Mathematical modeling of the demographic dividend (DD) capture applied in economy.

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Abstract

The purpose of this work is to develop tools and techniques for modeling the capture of the Demographic Dividend. We presented the ordinary differential equation (ODE) system modeling the variation of economically dependent and economically non dependent populations. The system uses natality, natural mortality, infant mortality, migration (incoming and outgoing), and transfers. The mathematical study of this ODE system shows the existence of an equilibrium point whose stability depends on a certain number of system parameters. Numerical simulations of the resulting model were performed using scenarios approach.

Key words: Demographic Dividend, capture, economically dependent, economically non dependent, mathematical study.

1. Introduction

The study of the issue of the demographic dividend has always been a field tumultuous research in economics, Cardona et al. [4], Elhadary et al. [6], Groth et al. [7], Groth et al. [8], Matytsin et al. [10], May et al. [11], May et al. [12], United Nations [14] and Turbat [15]. Capturing a demographic dividend (DD) on International Development in Sub-Saharan Africa (SSA) is a subject of major interest in recent years in Groth et al. [8].

Today, the objective is to reproduce a process of a similar SD in SSA like the countries East Asia benefited from it at the end of the 20th century. These necessary conditions and sufficient will be needed to improve not only the dependency ratio demographic (DDR) but also, and above all, the economic dependency ratio (EDR) and the Burden Dependency Ratio (BDR). Sub-Saharan African countries must also design and implement sound economic policies and improve good governance, in addition to the formulation and implementation of policies adequate ticks in the areas of population, education, health and gender equity. Since the early 1990s, as in many Eastern European countries, Russia has experienced three transitions political, economic and demographic according to Matytsin et al. [10]. Demographically,

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7 Russia has entered the transition period with a predominantly young population and
8 a low rate of dependency, and has benefited from large cohorts of people entering
9 the market work. Thus, the authors examined the impact of demographic trends
10 on regional economic convergence in Russia. They made a comparison demographic
11 trends in Russia with those in Europe, and explore the link between demographic
12 change and growth for the regions of Russia from 1996 to 2011. Matytsin et al. [10]
13 econometrically studied the conditions of regional convergence of the demographic
14 dividend over this period.

15
16 Bloom et al. [3] stipulated that managing rapid and stimulating population growth
17 uolling economic growth are among the most pressing policy challenges for Sub-
18 Saharan Africa. They studied the 40-year empirical estimate of the impact of family
19 planning programs on per capita income arising from demographic dividend (DD).
20 Through a causal chain linked to the decrease in fertility, there is an increase in per
21 capita income. To achieve this, authors developed a model allowing to determine
22 the impact of the "response unmet need" (MUN) of modern contraceptive meth-
23 ods on the fertility and therefore on the age structure of the subsequent population.
24 The potential additional economic growth for countries such as Kenya, Nigeria and
25 Senegal is estimated using the age structure projected by MUN and the empirical
26 statements of DD. Aiyar et al. [2] exploited the variation in the age structure of
27 the population through the Indian states for the identification of the demographic
28 dividend. They focus on the significant and significant impact of the growth rate
29 of the ratio of working age from india. working age from India. The demographic
30 dividend could add around 2 points percentage per year to the growth of India's
31 GDP per capita over the years next two decades according to the authors.

32
33 In Turbat [15], it explains the reason for the population increase rapid growth of
34 a possible quadrupling of the current SSA population by the end of century (unless
35 fertility drops sharply in the near future). The rapid population growth and the onset
36 of a demographic transition in SSA sparked discussions on prospects for SSA to open
37 a window demographic opportunity and capture a first demographic dividend. In
38 particular, the evaluation of the population of young dependents and the employment
39 adult workforce is a necessity. Inspection of the conditions required for triggering of
40 the rapid and significant decline in fertility in the eastern region also necessary.

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42 The demographic dividend is the acceleration of economic growth which would result
43 from changes in the age structure of the population due to the consequent decline
44 in fertility and mortality, see Groth et al. [7]. According to this argument, which
45 is today a central element of the debate on African emergence, the demographic
46 changes underway (in particular the decline in fertility and the temporary reduction
47 in the rate of economic dependence associated with it) could apply support the con-
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continent's development efforts by creating conditions conducive to savings, productive investment and improvement of human capital, Dramani et al. [5]. The demographic dividend theory is based on the fact that Africa has the opportunity today to emerge economically in part thanks to the impetus of the demographic dividend. By creating favorable conditions for savings and investment, the dividend could give new impetus to growth, driving innovation, offering the continent a clear competitive advantage and enabling it to achieve its development goals. In other words, the demographic dividend is measured by the growth of the ratio support which represents the proportion of workers in relation to consumption teurs. The demographic dividend first takes place following the rapid decline fertility, then the increase in the support ratio or the decrease in the fertility rate. dependence, then increased savings and investment, and finally improving economic growth and living conditions.

