Collatz conjecture($3X+1$) There is a hidden theorem ω_1 : If α Establishment, necessity ($3\alpha+1$) Established

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Collatz conjecture($3X+1$)There is a hidden theorem ω_1 :

If α Establishment, necessity ($3\alpha + 1$) Established

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Absrtact : From a number theory “Collatz conjecture ($3X+1$)”, Human beings use a large amount of computer data, so far no counterexample has been found. Does mathematical logic support " Collatz conjecture ($3X+1$)”?

Collatz conjecture ($3X+1$)There is a hidden theorem ω_1 : If x holds, it must be ($3X+1$).

In reality, human beings will only ($3X+1$) deduce that x holds.

Example: an integer a , and $a = 3b + 1$, $b \in \mathbb{N}$. If $b \rightarrow 3x + 1$ holds. There must be: $a \rightarrow 3x + 1$ is established.

In this way, there is no need to deduce ($3b + 1$).

Key words: Recursive reduction; Mathematical complete induction; Extreme law; ($3x + 1$) simplified

1 nouns and definitions, citing theorems

1.1 $\Phi 1$ Collatz conjecture($3X+1$)^[1]Definition (traditional expression): a positive integer x , if odd, is multiplied by 3 plus 1 . If even, the even factor 2^n is eliminated. In this way, it returns to 1 after several times.

1.2 $\Phi 2$ Collatz conjecture($3X+1$)Definition (unified expression): a positive integer x (including even number), multiplied by 3 and then 1 . If it is an even number, eliminate the precipitation even factor 2^n and unify it to an odd number. In this way, it returns to 1 after

several times.

Cause: an even number of 8, processed by $(3x + 1)$: $3 \times 8+1=25$.

Can't humans face the positive integer **25**? Can't $(3x + 1)$ handle **25**?

Since **25** can be processed, any integer in the first step can be processed by $(3x + 1)$.

No conflict between Φ_1 and Φ_2 : $\Phi_1 \in \Phi_2$

1.3 Φ_3 Extreme rule: in the proof process, it is not allowed to appear $(3x + 1)$ holds), and the final conclusion under extreme conditions is obtained.

2 Logical argument

2.1 **Theorem** ω_1 : { $y \in \mathbb{N}$. Known: $y \Rightarrow (3X+1)$ established. }

There must be: $3y+1 \Rightarrow (3X+1)$ established.

prove:

2.1.1 Press Φ_2 define and select natural numbers

{ $\Phi_2 \Rightarrow 2 \rightarrow 7 \rightarrow 22 \rightarrow 11 \rightarrow 34 \rightarrow 17 \rightarrow 52 \rightarrow 13 \rightarrow 40 \rightarrow 5 \rightarrow 16 \rightarrow 8 \rightarrow 4 \rightarrow 2 \rightarrow 1. \Rightarrow (3X+1)$ established.

$\Phi_2 \Rightarrow 3 \rightarrow 10 \rightarrow 5 \rightarrow 16 \rightarrow 8 \rightarrow 4 \rightarrow 2 \rightarrow 1. \Rightarrow (3X+1)$ established.

$\Phi_2 \Rightarrow 4 \rightarrow 13 \rightarrow 40 \rightarrow 5 \rightarrow 16 \rightarrow 8 \rightarrow 4 \rightarrow 2 \rightarrow 1. \Rightarrow (3X+1)$ established.

$\Phi_2 \Rightarrow 5 \rightarrow 16 \rightarrow 8 \rightarrow 4 \rightarrow 2 \rightarrow 1. \Rightarrow (3X+1)$ established.

$\Phi_2 \Rightarrow 6 \rightarrow 19 \rightarrow 58 \rightarrow 29 \rightarrow 88 \rightarrow 11$ (See: **2**) $\Rightarrow (3X+1)$ established.

.....

