Interaction-driven breakdown of dynamical localization in a kicked quantum gas

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Letter

Keywords: quantum interference, dynamical localization, quantum chaos

Posted Date: October 25th, 2021

DOI: <https://doi.org/10.21203/rs.3.rs-958231/v1>

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Version of Record: A version of this preprint was published at Nature Physics on September 26th, 2022.
See the published version at <https://doi.org/10.1038/s41567-022-01724-7>.

Interaction-driven breakdown of dynamical localization in a kicked quantum gas

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Quantum interference can terminate energy growth in a continually kicked system, via a single-particle ergodicity-breaking mechanism known as dynamical localization. The effect of many-body interactions on dynamically localized states, while important to a fundamental understanding of quantum decoherence, has remained unexplored despite a quarter-century of experimental studies. We report the experimental realization of a tunably-interacting kicked quantum rotor ensemble using a Bose-Einstein condensate in a pulsed optical lattice. We observe signatures of a prethermal localized plateau, followed for interacting samples by interaction-induced anomalous diffusion with an exponent near one half. Echo-type time reversal experiments establish the role of interactions in destroying reversibility. These results quantitatively elucidate the dynamical transition to many-body quantum chaos, advance our understanding of quantum anomalous diffusion, and delimit some possibilities for protecting quantum information in interacting driven systems.

Ergodicity breaking in quantum matter and relaxation dynamics of thermalizing phases are two aspects of a central question of non-equilibrium many-body physics: how and when do isolated quantum systems thermalize? A growing body of theoretical and experimental work suggests that ergodicity can be avoided or hindered by a variety of mechanisms, including many-body localization [1, 2], quantum many-body scarring [3, 4], and prethermalization [5–9]. Even without ergodicity breaking, the expected emergence of quantum chaos upon the addition of interactions to driven systems is not well understood. For example, one ubiquitous but incompletely-understood feature of the interface between localized and ergodic regimes is anomalous diffusion [10, 11], which can potentially serve as an indicator of entanglement spreading [12]. A general predictive understanding of such phenomena remains an open challenge to theory and experiment.

The quantum kicked rotor (QKR) [15–17] is a paradigmatic model of dynamical ergodicity breaking. While strong, repeated kicking drives a classical rotor into chaotic diffusion, the corresponding quantum rotor stops absorbing energy after a finite time, signaling the onset of dynamical localization. Despite the complete absence of disorder, this phenomenon can be understood as a manifestation of Anderson localization in momentum

Fig. 1. Experimentally realizing an interacting quantum kicked rotor. (a) Schematic of BEC in single pulsed optical lattice. (b) Experimental sequence. After setting the scattering length the trap is removed and kicking is applied with period T , pulse width τ , and amplitude V_0 for n cycles. The atoms are imaged after a time-of-flight expansion [13]. (c-d) Measured axial momentum distribution versus kick number n for noninteracting (c) and interacting (d) samples, revealing collisional momentum redistribution.

space [18, 19]. Although the QKR is a natural context for experimental probes of the interplay between many-body interactions and dynamical localization, the quantum thermodynamics of interacting kicked rotors have not previously been experimentally explored. Depending on how interactions are introduced into the model, theoretical studies have predicted a variety of novel dynamical phenomena ranging from anomalous diffusion [20–23] to classical prethermalization [24] to many-body dynamical localization [25, 26].

Here we report the first experimental study of dynamical localization in the presence of tunable contact interactions, which are nonlocal in momentum space. These experiments investigate a ^7Li Bose-Einstein condensate (BEC) kicked n times at period T by a far-detuned optical lattice of spacing $d = 532$ nm and depth V_0 for duration τ (see Fig. 1). We report momentum and energy in units of $k_L = \pi/d$ and $E_R = \hbar^2 k_L^2 / 2m$ with m the

