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Supply Chain Networking Models under Fuzzy Uncertainty

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Abstract— In decision-making model, the techniques of numerical analysis have been widely adopted. It is rare for someone to solve a linear program by hand — except perhaps in a class-room. Large-scale simulations would be all but impossible without the aid of a computer. For many people, numerical techniques have superseded analytic techniques as a tool for solving mathematical problems. This paper proposed Generalized LU-Exponential Trapezoidal Fuzzy Number and their ranking based on numerical integration. In this ranking method, the values are calculated with left and right spreads at some α –level of generalized LU-exponential trapezoidal fuzzy numbers and Weddle’s rule for numerical integration. To illustrate the proposed methods, a fuzzy four dimensional transportation problem (FDTP) is proposed and solved. This ranking approach is very simple and useful for the real life inequality based decision making problems.

Keywords— Generalized LU-Exponential Trapezoidal Fuzzy Number, Ranking Method, Numerical Integration, Fuzzy FDTP.

1. INTRODUCTION

Numerical analysis uses computation as a tool to investigate mathematical models. At its most basic, this might mean computing an answer, such as the optimal value of a linear program. Beyond this, one might want error estimates or sensitivity. One might even wish to analyze the effects of randomness in the data. Numerical analysis can also be used as an experimental tool to reveal properties of models that may be inaccessible by analytic means. Numerical integration is the approximate computation of integral using numerical techniques. The numerical computation of an integral is sometimes called quadrature. There are a wide range of methods available for numerical integration. The most straight forward numerical integration technique uses the Newton-Cotes formulas (also called quadrature formulas), which approximate a function tabulated at a sequence of regularly spaced intervals by various degree polynomials. If the end points are tabulated, then the 2-, 3- and 6-point formulas are called the trapezoidal rule, Simpson's rule and Weddle’s rule respectively.

Fuzzy set theory was first proposed by Zadeh [1] in 1965, is a powerful tool to deal with real life situations with uncertainties. Dubois and Prade [3] in 1987 defined any of the fuzzy number as a fuzzy subset of the real line. The comparison of two fuzzy numbers is tedious to compute since fuzzy numbers are represented by possibility distribution. Ranking function is an efficient tool for ordering fuzzy numbers. Thus, ranking of fuzzy numbers is one of the most important parts in fuzzy set theory as well as decision making problem in fuzzy environment. Yager [2] in 1981 proposed a procedure for ordering fuzzy subsets of the unit interval. Liou and Wang [5] in 1992 introduced a method of ranking of fuzzy numbers with integral value. Heilpern [4] in 1997 proposed the expected value of a fuzzy number and Heilpern [6] in 1997 present the theoretical background of

fuzzy numbers connected with the possibility and Dempster-Shafer theories. A ranking method based on the preference function which measures the fuzzy numbers point by point was proposed by Modarres and Nezhad [7] in 2001. Wang and Lee [9] proposed a ranking fuzzy numbers with centroid and original points. Abbasbandy and Hajjari [10] introduced a new approach for ranking of trapezoidal fuzzy numbers based on the left and right spreads at some α –levels of trapezoidal fuzzy numbers. Kumar and Gupta [11] in 2011 proposed two new methods for solving fuzzy assignment problems and fuzzy travelling salesman problems with different membership functions. Kumar et al. [12] in 2011 introduced ranking non-normal p-norm trapezoidal fuzzy numbers. Saeidifar [13] proposes a new ranking method for fuzzy numbers, which uses a defuzzification of fuzzy numbers and a weighting function. Kaur and Kumar [14] in 2012 proposed a new approach for solving fuzzy transportation problems using generalized trapezoidal fuzzy numbers. Xu et al. [15] in 2012 introduced modified ranking generalized fuzzy numbers. Vincent et al. [16] in 2013 proposed ranking generalized fuzzy numbers in fuzzy decision making based on the left and right transfer coefficients and areas. Zhang et al. [17] in 2014 introduced a new method for ranking fuzzy numbers based on a fuzzy probabilistic preference relation. Rezvani [18] in 2015 proposed ranking generalized exponential trapezoidal fuzzy numbers based on variance.

The paper is developed in such a way that the cargos are transported from sources to destination with different mode of transport via different routes i.e., four dimensional concepts are taken into account to formulate the model. The 2D transportation problem/classical transportation problem proposed to find the minimum transportation cost of a single commodity or multi commodities from a group of supply centers (sources) $i = 1, 2, \dots, M$ to a group of receiving centers (destinations) $j = 1, 2, \dots, N$. Each source has a certain supply and each destination has a certain demand. Actually, the transportation system runs with different mode of transport like air, train, truck etc. Keeping this concept in mind the extended version of 2D transportation problem i.e., the solid transportation problem had been developed by Haley [27] in 1962 where the cargos are transported from sources to destination via different mode of transports.

Figure-1: 2D transportation problem/classical transportation problem

Figure-2: 3D transportation problem/Solid transportation problem

Figure-3: 4D transportation problem

2. Literature Review, Research gap and contributions:

The classical transportation problem (TP) was first developed by Hitchcock [19] in 1941. The solid transportation problem (STP) was put forward by Shell [20] in 1955. Chen and Chen [8] in 2007 discussed the fuzzy risk analysis on the ranking of generalized trapezoidal fuzzy numbers. Bai and Liu [26] in 2016 discussed robust optimization of supply chain network design in fuzzy decision system. Baidya et al. [23] in 2016 solved multi-stage multi-objective solid transportation problem for disaster response operation with type-2 triangular fuzzy variables. Fathian et al. [24] in 2018 proposed location and transportation planning in supply chains under uncertainty and congestion by using an improved electromagnetism-like algorithm. Khorshidian et al. [25] in 2019 developed an intelligent truck scheduling and transportation planning optimization model for product portfolio in a cross-dock. Baidya et al. [31] studied an STP with breakable items with hybrid and fuzzy safety factors using LINGO and Genetic Algorithm. Das et al. [31] in 2016 developed a breakable multi-item multi

stage solid transportation problem under budget with gaussian type-2 fuzzy parameters. Sengupta et al. [31] in 2018 proposed a gamma type-2 defuzzification method for solving a solid transportation problem considering carbon emission. Roy and Midya [30] in 2019 developed multi-objective fixed-charge solid transportation problem with product blending under intuitionistic fuzzy environment. Mishra and Kumar [32] in 2020 proposed JMD method for transforming an unbalanced fully intuitionistic fuzzy transportation problem into a balanced fully intuitionistic fuzzy transportation problem. Baykasoğlu and Subulan [33] in 2019 developed a direct solution approach based on constrained fuzzy arithmetic and metaheuristic for fuzzy transportation problems. Bera and Mondal [34] in 2020 proposed Credit linked two-stage multi-objective transportation problem in rough and bi-rough environments.

This paper defines generalized LU-exponential trapezoidal fuzzy numbers and their ranking based on the approach proposed by Kumar et al. [12] in 2011 and numerical integration. The approach proposed by Kumar et al. [12] was developed with respect to ranking function $I_T^\alpha(\tilde{\xi}): F(R) \rightarrow R$, (where $F(R)$, a set of fuzzy numbers is defined on set of real numbers and $\alpha \in [0,1]$ is degree of optimism, which maps each fuzzy number into the real line, where a natural order exists) to compare both normal and non-normal p-norm trapezoidal fuzzy numbers. In this proposed ranking method, the most popular quadrature formula i.e., Weddle's rule and left and right spreads at some α –level of generalized LU-exponential trapezoidal fuzzy numbers are considered to find the ranking of generalized LU-exponential trapezoidal fuzzy numbers with appropriate membership function instead of definite integration used by Kumar et al. [12] to find ranking function $I_T^\alpha(\tilde{\xi})$. Looking the difficulty to handle the integration part of left and right spreads at some α –level of generalized LU-exponential trapezoidal fuzzy numbers, numerical integration is taken into account in place of direct integration to calculate the ranking of generalized LU-exponential trapezoidal fuzzy numbers.

Table 1: Summary of literature review for transportation problem

Authors' Name	Year of Publication	Type of TP (dimension)	Item quality	Type of Environment
Hitchcock [19]	1941	Two	General	Crisp
Shell [20]	1955	Three	General	Crisp
Haley [27]	1962	Three	General	Crisp
Kaur and Kumar [14]	2012	Two	General	Fuzzy
Baidya et al. [31]	2015	Three	Breakable	Hybrid and Fuzzy
Das et al. [29]	2016	Three	Breakable	Type-2 fuzzy
Halder et al. [21]	2017	Four	General	Type-2 fuzzy
Bera et al. [28]	2018	Four	General	Rough
Sengupta et al. [31]	2018	Three	General	Type-2 fuzzy
Roy and Midya [30]	2019	Three	General	Fuzzy
Aktar et al. [22]	2020	Four	Breakable	Type-2 fuzzy-random
Baykasoğlu and Subulan [33]	2019	Two	General	Fuzzy
Bera and Mondal [34]	2020	Two	General	Rough and bi-rough fuzzy
Mishra and Kumar [32]	2020	Two	General	Fuzzy

Present investigation	2021	Four	Breakable	Generalized LU-exponential trapezoidal fuzzy
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3. Motivation, Problem Description and Methodologies

3.1. Motivation:

Roadways is known to be the most reliable modes of transport; facilitating easy and fast movement of traffic with door to door services, also having comparatively lesser construction costs than railways. Now-a-days, sources and destinations are connected by different routes. But as we break through reality we all know that not all roads are as smooth as they sound. Certain roads exist which contain potholes, cracks, or uneven surfaces that are unsafe to drive on; such roads are termed as ‘Bad Roads’. Most of the good and safe roads in India or anywhere else in the world are turned into bad roads because of being left under construction or due to natural calamities like floods, landslides or even because of simple causes like heavy rains and accidents. Even though we may ignore such roads and drive past them and need to acknowledge the fact that they are very hazardous for all human beings to transport the commodities from sources to destinations. Every day we read the paper, and we see that there are at least two to three articles about accidents caused due to the extreme conditions of roads.