$\Phi_2 \Rightarrow (a-1) \rightarrow 3(a-1)+1 \rightarrow \dots \rightarrow 2 \rightarrow 1 \Rightarrow (3X+1)$ established. } Recorded as: A_n

Continuous natural sequence: **2, 3, 4, 5, ... , (a-1)**. Yes, press Φ_2 define the selected natural number,

$\Rightarrow ((3X+1) \text{ established.}) \in \{ \mathbf{2, 3, 4, 5, \dots, (a-1)} \}$

According to the definition of $(3x + 1)$, y satisfies: $y \in \{ \mathbf{2, 3, 4, 5, \dots, (a-1)} \}$

A_n : $\Phi_2 \Rightarrow y \rightarrow (3y+1) \rightarrow \dots \rightarrow 1. \Rightarrow (3X+1)$ established.

$\therefore 3y+1 \Rightarrow (3X+1)$ established.

Theorem: ω_1 QED

Friendly tips:

【 $\Phi 2 \Rightarrow 2 \rightarrow 7 \rightarrow 22 \rightarrow 11 \rightarrow 34 \rightarrow 17 \rightarrow 52 \rightarrow 13 \rightarrow 40 \rightarrow 5 \rightarrow 16 \rightarrow 8 \rightarrow 4 \rightarrow 2 \rightarrow 1. \Rightarrow (3X+1)$ established.
 $\{2,7,22,11,34,17,52,13,40,5,16,8,4,2\} \Rightarrow (3X+1)$ established. 】

2.2 Theorem ω_2 : { $0 < \forall u \in \mathbb{N}$. Get: $(3^u-1) \Rightarrow (3X+1)$ established. }

prove:

Theorem $\omega_1 \Rightarrow$

{ $\because 3 \Rightarrow (3X+1)$ established. $\therefore 3 \times 1+1 \Rightarrow (3X+1)$ established.
 $\because 3 \times 1+1 \Rightarrow (3X+1)$ established. $\therefore 3(3+1)+1 \Rightarrow (3X+1)$ established.
 $\because 3(3+1)+1 \Rightarrow (3X+1)$ established. $\therefore 3(3^2+3^1+1)+1 \Rightarrow (3X+1)$ established.
 $\because 3(3^2+3^1+1)+1 \Rightarrow (3X+1)$ established. $\therefore 3(3^3+3^2+3^1+1)+1 \Rightarrow (3X+1)$ established.
.....
 $\because 3(3^{u-2}+3^{u-3}+3^{u-4}+\dots+3^3+3^2+3^1+1)+1 \Rightarrow (3X+1)$ established.
 $\therefore 3^{u-1}+3^{u-2}+3^{u-3}+3^{u-4}+\dots+3^3+3^2+3^1+1 \Rightarrow (3X+1)$ established.
 $\therefore 2 \times (3^{u-1}+3^{u-2}+3^{u-3}+3^{u-4}+\dots+3^3+3^2+3^1+1) \Rightarrow (3X+1)$ established.
 $\therefore (3-1) \times (3^{u-1}+3^{u-2}+3^{u-3}+3^{u-4}+\dots+3^3+3^2+3^1+1) \Rightarrow (3X+1)$ established.
 $\therefore (3^u-1) \Rightarrow (3X+1)$ established. } Theorem: ω_2 QED

【 $((3^{u-1}+3^{u-2}+3^{u-3}+\dots+3^3+3^2+3+1)$ Is the superposition form of $(3x+1)$. 】

2.3 Theorem ω_3 :

{(Theorem ω_1) + Collatz conjecture $(3X+1)$ } Ultimate dynamic action in: $(3x+1) \rightarrow 1$

prove:

Natural number sequence: 1, 2, 3, 4, 5, ... , (a-1), a, ...

Press $\Phi 2$ definition selection:

A_n : { 2, 3, 4, 5, ... , (a-1) } $\Rightarrow$ Collatz conjecture: $(3X+1)$ established.

【 It can also be expressed here:

Step 1: take positive integer 2 $\Rightarrow$ Collatz conjecture: $(3X+1)$ established.

Step k: if { 2, 3, 4, 5, ... , (a-1) } $\Rightarrow$ Collatz conjecture: $(3X+1)$ established.

Step (K + 1): how to verify a $\Rightarrow$ Collatz conjecture: $(3X+1)$ established. 】

Known figures: { 2, 3, 4, 5, ... , (a-1) } $\Rightarrow$ Collatz conjecture: $(3X+1)$ established.