Fig. 2. **Observing the interaction-induced emergence of quantum chaos.** (a) Energy versus kick number for varying a . Blue horizontal shaded region indicates the measured single-rotor localization energy of $E_{loc} = 2.5(4)E_R$. Here $V_0 = 64E_R$, $T = 1.2 \mu s$ and $\tau = 300 ns$ ($K \approx 2.3$ and $\hbar \approx 1.5$). The inset contrasts interaction-induced delocalization and anomalous diffusion with classical diffusion caused by random kicking, the latter achieved by adding random offsets to the average kick spacing T drawn uniformly from the interval $[-T/4, T/4]$. The solid curve is noninteracting quantum theory and the dotted line is a diffusion curve $4Dn/\hbar^2$ with $D \approx 0.19$ extracted from the classical standard map [14]. The red dot-dashed line is a subdiffusive $\sqrt{n}$ law serving as a guide to the eye. (b) Momentum-space IPR with transverse dimensions integrated out. The shaded regions are predictions for two exponentially localized distributions with $1/e$ localization length $k_{loc} = \sqrt{E_{loc}} \approx 1.6(1)k_L$ [13]. (c-e) Normalized smoothed momentum space densities at various n . (f-h) The same densities on a logarithmic scale. The orange dotted and purple dashed lines are exponentially localized curves $\exp(-k/k_{loc})$ with amplitudes normalized to match the peak of the measured distributions at the given n . (i) Deviation from exponential localization over time based on integrated ratio between measured and exponential distributions with error bars computed from uncertainty in k_{loc} [13].

mass of 7Li . The single-particle QKR is defined by the 1- cycle Floquet map $U = e^{-i\hbar k^2/2}e^{-iK \cos z/\hbar}$ describing a sharp cosine potential impulse followed by free evolution. Here k and z are momentum and position respectively, $K = \hbar V_0 \tau / 2\hbar$ is the stochasticity parameter characterizing kicking strength and $\hbar = 8E_R T / \hbar$ is an effective Planck's constant determined by the kick period. Absorption imaging after free expansion is used to measure the momentum distribution; see the supplementary text for a full analysis of systematic effects in this procedure, such as dynamics transverse to the lattice direction. Interatomic interactions are varied by tuning the s -wave scattering length a (reported in units of the Bohr radius a_0) using a magnetic Feshbach resonance. While the kicking primarily couples discrete momentum states along a single dimension, the atoms are entirely unconfined between kicks; scattering between momentum modes thus couples the system to a bath of transverse free-particle states.

The main result of this work is presented in Fig. 2a.

While a noninteracting sample exhibits dynamical localization, saturating to a finite energy for over 800 kicks, interacting samples clearly demonstrate the destruction of the dynamically localized plateau with increasing scattering length. At intermediate interaction strength ($a = 240a_0$), we observe saturation to the same energy as non-interacting samples for approximately 300 kicks, suggesting the existence of a reasonably long-lived prethermal state. In contrast, the $760a_0$ trace exceeds this localized energy after around 100 kicks; whether a quasiequilibrium dynamical state is truly established in this stronger-interacting sample is less clear. Fig. 2b shows another aspect of the same evolution, plotting the momentum space inverse participation ratio (IPR) versus kick number. The IPR characterizes the number of states over which the system is distributed, thereby also probing how collisional momentum redistribution washes out the originally discrete momentum modes, a process less easily inferred from energy measurements. While the $240a_0$ data exhibit a clear steady-state behavior for 100

kicks, the $760a_0$ IPR decreases monotonically for almost the entire experiment.

A second key result of these measurements is that the observed delocalizing dynamics clearly exhibit anomalous diffusion: it appears that even interacting quantum kicked rotors absorb energy much more slowly than classical rotors. The inset of Fig. 2a compares the nature of the observed interaction-induced subdiffusive delocalization with linear energy growth in the classically chaotic model. We experimentally simulate classical dynamics by adding stochastic fluctuations to the kicking period T , making use of the known sensitivity of dynamical localization to timing noise [27]. These experimental data agree both with single-particle quantum numerics and with the linear energy growth predicted by the classical standard map [14], and stand in clear contrast to the measured interaction-induced anomalous diffusion away from the dynamically localized state. The dot-dashed red line indicates a $\sqrt{n}$ energy growth, and fitting the late-time data to n^α yields anomalous diffusion exponents α in the range $[0.4, 0.6]$. For reference, 1D Gross-Pitaevskii simulations on a ring [22] predict $\alpha \in [0.5, 0.8]$, though a direct quantitative comparison to theory is challenging due to the significant condensate depletion and the three-dimensional nature of the experiment. Theoretical studies of the effect of local nonlinearity on real-space Anderson localization instead suggest $\alpha \in [0.3, 0.4]$ [28, 29], but the long-range nature of contact interactions in momentum space similarly complicates comparison. This clear observation of anomalous diffusion in the interacting quantum kicked rotor raises a variety of fascinating questions for future exploration. What correlations are responsible for the anomalous diffusion dynamics? What feature of the interacting system prevents the interacting QKR from heating classically? What theoretical framework is appropriate for quantitatively predicting wavefunction evolution in this regime? What are the universality properties of the subdiffusive exponent?