The capture of the demographic dividend according to the NTA methodology is on a high cost data intensive computing process. This phenomenon complex cannot be assimilated only to sums of surplus or deficit of life cycle of individuals or households but rather to emerging behaviors and self-organizing across several scales resulting from the cycle deficit life of individuals and households in a country. In other words the dividend demographic is measured by the growth of the support ratio which represents the portion of workers versus consumers. The demographic dividend occurs first as a result of the rapid decline in fertility, then of the the support ratio or the decrease in the dependency ratio, then the increase in savings and investment, and finally improved economic growth and living conditions.

The methodology of the NTA network is a complete system of accounts which measures the business cycle and economic support systems. It also includes methods to examine the economic benefits of the first dividend demographic. Intergenerational transfer accounts are a new tool for analyzing exchanges between individuals from the same company. Those exchanges concern both the care of the youngest and the oldest in society, either by their parents, or by society itself through its governance mechanisms.

The first objective of the National Transfer Accounts- NTA, in English) is to develop and implement an account system measuring economic flows between age groups in a manner consistent with the System of National Accounts (SCN). These accounts bring a new perspective both microeconomic and demographic which makes it possible to put into perspective the large macroeconomic aggregates such as the Product Gross Domestic (GDP. Additionally in Andrew et al. [1], these accounts have been largely developed by Professors Ronald Lee of the University of Berkeley and Andrew Mason from the University of Hawaii, who are also Global Project Coordinators for the preparation of these accounts worldwide, which currently brings together close

to a forty countries including five in Africa and including Senegal. This contribution is drawn in large part from the work carried out within the network of NTAs, in particular those carried out for African countries on the NTAs. She details, the interest of intergenerational transfer accounts, then the methodology of its development and provides examples of use.

The goal of the NTA project is to improve understanding of how growth of the population and the change in the age structure influences the development economic, gender, generational equity, public finances and other aspects important aspects of macroeconomics. The project is made up of research teams based in universities, research institutions and government agencies tales in more than 50 countries around the world, with the role of building making it possible to measure how people at each year of age produce, consumement, share resources and save for their future. National accounts have two main purposes: to model and study the activity economy of a given country for a specific period (most of the time one year) on the one hand, and forecasting the evolution of an economic situation on the other hand. Accountability national government can thus be a forecasting tool to help a government find solutions, or to boost consumption for example. National Transfer Accounts (NTA) are based on an accounting identity such that, at each age a , the resources must be equal to the uses which are in United Nations [14]:

$$Y^L(a) + Y^k(a) + Y^R(a) = C(a) + S(a) + T^V(a), \quad (1.1)$$

The terms of the first member of the equation (1.1) corresponds to the outgoing flow and the terms of the second member corresponds to the incoming flow. The sum of labor income $Y^L(a)$, capital income $Y^K(a)$ and transferts received $T^R(a)$ must be equal to the sum of private and public consumption $C(a)$, savings $S(a)$ and transfers paid $T^V(a)$. This accounting identity allows thus puts to show the deficit of life cycle $D(a) = C(a) - Y^L(a)$ introduced by Dramani et al. [5]. This deficit corresponds, for a given age a , to the difference between private and public consumption and labor income:

$$C(a) - Y^L(a) = [Y^k(a) - S(a)] + [Y^R(a) - T^V(a)] \quad (1.2)$$

The lifecycle deficit $D(a)$ gives rise to reallocations of resources between ages which take the form of either net public or private transfers $T^N(a) = T^R(a) - T^V(a)$, or reallocations of assets $Y^K(a) - S(a)$. For each component of the lifecycle deficit, the methodology adopted includes three distinct stages, see United Nations [14].

The demographic support ratio is the ratio of the likely population- economically active to the elderly population, which is more likely economically inactive. It is therefore an indicator of the number of people tives which may be brought to economically support the working population. It is also a general indication of the age

structure of the population. Evolution of the demographic support ratio depends on past and present developments in the mortality and fertility as well as net migration flows. Unlike the demographic support ratio, the economic support ratio does not only reflect the mathematical relationship between the different age groups, but takes into account the economic situation and the needs of individuals. And that makes a huge difference both in terms of the number of workers and level of the number of consumers. If we compare with the support ratio demographic which can only be influenced by a modification of the age limits, the number of variables having an impact on the evolution of the economic support ratio turns out to be much higher: fall in the unemployment rate, faster integration of young graduates in the labor market, strengthening the participation rate of women in the labor market, etc. The economic support ratio therefore has the privilege to reflect both the effect of the age structure and the age profiles of consumption and income generation. It gives a richer and more precise description of the reality.

In the NTA model, the life cycle is calculated from the deficit by age group. The ratio economic support is defined as efficiency from producer to consumer:

$$r(t) = \frac{L(t)}{N(t)} = \frac{\sum_a \gamma(a, t)N(a, t)}{\sum_a \alpha_1(a, t)N(a, t)}, \quad (1.3)$$

$a \in [0, a_M]$ et $t > 0$,

Where a_M denotes the maximum age that is assumed to be finite. $N(a, t)$: the total number of people economically dependent (D) and non-dependent (ND) lations of age a to time t ; $\gamma(a, t)$: production rate of an age group a at time t ; $\alpha_1(a, t)$: rate consumption of an age group a at time t .

