does not change the nature of the energy transmitted by photons and electromagnetic waves, while the energy density corresponds to the number of photons in space and time. In the whole space and time range of electromagnetic waves and photon motion, N is the total number of photons, and N_j is the number of photons with momentum p_j . Let $|C_{Pj}|^2 = N_j/N$, which represents the photon distribution rate with momentum p_j . Since the photons move throughout the entire space-time range with a constant total number, the sum of the

distribution rates equals 1. Then, Formula (16) can be transformed into Formula (17).

$$\psi = \sum C_{pj} e^{i(p_j \cdot r - E_j t)/\hbar}, \quad j=1, 2, 3 \dots \quad (17)$$

where ψ represents the total momentum photon distribution rate amplitude at time t in space r . Formula (17) expresses the function relation of the distribution rate amplitude, which periodically changes with time and the spatial position when photons move in space, i.e., the wave function of photon motion. Moreover, the modular square of the wave function equals the photon distribution density at time t in position r .

Wave function of physical particles

As a particle, photons have zero rest mass, while the photon velocity at all frequencies in a vacuum equals the speed of light c . Energy transmitted by photons has periodic changes and fluctuations in space and time directly related to the number of photons. Additionally, the energy fluctuation causes the photon distribution rate amplitude to change periodically with time and space. The photon distribution density also fluctuates, as represented by the wave function (17).

A moving physical particle has energy; thus, it is a particle that transmits energy. Regarding energy transfer, physical particles are similar to photons, and their rest mass is not zero, with a moving speed less than that of photons. According to Formula (8), the energy quanta transmitted by the physical particle are proportional to the number of the quality quanta contained. Therefore, the distribution densities of the energy and quality quanta at time t and position r are equal.

When the energy transmitted by a particle equals that of a superimposed photon of the same frequency, the energy and momentum of the latter in Formula (17) can be transformed into those of the former. In addition, $|C_{pj}^0|^2$ represents the distribution rate of the particles' quality quanta with the momentum p_j in a multi-particle system, where C_{pj}^0 represents the distribution rate amplitudes of the quality quanta.

Regardless of the interaction between particles, the wave function of the superimposed photons becomes that of the multi-particle system. Therefore, the multi-particle system is a quantum objective system (QOS), and Formula (18) is its wave function.

$$\psi = \sum C_{pj}^o e^{i(p_j \cdot r - E_j t)/\hbar}, \quad j=1, 2, 3 \dots (18)$$

where ψ represents the distribution amplitude of the quality quanta of the particles at time t in space r . Meanwhile, the modular square of the wave function ψ equals the distribution density of the particles' quality quanta at time t and position r .

In addition, if all particles have the same quality, ψ represents the amplitude of the distribution rate of the particles at time t in space r , which changes periodically with time and space. Moreover, the modular square of the wave function ψ equals the distribution density of the particles at time t and position r . Formula (18) is the QOS wave function.

Since the total number and energy of particles in space are constant, the particle distribution rate equals 1, that is,

$$\sum |C_{pj}^o|^2 = 1, \quad j=1, 2, 3 \dots (19)$$

Origin of probability in quantum mechanics

Formula (18) is the wave function of a QOS composed of all state particles with the same quality. Regardless of the particle interaction, each particle has its energy, momentum, and state of motion. Considering a particle in the QOS, we do not know which one of the many in the system it is before measurement. Thus, it may be any one of the QOS. To facilitate a comprehensive study of this particle, we built a mathematical system artificially, called a quantum mechanics system (QMS). In the QMS, we study only one particle, which may be any particle in the QOS before measurement. The particle may correspond to all state particles in the QOS, meaning that the particle in the QMS has the possibility of all state particle distribution density at time t in space r in the QOS. We refer to the distribution rate of the particles in the QOS as the probability of the particle occurrence at time t in space r in the QMS, which is the origin of the Born probability hypothesis⁶ in quantum mechanics.