Figure-4: Images of the bad roads across India

In recent decades, very few researchers Halder et al. [21] in 2017 and Aktar et al. [22] in 2020 have used this assumption and STP becomes FDTP with source, destination, conveyance and path. This has motivated to formulate and solve multi-item four dimensional solid transportation problems in both crisp and fuzzy environments.

3.2. Problems Description and Methodologies:

This paper proposed generalized LU-exponential trapezoidal fuzzy number and their ranking based on numerical integration and also two four dimensional solid transportation problems have been developed in deterministic and fuzzy environment respectively. Considering transportation paths, distances, availabilities, demands, conveyances capacities, formulate the solid transportation models under crisp and fuzzy environments to transport q^{th} item through p^{th} path from i^{th} source to j^{th} destination using k^{th} conveyances. The fuzzy uncertainty has been removed by using the proposed ranking method i.e., using defuzzification technique for generalized LU-exponential trapezoidal fuzzy number. The models are illustrated numerically with LINGO.13.0 optimization software and the obtained optimal results are compared.

3. FUNDAMENTAL CONCEPTS

3.1. Generalized Exponential Trapezoidal Fuzzy Numbers: Generally, a generalized fuzzy number $\tilde{\xi}$ is described as any fuzzy subset of the real line R , whose membership function $\mu_{\tilde{\xi}}$ satisfies the following conditions,

- (i) $\mu_{\tilde{\xi}}$ is a continuous mapping from R to the closed interval $[0,1]$,
- (ii) $\mu_{\tilde{\xi}}(x) = 0, -\infty < x \leq \xi_1$,
- (iii) $\mu_{\tilde{\xi}}(x) = L(x)$ is strictly increasing on $\xi_1 \leq x \leq \xi_2$,
- (iv) $\mu_{\tilde{\xi}}(x) = w, \xi_2 \leq x \leq \xi_3$,
- (v) $\mu_{\tilde{\xi}}(x) = R(x)$ is strictly decreasing on $\xi_3 \leq x \leq \xi_4$,
- (vi) $\mu_{\tilde{\xi}}(x) = 0, \xi_4 \leq x < \infty$

Where $0 < w \leq 1$ and ξ_1, ξ_2, ξ_3 and ξ_4 are real numbers. We call this type of generalized fuzzy number a trapezoidal fuzzy number, and it is denoted by $\tilde{\xi} = (\xi_1, \xi_2, \xi_3, \xi_4; w)$. When $w = 1$, this type of generalized fuzzy number is called normal fuzzy number and is represented by $\tilde{\xi} = (\xi_1, \xi_2, \xi_3, \xi_4)$.

However, these fuzzy numbers always have a fix range as $[\xi_3, \xi_4]$. Here, we define its general form as follows:

$$\mu_{\tilde{\xi}}(x) = \begin{cases} 0 & \text{if } x < \xi_1 \\ we^{-\frac{(\xi_2-x)}{(\xi_2-\xi_1)}} & \text{if } \xi_1 \leq x \leq \xi_2 \\ w & \text{if } \xi_2 \leq x \leq \xi_3 \\ we^{-\frac{(x-\xi_3)}{(\xi_4-\xi_3)}} & \text{if } \xi_3 \leq x \leq \xi_4 \\ 0 & \text{if } \xi_4 < x \end{cases}$$

Where $0 < w \leq 1$, ξ_1, ξ_2 are real numbers and ξ_3, ξ_4 are *+ve* real numbers. This type of fuzzy number is known as generalized exponential fuzzy number and denoted by $\tilde{\xi} = (\xi_1, \xi_2, \xi_3, \xi_4; w)_E$.

3.2. Numerical Integration or Quadrature

The problem of numerical integration is to evaluate $I = \int_a^b f(x) dx$. The problem arises when the integration can't be carried out either due to the fact that $f(x)$ is not integrable or when $f(x)$ is known only at a finite number of points. So many methods are exists in the literature for numerical integration. Newton-Cotes quadrature formulae is one of the most popular and effective formulae for numerical integration or quadrature.

Let $y = f(x)$ be a real valued function defined in $[a, b]$. Also we assume that the analytical form of $f(x)$ is known but the function $f(x)$ is not integrable in $[a, b]$. Let $I = \int_a^b f(x) dx$ where $y_i = f(x_i), i = 0, 1, 2, \dots, n$. Also let the interval $[a, b]$ be divided into n equal sub-intervals such that $a = x_0 < x_1 < x_2 < \dots < x_n = b$ and $x_i = x_0 + ih$ ($i = 0, 1, 2, \dots, n$); h is the spacing. Hence integral I becomes $I = \int_{x_0}^{x_n} f(x) dx$. Now by Newton's forward difference interpolation, we have

$$y = f(x) = y_0 + s\Delta y_0 + \frac{s(s-1)}{2!}\Delta^2 y_0 + \dots, \text{ where } x = x_0 + sh.$$

$$\begin{aligned} \therefore I &= \int_{x_0}^{x_n} f(x) dx = \int_{x_0}^{x_n} \left[y_0 + s\Delta y_0 + \frac{s(s-1)}{2!}\Delta^2 y_0 + \dots \right] dx \\ &= nh \left[y_0 + \frac{n}{2}\Delta y_0 + \frac{n(2n-3)}{12}\Delta^2 y_0 + \frac{n(n-2)^2}{24}\Delta^3 y_0 + \dots \right] \end{aligned} \quad (1)$$

The above formula (1) is known as Newton-Cotes quadrature formulae of numerical integration or quadrature. Substituting $n = 6$ in equation (1) for the interval $[x_0, x_6]$ and neglecting higher order differences, we have

$$\int_{x_0}^{x_6} f(x) dx = \frac{3h}{10} [y_0 + 5y_1 + y_2 + 6y_3 + y_4 + 5y_5 + y_6] \quad (2)$$

For the interval $[x_6, x_{12}]$, we have

$$\int_{x_6}^{x_{12}} f(x) dx = \frac{3h}{10} [y_6 + 5y_7 + y_8 + 6y_9 + y_{10} + 5y_{11} + y_{12}] \quad (3)$$

Similarly, for the interval $[x_{n-6}, x_n]$, we have

$$\int_{x_{n-6}}^{x_n} f(x) dx = \frac{3h}{10} [y_{n-6} + 5y_{n-5} + y_{n-4} + 6y_{n-3} + y_{n-2} + 5y_{n-1} + y_n] \quad (4)$$

Adding all these integrals (2), (3) and (4), we have

$$\begin{aligned} \int_{x_0}^{x_n} f(x) dx &= \int_{x_0}^{x_6} f(x) dx + \int_{x_6}^{x_{12}} f(x) dx + \cdots + \int_{x_{n-6}}^{x_n} f(x) dx \\ &= \frac{3h}{10} [(y_0 + y_n) + 5(y_1 + y_7 + \cdots + y_{n-5}) + (y_2 + y_8 + \cdots + y_{n-4}) + 6(y_3 + y_9 + \cdots + y_{n-3}) + (y_4 + y_{10} + \cdots + y_{n-2}) + 5(y_5 + y_{11} + \cdots + y_{n-1}) + 2(y_6 + y_{12} + \cdots + y_{n-6})] \end{aligned} \quad (5)$$

The above formula (5) is known as Weddle's rule and is used when n is a multiple of 6. The Weddle's rule is very effective to find the numerical integration than Trapezoidal rule, Simpson's $\frac{1}{3}$ rule, Simpson's $\frac{3}{8}$ rule, Boole's rule because the number of sub-intervals ($n = 6$) in Weddle's rule is greater than the others.

4. Proposed Generalized LU-Exponential Trapezoidal Fuzzy Number (GLUETrFN), Arithmetic Operations and Ranking

Definition-4.1: A fuzzy number $\tilde{\xi} = (\xi_1, \xi_2, \xi_3, \xi_4; w)_{LU}$ is said to be a Generalized LU-exponential trapezoidal fuzzy number if its membership function is as follows:

$$\mu_{\tilde{\xi}}(x) = \begin{cases} 0 & \text{if } x < \xi_1 \\ w \frac{x}{x + e^{\frac{(x-\xi_2)}{(\xi_1-\xi_2)}}} = L(x) & \text{if } \xi_1 \leq x \leq \xi_2 \\ w & \text{if } \xi_2 \leq x \leq \xi_3 \\ w \frac{x}{x + e^{\frac{(x-\xi_3)}{(\xi_4-\xi_3)}}} = U(x) & \text{if } \xi_3 \leq x \leq \xi_4 \\ 0 & \text{if } \xi_4 < x \end{cases}$$

where all the characteristics of generalized LU-exponential trapezoidal fuzzy number are same as generalized exponential trapezoidal fuzzy numbers defined in section-2.2.

Definition-4.2: Let $\tilde{\xi} = (\xi_1, \xi_2, \xi_3, \xi_4; w_1)_{LU}$ and $\tilde{\eta} = (\eta_1, \eta_2, \eta_3, \eta_4; w_2)_{LU}$ are two GLUETrFNs and $\rho \in \mathbb{R}$.

Then the arithmetic operations on $\tilde{\xi}$ and $\tilde{\eta}$ are given by

- (i) $\rho \geq 0, \rho \tilde{\xi} = (\rho \xi_1, \rho \xi_2, \rho \xi_3, \rho \xi_4; w_1)_{LU}$,
- (ii) $\rho < 0, \rho \tilde{\xi} = (\rho \xi_4, \rho \xi_3, \rho \xi_2, \rho \xi_1; w_1)_{LU}$
- (iii) $\tilde{\xi} + \tilde{\eta} = (\xi_1 + \eta_1, \xi_2 + \eta_2, \xi_3 + \eta_3, \xi_4 + \eta_4; w = \min\{w_1, w_2\})$,

$$(iv) \tilde{\xi} - \tilde{\eta} = (\xi_1 - \eta_4, \xi_2 - \eta_3, \xi_3 - \eta_2, \xi_4 - \eta_1; w = \min\{w_1, w_2\}),$$

(v) $\tilde{\xi} \otimes \tilde{\eta}$ The multiplication between $\tilde{\xi}$ and $\tilde{\eta}$ is denoted by $\tilde{\xi} \times \tilde{\eta}$ and is defined as

$$(vi) \tilde{\xi} \times \tilde{\eta} = (\xi_1 \eta_1, \xi_2 \eta_2, \xi_3 \eta_3, \xi_4 \eta_4, \min(w_1, w_2)) \text{ where } \xi_1, \xi_2, \xi_3, \xi_4 \geq 0 \text{ and } \eta_1, \eta_2, \eta_3, \eta_4 \geq 0.$$

.

