A Data Fusion Method in Wireless Sensor Network Based on Belief Structure

Chenfeng Long (elong@hunau.edu.cn)
Hunan Agricultural University
Xinxing Liu
Hunan Agricultural University
Yakun Yang
Hunan Agricultural University
Tao Zhang
Hunan Agricultural University
Siqiao Tan
Hunan Agricultural University
Xiaoyong Tang
Changsha University of Science and Technology
Gelan Yang
Hunan City University

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A data fusion method in wireless sensor network based on belief structure

Chenfeng Long[1⊠], Xinxing Liu[1], Yakun Yang[1], Tao Zhang[1], Siqiao Tan[1], Xiaoyong Tang[2⊠], Gelan Yang[3]

Correspond Authors

1. Chenfeng Long, e-mail: elong@hunau.edu.cn

2. Xiaoyong Tang, e-mail: tang_313@163.com

Abstract

Considering the issue with respect to the high data redundancy and high cost of information collection in wireless sensor nodes, this paper proposes a data fusion method based on belief structure to reduce attribution in multi-granulation rough set. By introducing belief structure, attribute reduction is carried out for multi-granulation rough sets. From the view of granular computing, this paper studies the evidential characteristics of incomplete multi-granulation ordered information systems. On this basis, the positive region reduction, belief reduction and plausibility reduction are put forward in incomplete multi-granulation order information system, and analyze the consistency in the same level and transitivity in different levels. The positive region reduction and belief reduction are equivalent, and the positive region reduction and belief reduction is unnecessary and sufficient conditional plausibility reduction in the same level; if the cover structure order of different levels are the same, the corresponding equivalent positive region reduction. The algorithm proposed in this paper not only performs three reductions, but also reduces the time complexity largely. The above study fuses the node data which reduces the amount of data that needs to be transmitted and effectively improves the information processing efficiency.

Key words：wireless sensor network; granular computing; rough set; Dempster–Shafer theory; reduction

1 Introduction

Wireless Sensor Network (WSN) is a multi-hop self-organizing system formed by wireless sensor nodes communicating with each other. However, due to the short communication distance of sensor nodes, limited information processing capabilities, and the inability of a single node to provide all information, sensor nodes can only be deployed in an overlapping manner to obtain complete information about the object within the monitoring range. But this will lead to a series of problems such as high circuit complexity, high data redundancy, and high node energy consumption. To effectively solve the above problems, this paper studies rough set theory and evidence theory which can be used for data analysis and provide strong support data fusion technology in the real world of imprecise research12.

The basic structure of rough set theory is an approximation space consisting of a universe of discourse, in which lower and upper approximations are defined to approximate a undefinable set by using equivalence relations1-8. Research on rough set mainly focus on attribute reduction9 to fuse information on sensor nodes. From the perspective of granular computing, three extensions of rough set model have been proposed in terms of the characters of data, respectively, multi-granulation rough set based on multi-scale, multi-level and multi-angle10-. A general concept of multi-granulation rough set based on multi-scale describes that an attribute of an object can only take one value in a single scale information system where the object information is reflected at a fixed scale. We call such a single scale information system as the classic Pawlak's information system. However, in practical, an object could take on as many different hierarchical values under the same attribute with respect to different scales. And, there do exist special relationships among these hierarchical levels. One example is that the examination results of English for students can be recorded as natural numbers between 0 and 100, and it can also be graded as "Excellent", "Good", "Medium", "Bad", "Unacceptable" . Sometimes, if needed, it might be graded into two values, "Passed" and "Failed". A hierarchy of such obtained information granules can be organized to a system which is called multi-scale information system.

The evidence theory represents the uncertainty through the belief and plausibility function derived by the mass function which the core concept is belief structure and evidence structure20-. Recently, the combination of evidence theory and rough set model become one of the research hotspots. As introduced in Yao et al.(1998)23,the adequate condition for belief structure exactly exist in the classic rough set. On the above basis, this study was extended to covering rough set by Chen et al.(2015)2425,who successfully employ the belief function and the plausibility function to describe the upper and lower approximations of the covering rough set, which means the numerical features of the rough set can be characterized by evidence theory. In particular, from the perspective of information fusion, Lin et al.(2015)26 explores the relationship between evidence theory and classical multi-granulation rough sets, which shows that, in general, the classic optimistic multi-granulation rough set does not have its corresponding belief structure.

By introducing belief structure, this paper firstly studies the evidential characteristics of multi-granulation rough set based on multi-scale. On this basis, the positive region reduction, belief reduction and plausibility reduction are put forward in incomplete multi-granulation ordered information system, then, analyse the consistency in the same level and transitivity in different level, which can reduce data redundancy and circuit complexity, and save node limited resources through data fusion.

2 Method

This study put forward the positive region reduction, belief reduction and plausibility reduction in terms of reducing data redundancy of WSN, and proposes an algorithm to reduce the time complexity of attribute reduction. This section firstly introduces the basic preliminaries of WSN information processing, multi-granulation rough set and belief structure. On this basis, the above three reductions are proposed and will be conducted in Section 3.

2.1 Preliminaries

2.1.1 WSN information processing

WSN is mainly composed of nodes, sensor network and users which the core task of nodes is to data perceiving and processing[27-29]. According to a certain standard, n nodes can form m clusters and the cluster header is selected in each cluster, which can also represent this cluster at a higher level. Meanwhile, the same mechanism is also applied between cluster headers to form a hierarchical structure1.

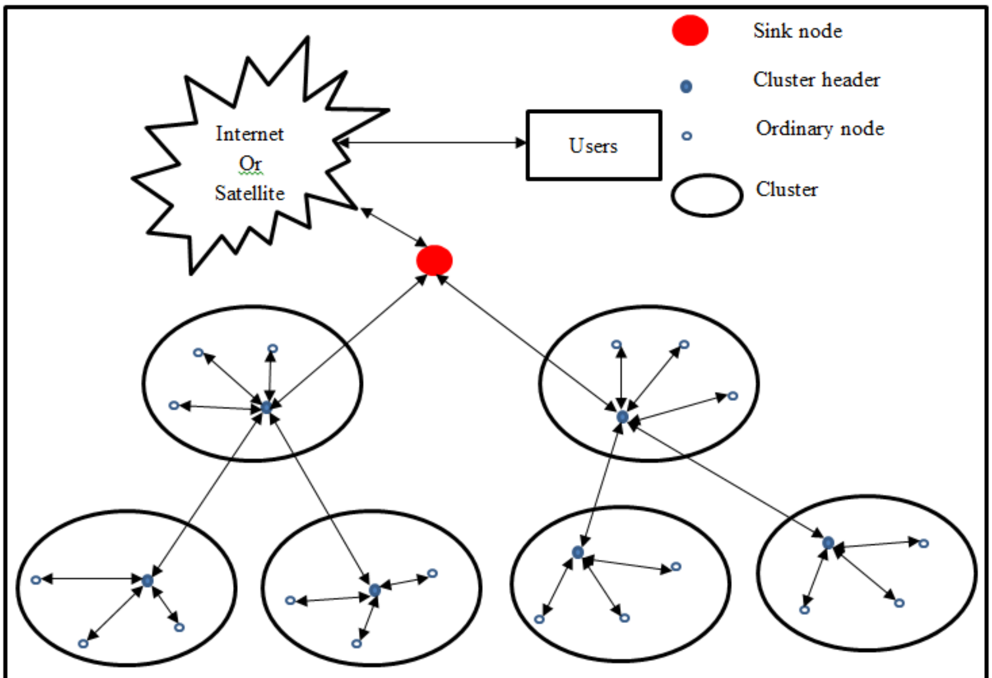

Fig.1 The hierarchical structure of WSN

This hierarchical routing structure focus on data which makes the node only interact with their neighbors within a certain range through localized principle. And the cluster header will perform data fusion in the cluster so that the sensor node only automatically obtains and transmits effective information. This is also the key to WSN information fusion. In this case, the multi-granulation rough set based on multi-scale can fuse data in the cluster, which ensure that a small amount of effective information is transmitted between the cluster header and sink node with respect to effectively balance information processing, energy consumption and system performance.

