Fizeau Interferometer to Measure Refractive Index of Air With Permanent Vacuum Prism Etalon

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Abstract: Over the years much research has been done into the measurement of the speed of light. The speed of light has had many applications in measurements since it has been determined with the required accuracy. The refractive index of the medium in which the measurements are performed is critical in order to use the speed of light for accurate measurements. In dimensional metrology, most measurements are performed in air, and therefore, the refractive index of the air must be determined to a higher accuracy than the dimensional measurements performed. The most accurate way to determine the refractive index is to use a refractometer.

Previous research by the authors looked at using a prism as the vacuum cell instead of a cylinder. The main advantage was that it eliminates the use of a vacuum pump. In this research, a new prism was designed to overcome difficulties with the previously used prism. A new method to use an optical flatness interferometer with a Fizeau interferometer layout to measure the air/vacuum ratio was researched and the results are presented here. The results achieved in the measurement method of the refractive index of air were accurate to 6×10^{-8}, which compares well with other refractometers with the added described advantage.

1. Introduction

The speed of light was first measured by Galileo Galilei [1] in 1638, whose first attempts did not involve explosions, and by Olaf Romer in 1676, who used Jupiter's orbit around the earth to determine the speed of light. Since then, many other scientists have measured the speed of light using different methods until, in 1975, the General Conference on Weights and Measures (CGPM) fixed the speed of light at exactly 299 792 459 metre per second (m/s) [2]. In 1862, Foucault determined the speed of light accurately enough to realise that light travels slower in water than in air, where the medium in which light travel became relevant [1]. It is then that the definition for the speed of light was fixed in a vacuum, and the refractive index was required to make the correction when measuring the speed of light in any other medium.

The refractive index is shown in Equation 1:

$$n = \frac{c}{v'}$$

[Eq. (1)]

Where n is the refractive index, c is the velocity of light (V.O.L.) in a vacuum, and v' is the V.O.L. in a particular medium, in this case air. This research concentrates on the refractive index of air.

Much research has been done to measure the refractive index of air using refractometers [3–11]. This research proves the accuracy of the refractometers but has limitations as described by Bonsch and Potulski and Kruger and Chetty [3,4,9].

In previous research by the authors [10], a hollow prism with a permanent vacuum was used to compare the refractive index of air to that of a vacuum, comparable to Renkens and Schellekens [11]. The results showed that it is possible to measure the refractive index of air using this prism refractometer, but the prism was a commercial design and not ideal for highly accurate refractive index measurements. The aim of the current study was to design, build and test a new prism refractometer using a specially designed prism and to then compare the results to the Edlén weather station method. During the research a new method to measure the vacuum-air ratio was investigated and a comparison between this method and the previous method of Kruger and Chetty are presented.

2. Refractive index measurements and refractometers

The most common method to determine the refractive index is to measure the environmental conditions, air temperature, air pressure, relative humidity and CO_2. The refractive index is then calculated using equations from Edlén [12,13] or Ciddor [14]. This method is called the weather station method.

The modified Edlén equation (Equation 2) is mainly used by metrologists in laboratories and by commercial manufacturers of laser measurement systems, such as Renishaw [15], Keysight (formerly known as Hewlett Packard and Agilent) [16] and Zygo [17]. Hence, the authors decided to also use the modified Edlén equation in this research. The different manufacturers all use equations based on the modified Edlén equation but with slight variations.

$$(n-1)_{tp} = \frac{p(n-1)_s}{93\ 214{,}60} . \frac{1+10^{-8}p(0{,}5953-0{,}009876t)}{(1+0{,}0036610_t)} .$$

[Eq. (2)]

Here $(n-1)$ is the refractivity of standard air, the temperature t is expressed in degrees Celsius, the air pressure p in torr, and standard air at 1 atmosphere and 15 °C.

The more accurate method to determine the refractive index is with a refractometer, which measures the refractive index directly by comparing an optical path in the air to the same optical path in a vacuum. Much research has been published on this [3–11], and the conclusions of these are well captured by Bonsch [3]. Kruger and Chetty's [4,9] investigation into these focused on refractometers that could be used for both their highly accurate refractive index measurements and their simplistic design for everyday applications.