Definition-4.3: The ranking function $I_T^\alpha(\tilde{\xi}): F(R) \rightarrow R$, where $F(R)$ is a set of real fuzzy numbers and the degree of optimism $\alpha \in [0,1]$, with a natural order which maps each fuzzy number into the real number.

Let $\tilde{\xi} = (\xi_1, \xi_2, \xi_3, \xi_4; w_1)_{LU}$ and $\tilde{\eta} = (\eta_1, \eta_2, \eta_3, \eta_4; w_2)_{LU}$ are two generalized LU-exponential trapezoidal fuzzy numbers, then

(i) $\tilde{\xi} \succsim \tilde{\eta}$ if and only if $I_T^\alpha(\tilde{\xi}) > I_T^\alpha(\tilde{\eta})$, (ii) $\tilde{\xi} \precsim \tilde{\eta}$ if and only if $I_T^\alpha(\tilde{\xi}) < I_T^\alpha(\tilde{\eta})$, (iii) $\tilde{\xi} \cong \tilde{\eta}$ if and only if $I_T^\alpha(\tilde{\xi}) = I_T^\alpha(\tilde{\eta})$.

Remark-4.1: For a pessimistic decision maker $\alpha = 0$, for an optimistic decision maker $\alpha = 1$ and for a moderate decision maker $\alpha = 0.5$.

4.1. Method to find the values of $I_T^\alpha(\tilde{\xi})$ and $I_T^\alpha(\tilde{\eta})$: Let $\tilde{\xi} = (\xi_1, \xi_2, \xi_3, \xi_4; w_1)_{LU}$ and $\tilde{\eta} = (\eta_1, \eta_2, \eta_3, \eta_4; w_2)_{LU}$ are two generalized LU-exponential trapezoidal fuzzy numbers then use the following steps to find the values of $I_T^\alpha(\tilde{\xi})$ and $I_T^\alpha(\tilde{\eta})$:

Step-1: Find $w = \min\{w_1, w_2\}$.

Step-2: $I_T^\alpha(\tilde{\xi}) = \frac{1}{2} \int_0^w [L^{-1}(x) + R^{-1}(x)] dx$, where $L^{-1}(x) = (1 - \alpha) \left[\xi_2 + (\xi_1 - \xi_2) \frac{x}{x + e^{\frac{x}{w}}} \right]$ and $R^{-1}(x) = \alpha \left[\xi_3 + (\xi_4 - \xi_3) \frac{x}{x + e^{\frac{x}{w}}} \right]$.

$$\begin{aligned} \Rightarrow I_T^\alpha(\tilde{\xi}) &= \frac{1}{2} \int_0^w \left[(1 - \alpha) \left[\xi_2 + (\xi_1 - \xi_2) \frac{x}{x + e^{\frac{x}{w}}} \right] + \left[\xi_3 + (\xi_4 - \xi_3) \frac{x}{x + e^{\frac{x}{w}}} \right] \right] dx \\ &= \frac{1}{2} \int_0^w \left[(1 - \alpha) \xi_2 + (1 - \alpha)(\xi_1 - \xi_2) \frac{x}{x + e^{\frac{x}{w}}} + \alpha \xi_3 + \alpha(\xi_4 - \xi_3) \frac{x}{x + e^{\frac{x}{w}}} \right] dx \\ &= \frac{1}{2} \int_0^w \left[((1 - \alpha)(\xi_1 - \xi_2) + \alpha(\xi_4 - \xi_3)) \frac{x}{x + e^{\frac{x}{w}}} + (1 - \alpha)\xi_2 + \alpha\xi_3 \right] dx, \\ &= \frac{((1 - \alpha)(\xi_1 - \xi_2) + \alpha(\xi_4 - \xi_3))}{2} \int_0^w \frac{x}{x + e^{\frac{x}{w}}} dx + \frac{((1 - \alpha)\xi_2 + \alpha\xi_3)}{2} \int_0^w dx \\ &= \frac{((1 - \alpha)(\xi_1 - \xi_2) + \alpha(\xi_4 - \xi_3))}{2} I_1 + \frac{((1 - \alpha)\xi_2 + \alpha\xi_3)}{2} I_2, \end{aligned} \tag{6}$$

Where $I_1 = \int_0^w \frac{x}{x + e^{\frac{x}{w}}} dx$ and $I_2 = \int_0^w dx$.

Here, we can find the value of I_2 easily and which is w but the value of I_1 is difficult because the integrand is a non-integrable function. To find the value of I_1 we may apply Weddle's rule (Newton-Cotes Quadrature formulae) for numerical integration or Quadrature.

Let $y = f(x) = \frac{x}{x+e^w}, \forall x \in [0, w], a = x_0 = 0, b = x_n = w, n = 12$ and $h = \frac{x_n - x_0}{n} = \frac{w-0}{12} = \frac{w}{12}$. Let us compute the integral I_1 using the following table of values for x and y :

Table-2: Values for Arguments (x) and Entries (y)

x	$x_0 = 0$	$x_1 = \frac{w}{12}$	$x_2 = \frac{w}{6}$	$x_3 = \frac{w}{4}$	$x_4 = \frac{w}{3}$
y	$y_0 = 0$	$y_1 = \frac{w}{w + 12e^{\frac{1}{12}}}$	$y_2 = \frac{w}{w + 6e^{\frac{1}{6}}}$	$y_3 = \frac{w}{w + 4e^{\frac{1}{4}}}$	$y_4 = \frac{w}{w + 3e^{\frac{1}{3}}}$
x	$x_5 = \frac{5w}{12}$	$x_6 = \frac{w}{2}$	$x_7 = \frac{7w}{12}$	$x_8 = \frac{2w}{3}$	
y	$y_5 = \frac{5w}{5w + 12e^{\frac{5}{12}}}$	$y_6 = \frac{w}{w + 2e^{\frac{1}{2}}}$	$y_7 = \frac{7w}{7w + 12e^{\frac{7}{12}}}$	$y_8 = \frac{2w}{2w + 3e^{\frac{2}{3}}}$	
x	$x_9 = \frac{3w}{4}$	$x_{10} = \frac{5w}{6}$	$x_{11} = \frac{11w}{12}$	$x_{12} = w$	
y	$y_9 = \frac{3w}{3w + 4e^{\frac{3}{4}}}$	$y_{10} = \frac{5w}{5w + 6e^{\frac{5}{6}}}$	$y_{11} = \frac{11w}{11w + 12e^{\frac{11}{12}}}$	$y_{12} = \frac{w}{w + e^1}$	

Now, using Weddle's rule we can write

$$\begin{aligned}
 I_1 &= \int_0^w \frac{x}{x+e^w} dx = \int_0^w f(x) dx = \frac{3h}{10} [(y_0 + y_{12}) + 5(y_1 + y_7) + (y_2 + y_8) + 6(y_3 + y_9) + (y_4 + y_{10}) + \\
 &5(y_5 + y_{11}) + 2(y_6)] \\
 &= \frac{w}{40} [(0 + \frac{w}{w+e^1}) + 5(\frac{w}{w+12e^{\frac{1}{12}}} + \frac{7w}{7w+12e^{\frac{7}{12}}}) + (\frac{w}{w+6e^{\frac{1}{6}}} + \frac{2w}{2w+3e^{\frac{2}{3}}}) + 6(\frac{w}{w+4e^{\frac{1}{4}}} + \frac{3w}{3w+4e^{\frac{3}{4}}}) + (\frac{w}{w+3e^{\frac{1}{3}}} + \frac{5w}{5w+6e^{\frac{5}{6}}}) + \\
 &5(\frac{5w}{5w+12e^{\frac{5}{12}}} + \frac{11w}{11w+12e^{\frac{11}{12}}}) + 2(\frac{w}{w+2e^{\frac{1}{2}}})] = \frac{w^2}{40} (\frac{w}{w+e^1} + 5(\frac{1}{w+12e^{\frac{1}{12}}} + \frac{7}{7w+12e^{\frac{7}{12}}}) + (\frac{1}{w+6e^{\frac{1}{6}}} + \frac{2}{2w+3e^{\frac{2}{3}}}) + 6(\frac{1}{w+4e^{\frac{1}{4}}} + \frac{3}{3w+4e^{\frac{3}{4}}}) + (\frac{1}{w+3e^{\frac{1}{3}}} + \frac{5}{5w+6e^{\frac{5}{6}}}) + 5(\frac{5}{5w+12e^{\frac{5}{12}}} + \frac{11}{11w+12e^{\frac{11}{12}}}) + \frac{2}{w+2e^{\frac{1}{2}}})
 \end{aligned}$$

Now putting the values of I_1 and I_2 in (6), we get

$$\begin{aligned}
 \Rightarrow I_T^\alpha(\xi) &= \frac{((1-\alpha)(\xi_1 - \xi_2) + \alpha(\xi_4 - \xi_3))}{2} (\frac{w^2}{40} [\frac{w}{w+e^1} + 5(\frac{1}{w+12e^{\frac{1}{12}}} + \frac{7}{7w+12e^{\frac{7}{12}}}) + (\frac{1}{w+6e^{\frac{1}{6}}} + \frac{2}{2w+3e^{\frac{2}{3}}}) + 6(\frac{1}{w+4e^{\frac{1}{4}}} + \frac{3}{3w+4e^{\frac{3}{4}}}) + (\frac{1}{w+3e^{\frac{1}{3}}} + \frac{5}{5w+6e^{\frac{5}{6}}}) + 5(\frac{5}{5w+12e^{\frac{5}{12}}} + \frac{11}{11w+12e^{\frac{11}{12}}}) + \frac{2}{w+2e^{\frac{1}{2}}})] + \frac{((1-\alpha)\xi_2 + \alpha\xi_3)}{2} w
 \end{aligned} \tag{7}$$