2.1.2 Multi-granulation rough set based on multi-scale

Definition 117. Let $U=\{x_1,x_2\ ,\cdots,x_n\}$ be a nonempty finite set of objects called the universe of discourse, $AT^k=\{a_1^k,a_2^k,\cdots a_m^k\}$ be a set of attribution and a_j be the attribute of multi-granulation. For each object in U, the attribute a_j can take different value on the different levels of granulations. If f is the attribute value surjective function of different levels (that is, for every k representing the number of levels with a value of positive integer, there exists $x\epsilon U$ such that $f(x)=k$) and V^k is the domain of the attribute a^k, then, the quaternary $MGIS=(U,AT^k,f^k,V^k)$ is called a multi-granulation information system.

From the above definition, the multi-scale information system will degenerate into the classic Pawlak information system when the number of granular levels is $k=1$. For convenience of description, the following simplifies the multi-granulation information system based on multi-scale as a multi-granulation information system.

Definition 217. Let $MGIS=(U,AT^k,f^k,V^k)$ be a multi-granulation information system which arbitrary attribute a_j has I levels of granulations. We further define the attribute of a_j on the $k-th$ levels of granulations $a_j^k\!:U\rightarrow V_j^k$ represents a surjective function and V_j^k is the domain of the k-th scale attribute a_j^k (that is, for any $1\leq k\leq I$, there exists $x\epsilon U$ such that $a_j^k(x)=*$, where $(*)$ means variable quantity). And the surjective function $g_j^{k,k+1}\!:V_j^k\rightarrow V_j^{k+1}$

(if there exists $k+1$) is called the granular transformation function with variable quantity $(*)$ as defined as follows:

$$a_j^{k+1}(x) = \begin{cases} * & ,if\ \ a_j^k = * \\ g_j^{k,k+1}\left(a_j^k(x)\right), & if\ \ a_j^k \neq * \end{cases}$$

On the basis of Definition 2, clearly, the value of an object between different levels of granulations is not arbitrary and depended on the value of the lower level in a multi-granulation information system, which means the value of $a_j^{k+1}(x)$ is determined by $a_j^k(x)$.

Definition 317. Let $MGIS = (U, AT^k, f^k, V^k)$ be a multi-granulation information system which arbitrary attribute a_j has I levels of granulations. For any $1 \leq k \leq I$, the multi-granulation information system $MGIOIS$ can be called multi-granulation ordered information system if the attribute value range of any levels of granulations is all partial ordering.

And $MGIOIS^{*\geq}$, a multi-granulation ordered information system with variable values $(*)$ and null values is collectively referred to as an incomplete multi-granulation ordered information system.

Definition 417. Let $MGIOIS^{*\geq} = (U, AT^k, f^k, V^k)$ be an incomplete multi-granulation information system which has I levels of granulations. For any $1 \leq k \leq I$, if attribute set $A^k \subseteq AT^k$ and two arbitrary elements $x, y\epsilon U$, then, there exists $R_{AT^k}^{*\geq} = \{(x,y)\epsilon U \times U | \forall a^k \epsilon AT^k, (f(x,a^k) \leq f(y,a^k)\ )\ \vee\ (f(x,a^k) = *)\ \vee\ (f(y,a^k) = *)\}$ can be called an incomplete multi-granulation ordered information system dominance relationship.

On the above basis, if the pair $(x,y)\epsilon R_{AT^k}^{*\geq}$, then, $[X]_{AT^k}^{*\geq}$ means that y is finer than x or x is coarser than y. The relationship $R_{AT^k}^{*\geq}$ can be considered as a kind of surjection from U to $P(U)$ where $P(U)$ is a power set. $U/_R = \{y\epsilon P(U) | x\epsilon U\}$ is a covering of universe of discourse.

Definition 517. Let $MGIOIS^{*\geq} = (U, AT^k, f^k, V^k)$ be an incomplete multi-granulation ordered information system which has I levels of granulations. For every $X \subseteq \mathrm{U}$ and any $1 \leq k \leq I$, the lower and upper approximations of X in the $k-th$ levels of granulation are defined as $\underline{R_{AT^k}^{*\geq}}(X) = \{x\epsilon U | [x]_{AT^k}^{*\geq} \subseteq \mathrm{X}\}$ and $\overline{R_{AT^k}^{*\geq}}(X) = \{x\epsilon U | [x]_{AT^k}^{*\geq} \cap \mathrm{X} \neq \emptyset\}$, respectively.

From the Definition 5, the relationship between the lower and upper approximations in the same levels of granulations has been clearly proved. $\overline{R_{AT^k}^{*\geq}}(X)$ and $\underline{R_{AT^k}^{*\geq}}(X)$ satisfy the following properties, which will be the theoretical foundation for the further discussions in this paper.

Proposition 117. Let $MGIOIS^{*\geq} = (U, AT^k, f^k, V^k)$ be an incomplete multi-granulation ordered information system which has I levels of granulations. For any $1 \leq k \leq I$ and two elements $X, Y \subseteq \mathrm{U}$, we denote the complement of X in U as $\sim U$, i.e. $\sim U = U - X = \{x\epsilon U | x \notin X\}$, then,

$\underline{R_{AT^k}^{*\geq}}(X) =$

$\sim \overline{R_{AT^k}^{*\geq}}(\sim X)$

$\overline{R_{AT^k}^{*\geq}}(X) =$

$\sim \underline{R_{AT^k}^{*\geq}}(\sim X)$

$\underline{R^{*\geq}_{AT^k}}(\emptyset) = \emptyset, \quad \overline{R^{*\geq}_{AT^k}}(\emptyset) =$

$\emptyset$

$\underline{R^{*\geq}_{AT^k}}(U) = U, \ \overline{R^{*\geq}_{AT^k}}(U) =$

U

$\underline{R^{*\geq}_{AT^k}}(X) \subseteq X \subseteq$

$\overline{R^{*\geq}_{AT^k}}(X)$

$\underline{R^{*\geq}_{AT^k}}(X \cap Y) = \underline{R^{*\geq}_{AT^k}}(X) \cap$

$\underline{R^{*\geq}_{AT^k}}(Y)$

$\overline{R^{*\geq}_{AT^k}}(X \cup Y) = \overline{R^{*\geq}_{AT^k}}(X) \cup$

$\overline{R^{*\geq}_{AT^k}}(Y)$

Theorem 1. Let $MGIOIS^{*\geq} = (U, AT^k, f^k, V^k)$ be an incomplete multi-granulation ordered information system which has I levels of granulations. For any $1 \leq k \leq I$ and every $x \epsilon U$, denote attribute subset $A^k \subseteq AT^k$, then $[x]^{*\geq}_{A^k} \subseteq [x]^{*\geq}_{A^{k+1}}$, where $1 \leq k \leq I-1$.