Therefore, for the particle in the QMS, Formula (18) is the wave function. The particle wave function in the QMS is represented as Formula (20) to distinguish the QOS wave function.

$$\psi = \sum C_{pj} e^{i(p_j \cdot r - E_j t)/\hbar}, \quad j=1, 2, 3 \dots (20)$$

If the wave function of the QMS is continuous, Formula (20) can be expressed in integral form (21).

$$\psi = \frac{1}{(2\pi\hbar)^{3/2}} \iiint_{-\infty}^{+\infty} C_p e^{i(\mathbf{p}\cdot\mathbf{r}-Et)/\hbar} d\mathbf{p}_x d\mathbf{p}_y d\mathbf{p}_z \quad (21)$$

where ψ represents the probability amplitude of a particle at time t in space $\mathbf{r}$, which changes periodically with time and space. Meanwhile, the modular square of the wave functions represents the probability density of the particle appearing at position $\mathbf{r}$ at time t . The wave function is exactly the same as de Broglie⁷ wave function. From a mathematical point of view, Formula (20) and (21) indicate that the wave function of the particle in the QMS is always on a linear superposition state in all states of the particles in the QOS, and this is the physical nature of the state superposition principle of the wave function.

Particles in different states in the QOS have different momenta and energy, indicating different moving velocities. A particle swarm composed of multiple particles expands in space over time. Therefore, the continuous expansion of the space volume occupied by the QOS is the physical nature of the continuous expansion of a particle's wave function in the QMS.

Since the particle always exists in the whole space, the particle probability in that space equals 1.

$$\sum |C_{pj}|^2 = 1, \quad j=1, 2, 3\cdots \quad (22)$$

$$\iiint_{-\infty}^{+\infty} |\psi|^2 dx dy dz = 1 \quad (23)$$

The QOS and QMS wave functions follow Schrödinger's formula (24)⁸.

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 \psi + U\psi \quad (24)$$

Since the QOS has no concept of probability, no randomness exists. However, the QMS introduces probability through artificial mathematical operations; thus, randomness is introduced.

Physical interpretation of the wave function

Revision of the uncertainty principle and physical nature

In a QMS, Heisenberg's uncertainty relation, shown in Formula (25), is deduced based

on the state superposition principle of the wave function, Born's probability hypothesis, and the non-commutation relation of operators.

$$\Delta x \cdot \Delta p \geq \hbar/2 \quad (25)$$

where Δx and Δp are the uncertainties of the position and momentum of a particle, respectively. The physical meaning of the uncertainty relation is that a particle's position and momentum cannot be determined simultaneously. It indicates that the particle does not have a defined trajectory of motion.

Based on the essence of the state superposition principle in a QMS and the origin of the Born probability, $\Delta x = x - \bar{x}$ and $\Delta p = p - \bar{p}$ in Formula (25), where x and p are the measured values of the determined particle position and momentum, respectively; $\bar{x}$ and $\bar{p}$ are the average values of all possible positions and momenta in the QMS, which can be calculated by Formulas (26) and (27), respectively, and correspond to the average value of all particle positions and momenta in the QOS, rather than that of the measured particle. Therefore, Δx and Δp indicate the deviation degree of the particle from the system (namely, the standard deviation) instead of being the measurement errors of the particle position and momentum. Use of the system's standard deviation to indicate the measurement error of the position and momentum of a particle is not appropriate.

$$\bar{x} = \iiint_{-\infty}^{+\infty} \psi^* \hat{x} \psi \, dx dy dz \quad (26)$$

$$\bar{p} = \iiint_{-\infty}^{+\infty} \psi^* \hat{p} \psi \, dx dy dz \quad (27)$$

where $\hat{x}$ and $\hat{p}$ are the position and momentum operators, respectively.

In a QMS, a particle can be measured by calculating its wave function with the operator. However, the wave function is expressed by the superposition state of particles with various states in the QOS; the position and momentum obtained by each measurement may be different; and the position and momentum of another particle are used to represent those of the particle to be measured. Therefore, in Formula (25), the position and momentum of a specified particle should not be expressed with those of another particle.