Similarly,

$$\begin{aligned}
 I_T^\alpha(\eta) &= \frac{((1-\alpha)(\eta_1 - \eta_2) + \alpha(\eta_4 - \eta_3))}{2} (\frac{w^2}{40} [\frac{w}{w+e^1} + 5(\frac{1}{w+12e^{\frac{1}{12}}} + \frac{7}{7w+12e^{\frac{7}{12}}}) + (\frac{1}{w+6e^{\frac{1}{6}}} + \frac{2}{2w+3e^{\frac{2}{3}}}) + 6(\frac{1}{w+4e^{\frac{1}{4}}} + \frac{3}{3w+4e^{\frac{3}{4}}}) + (\frac{1}{w+3e^{\frac{1}{3}}} + \frac{5}{5w+6e^{\frac{5}{6}}}) + 5(\frac{5}{5w+12e^{\frac{5}{12}}} + \frac{11}{11w+12e^{\frac{11}{12}}}) + \frac{2}{w+2e^{\frac{1}{2}}})] + \frac{((1-\alpha)\eta_2 + \alpha\eta_3)}{2} w
 \end{aligned} \tag{8}$$

Remark-4.2: If $\tilde{\xi} = (\xi_1, \xi_2, \xi_3, \xi_4; w_1)_{LU}$ be a generalized LU-exponential trapezoidal fuzzy numbers then the ranking value of the fuzzy variable $\tilde{\xi}$ is denoted by $I_T^\alpha(\tilde{\xi})$ and defined as

$$I_T^\alpha(\tilde{\xi}) = \left(\frac{((1-\alpha)(\xi_1-\xi_2)+\alpha(\xi_4-\xi_3))}{2} \left(\frac{w^2}{40} \left[\frac{w}{w+e^1} + 5 \left(\frac{1}{w+12e^{\frac{1}{12}}} + \frac{7}{7w+12e^{\frac{1}{12}}} \right) + \left(\frac{1}{w+6e^{\frac{1}{6}}} + \frac{2}{2w+3e^{\frac{2}{3}}} \right) + 6 \left(\frac{1}{w+4e^{\frac{1}{4}}} + \frac{3}{3w+4e^{\frac{3}{4}}} \right) + \left(\frac{1}{w+3e^{\frac{1}{3}}} + \frac{5}{5w+6e^{\frac{5}{6}}} \right) + 5 \left(\frac{5}{5w+12e^{\frac{5}{12}}} + \frac{11}{11w+12e^{\frac{11}{12}}} \right) + \frac{2}{w+2e^{\frac{2}{2}}} \right] \right) + \frac{((1-\alpha)\xi_2+\alpha\xi_3)}{2} w \right)$$

Let us consider,

$$P_w = \frac{w^2}{40} \left[\frac{w}{w+e^1} + 5 \left(\frac{1}{w+12e^{\frac{1}{12}}} + \frac{7}{7w+12e^{\frac{1}{12}}} \right) + \left(\frac{1}{w+6e^{\frac{1}{6}}} + \frac{2}{2w+3e^{\frac{2}{3}}} \right) + 6 \left(\frac{1}{w+4e^{\frac{1}{4}}} + \frac{3}{3w+4e^{\frac{3}{4}}} \right) + \left(\frac{1}{w+3e^{\frac{1}{3}}} + \frac{5}{5w+6e^{\frac{5}{6}}} \right) + 5 \left(\frac{5}{5w+12e^{\frac{5}{12}}} + \frac{11}{11w+12e^{\frac{11}{12}}} \right) + \frac{2}{w+2e^{\frac{2}{2}}} \right],$$

$$\text{Then } I_T^\alpha(\tilde{\xi}) \text{ becomes } I_T^\alpha(\tilde{\xi}) = \frac{((1-\alpha)(\xi_1-\xi_2)+\alpha(\xi_4-\xi_3))}{2} P_w + \frac{((1-\alpha)\xi_2+\alpha\xi_3)}{2} w.$$

5. EXAMPLES

In this section, six sets of generalized LU-exponential trapezoidal fuzzy numbers are compared using the proposed approach.

Example-5.1: Let $\tilde{\xi} = (0.2, 0.4, 0.6, 0.8; 0.35)_{LU}$ and $\tilde{\eta} = (0.1, 0.2, 0.3, 0.4; 0.7)_{LU}$ be two generalized LU-exponential trapezoidal fuzzy numbers

Here $w = \min\{0.35, 0.7\} = 0.35$. Also assume that $\alpha = 0.25$.

Therefore, (7) and (8) respectively becomes,

$$I_T^\alpha(\tilde{\xi}) = \frac{((1-0.25)(0.2-0.4)+0.25(0.8-0.6))}{2} \left(\frac{(0.35)^2}{40} \left[\frac{0.35}{0.35+e^1} + 5 \left(\frac{1}{0.35+12e^{\frac{1}{12}}} \left(\frac{1}{0.35+6e^{\frac{1}{6}}} + \frac{2}{2 \times 0.35+3e^{\frac{2}{3}}} \right) + 6 \left(\frac{1}{0.35+4e^{\frac{1}{4}}} + \frac{7}{7 \times 0.35+12e^{\frac{1}{12}}} \right) + \frac{3}{3 \times 0.35+4e^{\frac{3}{4}}} \right) + \left(\frac{1}{0.35+3e^{\frac{1}{3}}} + \frac{5}{5 \times 0.35+6e^{\frac{5}{6}}} \right) + 5 \left(\frac{5}{5 \times 0.35+12e^{\frac{5}{12}}} + \frac{11}{11 \times 0.35+12e^{\frac{11}{12}}} \right) + \frac{2}{0.35+2e^{\frac{2}{2}}} \right] \right) + \frac{((1-0.25)0.4+0.25 \times 0.6)}{2} \times 0.35$$

$$= -0.000153[0.114 + 5(0.0747 + 0.2922) + (0.1346 + 0.3056) + 6(0.1823 + 0.3152) + (0.2204 + 0.3215) + 5(0.2506 + 0.3250) + 0.5482] + 0.07875$$

$$= -0.000153[0.114 + 1.8345 + 0.4402 + 2.985 + 0.5419 + 2.878 + 0.5482] + 0.07875$$

$$= -0.000153 \times 9.3418 + 0.07875 = 0.07732$$

$$I_T^\alpha(\tilde{\eta}) = \frac{((1-0.25)(0.1-0.2)+0.25(0.4-0.3))}{2} \left(\frac{(0.35)^2}{40} \left[\frac{0.35}{0.35+e^1} + 5 \left(\frac{1}{0.35+12e^{\frac{1}{12}}} + \frac{7}{7 \times 0.35+12e^{\frac{1}{12}}} \right) + \left(\frac{1}{0.35+6e^{\frac{1}{6}}} + \frac{2}{2 \times 0.35+3e^{\frac{2}{3}}} \right) + 6 \left(\frac{1}{0.35+4e^{\frac{1}{4}}} + \frac{3}{3 \times 0.35+4e^{\frac{3}{4}}} \right) + \left(\frac{1}{0.35+3e^{\frac{1}{3}}} + \frac{5}{5 \times 0.35+6e^{\frac{5}{6}}} \right) + 5 \left(\frac{5}{5 \times 0.35+12e^{\frac{5}{12}}} + \frac{11}{11 \times 0.35+12e^{\frac{11}{12}}} \right) + \frac{2}{0.35+2e^{\frac{2}{2}}} \right] \right) + \frac{((1-0.25)0.2+0.25 \times 0.3)}{2} 0.25 = -0.000715 + 0.028125 = 0.0274$$

$$\therefore I_T^\alpha(\tilde{\xi}) = 0.07732 \succ 0.0274 = I_T^\alpha(\tilde{\eta}) \Rightarrow \tilde{\xi} > \tilde{\eta}.$$

Example-5.2: Let $\tilde{\xi} = (0.15, 0.25, 0.45, 0.55; 1)_{LU}$ and $\tilde{\eta} = (0.15, 0.35, 0.35, 0.55; 1)_{LU}$ be two generalized LU-exponential trapezoidal fuzzy numbers

Here $w = \min\{1, 1\} = 1$. Also assume that $\alpha = 0.6$.

Therefore, (7) and (8) respectively becomes,

$$I_T^\alpha(\tilde{\xi}) = 0.00025[0.269 + 1.584 + 0.379 + 2.5476 + 0.459 + 2.419 + 0.465] + 0.185 = 0.187$$

$$I_T^\alpha(\tilde{\eta}) = 0.001[0.269 + 1.584 + 0.379 + 2.5476 + 0.459 + 2.419 + 0.465] + 0.35 = 0.3581.$$

$$\therefore I_T^\alpha(\tilde{\xi}) = 0.187 \prec 0.3581 = I_T^\alpha(\tilde{\eta}) \Rightarrow \tilde{\xi} < \tilde{\eta}.$$

Example-5.3: Let $\tilde{\xi} = (0.15, 0.25, 0.45, 0.55; 1)_{LU}$ and $\tilde{\eta} = (1, 1, 1, 1; 1)_{LU}$ be two generalized LU-exponential trapezoidal fuzzy numbers

Here $w = \min\{1, 1\} = 1$. Also assume that $\alpha = 0.55$.

Therefore, (7) and (8) respectively becomes,

$$I_T^\alpha(\tilde{\xi}) = 0.000125[0.269 + 1.584 + 0.379 + 2.5476 + 0.459 + 2.419 + 0.465] + 0.18 = 0.1810$$

$$I_T^\alpha(\tilde{\eta}) = 0 + \frac{((1-0.55) \times 1 + 0.55 \times 1)}{2} 1 = 0.5$$

$$\therefore I_T^\alpha(\tilde{\xi}) = 0.1810 \lesssim 0.5 = I_T^\alpha(\tilde{\eta}) \Rightarrow \tilde{\xi} < \tilde{\eta}.$$

Example-5.4: Let $\tilde{\xi} = (-0.55, -0.35, -0.35, -0.15; 0.95)_{LU}$ and $\tilde{\eta} = (0.15, 0.35, 0.35, 0.55; 0.85)_{LU}$ be two generalized LU-exponential trapezoidal fuzzy numbers

Here $w = \min\{0.95, 0.85\} = 0.85$. Also assume that $\alpha = 0.95$.

