Predicting the Isotropic Volumetric Compression Response of Hydrating Cemented Paste Backfill (CPB)

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Abstract

Deep and high-stress mining results in stress transfers onto the previously placed backfill, and mines have recorded several MPa induced backfill stress. Understanding the backfill-rock mass interaction is therefore critical. Previous work considered tabular ore bodies undergoing primarily one-dimensional compression and showed how the backfill reaction curves could be estimated from oedometer laboratory test results. This work considers massive orebodies and develops a similar approach based on isotropic compression curves. Isotropic compression tests exceeding 6 MPa are carried out on samples with 3.0% to 11.1% binder content, tested at 1-day cure time to 28-day cure time. The compression curve is characterized in three stages: initial elastic compression up to a yield point, followed by a transition stage to the start of a final stage with a linear post-yield compression line in $\varepsilon_v - \log(p')$ space. Because these isotropic compression tests are rare (the reported results are the first for Cemented Paste Backfill), attempts are made to relate the isotropic compression test parameters to parameters from the more commonly used Unconfined Compression Strength (UCS) tests. Unifying equations as functions of binder content and cure time are found to determine the initial yield stress and the peak strength from UCS tests. These are then related to the corresponding parameters in isotropic compression. Finally, the slope of the post-yield compression line is found as a function of UCS, thereby enabling complete reconstruction of the isotropic compression response based on parameters from carefully controlled UCS tests, as functions of binder content and cure time. Although the calibrated parameters are specific to the studied mine's materials, the framework is general and applicable to other mines' CPBs.

Author keywords: Cemented paste backfill; underground mining support; isotropic consolidation; isotropic hardening law; laboratory tests.

1. Introduction

The economic and environmental sustainability of bulk underground mining operations depends on timely backfilling of the large voids created by mining. This ensures regional stability of the surrounding rock mass, and when the mine's own tailings are reused as backfill it also reduces the need for surface disposal of mine tailings and the attendant potentially adverse environmental impacts. Backfilling is particularly important in deep and high-stress mining for controlling progressive host rock closures resulting from ongoing mining, by partially transferring the load from the host rock onto the backfill. Cemented Paste Backfill (CPB), a mixture of mine tailings, water and binder, is the most rapid and efficient backfilling form that has gained increasing popularity in the mining industry over the past two decades.

An early attempt at measuring field stresses to understand the energy transfer from mining onto CPB was carried out by Hassani et al. (2001) at Chimo mine. They used stress measurements in the host rock and the backfill, coupled with displacement measurements in the host rock and quantified the energy absorbed by backfill during mining an adjacent stope. In their study, the induced stresses approached 1 MPa. Other subsequent studies have measured induced stresses in excess of 4 MPa and closure strains up to 15% (Thompson et al. 2009; Thompson et al. 2011; Thompson et al. 2012; Counter 2014; Raffaldi et al. 2019). While these induced stresses are smaller than the pre-existing field stresses, they are nonetheless much higher than confining stresses that are typically explored in mine backfill strength studies conducted under controlled laboratory conditions.

Unfortunately, the literature contains no data on the response of CPB to high confining stress. Uniaxial compressive strength (UCS) of CPB has been studied quite extensively (e.g., Kesimal et al. (2005); Klein and Simon (2006); Orejarena and Fall (2010); Yi et al. (2015); Gorakhki and Bareither (2017); and Xu et al. (2018)), in part because it is the least resource intensive, but also because there are many limit equilibrium design methods that depend on UCS as a key input (e.g., Mitchell et al. (1982) and Potvin et al. (2005)). Less attention has been paid to tests that study mechanical properties and effects of cemented bond breakage on the behavior of cured

CPB under more complex loading conditions (Pierce et al. 1998; Yilmaz et al. 2010; Yilmaz et al. 2015; Fang and Fall 2020; Jafari et al. 2020a; Jafari et al. 2020b). Therefore, new information about CPB's response to high confining pressures is needed. Jafari et al. (2020a) showed how laboratory oedometer test results could be used to understand the backfill-rock mass interaction for tabular ore bodies undergoing predominantly one-dimensional compression. In this work a similar approach is used to understand how backfill in massive orebodies would respond to predominantly isotropic loading.