They researched the development of refractometers, and developed a robust refractometer that uses a commercial laser displacement interferometer together with a vacuum cylinder accurate to parts in the 10^{-8}. This refractometer is very accurate and can be used daily because of its simplistic design; however, it requires a vacuum pump to perform the refractive index measurements. The vacuum pump must be close to the refractometer to achieve a good vacuum, which led to the heat from the pump affecting the air temperature of the laser measurements. Another limitation to this refractometer is that repeatedly drawing a vacuum can lead to dust particles entering the vacuum cylinder, and these can affect the measurements through diffraction of the laser beam [10].

A further challenge with the traditional tube refractometer is the change in the optical thickness of the side windows in the tube refractometer. Bonsch and Potulski [3] measured this to be 8 nm, and although very small, it added to the inaccuracy of these tube refractometers.

Considering the reasons above, the aim of this research was to design, build and test a simple air refractometer that can be used in everyday laser-based measurements without a vacuum pump and to eliminate the errors in tube refractometers described by Bonsch and Potulski [3].

3. Design of prism refractometer

Previous research by Kruger and Chetty into an innovative refractometer used a hollow prism as the vacuum etalon instead of a vacuum cylinder and proved the use of the prism successful [10]. This was a big improvement in not just the simplicity of the system but also in using a permanent vacuum, thereby reducing the amount of heat generated that negatively impacts the length measurements.

The prism used previously was made for the refractive index measurements of liquids, as described by Daimon and Masumura [18], and is manufactured by 3B Scientific [19] (Figure 1). Because these prisms are designed to measure the refractive index of liquids to less accuracy than is required for the refractive index of air, it is not designed and built to the required accuracy.

Fig. 1. Hollow prism from 3B Scientific.

There were two main concerns with this prism. Firstly, it had only one entry where the vacuum pump could be attached, which made the measurement of the vacuum in the prism inaccurate. Kruger [9] pointed out that to measure the vacuum in the etalon accurately, the vacuum gauge must be connected to a second entry of the etalon. The second concern was that the side windows of the prism where the light passes through were not of high optical quality. These two concerns were the focus in the design of the new prism.

Figure 2 shows the newly designed prism. The prism was designed with two entries, one where the vacuum pump connects and the second for the gauge. The second design modification is that the two side windows were made to be flat and parallel. This is to ensure the least beam deflection of the laser light passing through the prism. However, the windows were not manufactured perfectly and corrections for it had been made and will be discussed in the results. The sides were also made higher, allowing both laser beams to pass through the same amount of optics, similar to the design of the vacuum cylinder described by Chetty and Kruger [4].

Fig. 2. Newly designed prism.

The last design change incorporated into the new prism is the angle of the top corner of the prism. To achieve the maximum vacuum to air ratio, this angle must be as large as possible. However, a too large angle will lead to total reflection of the laser on the side windows of the prism. Therefore, it was decided to use the Brewster angle for the windows [1,20]. For quartz medium, $n = 1{,}5$ and vacuum n approximating 1, the Brewster angle is approximately 56 degrees for each window, resulting in a 122-degree angle at the apex of the prism.

$$Tan\theta = \frac{n1}{n2}$$

[Eq. (3)]

The optical layout is comparable to the design of the previous prism refractometer by Chetty and Kruger [4,9]. Figure 3 shows a schematic layout of the prism refractometer. Any laser system could theoretically be used, but in this design a commercial dual-frequency laser Zygo ZMI 2000 interferometer is used. The frequency difference of the two laser beams is about 20 MHz [17]. Keysight angle interferometer 10770A optics is used to split the two laser beams in the polarising beam splitter. It separates the two beams, f1 and f2, to be parallel and 32,62 mm apart [16].

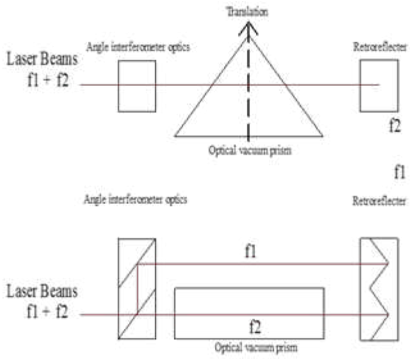

Fig. 3. Schematic diagram of the prism refractometer using laser displacement interferometer and of the translation stage

The prism is positioned in such a way that the one frequency of the laser beam (f2) passes through the vacuum chamber of the prism and the other frequency (f1) passes through the air above the prism, as shown in Figure 3. With the use of a translation stage, the prism is moved through the one laser beam (f2) so that the air is replaced with a vacuum. As discussed previously, this new prism was designed to be higher so that both laser beams pass through the same amount of optics.