Proof. On the basis of granular transformation function from Definition 2, it is easy to see that for any a_j^k, there exists $g_j^{k,k+1}\left(a_j^k(y)\right) \geq g_j^{k,k+1}\left(a_j^k(x)\right)$ such that $\left\{y \epsilon U \middle| \forall a_j^k \epsilon A^k, g_j^{k,k+1}\left(a_j^k(y)\right) \geq g_j^{k,k+1}\left(a_j^k(x)\right)\right\}$. Thus, we have $[x]^{*\geq}_{A^k} \subseteq [x]^{*\geq}_{A^{k+1}}$.

Theorem 1 represents the relationship of the attribute set AT in different levels of granulations, i.e. $R^{*\geq}_{AT^{k+1}}$ is subdivided by the relationship $R^{*\geq}_{AT^k}$ defined on the attribute set AT, and thus obtain the relationship between the upper and lower approximations in different levels of granulations.

Proposition 2. Let $MGIOIS^{*\geq} = (U, AT^k, f^k, V^k)$ be an incomplete multi-granulation ordered information system which has I levels of granulations. For any $1 \leq k \leq I$ and every $x \epsilon U$, denote attribute subset $A^k \subseteq AT^k$, then,

(1) $R^{*\geq}_{AT^{k+1}}(X) \subseteq R^{*\geq}_{AT^k}(X)$ where $1 \leq k \leq I-1$.

(2) $R^{*\geq}_{AT^k}(X) \subseteq R^{*\geq}_{AT^{k+1}}(X)$ where $1 \leq k \leq I-1$.

Proof. (1) If for every $x \epsilon U$, there exists $x \epsilon R^{*\geq}_{AT^{k+1}}(X)$ such that $[x]^{*\geq}_{A^{k+1}} \subseteq X$.

With theorem 1, if $[x]^{*\geq}_{A^k} \subseteq$ X,then $x \epsilon R^{*\geq}_{AT^k}(X)$.

Thus, we have proved that $R^{*\geq}_{AT^{k+1}}(X) \subseteq R^{*\geq}_{AT^k}(X)$.

(2) If for every $x \epsilon U$, there exists $x \epsilon R^{*\geq}_{AT^k}(X)$ such that $[X]^{*\geq}_{A^k} \cap X \neq \emptyset$.

With theorem 1, if $[X]^{*\geq}_{A^k} \cap X \neq \emptyset$, then $x \epsilon R^{*\geq}_{AT^k}(X)$.

Thus, we have proved that $R^{*\geq}_{AT^k}(X) \subseteq R^{*\geq}_{AT^{k+1}}(X)$.

From the Proposition 2, clearly, the lower approximation in the $(k+1)-th$ levels of granulations of X further subdivides the same in the $k-th$ and the upper approximation in the $(k+1)-th$ levels of granulation of X further subdivides the same in the $k-th$. Moreover, we have the corresponding hierarchical sequence of approximations as follows.

$$\underline{R^{*\geq}_{A^{k+1}}}(X) \subseteq \underline{R^{*\geq}_{A^{k}}}(X) \subseteq \cdots \subseteq \underline{R^{*\geq}_{A^{2}}}(X)$$

$$\subseteq \underline{R^{*\geq}_{A^{1}}}(X) \tag{1}$$

$$\overline{R^{*\geq}_{A^{1}}}(X) \subseteq \overline{R^{*\geq}_{A^{2}}}(X) \subseteq \cdots \overline{R^{*\geq}_{A^{k}}}(X)$$

$$\subseteq \overline{R^{*\geq}_{A^{k+1}}}(X) \tag{2}$$

In the following, employ the example 1 to illustrate $\overline{R^{*\geq}_{A^{1}}}(X) \subseteq \overline{R^{*\geq}_{A^{2}}}(X) \subseteq \cdots \overline{R^{*\geq}_{A^{k}}}(X) \subseteq \overline{R^{*\geq}_{A^{k+1}}}(X)$ for understanding the above eqs more conveniently.

Example 1. Table 1 is an incomplete multi-granulation ordered information system table of Rape pests detected by WSN in a certain period of time, where granularity $I = 3$, $U = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8\}$ and $AT^k = \{a_1^k, a_2^k, a_3^k, a_4^k\}$. $X_i (i = 1,2,\cdots 8)$ represents different clusters which stand for Cabbage butterfly, Aphids, Cabbage bug and Cricket, respectively. And AT^k is the attribute set where k=1,2,3, which is the different levels of granulations. And $*$ is the missing value. From a multi-granular information system structure, we obtain the hierarchical sequence of attributes in different levels of granulations as follows.

(1) The sequence of the value in the first level of granulation of the quantity of pests is $\{1 < 2 < \cdots 100\}$.

(2) The sequence of the value in the second level of granulation the grade of the quantity is $\{r < f < m < l < a\}$ where "r", "f", "m", "l" and "a" represent "rarely", "few" "medium", "lot" and "abundance", respectively.

(3) The sequence of the value in the third level of granulation of the grade of risk is $\{ S < F\}$ where "S" and "F" represent "Seconds" and "Firsts", respectively.

Notes: For facilitating reduction, in the first level of granulation, the quantity will be calculated as 90 when it is between 80 and 100 or 70 when it is between 60 and 80, and it can deduce the rest from this.

Table 1 A multi-granulation incomplete ordered information system with three levels of granulations

U	a_1^1	a_1^2	a_1^3	a_2^1	a_2^2	a_2^3	a_3^1	a_3^2	a_3^3	a_4^1	a_4^2	a_4^3
x_1	90	a	F	90	a	F	$*$	$*$	$*$	90	a	F
x_2	70	l	F	90	a	F	70	l	F	90	a	F
x_3	$*$	$*$	$*$	30	f	F	10	r	S	10	r	S
x_4	50	m	F	50	m	F	$*$	$*$	$*$	70	l	F
x_5	30	f	F	50	m	$*$	30	f	F	50	m	F
x_6	70	l	F	70	l	F	70	l	F	70	l	F
x_7	10	r	S	70	l	F	10	r	S	50	m	F
x_8	$*$	$*$	$*$	90	a	F	90	a	F	90	a	F

For the above levels of granulations, the system is decomposed into three decision tables which are described as table 2,3,4, respectively.

Table 2 The incomplete ordered information table with the first level of granulations

U	a_1^1	a_2^1	a_3^1	a_4^1
x_1	90	90	$*$	90
x_2	70	90	70	90
x_3	$*$	30	10	10
x_4	50	50	$*$	70
x_5	30	50	30	50
x_6	70	70	70	70
x_7	10	70	10	50
x_8	$*$	90	90	90

From Table 2, we can derive that the value of the first level of granulation as follows.