Based on the above analysis, the mathematical treatment of the state superposition

and the introduction of probability assumptions in the QMS make the measurement object random; thus, the measurement is not necessarily the objective state of the specified particle. Although the uncertainty relation of Formula (25) is obtained through mathematical deduction, its physical meaning obviously cannot be expressed as not being able to accurately determine the position and momentum of a particle at the same time.

The measurement of a particle in state j in the QOS requires applying an operator corresponding to the measured physical quantity to its wave function. When the particle moves in the X direction, the wave function is Formula (28). Formula (29) is obtained by applying any operator $\hat{F}$ to Formula (28).

$$\psi_j = e^{i(p_j \cdot x - E_j t)/\hbar} \quad (28)$$

$$\hat{F}\psi_j = F_j e^{i(p_j \cdot x - E_j t)/\hbar} \quad (29)$$

In Formula (29), F_j is the physical quantity eigenvalue of the particle in state j corresponding to the operator $\hat{F}$, indicating that the real number of the physical quantity F determines the measured value.

Formula (29) shows that $\hat{F}$ is an arbitrary operator, which indicates that Formulas (30) and (31) can be used to obtain the determined positive real values by measuring any physical quantity.

$$p_j x = nh, n=1, 2, 3 \dots \quad (30)$$

$$E_j t = nh, n=1, 2, 3 \dots \quad (31)$$

$n \geq 1$; thus, $p_j x \geq h$ and $E_j t \geq h$, which can be represented as

$$px \geq h, \quad px = nh, n=1, 2, 3 \dots \quad (32)$$

$$Et \geq h, \quad Et = nh, n=1, 2, 3 \dots \quad (33)$$

When the relationship between (32) and (33) is satisfied, all the physical quantities of the particle are determined, including its momentum and position. The geometric image consisting of these determined positions and momentums is the trajectory of a particle motion. Outside the range determined by these two relationships, all physical quantities of the particle are uncertain.

The certainty relation is as follows:

$$\begin{cases} px = nh \\ Et = nh \end{cases}, n=1, 2, 3 \dots \quad (34)$$

The uncertainty relation is as follows:

$$\begin{cases} (n-1)h < px < nh \\ (n-1)h < Et < nh \end{cases}, n=1, 2, 3 \dots \quad (35)$$

Physical mechanism of wave-function collapse

According to the origin of the probability and the nature of the state superimposition principle of the wave function in a QMS, for a particle with a stable state, the eigenfunction obtained by multiple measurements is the same, and measurements cause the superimposition state of the wave function to collapse to the same eigenstate. When Formula (34) is satisfied, the measurement obtains a definite engenvalue, and all physical quantities have their definite values. When Formula(35) is satisfied, the measurement does not have a eigenvalue and the particle is on an uncertain state. For a particle changing states, the eigenfunctions obtained by multiple measurements may be different, and every measurement makes the superimposition state of the wave function collapse to a possible different eigenstates. When Formula (34) is satisfied, the measurement obtains a defined eigenvalue. When Formula (35) is satisfied, the measurement does not have a eigenvalue, and the particle is on the superimposition state (quantum jump state) between the two determining states.

Physical mechanism of the quantum tunnelling effect

In a QMS, a particle's wave function in a finite deep potential trough is obtained by solving the Schrödinger equation. Then, the probability density of the particle in space is obtained. The results show that when the particle energy is lower than the energy of the potential well, the particle has a certain probability of passing through the potential well. However, the physical mechanism of the quantum tunnelling effect is unclear.

The origin of probability indicates that the probability density of a particle in the QMS at time t and position r is the distribution density of the particles in the j^{th} state in the corresponding QOS at time t and position r . Therefore, the energy of a particle in the QMS is the average energy of the particle calculated by Formula (36).

$$\bar{E} = \iiint_{-\infty}^{+\infty} \psi^* \hat{H} \psi \, dx dy dz \quad (36)$$

The average energy equals that of all particles in the QOS. The part particle energy of all particles in the system must be higher than the average energy $\bar{E}$. In addition, the energy of this group of particles should be partly higher than the energy U of the finite deep potential trough, and the particles should be able to cross the energy barrier. Therefore, for a particle in the QMS, when its energy eigenvalue is higher than the average energy and energy U of the finite deep potential trough, it will cross the energy barrier. However, for the infinite deep potential trough, since no particles with infinite energy exist in the QOS and the particle energy eigenvalue in the QMS is not infinite, the particles cannot cross the infinite energy barrier.