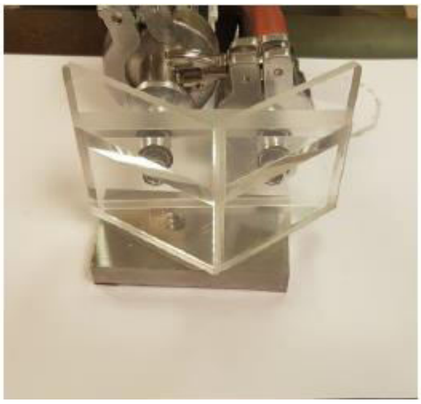

Fig. 4. Prism refractometer setup on a translation stage

Figure 4 shows the prism positioned on the translation stage where a displacement laser from Keysight is used to measure the stage movement accurately. The distance the translation stage is moved, Lt is used to calculate the amount of the vacuum inside the prism being replaced, L and θ the half-angle of the prism by using Equation 4.

$$L = 2 * (tan\theta * Lt)$$

[Eq. (4)]

The angle of the prism together with the distance of travel of the translation stage is needed to calculate the distance of air replaced within the vacuum. From this, the refractive index can be calculated in the same way as for the tube refractometer [4,21].

$$n = \left(\frac{L + l}{L}\right)$$

[Eq. (5)]

By substituting equation (4) into (5), equation (6) is used to calculate the refractive index

$$n = \frac{2 * (tan\theta * Lt) + l}{2 * (tan\theta * Lt)}$$

[Eq. (6)]

The angle of the prism is measured using a Zeiss Prismo Coordinate Measuring Machine (CMM). The accuracy of the CMM measurement is calculated from the CMM specification to be 0,5 µm [22].

The prism was purposefully designed for the refractive index measurements, however the optical flatness and parallelism of the side windows of the prism are not perfect and correction had to be made. These measurements are necessary to determine the wavefront of the optical path of the prism while the prism is traversed across the laser beam. To determine this, measurements were taken with air inside the prism. It was assumed that the air inside and outside the prism was at the same temperature, pressure and humidity. These readings were used as a baseline measurement and were added to the final measurements made when there was a vacuum inside the prism.

The results of the prism refractometer were compared to the weather station method, where the air temperature, pressure and humidity were measured and the refractive index calculated using the modified Edlén equation [13,14]. In Table 1, the V.O.L. compensation was calculated as most manufacturers of laser displacement systems use the V.O.L. in their software calculations [15–17]. The relation between the V.O.L. and the refractive index is shown in equation (6) and is the inverse of the refractive index *n*.

$$\text{V.O.L.} = n^{-1}$$

[Eq. (7)]

In Kruger and Chetty [10] this method is proved to be accurate for use in dimensional metrology. This method was also researched by Renkens and Schellekens [11]. This method was also filed for a US patent in by John C. Tsai [23] and its layout is shown Figure 5.

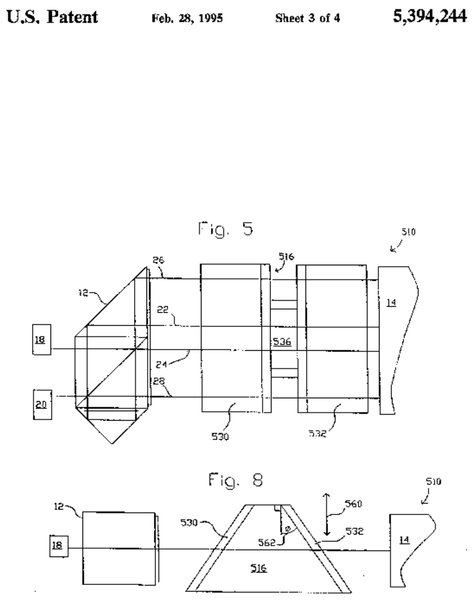

Fig. 5. Permanent vacuum refractometer setup on a translation stage

The research by Kruger and Chetty [10] improved on the method of Renkens and Schellekens [11] by adding an extra distance laser interferometer to continuously monitor the zero position. This was a major improvement in the long-term stability of the refractometer.