For further insight into the dynamics of kicked interacting quantum systems we examine the evolution of the momentum distribution, shown in Figs. 2c-e. We observe a clear distinction between the noninteracting samples, which settle at a sharply-peaked dynamically-localized momentum distribution, and the interacting samples, which gradually smear out in momentum space due to scattering. Plotting these same densities on a logarithmic scale in Fig. 2f-h illuminates the destruction of dynamical Anderson localization by assessing the departure from exponentially-localized Floquet states. The smeared-out lower-energy modes actually appear to maintain the expected localization length, and thus do not trivially indicate a departure from exponential localization. This observation is also reflected in the fact that two predictions based on exponentially localized distributions bound the measured IPR in Fig. 2b. Instead, the departure from exponential localization manifests in the emergence of increased relative population in the tails of the distribution. It is interesting to note that recent

Fig. 3. Effect of interactions on reversibility in Loschmidt echo experiments. (a) Measured Loschmidt echo fidelity F for a range of scattering lengths $a = [0, 1500] a_0$ for $N = 6$ (blue circles) and $N = 10$ (purple triangles), where N indicates the total number of kicks; a first set of $N/2$ kicks propagates the system *forward* in time and a second time-reversal set of $N/2$ kicks propagates it *backwards*. (b) Measured fidelity F at three different interaction strengths as a function of total number of kicks N in a Loschmidt echo experiment. (c-e) Averaged absorption images of a BEC after the first n kicks of an $N = 10$ Loschmidt echo protocol, for three different a .

theory suggests that even many-body dynamically localized phases are expected to exhibit universal power-law decaying tails [30]. In Fig. 2i we quantitatively characterize the overall deviation from exponential localization [13], revealing a break time near 200 kicks for both interaction strengths. These findings provide a second experimental signature of the destruction of the dynamically localized state by interactions, now both at the level of macroscopic observables and squared wavefunctions.

The onset of energy delocalization due to interactions indicates a transition to the regime of many-body quantum chaos, which can be probed directly by studying time-reversal dynamics [31, 32]. In Fig. 3 we probe the onset of chaotic dynamics by measuring the effect of interactions on a Loschmidt echo time-reversal protocol [33, 34]. The echo is realized using quantum resonances [35] which occur for $\hbar\omega = 2\pi q$ with q rational; in particular for $q = 2$ ($T \approx 9.95 \mu\text{s}$), the free evolution in U largely vanishes and effective time reversal can be achieved by setting q to 1 for a single kick halfway through the sequence [13, 33, 34]. This procedure would create exact time reversal for a single zero-quasimomentum state in the absence of interactions. Due to finite quasimomentum spread and non-reversed interactions, the reversal is imperfect, yielding a Loschmidt

fidelity $F = \left| \langle \psi | U_2^\dagger U_1 | \psi \rangle \right|^2$ where U_1 and U_2 are time-
 evolution operators differing by some perturbation. Per-
 haps surprisingly, F initially *increases* as the scattering
 length a is turned up from zero. In this regime U_1 and
 U_2 are primarily distinguished by the failure to reverse
 kinetic energy, and thus the increase can be explained by
 Thomas-Fermi reduction of the initial state momentum
 spread. Eventually, for large enough a , the interaction
 becomes the primary perturbation and F begins to de-
 crease with a , marking the transition to predominantly
 interaction-induced irreversibility. The decay of fidelity
 with total number of kicks in a Loschmidt echo exper-
 iment is shown in Fig. 3b. The use of Loschmidt echo
 techniques as a probe of many-body quantum chaos not
 only illuminates the origins of the delocalizing dynam-
 ics we observe, but opens up the possibility of extending
 these protocols to probe scrambling in many-body quan-
 tum chaotic systems [36].