Therefore, (7) and (8) respectively becomes,

$$I_T^\alpha(\tilde{\xi}) = 0.001626[0.2381 + 1.6347 + 0.3912 + 2.6358 + 0.4755 + 2.511 + 0.4822] + 0.14875 = -0.13514$$

$$I_T^\alpha(\tilde{\eta}) = 0.001626[0.2381 + 1.6347 + 0.3912 + 2.6358 + 0.4755 + 2.511 + 0.4822] + 0.14875 = 0.1623$$

$$\therefore I_T^\alpha(\tilde{\xi}) = -0.13514 \lesssim 0.1623 = I_T^\alpha(\tilde{\eta}) \Rightarrow \tilde{\xi} < \tilde{\eta}.$$

Example-5.5: Let $\tilde{\xi} = (0, 0.4, 0.5, 0.7; 0.6)_{LU}$, $\tilde{\eta} = (0.1, 0.4, 0.4, 0.8; 0.6)_{LU}$ and $\tilde{\tau} = (0.2, 0.7, 0.8, 0.9; 0.6)_{LU}$ be two generalized LU-exponential trapezoidal fuzzy numbers

Here $w = 0.6$. Also assume that $\alpha = 0.45$.

Therefore, (7) and (8) respectively becomes,

$$I_T^\alpha(\tilde{\xi}) = -0.000585 \times 8.8232 + 0.1335 = 0.1283$$

$$I_T^\alpha(\tilde{\eta}) = 0.0000675 \times 8.8232 + 0.1335 = 0.1341$$

$$I_T^\alpha(\tilde{\tau}) = -0.001035 \times 8.8232 + 0.2235 = 0.2144$$

$$\text{Here we see that } 0.1283 < 0.1341 < 0.2144 \Rightarrow I_T^\alpha(\tilde{\xi}) \lesssim I_T^\alpha(\tilde{\eta}) \lesssim I_T^\alpha(\tilde{\tau}) \Rightarrow \tilde{\xi} < \tilde{\eta} < \tilde{\tau}.$$

Example-5.6: Let $\tilde{\xi} = (0.11, 0.21, 0.41, 0.51; 0.25)_{LU}$ and $\tilde{\eta} = (-3, 0, 0, 3; 0.5)_{LU}$ be two generalized LU-exponential trapezoidal fuzzy numbers

Here $w = \min\{0.25, 0.5\} = 0.25$. Also assume that $\alpha = 0.81$.

Therefore, (7) and (8) respectively becomes,

$$I_T^\alpha(\tilde{\xi}) = 0.00004844 \times (0.3367 + 1.8812 + 0.4516 + 3.0669 + 0.5575 + 0.5636) + 0.0465 = 0.04697$$

$$I_T^\alpha(\tilde{\eta}) = 0.001453 \times 9.8219 + 0 = 0.01427$$

$$\therefore I_T^\alpha(\tilde{\xi}) = 0.04697 \gtrsim 0.01427 = I_T^\alpha(\tilde{\eta}) \Rightarrow \tilde{\xi} > \tilde{\eta}.$$

6. MATHEMATICAL MODEL FORMULATION:

6.1. Notations:

- ❖ M : number of sources ($i = 1, 2, \dots, M$)
- ❖ N : number of destinations ($j = 1, 2, \dots, N$)

- ❖ P : number of paths from each source to each destination ($p = 1, 2, \dots, P$)
- ❖ Q : number of different items ($q = 1, 2, \dots, Q$)
- ❖ K : number of conveyances ($k = 1, 2, \dots, K$)
- ❖ C_{ijkpq} : unit transportation cost per unit distance (km.) through p^{th} path for transportation of q^{th} item from i^{th} source to j^{th} destination using k^{th} conveyance (in rupees).
- ❖ A_{iq} : available quantity of q^{th} item at i^{th} source (in Kg).
- ❖ B_{jq} : required quantity of q^{th} item at j^{th} destination (in Kg).
- ❖ E_{pk} : capacity of k^{th} conveyance when going through p^{th} path (in Kg).
- ❖ ds_{ijp} : distance from i^{th} source to j^{th} destination through p^{th} path (in hundred of Km).
- ❖ y_{qk} : it is a binary auxiliary variable whose value is 1 if q^{th} item is transported from i^{th} source to j^{th} destination by k^{th} conveyance via p^{th} path and 0 otherwise, i.e. $y_{qk} = \begin{cases} 1 & \text{if } x_{ijkpq} > 0 \\ 0 & \text{otherwise} \end{cases}$.

Model-6.1.: Formulation of Breakable Crisp Multi-Item FDTP

Let us consider ‘M’ supply points (or sources), ‘N’ destination, K conveyances through P number of different paths connecting each sources to each destinations. Also we consider that Q be the number of items those are transported from plants to destination by different modes of conveyances. The crisp multi-item FDTP has been formulated as follows:

Objective function:

$$\text{Minimize } Z = \sum_{i=1}^M \sum_{j=1}^N \sum_{k=1}^K \sum_{p=1}^P \sum_{q=1}^Q C_{ijkpq} ds_{ijp} x_{ijkpq} \quad (1)$$

Constraints:

Plant capacity constraint: The sum of the number of units transported from the plant to the distribution centers by all vehicles must be less than or equal to availability of the plant. That is,

$$\sum_{j=1}^N \sum_{k=1}^K \sum_{p=1}^P x_{ijkpq} y_{qk} \leq A_{iq}, \text{ for } i = 1, 2, \dots, M, q = 1, 2, \dots, Q \quad (2)$$

Demand constraint: The sum of the number of units received by a distributor from different plants by all vehicles and the number of units back-ordered at that distributor must be greater than or equal to the demand of the distributor. That is,

$$\sum_{i=1}^M \sum_{k=1}^K \sum_{p=1}^P x_{ijkpq} (1 - \lambda_{ijkpq}) y_{qk} \geq B_{jq}, \text{ for } j = 1, 2, \dots, N, q = 1, 2, \dots, Q \quad (3)$$

Vehicle capacity constraint: The total load carried by any vehicle type, whether transported from each plant to all distributors, or from each distributor to all retailers or from each retailer to all customers should not exceed the available capacity of the respective vehicle type. That is,

$$\sum_{i=1}^M \sum_{j=1}^N \sum_{q=1}^Q x_{ijkpq} y_{qk} \leq E_{pk}, \text{ for } k = 1, 2, \dots, K, p = 1, 2, \dots, P \quad (4)$$

Non negativity restrictions:

$$x_{ijkpq} \geq 0 \text{ for } i = 1, 2, \dots, M, j = 1, 2, \dots, N, k = 1, 2, \dots, K, p = 1, 2, \dots, P, q = 1, 2, \dots, Q.$$

Model-6.2.: Formulation of Crisp Multi-Item FDTP without Breakability

$$\text{Minimize } Z = \sum_{i=1}^M \sum_{j=1}^N \sum_{k=1}^K \sum_{p=1}^P \sum_{q=1}^Q C_{ijkpq} ds_{ijp} x_{ijkpq} \quad (5)$$

Subject to the constraints (2), (4) and

$$\sum_{i=1}^M \sum_{k=1}^K \sum_{p=1}^P x_{ijkpq} y_{qk} \geq B_{jq}, \text{ for } j = 1, 2, \dots, N, q = 1, 2, \dots, Q \quad (6)$$

$$x_{ijkpq} \geq 0 \text{ for } i = 1, 2, \dots, M, j = 1, 2, \dots, N, k = 1, 2, \dots, K, p = 1, 2, \dots, P, q = 1, 2, \dots, Q.$$

Model-6.3.: Formulation of Breakable Fuzzy Multi-Item FDTP

Fuzzy set theory was presented by Zadeh [1] in 1965. The theory provided a mathematical approach for dealing with imprecise concepts and problems that have many possible solutions. In real world almost everywhere we see the uncertain events and fuzzy uncertainty is a growing concept. The transportation problem is the special application of linear programming problem which aims to assign the optimal amounts of a product to be transported from various supply points to various demand points so that the total transportation cost is a minimum. Considering unit transportation cost, availabilities, demands and conveyance capacities all are fuzzy in nature, the fuzzy multi-item FDTP has been formulated as follows:

Fuzzy Objective function:

$$\text{Minimize } Z = \sum_{i=1}^M \sum_{j=1}^N \sum_{k=1}^K \sum_{p=1}^P \sum_{q=1}^Q \tilde{C}_{ijkpq} ds_{ijp} x_{ijkpq} \quad (5)$$

Fuzzy Constraints:

Fuzzy Plant capacity constraint: The sum of the number of units transported from the plant to the distribution centers by all vehicles must be less than or equal to fuzzy availability of the source. That is,

$$\sum_{j=1}^N \sum_{k=1}^K \sum_{p=1}^P x_{ijkpq} y_{qk} \leq \tilde{A}_{iq}, \text{ for } i = 1, 2, \dots, M, q = 1, 2, \dots, Q \quad (6)$$

Fuzzy Demand constraint: The sum of the number of units received by a distributor from different plants by all vehicles and the number of units back-ordered at that distributor must be greater than or equal to the fuzzy demand of the distributor.

$$\sum_{i=1}^M \sum_{k=1}^K \sum_{p=1}^P x_{ijkpq} (1 - \lambda_{ijkpq}) y_{qk} \geq \tilde{B}_{jq}, \text{ for } j = 1, 2, \dots, N, q = 1, 2, \dots, Q \quad (7)$$

Vehicle capacity constraint: The total load carried by any vehicle type, whether transported from each plant to all distributors, or from each distributor to all retailers or from each retailer to all customers should not exceed the available fuzzy capacity of the respective vehicle type. That is,

$$\sum_{i=1}^M \sum_{j=1}^N \sum_{q=1}^Q x_{ijkpq} y_{qk} \leq \tilde{E}_{pk}, \text{ for } k = 1, 2, \dots, K, p = 1, 2, \dots, P \quad (8)$$