4. Method to use Fizeau interferometer to measure air refractive index together with prism vacuum etalon

The optical wave front of the side windows of the prims were measured using a Zygo GPI XP/D Phase-shifting interferometer [24]. During these measurements, the use of the shift in the fringes of the interferometer was investigated to measure the change in optical path length of the air/vacuum ratio.

The Zygo interferometer was designed to measure the optical quality, for example, flatness, parallelism and wave front, of optical components, and is based on a Fizeau interferometer setup [1]. The spacing and the angle of the fringes are used to measure the optical quality of the optics under investigation [24].

The orientation of the fringes in the interferometer was measured simultaneously in the vacuum chamber and the surrounding air. Then the same principle as in the previous method was used to calculate the air/vacuum ratio, but instead of using a laser displacement interferometer and a translation stage, the Zygo flatness interferometer was used to measure this variance in ratio. Figure 6 shows the prism placed between the optical layout of the flatness interferometer.

Fig. 6. Prism refractometer setup in Zygo flatness interferometer

An interferometer map, as shown in Figure 7, was generated, with the vacuum chamber at the bottom of the figure and air at the top. In this figure, there was air inside the vacuum chamber of the prism.

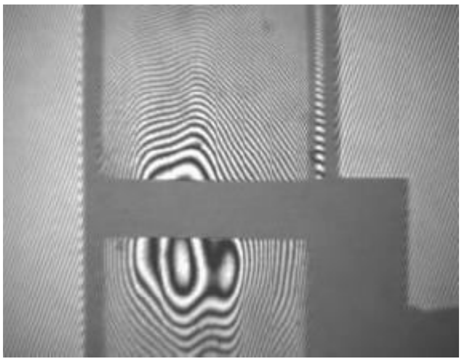

Fig. 7. Fringes generated with similar air inside and outside of the prism

The standard Zygo interferometer software [24] was used to select two areas for the measurement. Figures 8 and 10 show the prism with the areas selected with air inside the vacuum chamber. Figure 8 being the Fringe map and figure 10 the oblique plot. This was to calibrate the baseline, as described in Kruger and Chetty [10]. Thereafter a vacuum was drawn in the bottom chamber, and this fringe map is shown in Figures 9 and 11. The difference in the fringes from the air to the vacuum is clearly visible between Figures 8 and 9.