In conclusion, we have experimentally realized a many-
 body ensemble of quantum kicked rotors. Following the
 evolution of interacting samples over hundreds of kicks,
 we observe signatures of an initial prethermal state, fol-
 lowed by an interaction-induced breakdown of dynam-
 ical localization via anomalous diffusion, signaling the
 onset of many-body quantum chaos. Characterization
 of the departure from the dynamically localized state
 indicates subdiffusive energy growth with an exponent
 near 0.5, easily distinguishable from classical Joule heat-
 ing in a randomly kicked system, and reveals momentum
 space distributions which are not exponentially local-
 ized. Measuring Loschmidt echo time-reversal dynamics
 with a quantum resonance enabled us to directly probe

the role of interaction-induced irreversibility in driving
 a transition to many-body quantum chaos. Together,
 these results demonstrate and quantitatively illuminate
 the emergence of interaction-driven quantum chaos in a
 paradigmatic localized system.

We acknowledge helpful conversations with Adam
 Rançon, Norman Yao, and Thomas Schuster. During
 the performance of the research described here we be-
 came aware of related efforts underway in another ex-
 perimental group [37] which have reached similar con-
 clusions using a complementary approach. **Funding:**
 D.M.W. acknowledges support from the Air Force Of-
 fice of Scientific Research (AFOSR FA9550-20-1-0240),
 the Army Research Office (PECASE W911NF1410154),
 and the National Science Foundation (CAREER 1555313
 and QLCI OMA-2016245). D.M.W., R.S., and E.N-
 M. acknowledge support from the UCSB NSF Quantum
 Foundry through the Q-AMASE-i program (Grant No.
 DMR-1906325). V.G. was supported by US-ARO Con-
 tract No. W911NF1310172, NSF DMR-2037158, and the
 Simons Foundation. **Authors' contributions:** A.C.,
 R.S., H.M., E.Q.S., J.L.T., E.N-M., T.S. and H.E.K. con-
 tributed to operating the experiment and performing the
 measurements. A.C., R.S. and H.M. analyzed the data.
 A.C. conceptualized and performed the theoretical sim-
 ulations of rotor and spin models. V.G. and D.M.W.
 developed the idea for the experiment. D.M.W. super-
 vised the work. A.C., R.S., H.M., V.G. and D.M.W.
 wrote the manuscript. All authors contributed to dis-
 cussion and interpretation of the results. **Competing
 interests:** The authors declare no competing interests.
Data and materials availability: All data needed to
 evaluate the conclusions in this study are presented in
 the paper and in the supplementary materials.

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Methods

Experimental platform and sequence. The experiments begin with a Bose-Einstein condensate (BEC) of around 10^5 ^7Li atoms in a far-detuned optical dipole trap with trapping frequencies $\omega_{x,z}/2\pi \approx 40$ Hz and $\omega_y/2\pi \approx 56$ Hz, where z is the axis of the optical lattice, y is the direction of gravity, and x is the remaining orthogonal axis. The condensate is produced by optical evaporation at an s -wave scattering length of $a = 240a_0$, set by an applied magnetic field in the vicinity of the broad Feshbach resonance at 737 Gauss [38]. Immediately after evaporation, the fields are ramped to their desired value over 60-90 ms and maintained for the remainder of the experiment. The dipole trap is then extinguished and the BEC repeatedly subjected to a pulsed 1D optical lattice with lattice constant $d = 532$ nm, laser wave vector $k_L = \pi/d$, and recoil energy $E_R = \hbar^2 k_L^2 / 2m$ with m the mass of ^7Li . The full dynamics are then well described by the second-quantized Hamiltonian

$$\mathcal{H} = \int d^3r \hat{\Psi}^\dagger(\mathbf{r}, t) H(\mathbf{r}, t) \hat{\Psi}(\mathbf{r}, t) + \frac{g}{2} \int d^3r \hat{\Psi}^\dagger(\mathbf{r}, t) \hat{\Psi}^\dagger(\mathbf{r}, t) \hat{\Psi}(\mathbf{r}, t) \hat{\Psi}(\mathbf{r}, t) \quad (1)$$

$$H(\mathbf{r}, t) = \frac{p^2}{2m} + \frac{V_0}{2} \cos(2k_L z) I(x, y) \sum_n f_\tau(t - nT). \quad (2)$$