The majority of the studies on soil behavior under isotropic pressure have focused on uncemented sands (Lade et al. 1996; Jefferies and Been 2000; Consoli et al. 2005; Lade and Bopp 2005), with fewer studies on cemented sands and clays (summarized in **Table S1** in the **Supplementary Data**). There are no results for cemented silts, which is the common material in Cemented Paste Backfill. Furthermore, in all previous studies the experiments were carried out on "cured" samples (typically 28-day), however in mining the loading could occur much earlier due to ongoing proximate mining. Therefore, in the current work a range of binder contents and cure times is considered representative of practical mining conditions. Previous work (Jafari et al. 2020a) showed that the Unconfined Compressive Strength (UCS) can be determined as a function of binder content and cure time, and so it is also desired to relate the isotropic compression response to the UCS test results if possible, as the UCS is more conventional and readily carried out. Although the calibrated test results will be specific to the materials tested, it is desired that a framework be demonstrated by which other mine's materials can be similarly tested in a more efficient way with fewer resource-intensive tests.

2. Experimental program

2.1 Material description

The tailings used in this study were collected from the Williams mine located east of Marathon, Ontario, Canada, and mainly contain silt-size crushed rock with 14% fine sand as shown in **Figure 1**. Silicates were the main mineral constituent of the tailings, and small amounts of sulphur oxides were detected. It should be noted that the concentration of sulphur-bearing

minerals (particularly pyrite and pyrrhotite) is low as compared to other hard rock mine tailings reported in the literature, as will be considered subsequently. Atterberg limits were determined in accordance with ASTM D4318-05. The liquid limit (LL) of the tailings is 23%, the plastic limit (PL) is 18%, and the plasticity index (PI) is 5%. The specific gravity of tailings is 2.768 ± 0.009 determined using Helium Stereopycnometer in accordance with ASTM D5550-14. 2014 and the specific surface of tailings is $0.344 \text{ m}^2/\text{g}$ measured based on ASTM C204-18e1. The water collected from the mentioned mine's paste plant and ordinary Portland cement were other constituents of this CPB. More information about the chemical compositions of tailings and cement, and concentrations of ions in the process water may be found in Jafari et al. (2020c).

2.2 Specimen Preparation and Testing Procedure

The mine tailings were shipped from the mine in separate buckets. The water content of each bucket was first determined by mixing the bucket for 15 minutes to ensure that the material was well blended. It should be noted that the water contents used in this work are defined as mass water/mass solid, as opposed to the definition commonly used in the mining industry, which is mass water/total mass. Based on what has been used in the CPB preparation in the mentioned mine, the desired initial water content of 38% was considered in specimen preparation, and consequently, tailings, mine process water, and cement values were determined and mixed to obtain the desired water content representative of mining conditions. A range of cement content between 3.0 to 11.1% (weight of cement to dry tailings) was considered in this study which is consistent with the range of cement content commonly used in practice (Landriault 1995; Potvin et al. 2005). The specimens were cast into split moulds in three layers. The moulds were then sealed, and the specimens were cured in a humid container with 96% relative humidity and temperature of $22 \pm 1 \text{ }^\circ\text{C}$ for one day. Then, to prevent oxidization, after removing the top cap of the moulds, the specimens were submerged in the mine process water to cure until the desired testing time. The extracted specimens had 34.2 mm diameter, and the ratio of 2:1 between height and diameter was considered in specimen preparation.

UCS tests were conducted under water with a constant strain rate of 0.03 percent per minute to ensure fully drained conditions during testing. To conduct the isotropic compression test, a triaxial cell with capacity of 20 MPa was designed and fabricated (**Figure 2**). In these tests, the specimens were always back-pressurized (to about 600 kPa) to ensure the fully saturated condition. The volumetric change was measured using a frictionless rolling diaphragm type volume change device (VCD) with 0.01 ml resolution. Using the pore pressure transducer installed in the top cap, the pore pressure in the specimen was monitored, and drainage was allowed from the bottom platen.

3. Laboratory Test Results and Discussion

This section provides and briefly discusses the UCS and isotropic compression test results carried out in this study. The following abbreviations are used in the tables and figures: UCS: uniaxial compressive strength test; ISO: isotropic compression test; CC: Cement Content; and CT: Curing Time. The number following each abbreviation indicates the value of the parameter presented by the abbreviation.