Non negativity restrictions:

$$x_{ijkpq} \geq 0 \text{ for } i = 1, 2, \dots, M, j = 1, 2, \dots, N, k = 1, 2, \dots, K, p = 1, 2, \dots, P, q = 1, 2, \dots, Q.$$

Model-6.4.: Formulation of Fuzzy Multi-Item FDTP without breakability

$$\text{Minimize } Z = \sum_{i=1}^M \sum_{j=1}^N \sum_{k=1}^K \sum_{p=1}^P \sum_{q=1}^Q \tilde{C}_{ijkpq} ds_{ijp} x_{ijkpq} \quad (9)$$

Subject to the constraints (6), (8) and

$$\sum_{i=1}^M \sum_{k=1}^K \sum_{p=1}^P x_{ijkpq} y_{qk} \geq \tilde{B}_{jq}, \text{ for } j = 1, 2, \dots, N, q = 1, 2, \dots, Q \quad (10)$$

$$x_{ijkpq} \geq 0 \text{ for } i = 1, 2, \dots, M, j = 1, 2, \dots, N, k = 1, 2, \dots, K, p = 1, 2, \dots, P, q = 1, 2, \dots, Q.$$

6.2. Defuzzification of Model-6.3 using ranking function:

Defuzzification is the process of producing a quantifiable result in crisp logic, given fuzzy sets and corresponding membership degrees. It is the process that maps a fuzzy set to a crisp set. It is typically needed in fuzzy control systems. These will have a number of rules that transform a number of variables into a fuzzy result, that is, the result is described in terms of membership in fuzzy sets.

Let us assume that $\tilde{C}_{ijkpq} = (c_{ijkpq1}, c_{ijkpq2}, c_{ijkpq3}, c_{ijkpq4}; w_{ijkpqC})_{LU}$, $\tilde{A}_{iq} = (A_{iq1}, A_{iq2}, A_{iq3}, A_{iq4}; w_{iq})_{LU}$, $\tilde{B}_{jq} = (B_{jq1}, B_{jq2}, B_{jq3}, B_{jq4}; w_{jq})_{LU}$, $\tilde{E}_{pk} = (E_{pk1}, E_{pk2}, E_{pk3}, E_{pk4}; w_{pk})_{LU}$ are Generalized LU-exponential trapezoidal fuzzy number. Using the proposed defuzzification method (based on numerical quadrature formula) the proposed transportation problem was

$$\text{Minimize } Z = \sum_{i=1}^I \sum_{j=1}^J \sum_{k=1}^K \sum_{p=1}^P \sum_{q=1}^Q \left[\frac{((1 - \alpha_C)(c_{ijkpq1} - c_{ijkpq2}) + \alpha_C(c_{ijkpq4} - c_{ijkpq3}))}{2} P_{w_{ijkpqC}} \right. \\ \left. + \frac{((1 - \alpha_C)c_{ijkpq2} + \alpha_C c_{ijkpq3})}{2} w_{ijkpqC} \right] ds_{ijp} x_{ijkpq} \quad (9)$$

Subject to the constraints

$$\sum_{j=1}^N \sum_{k=1}^K \sum_{p=1}^P x_{ijkpq} y_{qk} \leq \frac{((1 - \alpha_A)(A_{iq1} - A_{iq2}) + \alpha_A(A_{iq4} - A_{iq3}))}{2} P_{w_{iq}} + \frac{((1 - \alpha_A)A_{iq2} + \alpha_A A_{iq3})}{2} w_{iq} \quad (10)$$

$$\sum_{i=1}^M \sum_{k=1}^K \sum_{p=1}^P x_{ijkpq} (1 - \lambda_{ijkpq}) y_{qk} \\ \geq \frac{((1 - \alpha_B)(B_{jq1} - B_{jq2}) + \alpha_B(B_{jq4} - B_{jq3}))}{2} P_{w_{jq}} + \frac{((1 - \alpha_B)B_{jq2} + \alpha_B B_{jq3})}{2} w_{jq} \quad (11)$$

$$\sum_{i=1}^M \sum_{j=1}^N \sum_{q=1}^Q x_{ijkpq} y_{qk} \leq \frac{((1 - \alpha_E)(E_{pk1} - E_{pk2}) + \alpha_E(E_{pk4} - E_{pk3}))}{2} P_{w_{pk}} + \frac{((1 - \alpha_E)E_{pk2} + \alpha_E E_{pk3})}{2} w_{pk} \quad (12)$$

Also, the deterministic form of the demand constraint (10) of the Model-6.4 is as follows:

$$\sum_{i=1}^M \sum_{k=1}^K \sum_{p=1}^P x_{ijkpq} y_{qk} \geq \frac{((1 - \alpha_B)(B_{jq1} - B_{jq2}) + \alpha_B(B_{jq4} - B_{jq3}))}{2} P_{w_{jq}} + \frac{((1 - \alpha_B)B_{jq2} + \alpha_B B_{jq3})}{2} w_{jq} \quad (11)$$

7. Numerical Experiment, Analysis of the results and Discussion

7.1. Numerical Experiment:

A company has received a contract to supply two types of gravel for two new construction projects location in towns A and B. The company has 2 gravel pits located in towns X and Y in two different routes. Construction engineers have estimated the required amounts of gravel which will be needed at these construction projects:

	Projects location in town A	Projects location in town B
Pit located of town X	$B_{11}=240$	$B_{12}=132$
Pit located of town Y	$B_{21}=100$	$B_{22}=80$

The required fuzzy amounts of gravel which will be needed at these construction projects:

	Projects location in town A	Projects location in town B
Pit located of town X	$\tilde{B}_{11} = (423,425,427,432; 0.7)_{LU}$	$\tilde{B}_{12} = (217,220,221,225; 0.73)_{LU}$
Pit located of town Y	$\tilde{B}_{21} = (134,147,170,182; 0.87)_{LU}$	$\tilde{B}_{22} = (219,230,235,340; 0.81)_{LU}$

The gravel required by the construction projects can be supplied by two pits. The amount of gravel which can be supplies by each pit is as follows:

	Projects location in town A	Projects location in town B
Pit located of town X	$A_{11}=447$	$A_{12}=369$
Pit located of town Y	$A_{21}=200$	$A_{22}=211$

The fuzzy amount of gravel which can be supplies by each pit is as follows:

	Projects location in town A	Projects location in town B
Pit located of town X	$\tilde{A}_{11} = (220,223,225,237; 0.71)_{LU}$	$\tilde{A}_{12} = (214,216,218,220; 0.55)_{LU}$
Pit located of town Y	$\tilde{A}_{21} = (314,315,317,320; 0.86)_{LU}$	$\tilde{A}_{22} = (151,163,174,176; 0.59)_{LU}$

The company has computed the delivery cost from each pit to each project site. These costs (in rupees) are shown below:

$C_{11111}=3.7$, $C_{21111}=5.45$, $C_{12111}=6$, $C_{22111}=4$, $C_{11211}=6$, $C_{21211}=3.3$, $C_{12211}=22$, $C_{22211}=4.2$, $C_{11121}=7$, $C_{21121}=3.51$, $C_{12121}=6.36$, $C_{22121}=9.65$, $C_{11221}=3.08$, $C_{21221}=5.07$, $C_{12221}=1.01$, $C_{22221}=7.88$, $C_{11112}=5.57$, $C_{21112}=6.93$, $C_{12112}=5.33$, $C_{22112}=5.38$, $C_{11212}=3.41$, $C_{21212}=7.48$, $C_{12212}=9.47$, $C_{22212}=5.37$, $C_{11122}=7.7$, $C_{21122}=8.75$, $C_{12122}=6.68$, $C_{22122}=5.76$, $C_{11222}=9.78$, $C_{21222}=10.37$, $C_{12222}=10.48$, $C_{22222}=7.56$.

These fuzzy unit transportation costs are shown below:

$\tilde{C}_{11111} = (12,13,15,17; 0.31)_{LU}$	$\tilde{C}_{21111} = (8,10,12,15; 0.73)_{LU}$	$\tilde{C}_{12111} = (13,15,16,19; 0.75)_{LU}$
$\tilde{C}_{22111} = (10,11,14,16; 0.66)_{LU}$	$\tilde{C}_{11211} = (13,17,20,21; 0.76)_{LU}$	$\tilde{C}_{21211} = (12,17,18,21; 0.19)_{LU}$
$\tilde{C}_{12211} = (11,12,13,14; 0.85)_{LU}$	$\tilde{C}_{22211} = (17,19,22,24; 0.91)_{LU}$	$\tilde{C}_{11121} = (14,16,18,20; 0.35)_{LU}$
$\tilde{C}_{21121} = (7,10,11,15; 0.52)_{LU}$	$\tilde{C}_{12121} = (17,19,25,26; 0.98)_{LU}$	$\tilde{C}_{22121} = (15,22,23,25; 0.88)_{LU}$
$\tilde{C}_{11221} = (22,26,27,29; 0.55)_{LU}$	$\tilde{C}_{21221} = (18,24,26,28; 0.65)_{LU}$	$\tilde{C}_{12221} = (19,20,25,29; 0.53)_{LU}$
$\tilde{C}_{22221} = (23,28,29,30; 0.37)_{LU}$	$\tilde{C}_{11112} = (13,15,17,19; 0.35)_{LU}$	$\tilde{C}_{21112} = (5,8,10,12; 0.79)_{LU}$
$\tilde{C}_{12112} = (10,15,16,19; 0.75)_{LU}$	$\tilde{C}_{22112} = (11,12,14,15; 0.61)_{LU}$	$\tilde{C}_{11212} = (14,17,21,22; 0.96)_{LU}$

$$\begin{aligned}
\tilde{C}_{21212} &= (12,17,18,21; 0.19)_{LU} & \tilde{C}_{12212} &= (11,12,13,14; 0.85)_{LU} & \tilde{C}_{22212} &= (17,19,22,24; 0.91)_{LU} \\
\tilde{C}_{11122} &= (13,15,19,21; 0.45)_{LU} & \tilde{C}_{21122} &= (17,19,21,25; 0.92)_{LU} & \tilde{C}_{12122} &= (16,18,21,24; 0.88)_{LU} \\
\tilde{C}_{22122} &= (12,15,18,22; 0.98)_{LU} & \tilde{C}_{11222} &= (20,21,24,25; 0.75)_{LU} & \tilde{C}_{21222} &= (15,16,22,23; 0.75)_{LU} \\
\tilde{C}_{12222} &= (12,16,20,22; 0.73)_{LU} & \tilde{C}_{22222} &= (20,21,25,26; 0.97)_{LU}
\end{aligned}$$