The key kick parameters are the lattice depth V_0 , effective pulse width τ , and kick period T . V_0 is calibrated through a standard Kapitza-Dirac diffraction technique. $f_\tau(t)$ denotes a unit amplitude pulse function beginning at $t = 0$ of width τ . The experimental pulse is approximated by a piecewise function with a linear rise and fall of 200 ns duration before and after a plateau of variable hold time. For the experimental data in the main text with $\tau = 300$ ns between the half-maximum points, this hold duration is 100 ns. The scattering length a determines the two-body coupling coefficient $g = 4\pi\hbar^2 a/m$. Here $I(x, y)$ denotes the transverse intensity profile of the lattice beams normalized to unity maximum; this is approximately Gaussian $I(x, y) \approx e^{-2(x^2+y^2)/\sigma^2}$ with a measured $1/e^2$ beam radius of $\sigma \approx 65$ μm . The total duration of kicking is at most 1 ms for our longest experiments, significantly shorter than the 4 ms it takes the BEC to fall under the influence of gravity through the lattice beam waist.

To measure the momentum distribution, we perform absorption imaging of the atoms after free expansion. The time-of-flight (TOF) duration is 3.5 ms for the delocalization data and 2 ms for the Loschmidt data. Due to the low mass of ^7Li and the breadth of the Feshbach resonance, coil inductance prevents sweeping the magnetic fields to the noninteracting regime for this expansion period. This means additional scattering occurs during expansion, which may lead to systematic errors in the

measured quantities (see supplementary section 6). For the energy, we are able to account for this scattering in our analysis due to the energy-conserving nature of the collisions. For metrics such as the IPR, this systematic is challenging to avoid. However by tracking the evolution of these observables as a function of kick number at a fixed TOF duration, we can largely attribute the qualitative observed dynamics to the evolution under the Hamiltonian (1) as opposed to the expansion. At large n , the majority of scattering happens during the kicking duration so expansion effects become negligible.

Delocalization data analysis. This section discusses the analysis behind Fig. 2. Because the momentum distributions of the interacting samples change significantly over the course of the delocalization experiments, the quantities shown in Fig. 2 are computed directly from raw or averaged images as opposed to fitting procedures. However, this can make measurable quantities such as energy sensitive to noise, especially near the edge of the camera sensor due to the quadratic weighting. To maximize the signal-to-noise ratio in our measurement, we analyze raw images using an adaptive region-of-interest (ROI). First, a single base ROI capturing all detectable atoms at all times is created for each interaction strength. The integrated density in this ROI is used to post-select images with total atom numbers falling within a $\pm 10\%$ window of the mean, in order to reduce variations in the interaction energy, which depends directly on atom density. For these data we take 10 images at each kick number, of which typically 4-7 are discarded by this post-selection procedure. The ROI boundaries at each kick number are then determined by the points at which the cumulative summed distributions of the averaged image outward from a center point reach a threshold value. The thresholds are set empirically and the boundaries obtained by the following procedure. First we compute the transverse bound by integrating out the entire axial direction to get the overall transverse distribution, find the point it crosses an 85% threshold and then expand the resulting boundary by a factor of 1.5 (1.2 in the delocalization data of supplementary section 1) to ensure all atoms are captured. We then compute an axial boundary going point by point along the transverse direction; at each transverse point we integrate over 10 neighboring transverse pixel rows to get a “local” axial distribution, find the point it crosses a 99.8% threshold and expand by a factor 1.15. Finally we smooth each ROI boundary and take a moving average across different kick numbers (4 on each side). Crucially, we have confirmed that the qualitative observation of delocalization is not significantly altered from the simple case where we use just the initial single base ROI across all shots. However, the details of the trends should be more accurately captured by the adaptive procedure because the signal-to-noise ratio over the ROI is optimized at each kick number. All measurable quantities are then calculated from the imaged densities within this region.