The growth of this ratio is considered as the demographic evolution of dividende in the cancellation of the methodology. He compares the speed between creation efficiency of efficient producers and consumers. We derive the definition of economic dependence from the methodology of the accounts national transfer (NTA). An individual is considered dependent if his labor income is not sufficient to satisfy its consumption. The equation of life cycle defined in the NTA methodology is expressed as follows:

$$C(a, t) - Y^L(a, t) = Y^A(a, t) - S(a, t) + T^+(a, t) - T^-(a, t), \quad (1.4)$$

For a dependent individual the term on the left is positive and the individual depends for its consumption of income from its assets and the transfer balance received which are represented by the term on the right. Equation (1.4) is dependent on time t and the age of the individual in the methodology.

$Y^L(a, t)$: represents labor income,

$C(a, t)$: represents public and private consumption,
 $Y^A(a, t)$: represents the income of workers of age a at time t ($Y^A(a, t) = Y^k(a, t) + Y^{p+}(a, t) - Y^{p-}(a, t)$),
 $Y^k(a, t)$: represents the capital income of age a at time t ,
 $Y^{p+}(a, t)$: represents the entry of income from the property of age a at time t ,
 $Y^{p-}(a, t)$: represents the exit of income from the property of age a at time t ,
 $S(a, t)$: represents savings,
 $T^+(a, t)$: represents the transfers of the entries of age a at time t ,
 $T^-(a, t)$: transfers of graduates of age a at time t , $T(a, t) = T^+(a, t) - T^-(a, t)$:
represents the net transfers of age a at time t ,
 $Y^A(a, t) - S(a, t)$: represents the asset-based reallocation.
The lifecycle deficit ($LCD = C(a, t) - Y^L(a, t)$), which is calculated by difference between consumption and income profiles, makes it possible to measure the contribution of active age groups to financing the consumption of inactive age groups. In this case, we consider that a non-dependent is an individual whose income of work ensures its consumption. In this case the left term of the equation(1.4) is negative. This is why it is important to propose models to better understand it and enable political decision-makers to take the right decisions to anticipate the social crises.

The document is organized as follows. In the second section, we will present and describe the dynamic model obtained. Thus, we will do the mathematicks necessary (study of stability around equilibria). In the third section, the Numerical results of the different theoretically established scenarios will be presented. Finally, the last section will concern the conclusion of the manuscript.

2. Mathematical model and analysis

2.1. The model presentation

The goal of this work is to make an asymptotic study on the evolution of the ratio economic support. This study will be based on the design of a permeable model. both to measure the dynamics of economic dependence. Some parameters of the model will be estimated from the results of the work on the methodology the NTA which allows measurement using official data on the population and economics of relevant demographic dividend indicators. Among these indicators we find the ratio of economic support which measures the speed at which a country creates more economic non-dependents than economic dependents. The model will make it possible to carry out a comparative study of the evolution of dependence economic between countries of the north and countries of the south. In the first section we will define our model

of economic dependence, make a theoretical study of it, in the second section we will do a mathematical study of the model, in the third section we will do simulations on the evolution of dependence economic through an estimation of the parameters from official data surveys or results of work on the NTA. These simulations will be made by country and will make it possible to monitor the asymptotic behavior of the support ratio according to demographic and economic criteria. The last section will allow to carry out a comparative study between northern and southern countries and to draw the perspectives.

Immigration notion: Immigration today refers to the entry into a country of foreign people who come there to stay or settle there. The word immigration comes from the Latin in-migrare which means "to enter a place".

Concept of emigration: The action of leaving one's country, one's region for economic, political, religious reasons.

Human migration (in and out): Displacement of people from one place to another, in particular from one country (emigration) to another (immigration) for political, social, economic or personal reasons, and which is the result either of an entire population, or of individuals integrating into a larger social phenomenon.

Flows: Flows are economic quantities which measure the exchanges carried out by economic agents during a given period (distinction between incoming and outgoing flows). The notion of Economic Flows is related to time.

Stocks: Stocks are economic quantities grouping together all populations (dependents and non-dependents).

Birth rate: The birth rate is measured from the crude birth rate which expresses the number of live births recorded during a year to an average population of the year (ie which increases the number of individuals).

Mortality: Mortality refers to the action of death on populations. This phenomenon results in death events (i.e. decreasing the number of individuals).

Infant mortality: Infant mortality is the number of children who die in the first year of their life; it is expressed as a rate per 1,000 live births for the year in question.

2.2. Model formulation

We will describe in this section a mathematical model of economic dependence. This model is based on a study of the stock and flows of dependents.