3.1 UCS Test Results

UCS tests were carried out on CPB specimens with four cement contents at five different curing times. Three tests were conducted for each combination of curing time and cement content, resulting in a total of 60 specimens being tested. The average and coefficient of variation of UCS measurements are presented in **Table 1**. It is noted that the coefficient of variation of UCS test results for each combination of cement content and curing time is less than 7%, and generally lower for higher cement contents.

The UCS test results indicate that, as expected, UCS and yield stress in the UCS tests (σ_{YS}^{UCS} , see **Section 3.1.1**) generally increase with the increase of curing time. Also, for all cement contents, the rate of increase in UCS seems much higher initially but declines over time (as can be seen in **Table 1**). This can be attributed to the process of cement hydration and the fact that over time, most of the cement particles have reacted, which would result in stiffening and progressive strength gaining of CPB. It should be noted that not all CPB material necessarily

144 behave this way. For instance, the presence of sulphide minerals within the CPB has a
145 deteriorative effect (chemical weathering) that leads to the reduction of UCS by curing time
146 (Benzaazoua et al. 1999; Benzaazoua et al. 2002; Kesimal et al. 2004; Li et al. 2019). Also,
147 Portland cement combined with fly ash, ground blast furnace slag, or other supplementary
148 cementitious materials can have multiple reaction phases leading to more discontinuous strength
149 gain with hydration relationships (e.g., Veenstra (2013)).

150 The results show that the cement content and curing time considerably affect CPB strength.
151 Here, a power function and a factor of the form $CT^B \cdot CC$ are proposed to capture both effect of
152 cement content and curing time on UCS and σ_{YS}^{UCS} (**Figure 3**). Similar power function and factor
153 are used by Consoli et al. (2012) to capture the effect of variation of cement content and void
154 ratio in cemented sands on the UCS of cemented sand. The best-fit power functions are shown
155 in **Figure 3**. This fitting approach works in part because the strength development with time is
156 a relatively monotonic, smooth and continuous function. It should be noted that this is not
157 always the case, as mentioned above. Therefore, the proposed fitting function is specific to the
158 studied backfill in terms of both its mathematical form as well as the calibrated parameters, and
159 other mine's materials will require similar laboratory testing programs to determine if the
160 mathematical formulations require adaptation or the material parameters require recalibration.

161 3.1.1 Yield stress in UCS tests (σ_{YS}^{UCS})

162 Yield is the process in which a material changes from a state of predominantly elastic
163 behavior to one of predominantly plastic behavior (Bieniawski 1967). It is generally accepted
164 that the cemented bonds control the yield behavior of cemented soils (Malandraki and Toll
165 2000). Determining this value is sometimes difficult due to the gradual onset of cementation
166 bond breakage (Rotta et al. 2003). This may be even more complicated because yield in
167 cemented or generally structured soils is known to depend on stress path, initial confining
168 pressure, and also drainage conditions (Smith et al. 1992; Coop and Atkinson 1993). For the
169 tested CPB specimens however, determining the yield stress was relatively straightforward.

The methods that define the yield stress can generally be classified into two main groups based on i) a clear change in stress-strain behavior (Anagnostopoulos et al. 1991; Coop and Atkinson 1993; Kavvadas et al. 1994; Rotta et al. 2003); and ii) a clear change in stiffness-strain behavior on the logarithmic or semilogarithmic scale (Malandraki and Toll 1996; Malandraki and Toll 2000; Malandraki and Toll 2001). In this study, both methods were used to determine the yield stress and essentially identical results were obtained; values from the second method are reported.

Figure 4 shows the ratio of yield stress to the strength of the material in the UCS test which is an indication of the domain of elastic behavior of the material based on applied stress. This ratio increases by curing time in the specimens with identical cement content; for example, for the specimens with 11.1% cement content, the ratio increases from 0.55 for the specimens with one-day curing time to 0.66 for the specimens with 28 days curing time, i.e. an approximately 20% increase. Also, the specimens with higher cement content show higher yield stress to strength ratios at the same curing time.