Since we do not observe any substantial atom loss during the kicking duration, we treat the imaged atomic densities as normalized distributions. For Figs. 2a-b, we compute the measured quantities from individual experimental runs and then average the results, with the reported error bar as the standard error of the mean. For Fig. 2i, we instead compute the averaged distributions first before computing the deviation from exponential localization; the errorbars are computed from a Monte Carlo simulation of the uncertainty in k_{loc} discussed later in this section. A smoothing filter is applied to the displayed densities in Figs. 2c-h for visual clarity, but not the subsequent calculation of the localization deviation in Fig. 2i.

To measure the energy, we compute the post-expansion spatial variance of the distribution in both the kicking z and transverse x directions of the image. Assuming cylindrical symmetry, the kinetic energy is then calculated as $m(\langle z^2 \rangle + 2\langle x^2 \rangle) / 2t_{\text{TOF}}^2$ with $t_{\text{TOF}} \approx 3.5$ ms (see section 2.4 for a discussion of possible corrections to the conversion of position to energy). For an accurate measurement of the interacting samples, inclusion of the transverse energy is necessary to account for energy-conserving scattering processes that occur both during the kicking and TOF. In addition, the inhomogeneous intensity profile of the beam $I(x, y)$ leads to a transverse energy oscillation in all samples including the noninteracting ones (see supplementary section 5). Since we are not interested in this effect, we remove it to leading order by subtracting off the noninteracting transverse energy from each trace, so that the noninteracting energy is purely the kinetic energy along the kicking direction. To compute the error bars on the interacting data, we add the error of the total interacting energy and noninteracting transverse energy in quadrature. The single-particle localization energy E_{loc} is estimated by averaging the noninteracting trace for $n \geq 100$, and the reported uncertainty is based on the standard deviation of those points. We note that this uncertainty is not only due to experimental imperfections, but also due to natural dynamical fluctuations, as evidenced by the results of noninteracting simulations like those shown in Fig. S3.

We compute an effective 1D momentum-space IPR by first integrating out the transverse dimension and then summing the squares of the subsequent normalized axial density. We confirmed that this qualitatively matches the result of directly integrating the squared 2D distribution while largely eliminating the beam-induced transverse oscillation. Specifically for computing this metric, we apply a smoothing filter to the normalized densities consistently across all 3 interaction strengths. This suppresses high-frequency background noise which sets a lower bound on the measurable IPR due to the squaring procedure. The measured values are compared to two predictions based on an exponentially localized distribution. The blue shaded region is obtained by numerically computing the IPR for the momentum space distribution $\exp(-|k|/k_{\text{loc}}) \sum_j \exp(-(k - 2k_{\text{L},j})^2/w^2)$, which models

a Gaussian comb with an exponential envelope. This is a reasonable expectation for a finite-size, localized noninteracting condensate occupying only discrete momentum modes. The width parameter w is measured from fitting the $n = 0$ noninteracting condensate and takes into account the momentum-space resolution of the TOF given the finite condensate spatial extent. The width of the region is based on Monte Carlo simulation of uncertainty in k_{loc} , where the resulting distribution is fit to a Gaussian to extract the mean and standard deviation. The green shaded region is calculated analytically for a pure exponential distribution of infinite extent and is given by $1/4k_{\text{loc}}$. Taking into account the finite width of the imaging region changes the distribution normalization and leads to the following correction factor $(1 - \exp(-2k_0/k_{\text{loc}}))/(1 - \exp(-k_0/k_{\text{loc}}))^2$; here $k_0 \approx 9.85k_L$ is the half-width of our images which yields a negligible correction factor of ≈ 1.006 . The width of the region is computed through linearized error propagation.