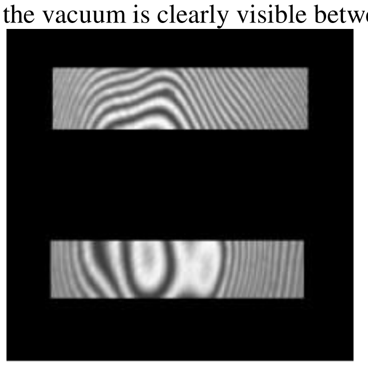

Fig. 8. Fringe map with air in the vacuum chamber of prism

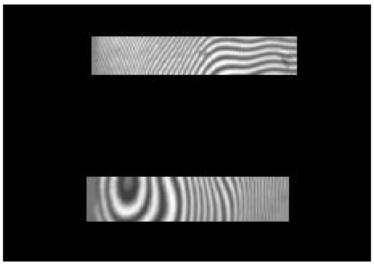

Fig. 9. Fringe map with a vacuum in the vacuum chamber

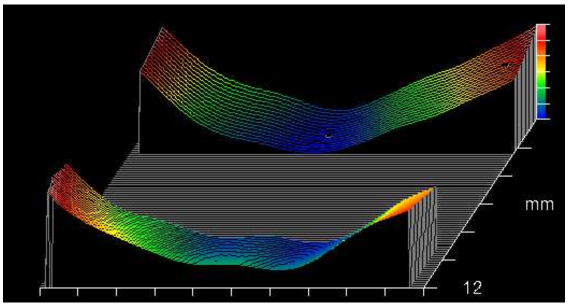

Fig. 10: Oblique plot with air in vacuum chamber

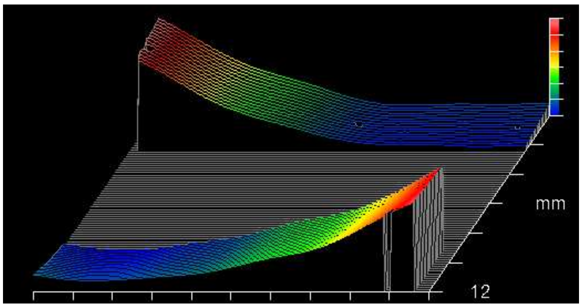

Fig. 11. Oblique plot under vacuum measuring the air/vacuum ratio

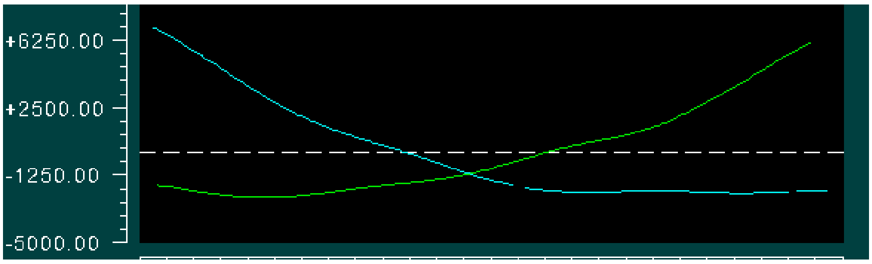

Fig. 12. Profile of two lines selected, one through air (blue line) and second through vacuum (green line)

The Zygo interferometer software was used to selects two horizontal profiles, one through the vacuum chamber and one through the surrounding air, to measure the differences in the air/vacuum ratio, as shown in Figure 12.

5. Results from Zygo flatness interferometer

This research focused on using the flatness interferometer with the newly designed prism. The translation stage was used to measure the refractive index of air to validate the results of the Zygo flatness interferometer. The method of using the translation stage was previously validated with the weather station method and found to be adequate to be used daily [10].

The angle of the prism was measured using a Zeiss Prismo CMM. The accuracy of the CMM measurement was calculated from the CMM specification to be 0,5 µm [22].

The prism was purposefully designed for the measurement of the refractive index of air, but the side windows were not perfect. Therefore, the first set of measurements was taken with air inside the prism (Figure 8). It was assumed that the air inside and outside the prism was at the same temperature, pressure, and humidity. These baseline measurements were added to the final measurements after a vacuum was drawn in the prism.

In Table 1, the V.O.L. calculation was used and not the refractive index as the V.O.L. calculation is mostly used by the manufacturers of the laser displacement systems in dimensional metrology [15-17].

The baseline results, readings in air were added to the readings under vacuum for a combined reading (column 5). The combined readings were used to calculate the V.O.L. and to compare to the weather station reading, while the difference in the two readings is reported in the last column. The worst difference between the two methods is $9{,}28 \times 10^{-8}$.

Numerous readings were taken over a period of a few weeks, and the standard deviation was calculated over all the differences between the two methods: The Zygo flatness interferometer and the translation stage compared to the weather station method. The worst difference is $3{,}15 \times 10^{-8}$ and is discussed in detail in the uncertainty section.

Table 1: Results for the prism refractometer data compared to the modified Edlén equation using weather station measurement calculations

Horizontal distance of prism		Prism readings								
Horizon -tal distance (mm)	Length air/ vacuum (mm)	Interfero-meter reading in air (mm)	Interfero-meter reading in vacuum (mm)	Interfero-meter reading combined mm)	Air temp . (°C)	Air pres. (mbar)	Air humid . (%RH)	V.O.L. Edlén eq.	V.O.L. prism	Diff. in V.O.L. between Edlén eq. and prism refrac.
		Zygo Flatness								

		interfero-meter								
25	74.267	0.00124	0.017339	0.017340	19.8	872	61	0,999766572	0,999766573	6,13E-08
		Laser displace-ment								
25	74.267	0.00242	0.015411	0.017157	20.5	865	41	0,99976883	0,999768923	9,28E-08

6. Uncertainty calculation for the Zygo flatness interferometer

Only the Zygo flatness interferometer method is discussed in the calculation of the uncertainty. The uncertainty for both the translation stage and the Edlén weather station methods are discussed in detail in previous research by Kruger and Chetty [4,9,11]. The uncertainty for the prism with the translation stage was calculated to be $8,4 \times 10^{-8}$ in the refractive index[11]. The uncertainty for the weather station method in this setup was previously calculated by Kruger and Chetty to be $5,9 \times 10^{-8}$ in the refractive index [4,11].