Stocks	Flows
Number of PED (D)	Births numbers (NB), proportion of economically dependent MEs, proportion of economically dependent MS, proportion of PEND who become dependent
Number of PEND (ND)	proportion of economically non-dependent MEs, proportion of economically non-dependent MS, proportion of PEDs who become non-dependent
Number of MS	Number of economically dependent MS, Number of MS economically non-dependent
Number of ME	Number of economically dependent ME, Number of ME economically non-dependent
Capital stocks (S)	Difference between average income of PENDs and average consumption of PENDs, Difference between average income of PEDs and average consumption of PEDs

We obtain the following system:

$$\begin{cases} \frac{dD}{dt} = -\alpha D - \tau D + \delta ND - \varepsilon MS + \sigma ME + (1 - \alpha_M)NB \\ \frac{dND}{dt} = -\alpha ND + \tau D - \delta ND - (1 - \varepsilon)MS + (1 - \sigma)ME, \end{cases} \quad (2.1)$$

where t is the continuous time,

$\frac{dD}{dt}$ and $\frac{dND}{dt}$ represent respectively the instantaneous variation of the PEDs and PENDs with respect to time t ; α ; τ ; δ ; ε ; σ ; α_M and β represents respectively the death rate in the population (it is assumed that the death rate is the same for PEDs and PENDs); a proportion of PEDs who become non-dependents; the proportion of PEND who become dependent; the proportion of emigrants who are dependents; the rate of immigrants who are dependent (entering the country of destination); models the infant mortality rate and the birth rate in the population;

By setting $NB = \beta(D + ND)$, then the system (2.1) becomes:

$$\begin{cases} \frac{dD}{dt} = -\alpha D - \tau D + \delta ND - \varepsilon MS + \sigma ME + \beta(1 - \alpha_M)(D + ND) \\ \frac{dND}{dt} = -\alpha ND + \tau D - \delta ND - (1 - \varepsilon)MS + (1 - \sigma)ME \end{cases}$$

By factoring, we obtain the following system:

$$\begin{cases} \frac{dD}{dt} = -(\alpha + \tau - \beta + \beta\alpha_M)D + (\delta + \beta - \beta\alpha_M)ND - \varepsilon MS + \sigma ME \\ \frac{dND}{dt} = \tau D - (\alpha + \delta)ND - (1 - \varepsilon)MS + (1 - \sigma)ME \end{cases} \quad (2.2)$$

2.3. Model mathematical analysis

In this section, we will do the mathematical study of the (2.2) system.

We are interested in the evolution over time of an economic system made up of

two categories: the dependent (number of economically dependent people) and the non-dependent (number of economically non-dependent people). Consider $x(t)$ the number of dependents (D) at time t and $y(t)$ the number of non-dependents (ND) at time t . The quantities x and y are therefore functions of $\mathbb{R}^+$ in $\mathbb{N}$. We propose to study the following system of equations: The dynamics of the model are described by the following ordinary differential equations:

$$\begin{cases} \frac{dx}{dt} &= -(\alpha + \tau - \beta + \beta\alpha_M)x(t) + (\delta + \beta - \beta\alpha_M)y(t) - \varepsilon MS + \sigma ME \\ \frac{dy}{dt} &= \tau x(t) - (\alpha + \delta)y(t) - (1 - \varepsilon)MS + (1 - \sigma)ME \end{cases} \quad (2.3)$$

Let's pose $C_1 = -\varepsilon MS + \sigma ME$ et $C_2 = -(1 - \varepsilon)MS + (1 - \sigma)ME$. Then the system (2.3) becomes:

$$\begin{cases} \frac{dx}{dt} &= -(\alpha + \tau - \beta + \beta\alpha_M)x(t) + (\delta + \beta - \beta\alpha_M)y(t) + C_1 \\ \frac{dy}{dt} &= \tau x(t) - (\alpha + \delta)y(t) + C_2 \end{cases} \quad (2.4)$$

Theorem 2.1. *Suppose that $A = \alpha - \beta + \beta\alpha_M$ and $C = C_1 + C_2$ are negative, and $(x(t), y(t))$ a solution of the system (2.4) with $x_0 > 0$ and $y_0 > 0$, then $N(t) = x(t) + y(t)$ is positive for all $t > 0$.*

Proof 2.1. *By taking the sum of the two equations of the system (2.4), we obtain:*

$$\frac{dx(t)}{dt} + \frac{dy(t)}{dt} = -(\alpha - \beta + \beta\alpha_M)(x(t) + y(t)) + C_1 + C_2$$

By factoring, we obtain the following differential equation:

$$\frac{dN(t)}{dt} = C - AN(t). \quad (2.5)$$

Consider the following homogeneous equation:

$$\frac{dN(t)}{dt} + AN(t) = 0.$$

The solution of this homogeneous equation is given by

$$N(t) = Ke^{-At}.$$

with K a constant. By varying this constant K we have: $K = Ce^{At} + C'$, C' is a constant.

$$N(t) = C + C'e^{-At}.$$

At $t = 0$ then $N(0) = C + C'$ this leads to $C' = N(0) - C$, then the general solution of the equation (2.5)

$$N(t) = C + (N(0) - C)e^{-At}.$$

from where

$$N(t) = N(0)e^{-At} - C(e^{-At} - 1).$$

Now $N(0)e^{-At} > 0$, $e^{-At} - 1 > 0$ and $-C > 0$. This results in $N(t) > 0$.