The percentages of breakability per unit distance are given below:

$$\begin{aligned}
\lambda_{11111} &= 0.04 & \lambda_{21111} &= 0.06 & \lambda_{12111} &= 0.02 & \lambda_{22111} &= 0.04 & \lambda_{11211} &= 0.03 & \lambda_{21211} &= 0.03 \\
\lambda_{12211} &= 0.07 & \lambda_{22211} &= 0.03 & \lambda_{11121} &= 0.042 & \lambda_{21121} &= 0.02 & \lambda_{12121} &= 0.05 & \lambda_{22121} &= 0.09 \\
\lambda_{11221} &= 0.035 & \lambda_{21221} &= 0.043 & \lambda_{12221} &= 0.01 & \lambda_{22221} &= 0.02 & \lambda_{11112} &= 0.03 & \lambda_{21112} &= 0.037 \\
\lambda_{12112} &= 0.05 & \lambda_{22112} &= 0.01 & \lambda_{11212} &= 0.03 & \lambda_{21212} &= 0.04 & \lambda_{12212} &= 0.06 & \lambda_{22212} &= 0.04 \\
\lambda_{11122} &= 0.04 & \lambda_{21122} &= 0.01 & \lambda_{12122} &= 0.02 & \lambda_{22122} &= 0.051 & \lambda_{11222} &= 0.03 & \lambda_{21222} &= 0.05 \\
\lambda_{12222} &= 0.07 & \lambda_{22222} &= 0.03
\end{aligned}$$

The distances from pits located in towns X and Y to two projects location in towns A and B in two different routes (in hundred of Km) are given in the following table:

	Projects location in town A		Projects location in town B	
	$p = 1$	$p = 2$	$p = 1$	$p = 2$
Pit located of town X	$ds_{111}=1.0$	$ds_{112}=1.30$	$ds_{121}=1.10$	$ds_{122}=1.50$
Pit located of town Y	$ds_{211}=1.05$	$ds_{212}=1.35$	$ds_{221}=1.07$	$ds_{222}=1.48$

The conveyances capacities to transport the gravels from 2 gravel pits located in towns X and Y to two construction projects location in towns A and B with two different types of vehicles are given below:

	Projects location in town A	Projects location in town B
Pit located of town X	$E_{11}=245$	$E_{12}=120$
Pit located of town Y	$E_{21}=80$	$E_{22}=290$

The fuzzy amount of conveyances capacities is as follows:

	Projects location in town A	Projects location in town B
Pit located of town X	$\tilde{E}_{11} = (140,142,153,164; 0.75)_{LU}$	$\tilde{E}_{12} = (98,110,115,124; 0.98)_{LU}$
Pit located of town Y	$\tilde{E}_{21} = (113,124,129,136; 0.87)_{LU}$	$\tilde{E}_{22} = (97,170,181,183; 0.79)_{LU}$

Problem is to schedule the shipment from each pit to each project location under crisp and fuzzy environments in such a manner so as to minimize the total transportation cost within the constraints imposed by pit capacities and project requirements.

7.2. Analysis of the results and Discussion:

When some items are produced in the firm then it's required to deliver to the customers so safely in such a way that the transportation cost has minimum. In practical, to solve fuzzy optimization problem or fuzzy decision making problem, we face some problems due to complexity of the function and in such cases numerical

integration play a vital role in computational Mathematics. We know that, breakability is a part of transportation. When breakable items are transported then due to breakability, the costing of transportation increases and to transport the breakable items need to take more care in transportation. In this paper, two such transportation problems are proposed under deterministic and fuzzy environments and as well as solved using LINGO 13.0 optimization solver (Generalized Reduced Gradient Technique). The optimal solutions of the proposed models are given below:

Result for Model-6.1. (With breakability):

$x_{11111} = 227.3$, $x_{11221} = 22.58$, $x_{12221} = 101.01$, $x_{11112} = 16.08$, $x_{11212} = 120$, $x_{12122} = 80$, $x_{22112} = 1.62$ and remaining variables are zero. The Transportation cost is $Z_{min} = \text{Rs. } 84605.61$.

Result for Model-6.2. (Without breakability):

$x_{11111} = 233$, $x_{11221} = 7$, $x_{12221} = 100$, $x_{11112} = 12$, $x_{11212} = 120$, $x_{12122} = 80$ and remaining variables are zero. The Transportation cost is $Z_{min} = \text{Rs. } 80434.36$.

Result for Model-6.3. (With breakability): Taking degree of optimism $\alpha_C = 0.79$, $\alpha_A = 0.25$, $\alpha_B = 0.12$, $\alpha_E = 0.07$.

$x_{11111} = 135.15$, $x_{12221} = 56.03$, $x_{11112} = 72.17$, $x_{22112} = 83$ and remaining variables are zero. The Transportation cost is $Z_{min} = \text{Rs. } 63549.29$.

Result for Model-6.4. (Without breakability): Taking degree of optimism $\alpha_C = 0.79$, $\alpha_A = 0.25$, $\alpha_B = 0.12$, $\alpha_E = 0.07$.

$x_{11111} = 129.74$, $x_{11112} = 70.53$, $x_{22111} = 55.46$, $x_{22112} = 82.18$ and remaining variables are zero. The Transportation cost is $Z_{min} = \text{Rs. } 59343.93$.

Here, the commodities are transported from sources to destination with different mode of transport i.e., conveyances with different paths from each source to each destination so as to minimize the total transportation cost. As we know, none of the researcher have been applied the concept of numerical integration to solve the fuzzy optimization problem for fruitful optimum solution, the proposed defuzzification technique for generalized LU-exponential trapezoidal fuzzy number is highly significant and useful to solve fuzzy optimization problem like fuzzy transportation problem, fuzzy inventory problem, fuzzy traveling salesman problem etc. The four optimal solutions of the model-6.1, model-6.2, model-6.3 and model-6.4 imply that the use of numerical integration i.e., application of proposed defuzzification technique to solve fuzzy decision making problem or fuzzy optimization problem is highly fruitful and effective. As we know that breakable items are so sensitive and need to take more care in the time of transportation and for this reason, need to pay more money in transportation. For this reason, the transportation cost (Rs. 63549.29, for fuzzy model and Rs. 84605.61, for crisp model) of the model with breakability is more than the transportation cost (Rs. 59343.93, for fuzzy model and Rs. 80434.36, for crisp model) of the model without breakability.

Sensitivity Analysis and Geometrical Meaning of the Model-6.2:

In this paper, the ranking method is so calculated with left and right spreads at some α –level of fuzzy numbers. In the ranking function $I_T^\alpha(\tilde{\xi})$, the degree of optimism α lies between 0 and 1. For the pessimistic decision

maker $\alpha = 0$, optimistic decision maker $\alpha = 1$ and moderate decision maker $\alpha = 0.5$. The sensitivity analysis of the Model-6.3 and Model-6.4 with respect to the degree of optimism α and breakability are presented below:

Table-3: Sensitivity analysis of the Model-6.3 with respect to the α with breakability

α	Optimal transportation schedule	Optimal Transportation Cost
0.0	$x_{11111} = 153.54$, $x_{12221} = 63.53$, $x_{11112} = 82.60$, $x_{22112} = 93.31$ and remaining variables are zero.	Rs. 61396.32
0.05	$x_{11111} = 145.88$, $x_{12111} = 61.02$, $x_{21112} = 79.05$, $x_{22112} = 89.02$ and remaining variables are zero.	Rs. 67604.08
0.10	$x_{11111} = 138.21$, $x_{11212} = 74.37$, $x_{22111} = 59.07$, $x_{22112} = 84.72$ and remaining variables are zero.	Rs. 66329.61
0.15	$x_{11111} = 130.55$, $x_{11112} = 70.24$, $x_{22111} = 55.84$, $x_{22112} = 80.43$ and remaining variables are zero.	Rs. 51816.15
0.20	$x_{11111} = 122.89$, $x_{11112} = 66.13$, $x_{22111} = 52.62$, $x_{22112} =$ and remaining variables are zero.	Rs. 49435.93
0.25	$x_{11111} = 115.23$, $x_{11112} = 62.01$, $x_{22111} = 49.39$, $x_{22112} = 71.85$ and remaining variables are zero.	Rs. 46992.92
0.30	$x_{11111} = 107.56$, $x_{11112} = 57.89$, $x_{22111} = 46.17$, $x_{22112} = 67.56$ and remaining variables are zero.	Rs. 44487.12
0.35	$x_{11111} = 99.9$, $x_{11112} = 53.78$, $x_{22111} = 42.94$, $x_{22112} = 63.27$ and remaining variables are zero.	Rs. 41918.54
0.40	$x_{11111} = 92.24$, $x_{22111} = 39.71$, $x_{21112} = 50.02$, $x_{22112} = 58.97$ and remaining variables are zero.	Rs. 42849.22
0.45	$x_{11111} = 84.57$, $x_{11112} = 45.54$, $x_{22111} = 36.49$, $x_{22112} = 54.68$ and remaining variables are zero.	Rs. 36593.01
0.50	$x_{11111} = 76.91$, $x_{22111} = 33.26$, $x_{21112} = 41.72$, $x_{22112} = 50.39$ and remaining variables are zero.	Rs. 36968.04
0.55	$x_{11111} = 69.25$, $x_{22111} = 30.04$, $x_{21112} = 37.57$, $x_{22112} = 46.1$ and remaining variables are zero.	Rs. 33909.29
0.60	$x_{11111} = 61.58$, $x_{11112} = 33.18$, $x_{12212} = 44.03$, $x_{12111} = 26.26$ and remaining variables are zero.	Rs. 34332.30
0.65	$x_{11111} = 53.92$, $x_{22111} = 23.58$, $x_{21112} = 29.28$, $x_{22112} = 37.52$ and remaining variables are zero.	Rs. 27555.51
0.70	$x_{11111} = 46.26$, $x_{22111} = 20.36$, $x_{21112} = 25.13$, $x_{22112} = 33.22$ and remaining variables are zero.	Rs. 24260.47
0.75	$x_{11111} = 38.59$, $x_{22111} = 17.13$, $x_{21112} = 20.98$, $x_{22112} = 28.93$ and remaining variables are zero.	Rs. 20886.66
0.80	$x_{11111} = 30.93$, $x_{11112} = 16.71$, $x_{22111} = 13.91$, $x_{22112} = 24.64$ and	Rs. 15975.85