3.2 Isotropic Compression Test Results

The variation of total volumetric strain (ε_{Tot}^v) with respect to effective isotropic pressure (p') are shown in **Figure 5** for specimens with 3.0, 5.3, 7.5, and 11.1 % cement contents. The summary of individual specimen test results is presented in **Table 2**. These results show that the typical isotropic compressive behavior of CPB is similar to other cemented soils. This means that CPB shows low volumetric strain change under low effective isotropic pressure up to the yield stress. Here, the pressure at which the material behavior begins to deviate from linearity is defined as the isotropic yield stress (σ_{YS}^{ISO}). Beyond σ_{YS}^{ISO} , the rate of volumetric change against the increase of isotropic pressure increases followed by linear behavior at higher pressures. **Figure 5** further shows that σ_{YS}^{ISO} increases with increasing cement content and curing time, and that the corresponding curves expand and become steeper.

4. General Equation of Volumetric Strain Variation versus Isotropic Pressure

A careful study of all isotropic compression tests shows that for all cement content and curing time combinations, CPB response under isotropic pressure can be defined by three stages of behavior, namely, elastic, transitional compression, and post-yield compression. These three stages have also been observed in cemented sands and clays under isotropic pressure (**Table S1**). To illustrate, **Figure 6a** highlights these as segments AB, BC, and CD for the ISO-CC7.5-CT28 specimen. Segment AB shows the pre-yield compression where CPB exhibits elastic behavior up to the yield stress (σ_{YS}^{ISO}) which can be defined by the bulk modulus of the material (shown as PrYC_S). Beyond the yield stress, cement bond breakage progresses, and the CPB response deviates from linear elastic behavior. Once bond breakage is started, the impact on the volumetric response is considerable, causing a concave response labelled as TC_S (segment BC). Finally, in segment CD, as the isotropic pressure increases, more cement bonds break and the volumetric response of CPB relies more on the movement and rearrangement of silt-size crushed rock and broken hydration products. This is called the post-yield compression segment and is labelled as PYC_S. **Figure 6b** shows the same interpretation as above except that the elastic volumetric strain corresponding to yield stress (ϵ_{YS}^v) is subtracted from the data; this alternative presentation prevents visual clutter when curves from multiple curing times are superimposed (**Figure 7**). Note the subsequent change of coordinates of points B, C, D, and particularly P_{ISO} which will be used in presenting the proposed equations in the following sections.

Both UCS and isotropic test results show that CPB behaves linearly in the elastic domain and therefore, this segment of isotropic compression response can be simply predicted by elastic properties of CPB. However, predicting CPB behavior after the yield stress is not as straightforward. Examining the results for all cement content and curing time combinations suggests that the nonlinear transitional segment (segment BC in **Figure 6a**) can be captured by

221 a third-degree polynomial, followed by a linear equation in the post-yield compression segment
 222 (segment CD):

$$\begin{cases} \varepsilon_{Tot}^v = a_1 \cdot \log p'^3 + a_2 \cdot \log p'^2 + a_3 \cdot \log p' + a_4 & \sigma_{YS}^{ISO} \leq p' \leq \sigma_C \\ \varepsilon_{Tot}^v = PYCLS \cdot \log p' + m & \sigma_C \leq p' \end{cases} \quad 1$$

223 This set of equations must result in a continuous and differentiable function over the three
 224 segments mentioned which is preferable in any constitutive equations to prevent issues related
 225 to singular corner regions. To fulfill these conditions, the value and derivative (slope) at B (yield
 226 stress) and C (end of the transitional compression segment) should be equal; so:

$$\begin{cases} a_1 \cdot \log(\sigma_{YS}^{ISO})^3 + a_2 \cdot \log(\sigma_{YS}^{ISO})^2 + a_3 \cdot \log(\sigma_{YS}^{ISO}) + a_4 = \varepsilon_{YS}^v & \text{(continuity at B)} \\ a_1 \cdot \log(\sigma_C)^3 + a_2 \cdot \log(\sigma_C)^2 + a_3 \cdot \log(\sigma_C) + a_4 = \varepsilon_C^v & \text{(continuity at C)} \\ PYCLS \cdot \log(\sigma_C) + m = \varepsilon_C^v & \text{(continuity at C)} \\ 3a_1 \cdot \log(\sigma_{YS}^{ISO})^2 + 2a_2 \cdot \log(\sigma_{YS}^{ISO}) + a_3 = S_{YS} & \text{(differentiable at B)} \\ 3a_1 \cdot \log(\sigma_C)^2 + 2a_2 \cdot \log(\sigma_C) + a_3 = PYCLS & \text{(differentiable at C)} \end{cases} \quad 2$$