$[x_1]_{AT^1}^{*\geq} = \{x_1, x_8\}$

$[x_2]_{AT^1}^{*\geq} = \{x_1, x_2, x_8\}$

$[x_3]_{AT^1}^{*\geq} = \{U\}$

$[x_4]_{AT^1}^{*\geq} = \{x_1, x_2, x_4, x_6, x_8\}$

$[x_5]_{AT^1}^{*\geq} = \{x_1, x_2, x_4, x_5, x_6, x_8\}$

$[x_6]_{AT^1}^{*\geq} = \{x_1, x_2, x_6, x_8\}$

$[x_7]_{AT^1}^{*\geq} = \{x_1, x_2, x_3, x_6, x_7, x_8\}$

$[x_8]_{AT^1}^{*\geq} = \{x_1, x_8\}$

Suppose $X = \{x_1, x_2, x_4, x_8\}$, then, the reduction of the lower and upper approximation are $\underline{R_{AT^1}^{*\geq}}(X) = \{x_1, x_2, x_8\}$ and $\overline{R_{AT^1}^{*\geq}}(X) = \{U\}$, respectively.

Table 3 The incomplete ordered information table with the second level of granulations

U	a_1^2	a_2^2	a_3^2	a_4^2
x_1	a	a	$*$	a
x_2	l	a	l	a
x_3	$*$	f	r	r
x_4	m	m	$*$	l
x_5	f	m	f	m
x_6	l	l	l	l
x_7	r	l	r	m
x_8	$*$	a	a	a

From Table 3, we can derive that the value of the second level of granulation as follows.

$[x_1]_{AT^2}^{*\geq} = \{x_1, x_8\}$

$[x_2]_{AT^2}^{*\geq} = \{x_1, x_2, x_8\}$

$[x_3]^{*\geq}_{AT^2} =$
$\{U\}$
$[x_4]^{*\geq}_{AT^2} =$
$\{x_1, x_2, x_4, x_6, x_8\}$
$[x_5]^{*\geq}_{AT^2} =$
$\{x_1, x_2, x_4, x_5, x_6, x_8\}$
$[x_6]^{*\geq}_{AT^2} =$
$\{x_1, x_2, x_6, x_8\}$
$[x_7]^{*\geq}_{AT^2} =$
$\{x_1, x_2, x_3, x_6, x_7, x_8\}$
$[x_8]^{*\geq}_{AT^2}$
$= \{x_1, x_8\}$

Suppose $X = \{x_1, x_2, x_4, x_8\}$, then, the reduction of the lower and upper approximation are $\underline{R^{*\geq}_{AT^2}}(X) = \{x_1, x_2, x_8\}$ and $\overline{R^{*\geq}_{AT^2}}(X) = \{U\}$, respectively. And for every $x \epsilon U$, we can obtain $[x]^{*\geq}_{AT^1} \subseteq [x]^{*\geq}_{AT^2}$.

Table 4 The incomplete ordered information table with the third level of granulations

U	a_1^3	a_2^3	a_3^3	a_4^3
x_1	F	F	*	F
x_2	F	F	F	F
x_3	*	F	S	S
x_4	F	F	*	F
x_5	F	*	F	F
x_6	F	F	F	F
x_7	S	F	S	F
x_8	*	F	F	F

From Table 4, we can obtain that the value of the third level of granulations as follows.

$[x_1]^{*\geq}_{AT^3} = [x_2]^{*\geq}_{AT^3} = [x_4]^{*\geq}_{AT^3} = [x_5]^{*\geq}_{AT^3} = [x_6]^{*\geq}_{AT^3} = [x_8]^{*\geq}_{AT^3} =$
$\{x_1, x_2, x_4, x_5, x_6, x_8\}$ (1)
$[x_3]^{*\geq}_{AT^3} = [x_7]^{*\geq}_{AT^3} =$
$\{U\}$

Suppose $X = \{x_1, x_2, x_4, x_8\}$, then, the reduction of the lower and upper approximation are $\underline{R^{*\geq}_{AT^3}}(X) = \emptyset$ and $\overline{R^{*\geq}_{AT^3}}(X) = \{U\}$, respectively. And for every $x \epsilon U$, we can obtain $[x]^{*\geq}_{AT^2} \subseteq [x]^{*\geq}_{AT^3}$.

The Example 1 illustrates that it is not arbitrary for the value of the same attribute of the same object in different levels of granulation and proves that the value of the higher level of granulations is determined by the lower, i.e. The "a" from the attribute a_1 of x_1 in the second level of granulations is determined by "90" from the value of the first level.

2.1.3 Evidence Structure

Definition 620-**.** Let Θ be a finite and nonempty set which is called the frame of discernment, where A is the arbitrary subset of Θ. If there exists a mapping function $m: 2^{\Theta} \rightarrow [0,1]$ that satisfies $m(\emptyset) = 0$ and $\sum_{X \subseteq \mathrm{U}} m(X) = 1$, then, we define that the function m is the basic probability assignment function or the mass function on 2^{Θ}.

The degree of evidence exactly to A is indicated by $m(A)$. If there exists $m(A) > 0$, then, we suppose that A is called the focal element of m and a family of all focal elements is viewed as the core. A pair of (F, m) is called a belief structure on the core. And we can obtain the other pair of belief and plausibility functions can be derived as in terms of the mass function as Definition 7.

Definition 720-**.** Let Θ be a finite and nonempty set which is called the frame of discernment, where A is the subset of Θ and m is the basic probability assignment function of the frame of discernment Θ. The belief function is a mapping $Bel(X)$ that satisfies $Bel(X) = \sum_{A \subseteq \mathrm{U}} m(A)$ and the plausibility is a mapping $Pl(X)$ that satisfies $Pl(X) = \sum_{A \cap X \neq \emptyset} m(A)$.

The Belief function $Bel(X)$ represents the true degree of trust for X while the plausibility function $Pl(X)$ indicates that it is no doubt with trust is not true for X. These two functions are based on the same belief structure are connected by the dual property, i.e. $Bel(X) = 1 - Pl(\sim X)$, where $\sim X$ is the complement of X. Also, the belief function can be defined by semi-additive measure as Definition 8.

Definition 820-**.** For the arbitrary subset 2^{Θ} of the frame of discernment, a mapping function $Bel: 2^{\Theta} \to [0,1]$ is belief function if it satisfies the following conditions.

(1) $Bel(\emptyset) = 0$,

(2) $Bel(\Theta) = 1$,

(3) $Bel(\cup_{i=1}^{m} X_i) \geq \sum_{\varphi \neq J(1,2,\cdots,m)} (-1)^{|U+1|} Bel(\cap_{i \in J} X_i)$

Theorem 2. Let $MGIOIS^{*\geq} = (U, AT^k, f^k, V^k)$ be an incomplete multi-granulation ordered information system which has I levels of granulations. For any $1 \leq k \leq I$, denote the subset $A^k \subseteq AT^k$. For every $X \subseteq U$, then, $Bel_{A^k}^{\geq}(X) = \frac{\left|\underline{R_{A^k}^{\geq}}(X)\right|}{|U|}$ and $Pl_{A^k}^{*\geq}(X) = \frac{\left|\overline{R_{A^k}^{*\geq}}(X)\right|}{|U|}$ are belief and plausibility function of the $k-th$ level of granulations, respectively, and the corresponding mass function is $m_{A^k}(X) = \begin{cases} \frac{\left|\{x \epsilon U \mid [x]_{AT^k}^{*\geq} = X\}\right|}{|U|} \\ 0, \quad where\ x \notin \mathrm{U} \end{cases}$.