Remark 1. i) $C = C_1 + C_2 = ME - MS$ is the difference between inbound and outbound migration. This difference is negative because the number of people who leave is much more important than those who enter.

ii) $A = \alpha - \beta + \beta\alpha_M$ is the difference between the number of deaths and the number of births. This difference is negative because the number of births is more important than the number of deaths.

Lemma 2.2. Let $C = C_1 + C_2 < 0$ and $A = \alpha - \beta + \beta\alpha_M < 0$. Suppose that $N(0) - \frac{C}{A} < 0$, then the set

$$\Omega = \{(x, y) \in \mathbb{R}_+^2 / x + y \leq \frac{C_1 + C_2}{\alpha - \beta + \beta_M}\}$$

is a positively invariant compact of the system (2.4).

Proof 2.2. Let the equation

$$\frac{dN(t)}{dt} = C - AN(t).$$

Then

$$\frac{dN(t)}{dt} = C - A\left(\int_0^t \frac{dN(s)}{ds} ds + N(0)\right)$$

so

$$\frac{dN(t)}{dt} = C - AN(0) - A\left(\int_0^t \frac{dN(s)}{ds} ds\right) \quad (2.6)$$

Let $y(t) = N(t)$, $\Phi = C - AN(0)$ and $\Psi = -A$. Applying Gronwall's lemma to the equation (2.6), we get:

$$\frac{dN(t)}{dt} \leq C - AN(0) + \int_0^t -A(C - AN(0))\exp\left(\int_s^t -Adu\right)ds$$

so

$$\frac{dN(t)}{dt} \leq C - AN(0) - A(C - AN(0))\left(\int_0^t e^{-As} e^{As} ds\right)$$

We have

$$\frac{dN(t)}{dt} \leq C - AN(0) - A(C - AN(0))e^{-At}(e^{At} - 1)$$

so

$$\frac{dN(t)}{dt} \leq C - AN(0) - A(C - AN(0))(1 - e^{-At})$$

and finally we have:

$$\frac{dN(t)}{dt} \leq (C - AN(0))e^{-At}$$

By integrating from 0 to t, we obtain:

$$N(t) - N(0) \leq (C - AN(0))\left(\frac{-1}{A}\right)(e^{-At} - 1)$$

then

$$N(t) \leq N(0) + (N(0) - \frac{C}{A})(e^{-At} - 1)$$

finally we have:

$$N(t) \leq \frac{C}{A} + (N(0) - \frac{C}{A})e^{-At}$$

Thus

$$N(t) \leq \frac{C}{A} \text{ avec } C = C_1 + C_2 < 0 \text{ and } A = \alpha - \beta + \beta\alpha_M$$

hence

$$N(t) \leq \frac{C_1 + C_2}{\alpha - \beta + \beta\alpha_M} \quad \square$$

So, we can conclude that all the trajectories of the system (2.4) are positive and bounded for all $t > 0$. The model being well posed, we will move on to the qualitative study of the model.

The equilibrium points of the model: By considering the system (2.4), this allows us to find the equilibrium points of this system.

$$\begin{cases} \frac{dx}{dt} = 0 \\ \frac{dy}{dt} = 0 \end{cases}$$

We will study the problem with C_1 and C_2 constants

$$\begin{cases} -(\alpha + \tau - \beta + \beta\alpha_M)x(t) + (\delta + \beta - \beta\alpha_M)y(t) + C_1 = 0 \\ \tau x(t) - (\alpha + \delta)y(t) + C_2 = 0 \end{cases}$$

The matrix formulation of the system is as follow:

$$\begin{cases} (\alpha + \tau - \beta + \beta\alpha_M)x(t) - (\delta + \beta - \beta\alpha_M)y(t) = C_1 \\ -\tau x(t) + (\alpha + \delta)y(t) = C_2 \end{cases}$$

The system can be written in matrix form:

$$\begin{pmatrix} \alpha + \tau - \beta + \beta\alpha_M & -(\delta + \beta - \beta\alpha_M) \\ -\tau & \alpha + \delta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} C_1 \\ C_2 \end{pmatrix}.$$

We call A the matrix of the coefficients of the system. The vector X is a solution of the system if and only if

$$AX = B,$$

$$\text{with } X = \begin{pmatrix} x \\ y \end{pmatrix}; B = \begin{pmatrix} C_1 \\ C_2 \end{pmatrix} \text{ and } A = \begin{pmatrix} \alpha + \tau - \beta + \beta\alpha_M & -(\delta + \beta - \beta\alpha_M) \\ -\tau & \alpha + \delta \end{pmatrix}$$