227 where

228 σ_{YS}^{ISO} yield stress in isotropic compression test

229 σ_C stress at the end of transitional compression segment

230 ε_{YS}^v total volumetric strain at yield stress

231 ε_C^v total volumetric strain at the end of transitional compression segment

232 m intercept of post-yield compression line in ε_{Tot}^v - $\log(p')$ space

233 $PYCLS$ post-yield compression line slope in ε_{Tot}^v - $\log(p')$ space

234 S_{YS} slope at yield stress in ε_{Tot}^v - $\log(p')$ space

235 Solving the above system of equations results in the following a_i coefficients:

$$a_1 = \frac{-(2m - 2\varepsilon_{YS}^v + PYCLS \cdot \log(\sigma_C) + PYCLS \cdot \log(\sigma_{YS}^{ISO}) - S_{YS} \cdot \log(\sigma_C) + S_{YS} \cdot \log(\sigma_{YS}^{ISO}))}{(\log(\sigma_C) - \log(\sigma_{YS}^{ISO}))^3}$$

$$\begin{aligned}
a_2 &= (3m \cdot \log(\sigma_c) - 3\log(\sigma_{YS}^{ISO}) \cdot \varepsilon_{YS}^v - 3\log(\sigma_c) \cdot \varepsilon_{YS}^v + 3m \cdot \log(\sigma_{YS}^{ISO}) + 2PYCLS \cdot \log(\sigma_c)^2 \\
&\quad + 2PYCLS \cdot \log(\sigma_{YS}^{ISO})^2 - 2S_{YS} \cdot \log(\sigma_c)^2 + S_{YS} \cdot \log(\sigma_{YS}^{ISO})^2 + 2PYCLS \cdot \log(\sigma_c) \\
&\quad \cdot \log(\sigma_{YS}^{ISO}) + S_{YS} \cdot \log(\sigma_c) \cdot \log(\sigma_{YS}^{ISO})) / (\log(\sigma_c) - \log(\sigma_{YS}^{ISO}))^3 \\
a_3 &= -(PYCLS \cdot \log(\sigma_{YS}^{ISO})^3 - S_{YS} \cdot \log(\sigma_c)^3 - 6\log(\sigma_c) \cdot \log(\sigma_{YS}^{ISO}) \cdot \varepsilon_{YS}^v + 6\log(\sigma_c) \cdot \log(\sigma_{YS}^{ISO}) \\
&\quad \cdot m + PYCLS \cdot \log(\sigma_c) \cdot \log(\sigma_{YS}^{ISO})^2 + 4PYCLS \cdot \log(\sigma_c)^2 \cdot \log(\sigma_{YS}^{ISO}) + 2S_{YS} \\
&\quad \cdot \log(\sigma_c) \cdot \log(\sigma_{YS}^{ISO})^2 - S_{YS} \cdot \log(\sigma_c)^2 \cdot \log(\sigma_{YS}^{ISO})) / (\log(\sigma_c) - \log(\sigma_{YS}^{ISO}))^3 \\
a_4 &= \varepsilon_{YS}^v + \log(\sigma_{YS}^{ISO}) \\
&\quad \cdot (\varepsilon_{YS}^v \cdot \log(\sigma_{YS}^{ISO})^2 - S_{YS} \cdot \log(\sigma_c)^3 - m \cdot \log(\sigma_{YS}^{ISO})^2 - 3\varepsilon_{YS}^v \cdot \log(\sigma_c) \cdot \log(\sigma_{YS}^{ISO}) \\
&\quad + 3m \cdot \log(\sigma_c) \cdot \log(\sigma_{YS}^{ISO}) + 2PYCLS \cdot \log(\sigma_c)^2 \cdot \log(\sigma_{YS}^{ISO}) + S_{YS} \cdot \log(\sigma_c)^2 \\
&\quad \cdot \log(\sigma_{YS}^{ISO})) / (\log(\sigma_c) - \log(\sigma_{YS}^{ISO}))^3
\end{aligned}$$

Solid lines in **Figure 7** show the predictions by **Equation 1** for CPBs with the tested cement contents and curing times, and both the transitional and post-yield compression segments are predicted reasonably well. It should be noted that no prediction of isotropic compression behavior is made for specimens with 3.0% cement content and 5.3% cement content at one-day curing time due to lack of at least three data points before the yield stress.