Proof. According to Definition 6, we can derive that $m_{A^k}(X)$ is called mass function and then only need to demonstrate that $Bel_{A^k}^{*\geq}$ satisfies three conditions of Definition 8. From the basic Definition 7, we take $Bel_{A^k}^{*\geq}(\emptyset) = 0$ and $Bel_{A^k}^{*\geq}(U) = 1$, respectively. Next, prove the condition (3) of Definition 8. Considering a collection $x_1, x_2, \cdots, x_n \subseteq \mathrm{U}$, then, we have

$$\frac{\left|\underline{R_{A^k}^{*\geq}}(X_1 \cup X_2 \cup \cdots \cup X_n)\right|}{|U|} \geq \frac{\left|\underline{R_{A^k}^{*\geq}}(X_1) \cup \underline{R_{A^k}^{*\geq}}(X_2) \cup \cdots \cup \underline{R_{A^k}^{*\geq}}(X_n)\right|}{|U|}$$

$$= \sum_i \frac{\left|\underline{R_{A^k}^{*\geq}}(X_i)\right|}{|U|} - \sum_{i<j} \frac{\left|\underline{R_{A^k}^{*\geq}}(X_i) \cap \underline{R_{A^k}^{*\geq}}(X_j)\right|}{|U|} + \cdots + (-1)^{n+1} \frac{\left|\underline{R_{A^k}^{*\geq}}(X_1) \cap \underline{R_{A^k}^{*\geq}}(X_2) \cap \cdots \cap \underline{R_{A^k}^{*\geq}}(X_n)\right|}{|U|}$$

$$= \sum_i \frac{\left|\underline{R_{A^k}^{*\geq}}(X_i)\right|}{|U|} - \sum_{i<j} \frac{\left|\underline{R_{A^k}^{*\geq}}(X_i \cap X_j)\right|}{|U|} + \cdots + (-1)^{n+1} \frac{\left|\underline{R_{A^k}^{*\geq}}(X_1) \cap \underline{R_{A^k}^{*\geq}}(X_2) \cap \cdots \cap \underline{R_{A^k}^{*\geq}}(X_n)\right|}{|U|}.$$

Hence, $Bel_{A^k}^{*\geq}(X)$ is a belief function. And $Pl_{A^k}^{*\geq}(X)$ is also a plausibility function due to the duality of the belief and plausibility functions.

With Theorem 3, there exactly exists the corresponding belief structure of multi-granulation rough set and it can be derived the consistency of belief structure in different levels of granulations from Theorem 2 as follows.

Proposition 3. Let $MGIOIS^{*\geq} = (U, AT^k, f^k, V^k)$ be an incomplete multi-granulation ordered information system which has I levels of granulations. For any $1 \leq k \leq I$, denote the subset $A^k \subseteq AT^k$. For $x \subseteq U$, $Bel_{A^k}^{*\geq}(X)$ and $Pl_{A^k}^{*\geq}(X)$ are belief and plausibility functions, respectively, and $P(X)=\frac{|X|}{|U|}$. By the above analysis, we have the properties as follows.

(1) $Bel^{*\geq}_{A^I}(X)\leq Bel^{*\geq}_{A^{I-1}}(X)\leq\cdots\leq Bel^{*\geq}_{A^1}(X)\leq P(X)$,

(2) $P(X)\leq Pl^{*\geq}_{A^1}(X)\leq Pl^{*\geq}_{A^2}(X)\leq\cdots\leq Pl^{*\geq}_{A^I}(X)$.

2.2 Reduction in incomplete multi-granulation ordered information system

First, the positive region reduction, belief reduction and plausibility reduction are put forward in incomplete multi-granulation ordered information system.

Definition 9. Let $MGIOIS^{*\geq}=(U,AT^k,f^k,V^k)$ be an incomplete multi-granulation ordered information system which has I levels of granulations. For any $1\leq k\leq I$, denote the subset $A^k\subseteq AT^k$, and the positive region, belief and plausibility reduction are developed in the $k-th$ level of granulations of information system as follows.

(1) If there exists $U/R^{*\geq}_{A^k}=U/R^{*\geq}_{AT^k}$, then, assume that A^k is a consistent set. Furthermore, if any true subset of A^k is not a consistent set, then, A^k can be defined as the positive region reduction.

(2) If there exists $Bel^{*\geq}_{A^k}(X)=Bel^{*\geq}_{AT^k}(X)$, where for every $X\epsilon\, U/R^{*\geq}_{AT^k}$, then, assume that A^k is a belief consistent set. Furthermore, if any true subset of A^k is not a belief consistent set, then, A^k can be defined as belief reduction.

(3) If $Pl^{*\geq}_{A^k}(X)=Pl^{*\geq}_{AT^k}(X)$, where for every $X\epsilon\, U/R^{*\geq}_{AT^k}$, then, assume that A^k is a plausibility consistent set. Furthermore, if any true subset of A^k is not a plausibility consistent set, then, A^k can be defined as plausibility reduction.

Based on Definition 9, belief reduction and plausibility reduction are the minimal attribute set to keep the degree of belief and plausibility. Next, analyze the consistency of three ways of reduction in the same level of granulations.

Theorem 3. Let $MGIOIS^{*\geq}=(U,AT^k,f^k,V^k)$ be an incomplete multi-granulation ordered information system which has I levels of granulations. For any $1\leq k\leq I$, denote the subset $A^k\subseteq AT^k$, and we have some properties in the $k-th$ level of granulations of information system as follows.

(1) If A^k is the consistency set if and only if it is the belief consistency set.

(2) If A^k is the positive region reduction if and only if it is the belief reduction.

(3) If A^k is the consistency set if and only if it is the plausibility consistency set.

(4) If A^k is the positive region reduction if and only if it is the plausibility reduction.

Proof (1) Suppose that A^k is the consistency set in the $k-th$ level of granulations of information system. Then, we have $U/R^{*\geq}_{A^k}=U/R^{*\geq}_{AT^k}$.

Clearly, A^k is also the belief consistency set in the same level with Definition 9.

If A^k is the belief reduction in the $k-th$ level of granulations. Then, we have $Bel^{*\geq}_{A^k}(X)=Bel^{*\geq}_{AT^k}(X),X\epsilon\, U/R^{*\geq}_{AT^k}$ (1).

According to Eqs.(1), one obtains $Bel^{*\geq}_{A^k}\left([X]^{*\geq}_{AT^k}\right)=Bel^{*\geq}_{AT^k}\left([X]^{*\geq}_{AT^k}\right)$ (2),

According to Eqs.(2), one obtains $\frac{\left|\underline{R^{*\geq}_{A^k}}\left([X]^{*\geq}_{AT^k}\right)\right|}{|U|}=\frac{\left|\underline{R^{*\geq}_{AT^k}}\left([X]^{*\geq}_{AT^k}\right)\right|}{|U|}$ (3).