The uncertainty calculation of the Zygo flatness interferometer method is as follows:

The uncertainty in the angle of the prism was calculated from the uncertainty of the Zeiss CMM used and was calculated to be 0,5 µm. The Tan function of this uncertainty was used to first calculate the uncertainty of the angle of the prism to be 0,004°, and then to calculate the length of the air replaced, resulting in a 0,47 µm uncertainty in the air/vacuum ratio. This resulted in an uncertainty of 4×10^{-8} in the V.O.L.

A lateral distance of 25 mm was used to measure the air/vacuum ratio in the interferometer. This distance was calibrated using a gauge block, as per the Zygo manual [24]. The uncertainty in the size of the gauge block was 20 nm, which is relatively small compared to the calibration of the interferometer lateral scale. Repeatable calibrations using various gauge blocks resulted in an uncertainty of 5 µm. This value resulted in an uncertainty of 8.6×10^{-9} in the V.O.L. reading.

The uncertainty in the measurement of the change in the air/vacuum ratio was calculated from the Zygo interferometer manual [24], and the repeatability of the readings in this setup was 2 nm, resulting in uncertainty of $7,2 \times 10^{-8}$ in the V.O.L.

The baseline measurements for the imperfection in the side windows of the prism, as described earlier, were taken with air inside the vacuum etalon and added to the readings under vacuum. This theoretically eliminated this uncertainty and only the repeatability in the readings were added to the total uncertainty. The repeatability was included in the uncertainty of the laser reading uncertainty.

Table 2: Uncertainty budget for prism refractometer at the 25 mm position only

Sym	Description	Value	Uncertainty	Sensitivity Coefficient	Uncertainty Contributor	Significance %
q	Angle of prism (rad)	0,97825	0,000019	$-490*10^{-6}$	$-20*10^{-9}$	0.2
Lt	"Translation" distance (um)	25000	21	$-9.2*10^{-9}$	$-110*10^{-9}$	95.1
l	Difference in laser reading, (profile height difference) including repeatability (um)	17.15	0,002	$13*10^{-6}$	$16*10^{-9}$	1.9
Combined uncertainty					$114*10^{-9}$	
Expanded uncertainty					$8,4*10^{-8}$	

In table 2 the uncertainties are presented is a typical uncertainty budget.

To combine the uncertainties, the root sum square method was used as described in the GUM [25]. The combined uncertainty was calculated to be 114×10^{-9} in the V.O.L. measurements.

7. Conclusion

The research was into a permanent vacuum prism refractometer using different methods to measure the air/vacuum ratio required to calculate the refractive index. Two methods were investigated: The translation stage method, which was discussed in previous research, and the use of an interferometer, normally used to measure optical flatness/wave front. The results for both methods were compared to the weather station method.

The results of the translation stage show an agreement of 6.13×10^{-8} with the Edlén weather station method, and the results of the interferometer show an agreement of 9.28×10^{-8} with the Edlén weather station method.

The results of both methods show that the accuracy is adequate to be used for refractive index measurements of air in high accurate dimensional metrology without the use of a vacuum pump.

In the method using a flatness interferometer to measure the refractive index, the largest contributor to the uncertainty is the lateral calibration of the camera, with a 95% significance. This interferometer is not designed for these measurements and a high accurate resolution camera will vastly improve this uncertainty contributor. It does however show that this method can perform these measurements with the advantage that it takes the readings directly, with no moving of a translation stage required, meaning the results are instantaneous and more points along the windows can be taken given a better average reading. Future research will be performed into a purpose-built system with higher accuracy cameras which will result in better lateral resolution and taking more points.

The research demonstrated a new method to measure the air/vacuum ratio in refractometers. It can lead to systems that are faster and cheaper, and with improved camera resolution, it will have an improved accuracy, which was the original objective of this research.

Disclosures

The authors declare no conflicts of interest.

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