Determinant of matrix A:

$$|A| = (\alpha + \delta + \tau)(\alpha - \beta + \beta\alpha_M)$$

Corollary 2.3. i) *If the determinant is zero ($|A| = 0$) then $\alpha = \beta(1 - \alpha_M)$ therefore there is no renewal in the population because the number of deaths is equal to the number of births. So the system does not admit a point of equilibrium.*

ii) *If the determinant is different to zero ($|A| \neq 0$) then $\alpha \neq \beta(1 - \alpha_M)$ therefore there is an evolution in the population because the number of deaths is different to the number of births. Then the system admits a point of equilibrium. So the matrix A is invertible.*

In the remainder of this manuscript, we consider the case where the determinant is non-zero. The invertible matrix denoted A^{-1} is given:

$$A^{-1} = \frac{1}{|A|} \begin{pmatrix} \alpha + \delta & \delta + \beta - \beta\alpha_M \\ \tau & \alpha + \tau - \beta + \beta\alpha_M \end{pmatrix}$$

and so

$$X = A^{-1}B = \frac{1}{|A|} \begin{pmatrix} (\alpha + \delta)C_1 + (\delta + \beta - \beta\alpha_M)C_2 \\ \tau C_1 + (\alpha + \tau - \beta + \beta\alpha_M)C_2 \end{pmatrix}$$

Hence

$$x = \frac{(\alpha + \delta)C_1 + (\delta + \beta - \beta\alpha_M)C_2}{|A|} \text{ and } y = \frac{\tau C_1 + (\alpha + \tau - \beta + \beta\alpha_M)C_2}{|A|}.$$

Corollary 2.4. *There is a single possible equilibrium point. This equilibrium is characterized by the coexistence of the coefficients. $\left(x = \frac{(\alpha + \delta)C_1 + (\delta + \beta - \beta\alpha_M)C_2}{|A|}, y = \frac{\tau C_1 + (\alpha + \tau - \beta + \beta\alpha_M)C_2}{|A|}\right)$ is the unique equilibrium point of the system (2.4).*

Economically, this equilibrium point is characterized by the coexistence of economically dependent and non-dependent people.

The Jacobian matrix: An equilibrium point is said to be stable if the determinant of the Jacobian matrix is positive and its trace is negative.

Consider the following system:

$$\begin{cases} \frac{dx}{dt} = -(\alpha + \tau - \beta + \beta\alpha_M)x(t) + (\delta + \beta - \beta\alpha_M)y(t) + C_1 = f(x, y) \\ \frac{dy}{dt} = \tau x(t) - (\alpha + \delta)y(t) + C_2 = g(x, y) \end{cases} \quad (2.7)$$

The equilibrium point study: Let $E = (x_0, y_0)$

$$J_E = \begin{pmatrix} -(\alpha + \tau - \beta + \beta\alpha_M) & \delta + \beta - \beta\alpha_M \\ \tau & -(\alpha + \delta) \end{pmatrix}$$

In the next paragraph, we introduce the notion of stability for the solutions of our system. We give proofs which relate the asymptotic stability of a point of equilibrium to the eigenvalues of linearization in the neighborhood of this equilibrium. To do this, we will use the Jacobian matrix associated to system (2.4).

The stability study of the equilibrium point: We will study their sign thanks to the determinant and the trace of the Jacobian matrix:

- The determinant $|J_E| = (\alpha + \tau + \delta)(\alpha - \beta + \beta\alpha_M)$ is negative.
- The trace is $Tr_E = -(\alpha + \tau - \beta + \beta\alpha_M) - (\alpha + \delta) = -(2\alpha + \tau + \delta - \beta + \beta\alpha_M)$, is negative and E is asymptotically unstable.

3. Numerical results

In this section, we perform simulations of different scenarios of the model to compare the numerical results and see the effect of the parameters on the results of the diagram, that is to say the curve of evolution according to time of the numbers of people economically dependent and non-dependent and the influence of the birth rate on populations. In this section, it is mainly a question of making simulations from the data taken in the methodology of the NTA network. These simulations are made on the basis of the proportions of economically dependent people who become non-dependent and of economically non-dependent people who become dependent. Thus, the following graphical results illustrate two dynamic evolutions over a period of 60 years (2005 to 2065): a curve which describes the evolution of economically dependent populations (D) and that of economically non-dependent populations (ND). For initial conditions, drawn from national survey data from Senegal in 2005 (ESPS2), we consider $x(0) = 10059731$ and $y(0) = 3297272$. In the simulations, we assume that the proportion of immigrants and emigrants remains low and does not influence the evolution of populations, but does not prevent this from being a hypothesis to be reviewed. Also, we have assumed in these different cases of simulations that the birth rate is high and the mortality is low. We were mainly interested in six cases. In the first four cases, two strong hypotheses should be emphasized: i) We assume, according to a chronogram, that a very low proportion of economically dependent people in the population transforms into economically independent people during the year. Thus, we respectively assign to τ the values 0.15, 0.2, 0.25 and 0.15, which assumes that only 15%, 20% and 25% of economically dependent people become economically independent during the year. ii) We assumed that a very small proportion of economically independent people in the population transform into economically dependent people during the same year. We also assign to δ the respective values 0.1, 0.2, 0.12 and 0.2 that is to say 10%, 20%, 12% and 20% of economically independent persons become economically dependent on the during the year. Thus, we are in the case of a country where the proportion of people leaving poverty is greater than that entering. The other parameters values during the simulations are $\alpha = 0.008$, $\alpha_M = 0.047$, $\varepsilon = 0.6$, $\sigma = 0.8$, $\beta = 0.037$, $ME = 263242$ and $MS = 585392$. The graphics (1) to (6) illustrate the results of various scenarios involved. For the figure (1), we note that there is an equilibrium point after 15 simulation steps, i.e. 15 years and that from this moment the proportion of economically non-dependent people is greater than that of economically dependent people in the population. This leads to the creation of wealth for the country. This can be explained by the fact that from 2020 this country will have total stability of its economy. From 2015 to 2060, this phenomenon models the economic and demographic evolution of countries