	remaining variables are zero.	
0.85	$x_{11111} = 23.27$, $x_{11112} = 12.59$, $x_{22111} = 10.68$, $x_{22112} = 20.34$ and remaining variables are zero.	Rs. 12779.39
0.90	$x_{12221} = 7.23$, $x_{21211} = 15.45$, $x_{21112} = 8.54$, $x_{22112} = 16.06$ and remaining variables are zero.	Rs. 10737.14
0.95	$x_{11111} = 7.94$, $x_{11112} = 4.36$, $x_{22111} = 4.23$, $x_{22112} = 11.76$ and remaining variables are zero.	Rs. 6198.116
1.00	$x_{11111} = 0.28$, $x_{11112} = 0.24$, $x_{22111} = 1$, $x_{22112} = 7.47$ and remaining variables are zero.	Rs. 2813.29

Figure-5: Pictorial Representation of Sensitivity analysis of the Model-6.3 with respect to the α with breakability

Table-4: Sensitivity analysis of the Model-6.4 with respect to the α without breakability

α	Optimal transportation schedule	Optimal Transportation Cost
0.0	$x_{11111} = 147.4$, $x_{11112} = 80.12$, $x_{22111} = 146.35$, $x_{22112} = 92.37$ and remaining variables are zero.	Rs. 79477.39
0.05	$x_{12221} = 139.08$, $x_{11212} = 76.13$, $x_{21211} = 140.04$, $x_{22112} = 88.12$ and remaining variables are zero.	Rs. 97785.30
0.10	$x_{11111} = 132.68$, $x_{11112} = 72.13$, $x_{22111} = 131.81$, $x_{22112} = 83.87$ and	Rs. 73544.48

	remaining variables are zero.	
0.15	$x_{12221} = 124.54$, $x_{11122} = 68.14$, $x_{12222} = 79.63$, $x_{21211} = 125.33$ and remaining variables are zero.	Rs. 96179.07
0.20	$x_{11111} = 117.97$, $x_{11112} = 64.14$, $x_{22111} = 117.27$, $x_{22112} = 75.38$ and remaining variables are zero.	Rs. 67225.47
0.25	$x_{11112} = 60.15$, $x_{22111} = 110$, $x_{21211} = 110.62$, $x_{22112} = 71.13$ and remaining variables are zero.	Rs. 65076.31
0.30	$x_{11111} = 103.26$, $x_{11112} = 56.15$, $x_{22111} = 102.73$, $x_{22112} = 66.88$ and remaining variables are zero.	Rs. 60520.34
0.35	$x_{12221} = 95.46$, $x_{11112} = 52.16$, $x_{12212} = 62.63$, $x_{21211} = 95.90$ and remaining variables are zero.	Rs. 67973.13
0.40	$x_{11111} = 88.54$, $x_{22111} = 88.19$, $x_{21112} = 48.16$, $x_{22112} = 58.38$ and remaining variables are zero.	Rs. 56830.60
0.45	$x_{11112} = 44.17$, $x_{21211} = 81.19$, $x_{22221} = 80.93$, $x_{22112} = 54.13$ and remaining variables are zero.	Rs. 59929.23
0.50	$x_{11111} = 73.83$, $x_{11112} = 40.18$, $x_{22111} = 73.65$, $x_{22112} = 49.88$ and remaining variables are zero.	Rs. 45951.78
0.55	$x_{11111} = 66.47$, $x_{11112} = 36.18$, $x_{22111} = 66.38$, $x_{22112} = 45.63$ and remaining variables are zero.	Rs. 42068.32
0.60	$x_{11111} = 59.12$, $x_{12121} = 59.12$, $x_{11212} = 32.19$, $x_{12212} = 41.38$ and remaining variables are zero.	Rs. 68281.22
0.65	$x_{11111} = 51.76$, $x_{22111} = 51.84$, $x_{21112} = 28.19$, $x_{22112} = 37.14$ and remaining variables are zero.	Rs. 36273.79
0.70	$x_{11111} = 44.41$, $x_{11112} = 24.2$, $x_{22111} = 44.58$, $x_{22112} = 32.89$ and remaining variables are zero.	Rs. 29838.79
0.75	$x_{12211} = 37.31$, $x_{11112} = 20.21$, $x_{21211} = 37.05$, $x_{22112} = 28.64$ and remaining variables are zero.	Rs. 27721.66
0.80	$x_{11111} =$, $x_{11112} = 16.21$, $x_{22111} = 30.04$, $x_{22112} = 24.39$ and remaining variables are zero.	Rs. 21203.13
0.85	$x_{22111} = 22.77$, $x_{21211} = 22.33$, $x_{21112} = 12.21$, $x_{22112} = 20.14$ and remaining variables are zero.	Rs. 17988.28
0.90	$x_{11111} = 14.98$, $x_{12212} = 15.89$, $x_{22111} = 15.50$, $x_{21112} = 8.22$ and remaining variables are zero.	Rs. 14031.10
0.95	$x_{12211} = 8.23$, $x_{11112} = 4.22$, $x_{12212} = 11.65$, $x_{21211} = 7.62$ and remaining variables are zero.	Rs. 8723.26
1.00	$x_{11111} = 0.27$, $x_{11112} = 0.23$, $x_{22111} = 0.96$, $x_{22112} = 7.39$ and remaining variables are zero.	Rs. 2773.49

Figure-6: Pictorial Representation of Sensitivity analysis of the Model-6.4 with respect to the α without breakability

In view of Transportation problems, optimism and pessimism can reflect distinguished distinction of different decision makers and also they can tend to be both compatible and enduring. It also implies that optimism and pessimism consistently influence how the decision maker responds to the decision environment. From the sensitivity analysis of the models 6.3 and 6.4 it is so much clear that the optimal cost and optimal solution are highly dependent on degree of optimism α ($0 \leq \alpha \leq 1$) and since the optimal costs with respect to α are fluctuating, so its decision maker's choice, what value of α they would like to consider for getting the optimal solution of the proposed models.

8. Conclusion and future extension of the work: This paper proposed a new type of fuzzy number i.e. generalized LU-exponential trapezoidal fuzzy number and its ranking with left and right spreads at some α –level of generalized LU-exponential trapezoidal fuzzy numbers. Since left and right spreads at some α –level of generalized LU-exponential trapezoidal fuzzy numbers is difficult to calculate due to non-integrable integrand and that's why the numerical integration i.e., Weddle's rule is used to find the left and right spreads at some α –level of generalized LU-exponential trapezoidal fuzzy numbers as well as to compare the two generalized LU-exponential trapezoidal fuzzy numbers. Also to validate the methodologies, the six sets of generalized LU-exponential trapezoidal fuzzy numbers are compared. The main advantage of the proposed approach is that the proposed approach provides the correct ordering of generalized LU-exponential trapezoidal fuzzy numbers and also the proposed approach is very simple and easy to apply in the real life problems. The

proposed methodology to find the ranking value of the proposed fuzzy number gives some numerical error since here we developed the methodology using Weddle's rule with six numbers of sub-intervals. In future, if we can increase the number of sub-interval in the numerical integration part than we can get the more accurate ranking value of generalized LU-exponential trapezoidal fuzzy numbers. We all know that, transportation plays a vital role to grow the economy and the path of the transportation is an imperatives part like source, destination and conveyances of any transportation problem. The paths from each source to each destination are taken into account to formulate crisp as well as fuzzy transportation problem. As the distances from sources to destinations through different paths are significant in transportation problem thus, in reality, the said concept is highly acceptable. When impreciseness and complexity of the function occurs in any optimization problem at a time then it is difficult to solve the problem to obtain the fruitful optimal solution. In practical sense, the proposed defuzzification technique for generalized LU-exponential trapezoidal is more significant and useful to overcome. The models in the paper are formulated by taking the different path of the transportation and fuzziness. Also here the fuzzy (type-1) model has been defuzzified using the proposed defuzzification technique for generalized LU-exponential trapezoidal fuzzy number and in future the models and the methodologies can be developed in the following manner:

- 1) Smooth transportation is possible when the routes of the transportation are safe and sound. So the proposed deterministic and fuzzy models can be formulated involving safety constraint.
- 2) In the fuzzy model formulation, type-1 generalized LU-exponential trapezoidal fuzzy number has been considered but in future we can formulate the model with type-2 generalized LU-exponential trapezoidal fuzzy number.
- 3) Breakability and restriction on transportation is an important concept in transportation problem. So in future we can formulate the models introducing % of breakability and restriction on transportation.
- 4) Global warming is a important topic in the world. Several meetings has been held among the different countries to control global warming & global warming is mainly due to emission of green house gasses i.e mainly due to carbon die oxide. Now worldwide regulation has been made to control carbon die oxide emission in production centre, transportation sector, logistic sector etc. If they emit CO_2 , that company has to paid tax for that. In future we can develop the models in such a way that, how firms manage carbon footprints in transportation problems under the carbon emission trading mechanism.
- 5) In practical, the cargo transportation is happened in step by step system i.e., the real life transportation is happened from sources to destination centers via different intermediate destination centers. Taking this concept, in future, multi-stage four dimensional solid transportation problems can be formulated.

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Informed consent: No humans are involved in this study thus, Informed consent is not applicable for the study.

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