It is worth mentioning that the reported isotropic test results for varieties of cemented soils (**Table S1**) and other cohesive porous materials such as concrete and rock (Fossum and Brannon 2004; Warren et al. 2004; Foster et al. 2005) suggest similar three-stage behavior. Therefore, the proposed equation is not necessarily limited to CPB and may be considered generally applicable to other cohesive porous materials.

5. Estimating Parameters for Isotropic Compression Behavior

An interesting feature of **Equation 1** is that the coefficients a_i are functions of parameters with identifiable physical meaning (strain and stress at the yield and end of the transitional compression segment, slope at the yield point, intercept and slope of the post-yield compression line) and not solely determined by curve fitting/regression. Furthermore, it is possible to establish empirical relationships to predict these parameters based on cement content and curing time, and therefore UCS. This approach is particularly useful because compared to isotropic

compression tests under high confining pressure, the UCS test is simpler, cheaper, and is widely used in many geotechnical and mining projects. In addition, the physical meaning behind the parameters used to calculate the a_i coefficients makes it possible to study and potentially quantify the effect of hydration on the isotropic compressive response of CPB under different combinations of cement content and curing time. This is examined in the following sections.

5.1 Estimating σ_{YS}^{ISO} , σ_C , and m Using UCS Test Results

Figure 8 shows that isotropic yield stress σ_{YS}^{ISO} can be predicted as a linear function of UCS yield stress σ_{YS}^{UCS} for all combinations of cement content and curing time. The pressure at which the volumetric strain of CPB deviates from linear behavior is on average 7% higher than that of the UCS yield stress.

The isotropic pressure at the end of transitional compression segment (σ_C) was determined for each test. Each σ_C is plotted against the corresponding average UCS in **Figure 9**. This figure shows that it is reasonable to assume a linear relationship between the value of UCS and pressure at the end of transitional compression segment. This means that σ_C can also be predicted based on UCS test results for the range of cement content and curing time studied here. It is worth mentioned that same linear relationship between yield stress in one dimensional test and UCS has been reported by Horpibulsuk et al. (2004) for cemented clay as well.

The intercept of post-yield compression line (m , see **Figure 6a**) is another parameter required to predict behavior of CPB under isotropic pressure. Rearranging the post-yield compression line equation, the intersection of this line with the effective isotropic pressure axis can be defined as P_{ISO} which is equal to $10^{-m/PYCLS}$ (**Figure 6a**). This means that by having P_{ISO} and $PYCLS$ the value of m can be predicted. **Figure 10** shows that P_{ISO} also varies linearly with the corresponding average UCS for each combination of cement content and curing time.

5.2 Estimating S_{YS} , ε_{YS}^v , and ε_C^v

The isotropic compressive behavior represented by the pre-yield compression section is elastic and controlled by the cement bonds, therefore the bulk modulus can be used to determine

the slope at yield stress (S_{YS}). The bulk modulus can be calculated either directly from isotropic compression tests or indirectly from other elastic properties considering linear elastic behavior observed for CPB. **Figure 11** presents bulk modulus calculated from isotropic compression tests conducted in this study. As expected, by the increase of cement content and curing time the bulk modulus increases as well. Also, considering linear elastic behavior of CPB in the pre-yield compression section, using bulk modulus and yield stress values, the value of volumetric strain at yield stress (ε_{YS}^v) can be calculated.

Figure 12 presents total volumetric strain at the end of the transitional compression segment (ε_C^v) for different cement contents and curing times. Examination of the data shows that the value of ε_C^v is not affected by the studied range of cement content and curing time (which, as mentioned, are representative of current engineering practice) and can be taken as independent from cement content and curing time. Interestingly, the a_i coefficients were found to be independent of ε_C^v .