According to the conjunction of Theorem 2 and Eqs(3), we can obtain $\underline{R^{*\geq}_{A^k}}\left([X]^{*\geq}_{AT^k}\right)=$

$\underline{R^{*\geq}_{AT^k}}\left([X]^{*\geq}_{AT^k}\right),x\epsilon U.$

By the definition of the lower approximation from Definition 7,we can get the following relationship: $\left\{y\middle|[y]^{*\geq}_{A^k}\subseteq[X]^{*\geq}_{AT}\right\}=\left\{y\middle|[y]^{*\geq}_{AT^k}\subseteq[X]^{*\geq}_{AT}\right\}\leftrightarrow[X]^{*\geq}_{A^k}\subseteq[X]^{*\geq}_{AT}$.

Since $[X]_{AT}^{*\geq} \subseteq [X]_{A^k}^{*\geq}$, thus $[X]_{A^k}^{*\geq} = [X]_{AT}^{*\geq}$.

Consequently, A^k is the consistency set in the $k-th$ level of granulations of information system.

(2) Similar to the proof of (1), it can be proved.

(3) Suppose that A^k is the consistency set in the $k-th$ level of granulations of information system. Then, we have $U/R_{A^k}^{*\geq} = U/R_{AT^k}^{*\geq}$.

Clearly, A^k is also the plausibility consistency set in the same level with Definition 9.

(4) Similar to the proof of (3), it can be proved.

As demonstrated above, we obtain the consistency of several reduction in the same level, which means the positive region reduction is equivalent to belief reduction. And it is also proved that the positive region reduction and belief reduction are adequate condition for plausibility reduction. Next, analyze the transitivity of the above three reduction in different levels.

Theorem 4. Let $MGIOIS^{*\geq} = (U, AT^k, f^k, V^k)$ be an incomplete multi-granulation ordered information system which has I levels of granulations. For any $1 \leq k \leq I$, denote the subset $A^k \subseteq AT^k$ and we have some properties as follows.

(1) For any $x\epsilon U$, we can obtain $[X]_{A^k}^{*\geq} = [X]_{A^{k+1}}^{*\geq}$, where A^k is the positive region reduction in the $k-th$ level of granulations of information system. Then, we define that A^{k+1} is the positive region reduction in the $(k+1)-th$ level of granulations.

(2) Reversely, if for $x\epsilon U$,we can obtain $[X]_{A^k}^{*\geq} = [X]_{A^{k+1}}^{*\geq}$, where A^{k+1} is the positive region reduction in the $(k+1)-th$ level of granulations of information system. Then, we define that A^k is the positive region reduction in the $k-th$ level of granulations.

(3) Conversely, if for $x\epsilon U$, we can obtain $[X]_{A^k}^{*\geq} = [X]_{A^{k+1}}^{*\geq}$, where A^{k+1} is not the positive region reduction in the $(k+1)-th$ level of granulations of information system. Then, we define that A^k is not the positive region reduction in the $k-th$ level of granulations.

Proof (1) If A^k is the positive region reduction in the $k-th$ level of granulations of information system, then, we get $[X]_{A^k}^{*\geq} = [X]_{AT^k}^{*\geq}$, where $\forall x\epsilon U$.

By $[X]_{A^k}^{*\geq} \subseteq [X]_{A^{k+1}}^{*\geq}$, we can obtain $[X]_{A^{k+1}}^{*\geq} \subseteq [X]_{AT^k}^{*\geq}$.

And then, $[X]_{A^{k+1}}^{*\geq} = [X]_{AT^k}^{*\geq}$ can be verified.

Hence, we get the result that A^{k+1} is the positive region reduction in the $(k+1)-th$ level of granulations.

(2) Similar to the proof of (1), it can be proved.

(3) If A^{k+1} is not the positive region reduction in the $(k+1)-th$ level of granulations, then, we get $[X]_{A^k}^{*\geq} \subset [X]_{A^{k+1}}^{*\geq}$.

By $[X]_{AT^k}^{*\geq} \subseteq [X]_{AT^{k+1}}^{*\geq}$, we can obtain $[X]_{A^k}^{*\geq} = [X]_{A^{k+1}}^{*\geq}$.

And then, $[X]_{AT^k}^{*\geq} \subset [X]_{A^k}^{*\geq}$ can be derived.

Hence, we can get the result that A^k is not the positive region reduction in the $k-th$ level of granulations.

It is demonstrated in Theorem 4 that, when the condition $[X]_{A^k}^{*\geq} = [X]_{A^{k+1}}^{*\geq}$ where for every $x\epsilon U$ are satisfied, reducibility has bidirectional transitivity between different levels of granulations and non-reducibility has only unidirectional transitivity between different levels of granulations, i.e. if the higher level of granulations has non-reducibility, then the lower level has irreducibility, too. Whereas it is impossible to judge whether the higher level can be reduced if the lower level has irreducibility. On the above analysis, the following inference can be put forward.

(1) Suppose that A^i is the positive region reduction in the $i-th$ level of granulations and

for every $x \epsilon U$,we have $[X]^{*\geq}_{A^k} = [X]^{*\geq}_{A^{k+m}}$,where $m > 0, k + m \leq I$ and for every I, there exists $k \leq i \leq k + m$. Then, $A^k, A^{k+1}, \cdots, A^{k+m}$ are the positive region reduction in the $k - th$, $(k+1) - th, \ldots, (k+m) - th$ levels of granulations of information system, respectively. Specially, when $i = 1$, incomplete multi-granulation ordered information system completely can be positive reduced.

(2) Conversely, suppose that A^i is not the positive region reduction in the $i - th$ level of granulations and for every $x \epsilon U$, we have $[X]^{*\geq}_{A^k} = [X]^{*\geq}_{A^{k+m}}$,where $m > 0, k + m \leq I$ and for every I, there exists $k \leq i \leq k + m$. Then, $A^k, A^{k+1}, \cdots, A^{k+m}$ are not the positive region reduction in the $k - th$, $(k+1) - th, \ldots, (k+m) - th$ levels of granulations of information system, respectively. Specially, when $i = 1$, incomplete multi-granulation ordered information system completely can't be positive region reduced.

In order to describe the relationship among the reduction above conveniently, given the relationship figures as follows.