If we can prove the number a $\Rightarrow$ Collatz conjecture: $(3X+1)$ established.

Get: any natural number $\Rightarrow$ Collatz conjecture: $(3X+1)$ established.

Take: { 2, 3, 4, 5, ... , (a-1) } = B_n

Take the adjacent natural number a after (a-1)

If: $a=2a_1$ $\because a > a_1 \in B_n$

$\therefore a \Rightarrow (3X+1)$ established.

Extreme rule $\Phi 3$: $a \neq 2a_1$

$$\therefore a = 2a_1 + 1 = 2^1 a_1 + 1 = 2^1 a_1 + 2^1 - 1 \quad (1)$$

$$3a + 1 = 6a_1 + 4 = 2(3a_1 + 2) \quad (2)$$

If: $a_1 = 2a_2$

$$\Rightarrow a = 4a_2 + 1$$

$$\because (2) \Rightarrow (3a_1 + 2) = 6a_2 + 2 = 2(3a_2 + 1) \Rightarrow (3a_2 + 1) < (4a_2 + 1) = a$$

$$\therefore (3a_2 + 1) \in \{2, 3, 4, 5, \dots, (a-1)\} \Rightarrow (3X+1) \text{ established.}$$

This status proves a: $\Rightarrow (3X+1)$ established.

Extreme rule $\Phi 3$: $a_1 \neq 2a_2 \Rightarrow a_1 = 2a_2 + 1$

$$(1) \Rightarrow a = 2^1 a_1 + 1 = 2(2^1 a_2 + 1) + 1 = 4a_2 + 3 = 2^2 a_2 + 3 = 2^2 a_2 + 2^2 - 1 \quad (3)$$

$$(3) \Rightarrow 3a + 1 = 12a_2 + 9 + 1 = 12a_2 + 10 = 2(6a_2 + 5)$$

$$3(6a_2 + 5) + 1 = 18a_2 + 16 = 2(9a_2 + 8)$$

If: $a_2 = 2a_3$

$$(9a_2 + 8) = 2(9a_3 + 4)$$

$$9a_3 + 4 = 9a_3 + 3 + 1 = 3(3a_3 + 1) + 1 \quad (4)$$

$$a = 8a_3 + 1 > (3a_3 + 1)$$

$$\because a > (3a_3 + 1) \Rightarrow (3a_3 + 1) \in B_n \Rightarrow (3a_3 + 1) \Rightarrow (3X+1) \text{ established.}$$

Theorem ω_1 : $3(3a_3 + 1) + 1 \Rightarrow (3X+1)$ established.

$$\therefore (4): \{3(3a_3 + 1) + 1\} \Rightarrow (3X+1) \text{ established.}$$

This status proves a: $\Rightarrow (3X+1)$ established.

Extreme rule $\Phi 3$: $a_2 \neq 2a_3 \Rightarrow a_2 = 2a_3 + 1$

$$(3) \Rightarrow a = 4a_2 + 3 = 4(2a_3 + 1) + 3$$

$$a = 8a_3 + 7 = 2^3 a_3 + 7 = 2^3 a_3 + 2^3 - 1 \quad (5)$$

$$(5) \Rightarrow 3a + 1 = 24a_3 + 22 = 2(12a_3 + 11)$$

$$3(12a_3 + 11) + 1 = 36a_3 + 34 = 2(18a_3 + 17)$$

$$3(18a_3 + 17) + 1 = 54a_3 + 52 = 2(27a_3 + 26)$$

Take: $(27a_3 + 26)$

If: $a_3 = 2a_4$

$$(27a_3 + 26) = 2(27a_4 + 13) \Rightarrow 27a_4 + 13 = 27a_4 + 12 + 1 = 3(9a_4 + 4) + 1 \quad (6)$$

$$(5) : a = 8a_3 + 7 = 16a_4 + 7 > (9a_4 + 4)$$

$$\because a > (9a_4 + 4) \in B_n \therefore (9a_4 + 4) \Rightarrow (3X+1) \text{ established.}$$

Theorem ω_1 : (6) Formula $(3(9a_4 + 4) + 1) \Rightarrow (3X+1)$ established.

This status proves a: $\Rightarrow (3X+1)$ established.