Figure 1: Evolution of dependents and non-dependents ($\tau = 15\%$ and $\delta = 10\%$)

such as Morocco, South Africa, Rwanda etc. We will describe the economic and demographic situation of Morocco: From 2015 to 2060, this phenomenon models the economic and demographic evolution of countries such as Morocco, South Africa, Rwanda etc. We will describe the economic and demographic situation of Morocco: Economic situation: In recent years, the Moroccan economy has been characterized by its macroeconomic stability coupled with low inflation. It is solid, based on exports, the growth of private investment and tourism. After an exceptional 2015 due to a good agricultural season, the rainfall deficit led to a drop in agricultural production in 2016 which led to a sharp economic slowdown. From 4.5% in 2015, economic growth slowed to 1.6% in 2016, driven mainly by investment. An acceleration is expected for 2017 due to a good agricultural season and the maintenance of the growth rate of non-agricultural activity. Unemployment (over 10%), on the rise in recent years, particularly affects the 15-24 age group and young graduates. The poverty rate remains one of the highest in the Mediterranean area, with 15% of the

population living below the poverty line. There are also strong regional disparities in terms of development. Demographic situation: The demography of Morocco is the set of data and studies concerning the population of Morocco at all times. These data are managed by the High Commission for Planning (HCP). The latest official census estimates the population of Morocco at 33.8 million, to which must be added the diaspora estimated at 4.5 million people. We also note that the decline in the birth rate still leads to rapid growth in the number of economically non-dependent people. Figure (2) shows that there is no equilibrium point, but we find that the

Figure 2: Evolution of dependents and non-dependents ($\tau = 20\%$ and $\delta = 20\%$)

proportion of economically dependent people is much larger than that of economically independent people in the population. This leads to severe poverty in the country. We note that there is a total lack of economic growth in this country due to a problem of employment, health, education etc. This phenomenon models the economic and demographic evolution of countries such as Senegal, Mauritania etc. As an economic situation: Senegal has long experienced growth rates among the

highest in the UEMOA (West African Economic and Monetary Union). Structural reforms have enabled a significant change in Senegal's economic landscape, in particular thanks to the privatization of many public enterprises in the agricultural sector and in infrastructure. The economy suffers from a faulty electricity grid, which is a major obstacle to the country's growth. However, economic growth was stable at 6.6% in 2016 (6.5% in 2015 and 4.4% in 2014). In 2016, the informal sector continued to represent a significant share of GDP. Senegal is still among the least developed countries (LDCs) according to the UN, with 50% of its population living below the poverty line. And as a demographic situation: The demography of Senegal is all the data and studies concerning the population of Senegal at all times. These data are managed by the National Agency for Statistics and Demography (ANSD). The population is estimated at 15,256,346 inhabitants (2017). In addition, it is also important to stress that the high birth rate always leads to an increase in the number of economically dependent people. We note in the graph (3) that there is an equi-

Figure 3: Evolution of dependents and non-dependents ($\tau = 25\%$ and $\delta = 12\%$)