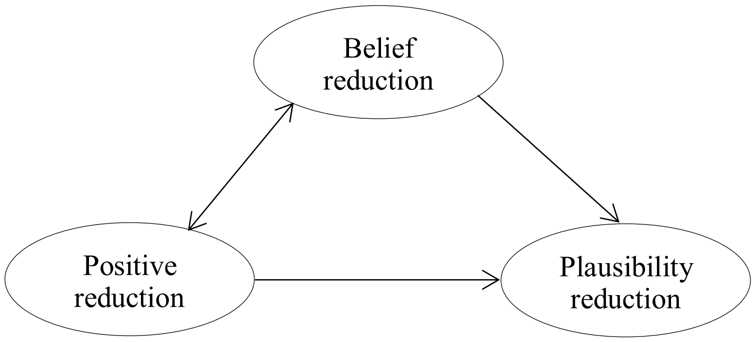

Fig.2 The consistency of several reductions in the same levels of granulations

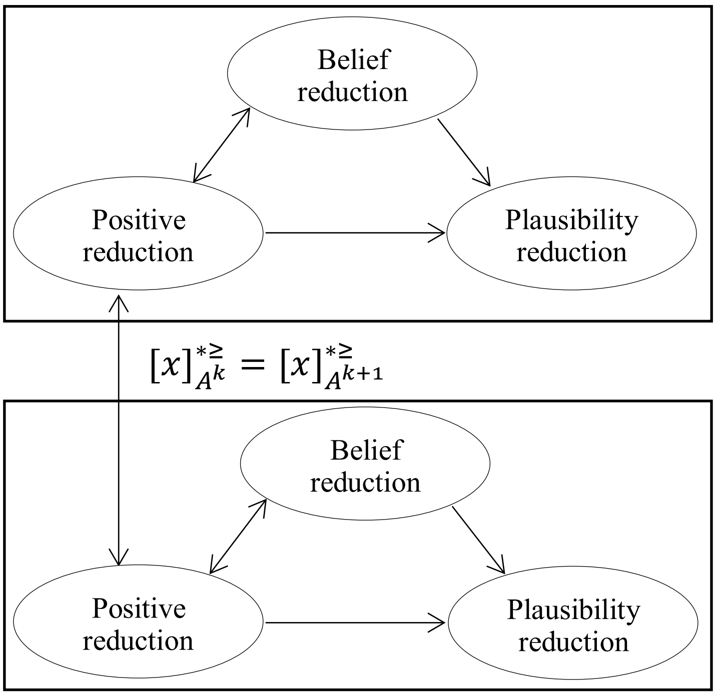

Fig.3 The transitivity of several reductions between k and k+1 level of granulations

Clearly, the Fig.2 is the consistency of several reductions in the same level of granulations. In this representation, each of nodes represents a kind of reduction and the unidirectional arrow is granted as a reduction of the end point from the starting point, i.e. the positive region reduction A must be the plausibility reduction at the mean time. The bidirectional arrow means the equivalent of the ends of the arrow. The Fig.3 represents the transitivity of several reductions in the different levels of granulations. When $[X]^{*\gtrless}_{A^k} = [X]^{*\gtrless}_{A^{k+1}}$ are satisfied, we can get that A is both the positive region reduction of the $k-th$ and $(k+1)-th$ levels of granulations.

3 Results and discussion

In this section, we will simulate numerically the relationship of three reductions of an incomplete multi-granulation ordered information system based on belief structure that is defined in Section 3, and give the notions of the significance to explain whether the corresponding attribute is dispensable or not. Based on this fact, we proposed an algorithm to find out reductions of an incomplete multi-granulation ordered information system.

3.1 Algorithm design

Definition 10. Let $MGIOIS^{*\geq} = (U, AT^k, f^k, V^k)$ be an incomplete multi-granulation ordered information system. If $AT^k = \{a_1^k, a_2^k, \cdots, a_m^k\}$, then, for every attribute $a_i^k \epsilon AT^k$, we define the significance degree of the attribute AT^k as follows.

$$sig_1(a_i^k, AT^k) = \frac{U}{R_{AT^k}} - \frac{U}{R_{AT^k-\{a_i^k\}}}$$

$$sig_2(a_i^k, AT^k) = Bel_{AT^k} - Bel_{AT-\{a_i^k\}}$$

$$sig_3(a_i^k, AT^k) = Pl_{AT^k} - Pl_{AT-\{a_i^k\}}$$

$sig_1(a_i^k, AT^k) > 0$ explains that the attribute a_i^k is not dispensable for AT^k and a_i^k should be included in positive region reduction. If $sig_1(a_i^k, AT^k) \leq 0$, it shows the attribute a_i^k is dispensable for AT^k and a_i^k should not be included in positive region reduction.

$sig_2(a_i^k, AT^k) > 0$ explains that the belief function of the attribute $a^k_{AT-\{a_i^k\}}$ is equal to AT^k, which means a_i^k should be included in belief reduction. If $sig_2(a_i^k, AT^k) \leq 0$, it shows the belief function of the attribute $a^k_{AT-\{a_i^k\}}$ is not equal to AT^k, and a_i^k should not be included in belief reduction. $sig_3(a_i^k, AT^k) > 0$ explains that the plausibility function of the attribute $a^k_{AT-\{a_i^k\}}$ is equal to AT^k, and a_i^k should be included in plausibility reduction. If $sig_1(a_i^k, AT^k) \leq 0$, it shows the plausibility function of the attribute $a^k_{AT-\{a_i^k\}}$ is equal to AT^k, and a_i^k should not be included in plausibility reduction.

The specific steps of the reduction will be given in the following. Since that it is same for reduction steps in different levels of granulations, just given the reduction process of a certain level of granulations as Algorithm 1 shows.

Algorithm 1 Reduction in incomplete multi-granulation ordered information system based on belief structure

Input: incomplete information system $IS^{*\geq} = (U, AT, f, V)$;

Output: let the positive region reduction be Red, belief reduction be $RedBel$, plausibility reduction be $RedPl$;

Step 1: let $Red = \phi, RedBel = \phi\ , RedPl = \phi$ and $AT` = AT$;

Step 2: according to Definition 9, calculate the positive region consistent set U/R_{AT}, belief consistent set Bel_{AT} and plausibility consistent set Pl_{AT} of the attribute AT;

Step 3: let $a_i \epsilon AT$, according to Definition 9, calculate the positive region consistent set $U/R_{AT-\{a_i\}}$, belief consistent set $Bel_{AT-\{a_i\}}$ and plausibility consistent set $Pl_{AT-\{a_i\}}$ of the attribute $AT - \{a_i\}$;

Step 4: let $sig_j(a_i, AT)$ be the significance and relative of the attribute a_i where $1 \leq |j| \leq 3$, $sig_1(a_i, AT) = U/R_{AT} - U/R_{AT-\{a_i\}}$, $sig_2(a_i, AT) = Bel_{AT} - Bel_{AT-\{a_i\}}$ and $sig_3(a_i, AT) = Pl_{AT} - Pl_{AT-\{a_i\}}$;

Step 5: if $sig_1(a_i, AT) > 0$, then suppose the attribute a_i is important, and add it into the positive region reduction set, and obtain $a_i \epsilon Red$, then go to Step 8, else go to Step 8 directly;

Step 6: if $sig_1(a_i, AT) > 0$, then suppose the attribute a_i is important, and add it into the belief reduction set, and obtain $a_i \epsilon Red_{Bel}$,then go to Step 8, else go to Step 8 directly;

Step 7: if $sig_1(a_i, AT) > 0$, then suppose the attribute a_i is important, and add it into the plausibility reduction set, and obtain $a_i \epsilon Red_{Pl}$, then go to Step 8, else go to Step 8 directly;

Step 8: let $AT` = AT` - \{a_i\}$, if $AT` = \ \phi$, then return step 8, else let $a_i = a_j$, where $i \neq j$, and return Step 3;

Step 9: output the positive region reduction Red, belief reduction Red_{Bel} and plausibility reduction Red_{Pl} as reduction.

Suppose that the size of U is n and the number of attributions is m, then, the time complexity of Algorithm 1 is $O(m * n)$. Table 5 shows the comparison of time complexity among different rough set measures, like covering rough set[25], traditional rough set[6] and multi-granulation rough set[18], illustrating that the calculating time can be reduced largely. And the following analysis of Example 1, which is analyze briefly in Section 2, is employed to illustrate our idea.