Quantum theory of double-slit and single-slit experiments

Based on wave theory, in double-slit interference and single-slit diffraction, the bright-dark stripe is formed by interference between two waves with the same frequency. The wave coherency theoretically yields Formula (37), consistent with the experimental results.

$$d\sin\theta = n\lambda \quad (37)$$

If photons or particles arrive at the receiving screen discontinuously one by one, the bright-dark stripe appears when the particle number increases to a certain extent. The results of the experiments cannot be explained either by the theory of fluctuations or by the theory of particles. Therefore, wave-particle duality is the basic theory of quantum mechanics.

Based on particle theory, in double- or single-slit experiments, photons or particles move along the X direction after passing through the slit. In addition, in a QMS, the position of a photon or particle is measured by applying the position operator to the wave function. Namely,

$$\hat{x}\psi = xe^{i(p\cdot x - Et)/\hbar} \quad (38)$$

where the measurement has a defined value x when $px = nh$ and $Et = nh$. In other words, only in this space and time can photons or particles be measured. If two photons or particles with the same momentum from different directions are fired towards the receiving screen as a measuring instrument, whether they come from a double slit, single slit or the obstacle's edge, the movement distance is $x_1 = n_1 h/p$

and $x_2 = n_2 h/p$, and the distance difference is $x_1 - x_2 = (n_1 - n_2)h/p$. Depending on the geometry of the path of two photons or particles, $x_1 - x_2 = d \sin \theta$, considering $n_1 - n_2 = n$ and $h/p = \lambda$. Thus, Formula (39) can be derived.

$$d \sin \theta = n \lambda \quad (39)$$

Formula (39) refers to the quantum theory of double-slit and single-slit experiments for photons or particles, where d is the distance between two photon or particle sources; θ is the deviation angle of the receiving point on the receiving screen; n is a natural number not less than zero; and λ is the wavelength of the photon or particle. The expression is derived from the new uncertainty principle, which is the same as Formula (37) obtained with the wave theory of light interference and diffraction.

In the range of $px = nh$, the photons or particles have a determined measurement value on the receiving screen, indicating that this area with photons or particles is a bright stripe. Correspondingly, in the range of $nh < px < (n + 1)h$, the photons or particles have no determined measurement value on the receiving screen, indicating that this area with no photons or particles is a dark stripe.

The derivation of Formula (39) shows that a photon or particle does not need to pass through two slits simultaneously nor do the two photons or particles need to reach the measuring point of the receiving screen simultaneously. As long as the relationship in (39) is satisfied, the results of the double-slit, single-slit, or diffraction experiments are the same regardless whether the photons and particles are viewed as waves or particles.

Conclusions

(1) The energy of the objective material world is discontinuous with an indivisible minimum unit of energy, namely, the energy quantum. The quantum is $\hat{e}$ instead of $h\nu$ as assumed by Planck, with a value of $6.62606896 \times 10^{-34}$ J. Moreover, the quality of the objective material world is discontinuous, and an indivisible minimum unit of quality exists, namely, the quality quantum $\hat{o}$, with a value of 7.372496×10^{-51} kg.

(2) The energy transmitted by a particle's motion has volatility, restricting the particle's distribution in space. This volatility can be represented by a wave function. Multiple particles constitute a QOS, the wave function of which is formed by

superposing the wave functions of all particles. In the QMS, a particle's wave function is formed by superposing the wave functions of all particles in the QOS. The probability of quantum mechanics originates from the distribution rate of particles in the QOS per unit volume at time t and near position r .

(3) Based on the revision of the energy quantum assumption and the origin of probability, new certainty and uncertainty relations are proposed, the physical mechanisms of wave-function collapse and the quantum tunnelling effect are explained, the quantum theoretical expression of double-slit and single-slit experiments is derived, and the physical nature of counterintuitive phenomena in quantum mechanics is revealed.

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