librium point after 9 simulation steps, i.e. 9 years and that from this moment the proportion of economically non-dependent people is much greater than that of people economically dependent in the population. This leads to strong wealth creation and the emergence of the country. We also note that from 2010 this country shows economic growth. From 2010 to 2060, this phenomenon models the economic and demographic development of countries such as Japan, South Korea, etc. Economically, South Korea (the thirteenth largest economy in the world) has experienced spectacular growth over the past thirty years. However, due to its strong integration into international trade and financial exchanges, the country is vulnerable to external shocks. In 2015, Korean economic growth stalled (2.6%) as the country faced an epidemic of Middle East Respiratory Syndrome (MERS) which affected consumption. Growth remained stagnant in 2016 (2.7%) and according to IMF projections it should strengthen slightly in 2017 (3%) thanks to a recovery in private consumption, to the strengthening of the real estate market. and fiscal and monetary incentives. South Korea's per capita income has risen from 100\$ in 1963 to nearly 30,000\$ today. The unemployment rate is falling, reaching its lowest level since 2002, but the number of irregular workers is high, inequalities are growing and social ties are deteriorating. South Korea Demographics is the collection of data and studies about the population of South Korea through all eras. These data are managed by the National Bureau of Statistics. The population is estimated at 51,755,792 inhabitants (2017) with a population growth of 0.54%/an. Therefore, we note a sharp decrease in the birth rate still leads to rapid growth in the number of economically non-dependent people. Figure (4) tells us that there is no point of equilibrium, but we find that the proportion of economically dependent people is much larger than that of economically non-dependent in the population. This leads to total poverty in the country. We also note an economic crisis in this country due to a very high unemployment rate. This phenomenon models the evolution of countries like Malawi, Burundi, Gambia etc. In 2015, Malawi's GDP growth was lower than expected (4%) due in part to severe flooding which affected agricultural production. Growth of 5% is expected for 2016, driven by more favorable harvests and the development of the tertiary sector through the introduction of new technologies. Domestic demand is nevertheless expected to be constrained by high inflation, manifested in rapidly rising food prices. The inflation rate is expected to slow from 20.1% in 2015 to 14% in 2016. Malawi continues to face challenges such as poverty, food insecurity and political fragility. Malawi Demography is the collection of data and studies pertaining to the population of Malawi at all times. The population is estimated at 15,805,240 inhabitants (2014) with a population growth of 3.180%/year. It is also important to stress that rapid changes in the birth rate always lead to rapid growth in the number of economically

Figure 4: Evolution of dependents and non-dependents ($\tau = 15\%$ and $\delta = 20\%$)

dependent people. For the remaining two scenarios, we consider the same values of the parameters of scenarios two and three. Thus, we will assume that the birth rate is low and the mortality always remains low. For the figure (5), we note that there is no equilibrium, but we find that the proportion of economically dependent people gradually decreases until 2065 while that of economically non-dependent people, we find a rapid increase until 2010 then gradually decreases until 2065. This presents the case of very poor countries with a very high life expectancy. This phenomenon models the economic and demographic evolution of countries such as Liberia, the Democratic Republic of Congo, Guinea etc. Liberia remains a poor country, with high unemployment and precarious employment rates and high social tensions. Not all disarmed ex-combatants have successfully reinserted themselves into civil society and may be tempted to form or join regional militias. The United Nations Mission in Liberia (UNMIL) withdrew completely at the end of June 2016, after 13 years in the country. Liberia Demography is the collection of data and studies pertaining to the

Figure 5: Evolution of dependents and non-dependents ($\tau = \delta = 20\%$, and $\beta = 0.032$)

population of Liberia at all times. The population is estimated at 4.5 million (World Bank 2015) with a population growth of 2.4%. We note that an increase in the birth rate leads to a gradual decrease in both populations. As for the figure (6), we note that there is an equilibrium point after 9 simulation steps, i.e. 9 years and that from this moment the proportion of economically non-dependent people increases gradually until 2015 then gradually decreases while that of economically dependent people gradually decreases. This leads to strong wealth creation and the emergence of the country. This is the case for the emerging countries. We also note that this country has a large aging population. From 2010 to 2060, this phenomenon models the economic and demographic evolution of countries like Canada. The Canadian economy is struggling to recover from the global recession. Growth was only 1.2% in 2016 (a very slight increase). The economy has suffered from the poor performance of the oil sector. In May 2016 massive fires hit the oil state of Alberta. Growth could reach 1.9% in 2017, supported by domestic demand and non-oil exports. The

Figure 6: Evolution of dependents and non-dependents ($\tau = 25\%$, $\delta = 12\%$ and $\beta = 0.032$)

demography of Canada is the collection of data and studies concerning the population of Canada at all times. These data are calculated in particular by Statistics Canada. The population is estimated at 36,286,425 (2016). This sharp drop in the birth rate is leading to rapid growth in the number of economically non-dependent people. Finally, we have presented different scenarios to understand the evolution of economically dependent people and those of economically non-dependent people and the role of the birth rate in the population according to the different proportions of dependent people who become non-dependent and those of non-dependent people. addicts who become addicts. Indeed, these different scenarios enabled us to confirm the analytical results. In addition, these simulations have clearly shown the role of the birth rate in the population for a given country.

4. Conclusion

In this manuscript, it is about studying the evolution of the dynamics of economically dependent and non-dependent populations. We have studied the theoretical aspect (stability of the equilibrium of the system) and the numerical aspect of the model. Thus, we were able to show theoretically that our model is well posed. The validation of the model was carried out by comparing the results obtained by it with several cases of simulations with different parameter values. The simulations made it possible to represent the dynamics of the two populations with different scenarios over a period of 60 years.

Conflict of interest

The authors declare that they have no conflict of interest.

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