5.3 Estimating Post-Yield Compression Line Slope (PYCLS)

Jafari et al. (2020b) studied the slope of the post-yield compression line (PYCLS) in the normalized void ratio-isotropic compression stress space. They noted that this slope increases with cement content and curing time, and proposed the following equation:

$$PYCLS = B_1 + B_2 \cdot \ln(CT) + B_3 \cdot CC \cdot \ln(CT) + B_4 \cdot CC^{B_5} ; 1 \leq CT \leq 28 \text{ \& } 3.0 \leq C \leq 11.1 \quad \mathbf{3}$$

in which CT and CC are curing time (days) and cement content (percentage) of the specimen, respectively. The prediction of **Equation 3** for the volumetric strain of CPB is presented in **Figure 13** with the values of coefficients B_i noted in **Figure 13a**. Alternatively, there is the possibility of predicting PYCLS using (logarithm of) UCS since the value of PYCLS increases with curing time and cement content. This is shown in **Figure 14**.

Having established relationships between the equation's parameters and UCS based on cement content and curing time, it is now possible to interpolate to intermediate cement content and curing time combinations, determine the corresponding UCS, and therefore the expected isotropic compression response.

6. Conclusions

A series of uniaxial compression tests and isotropic compression tests were utilized to study the impact of different combinations of cement content and curing time on CPB's isotropic compression behavior. The test results provide a better understanding of the effect of cementation on the isotropic compression response of this cemented silt-size crushed rock. Based on the test results a straightforward equation is proposed that can predict the isotropic compression response of CPB based on parameters with physical meaning – rather than solely curve-fitting – in the isotropic stress-volumetric strain space. In addition, other key experimental findings specific to the studied CPB are:

- UCS of this CPB with different cement content and curing time can be predicted using an exponential fitting function based on a $CT^B \cdot CC$ factor (**Figure 3**).
- The isotropic behavior of CPB can be predicted using three segments: the pre-yield compression, transitional compression, and the post-yield compression.
- In both uniaxial and isotropic compression tests CPB shows linear stress-strain behavior before reaching a yield stress. The yield stress in isotropic compression is on average 7% higher than in uniaxial compression.
- The slope of the pre-yield compression line can be determined using the bulk modulus, which can be predicted using a logarithmic fitting function based on curing time, with coefficients calibrated to the cement content (refer to **Figure 11**).
- The slope of the post-yield compression line can be predicted using a combined logarithmic-exponential fitting function based on cement content and curing time (**Figure 13**). Also, there is a logarithmic relationship between this slope and the corresponding UCS (**Figure 14**).

The findings in this paper provide a framework for predicting the isotropic compression behavior of the material studied, within the stated ranges of test parameters. The calibrated fitting functions are specific to the material tested, but the testing procedures and analysis

framework can be used as a basis to determine the isotropic compression behavior of similar backfill materials for other mines.

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Declarations

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Conflict of Interest

The authors declare that they have no conflict of interest.

Data Availability Statement

All data, models, and code generated or used during the study appear in the submitted article.

Author Contributions section

M. Jafari developed the experiment strategy, conducted the experiment, interpreted the data, and wrote the manuscript. M. Grabinsky was edited the manuscript. The senior responsible author (SRA) of the publication is M. Grabinsky.

ABBREVIATIONS

B_i	constant coefficients
CC	cement content
CT	curing time
e	void ratio at the end of each isotropic consolidation step
e_0	void ratio after curing

356	ε_C^v	total volumetric strain at the end of transitional compression segment
357	ε_{Tot}^v	total volumetric strain
358	ε_{YS}^v	total volumetric strain at yield stress
359	K	Bulk Modulus
360	m	intercept of post-yield compression line in ε_{Tot}^v - $\log(p')$ space
361	P_{ISO}	pressure corresponding to the intersection of post-yield compression line and pressure
362		axis
363	$PYCLS$	post-yield compression line slope in ε_{Tot}^v - $\log(p')$ space
364	p'	effective mean isotropic stress
365	S_{YS}	slope at yield stress in ε_{Tot}^v - $\log(p')$ space
366	σ_C	stress at the end of transitional compression segment
367	σ_{YS}^{ISO}	yield stress in isotropic compression test
368	σ_{YS}^{UCS}	yield stress in UCS test

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