Extreme rule $\Phi 3$: $a_3 \neq 2a_4 \Rightarrow a_3 = 2a_4 + 1$

$$(5): a = 8a_3 + 7 = 8(2a_4 + 1) + 7 = 16a_4 + 15 = 2^4 a_4 + 15 = 2^4 a_4 + 2^4 - 1 \quad (7)$$

... After n times of the same logic, we get:

According to $\{(1), (3), (5), (7), \dots, (n)\}$ logic, we get: $\Rightarrow a_1 \rightarrow a_2 \rightarrow a_3 \rightarrow \dots \rightarrow 15 \rightarrow 7 \rightarrow 3 \rightarrow 1$

$$(1) \rightarrow a = 2^1 a_1 + 2^1 - 1$$

$$(3) \rightarrow a = 2^2 a_2 + 2^2 - 1$$

$$(5) \rightarrow a = 2^3 a_3 + 2^3 - 1$$

$$(7) \rightarrow a = 2^4 a_4 + 2^4 - 1$$

.....

$$(n-6) \rightarrow a = 2^{(u-4)} \times 15 + 2^{(u-4)} - 1$$

$$(n-4) \rightarrow a = 2^{(u-3)} \times 7 + 2^{(u-3)} - 1$$

$$(n-2) \rightarrow a = 2^{(u-2)} \times 3 + 2^{(u-2)} - 1$$

$$(n) \rightarrow a = 2^{(u-1)} \times 1 + 2^{(u-1)} - 1$$

$$\Rightarrow a = 2^u - 1 \quad (8)$$

Operate a in sequence according to the principle of $(3x + 1)$: Check whether $(3x + 1)$ can be obtained from $(2^u - 1)$.

{ Take: $3a + 1 = 3(2^u - 1) + 1$

$$\Rightarrow 3 \times 2^u - 3 + 1 = 2(3 \times 2^{u-1} - 1) \quad (9)$$

Take: $3(3 \times 2^{u-1} - 1) + 1$

$$\Rightarrow 3^2 \times 2^{u-1} - 3 + 1 = 2(3^2 \times 2^{u-2} - 1) \quad (10)$$

Take: $3(3^2 \times 2^{u-2} - 1) + 1$

$$\Rightarrow 3^3 \times 2^{u-2} - 3 + 1 = 2(3^3 \times 2^{u-3} - 1) \quad (11)$$

Take: $3(3^3 \times 2^{u-3} - 1) + 1$

$$\Rightarrow 3^4 \times 2^{u-3} - 3 + 1 = 2(3^4 \times 2^{u-4} - 1) \quad (12)$$

$$\dots \text{The same logic recursively gets: } (3^{u-2} \times 2^2 - 1) \quad (13)$$

$$\text{Take: } 3(3^{u-2} \times 2^2 - 1) + 1 \quad (14)$$

$$\Rightarrow 3^{u-1} \times 2^2 - 3 + 1 = 2(3^{u-1} \times 2^1 - 1)$$

$$\text{Take: } 3(3^{u-1} \times 2^1 - 1) + 1 \quad (15)$$

$$\Rightarrow 3^u \times 2^1 - 3 + 1 = 2(3^u - 1)$$

$$\Rightarrow (3^u - 1) \}$$

$\therefore$ Theorem: $\omega_2 \Rightarrow (3^u - 1) \Rightarrow (3X+1)$ established.

$\therefore$ Operate $(2^u - 1)$ according to the principle of $(3x + 1)$ to obtain: $(3X+1)$ established.

$\therefore (2^u - 1) \Rightarrow (3X+1)$ established.

$a \Rightarrow (3X+1)$ established. 【 $\therefore (8): a = (2^u - 1)$ 】

Theorem: ω_3 QED

Collatz conjecture $(3X+1)$ QED

3 Conclusion

The definition of $3x + 1$ tells us how to do it.

Human checking calculation is carried out according to the definition rule of $3x + 1$.

This main condition has not been cited by mankind, which has plagued mankind for decades.

Human beings do not start with simple principles, but go to complex forms to solve problems. Of course, they go astray.

Scissors principle!

I thought about this problem for only three days and found a clever way.

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5 References cited:

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