In Fig. 2i, the plotted localization metric is $\int_{-k_0}^{k_0} \max[r(k) - 1, 0] dk / 2k_0$. Here, $r(k) = |\psi(k)|^2 / \exp(-k/k_{\text{loc}})$ is the ratio of the measured axial density denoted $|\psi(k)|^2$ and an exponential localization envelope. Here the maximum of $|\psi(k)|^2$ is set to unity. Taking the maximum of $r(k) - 1$ and 0 ensures that the result is only sensitive to regions of the distribution which decay more slowly than exponentially. That is, it interprets 0 as “at least exponentially” localized with respect to a given localization length, and thus characterizes departures from a given dynamically localized state in the traditional sense of exponentially localized wavefunctions. We note however that the system remaining exponentially localized but with a larger localization length would result in a non-zero value for this metric, which motivates the direct inspection of the distributions in Figs. 2f-h. The reported values and errorbars are extracted by propagating a Gaussian uncertainty in the measured k_{loc} through a Monte-Carlo simulation. We find that the resulting distributions interpolate between sharply peaked at 0 with a rapid fall-off when well-localized, to positively skewed with non-zero peak in the delocalized regime. We empirically find that a log-normal distribution fits the Monte Carlo result well, and we use this fit to extract the data reported in Fig. 2i. In particular, the markers indicate the mean of the distribution and the errorbars represent the interquartile range containing the central 50% of the distribution. Because of the skewness, we investigate the Monte Carlo simulated distributions in more detail in supplementary section 2.

Loschmidt experimental sequence and data analysis. Here we discuss the methods and analysis used to produce Fig. 3. The Loschmidt experiments begin similarly to the previously described sequence; for an N kick Loschmidt sequence, the BEC is first kicked $N/2$ times near quantum resonance at the parameters $V_0 \approx 50E_R$,

$\tau = 300$ ns, and $T = 9.93$ μ s. For this data, we adjusted the lattice depth V_0 for different interaction strengths to achieve the same amount of absorbed energy after the first $N/2$ kicks. This compensates for a decrease in energy absorption at the same lattice depth for higher interaction strengths, which we attribute to the increase of the Thomas-Fermi radius of the BEC relative to the lattice beam size. Neglecting this effect would artificially enhance the fidelity at very large interaction strengths due to a reduction in the effective stochasticity parameter K . We plot the zero mode fraction after the first $N/2$ kicks without time-reversal (denoted F' Gauss) in Fig. S6 to benchmark this kicking amplitude normalization procedure.

After the first $N/2$ kicks, we wait a half period $T/2$ to shift the wavefunction spatially by half a lattice spacing, causing the sign of subsequent kicks to be reversed. We then apply another sequence of $N/2$ kicks using the same lattice parameters to complete the echo sequence. The time series in Fig. 3c-e show absorption images averaged over 5 shots for each kick number n in a $N = 10$ experiment. Since we begin with a zero-momentum condensate mode, to measure the Loschmidt fidelity we simply need to count the fraction of atoms remaining in this mode. While atoms in other momentum modes coupled by the lattice are easily distinguished, atoms that have undergone scattering events into a smeared-out background distribution are not always well-separated. Thus, to extract the return fraction we fit the axial atomic distribution around the zero-momentum mode with a pair of Gaussians of varying width. The narrower Gaussian accounts for atoms remaining in the zero-momentum condensate after expansion, while the broader Gaussian measures the atoms that have been collisionally ejected from the condensate [39, 40]. In Fig. 3b, we show the fraction of atoms remaining in the narrow Gaussian and use this quantity as an estimate of the Loschmidt fidelity. Scattering during the expansion means that this necessarily underestimates the true fidelity, a possibility further addressed in supplementary section 6.

Noninteracting QKR numerics. One-dimensional simulations of the noninteracting kicked rotor problem for comparison with experimental data are executed in two ways. We either perform a split-step Fourier integration of the QKR Hamiltonian (2) (ignoring the transverse distribution $I(x, y)$) to model the finite-width pulse shapes, or iterate the QKR Floquet map described in the main text. The simulations are typically performed with periodic boundary conditions over a single lattice site (except when modeling the TOF readout; see supplementary section 4). We perform a Gaussian sampling of quasimomenta with standard deviation $\sim 0.1k_L$, in rough accordance with the measured BEC temperature of around 10-15 nK. For simulation of the stochastic kicking protocol, we use the same techniques and additionally average over 100 different realizations of the fluctuations (note this is slightly different than in the experiment where a

single kick period disorder realization is used).

Supplementary Files

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