Table 5 The comparison of time complexity of different rough set measures

Measure	MGRS based on belief structure	MGRS	Covering rough set	Traditional rough set
Time complexity	$O(m * n)$	$O(n^2)$	$O(m^2 * n^2)$	$O(m * n^2)$

3.2 Algorithm implementation

Example 1.1. It is an incomplete multi-granulation ordered information system table of Rape pests detected by WSN in a certain period of time, where granularity $I = 3$, $U = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8\}$ and $AT^k = \{a_1^k, a_2^k, a_3^k, a_4^k\}$. $X_i (i = 1,2, \cdots 8)$ represents different clusters which stand for Cabbage butterfly, Aphids, Cabbage bug and Cricket, respectively. And AT^k is the attribute set where $k = 1,2,3$, which is the different levels of granulations. Since that it is same for reduction steps in different levels of granulations, just given the reduction process of the first level of granulations as Algorithm 1 shows.

(1) Let $Red = \phi, Red_{Bel} = \phi$, $Red_{Pl} = \phi$ and $AT` = AT$;

(2) According to Definition 9, calculate the positive region consistent set U/R_{AT^1}, belief consistent set Bel_{AT^1} and plausibility consistent set Pl_{AT^1} of the attribute AT^1.

$$\frac{U}{R_{AT^1}} = \{a_1^1, a_2^1, a_3^1, a_4^1\} \qquad Bel_{AT^1} = \{a_1^1, a_2^1, a_3^1, a_4^1\} \qquad Pl_{AT^1} = \{a_1^1, a_2^1, a_3^1, a_4^1\}$$

(3) Calculate the positive region consistent set $U/R_{AT^1-\{a_1^1\}}$, belief consistent set $Bel_{AT^1-\{a_1^1\}}$ and plausibility consistent set $Pl_{AT^1-\{a_1^1\}}$ of the attribute $AT^1 - \{a_1^1\}$. And we can get $[x]^{*\geq}_{AT^1-\{a_1^1\}}$ from Table 2 as follows.

$[x_1]^{*\geq}_{AT^1-\{a_1^1\}} =$

$\{x_1, x_2, x_8\}$

$[x_2]^{*\geq}_{AT^1-\{a_1^1\}} =$

$\{x_1, x_2, x_8\}$

$[x_3]^{*\geq}_{AT^1-\{a_1^1\}} =$

$\{U\}$

$[x_4]^{*\geq}_{AT^1-\{a_1^1\}} =$

$\{x_1, x_2, x_4, x_6, x_8\}$

$[x_5]^{*\geq}_{AT^1-\{a_1^1\}} =$

$\{x_1, x_2, x_4, x_5, x_6, x_8\}$

$[x_6]^{*\geq}_{AT^1-\{a_1^1\}} =$

$\{x_1, x_2, x_6, x_8\}$

$[x_7]^{*\geq}_{AT^1-\{a_1^1\}} =$

$\{x_1, x_2, x_6, x_7, x_8\}$

$[x_8]^{*\geq}_{AT^1-\{a_1^1\}}$

$= \{x_1, x_8\}$

Thus, $\frac{U}{R_{AT^1-\{a_1^1\}}} = \{a_1^1, a_2^1, a_3^1, a_4^1\}$.

For every $X \epsilon\, U/R^{*\geq}_{AT^k}$, suppose $X = \{x_1, x_8\}$, and the reduction of the lower and upper approximation of $AT^1 - \{a_1^1\}$ are $\underline{R^{*\geq}_{AT^1-\{a_1^1\}}}(X) = \{x_8\}$ and $\overline{R^{*\geq}_{AT^1-\{a_1^1\}}}(X) = \{U\}$, respectively. Additionally, the reduction of the lower and upper approximation of AT^1 are $\underline{R^{*\geq}_{AT^1}}(X) = \{x_1, x_8\}$ and $\overline{R^{*\geq}_{AT^1}}(X) = \{U\}$, respectively.

Hence, $Bel^{*\geq}_{AT^1-\{a_1^1\}}(X)=\frac{\left|\underline{R^{*\geq}_{AT^1-\{a_1^1\}}}(X)\right|}{|U|}=\frac{1}{8}\neq Bel^{*\geq}_{AT^1}(X), Pl^{*\geq}_{AT^1-\{a_1^1\}}(X)=\frac{\left|\overline{R^{*\geq}_{AT^1-\{a_1^1\}}}(X)\right|}{|U|}=$ $1=Pl^{*\geq}_{AT^1}(X)$.

Consequently, $Bel_{AT^1}=\{a_1^1,a_2^1,a_3^1,a_4^1\}, Pl_{AT^1}=\{a_2^1,a_3^1,a_4^1\}$.

(4) Thus, we get $sig_1(a_1^1,AT^1)=\frac{U}{R_{AT^1}}-\frac{U}{R_{AT^1-\{a_1^1\}}}=\{\emptyset\}$, $sig_2(a_1^1,AT^1)=Bel_{AT^1}-Bel_{AT^1-\{a_1^1\}}=\{\emptyset\}$ and $sig_3(a_1^1,AT^1)=Pl_{AT^1}-Pl_{AT^1-\{a_1^1\}}=\{a_1^1\}$. Then, add the attribute a_1 into plausibility reduction Red_{Pl}.

(5) Similarly, $Bel^{*\geq}_{AT^1-\{a_2^1\}}(X)=\frac{1}{4}, Pl^{*\geq}_{AT^1-\{a_2^1\}}(X)=1, sig_1(a_2^1,AT^1)=\frac{U}{R_{AT^1}}-\frac{U}{R_{AT^1-\{a_2^1\}}}=\{a_2^1\}, sig_2(a_2^1,AT^1)=Bel_{AT^1}-Bel_{AT^1-\{a_2^1\}}=\{a_2^1\}$ and $sig_3(a_2^1,AT^1)=Pl_{AT^1}-Pl_{AT^1-\{a_2^1\}}=\{a_2^1\}$, then, add a_2 into $Red, Red_{Bel,}Red_{Pl}$.

$Bel^{*\geq}_{AT^1-\{a_3^1\}}(X)=\frac{1}{8}, Pl^{*\geq}_{AT^1-\{a_3^1\}}(X)=1, sig_1(a_3^1,AT^1)=\frac{U}{R_{AT^1}}-\frac{U}{R_{AT^1-\{a_3^1\}}}=\{\emptyset\}, sig_2(a_3^1,AT^1)=Bel_{AT^1}-Bel_{AT^1-\{a_3^1\}}=\{\emptyset\}$ and $sig_3(a_3^1,AT^1)=Pl_{AT^1}-Pl_{AT^1-\{a_3^1\}}=\{a_3^1\}$, then, add a_3 into Red_{Pl}.

$Bel^{*\geq}_{AT^1-\{a_4^1\}}(X)=\frac{1}{4}, Pl^{*\geq}_{AT^1-\{a_4^1\}}(X)=1, sig_1(a_4^1,AT^1)=\frac{U}{R_{AT^1}}-\frac{U}{R_{AT^1-\{a_4^1\}}}=\{a_4^1\}, sig_2(a_4^1,AT^1)=Bel_{AT^1}-Bel_{AT^1-\{a_4^1\}}=\{a_4^1\}$ and $sig_3(a_4^1,AT^1)=Pl_{AT^1}-Pl_{AT^1-\{a_4^1\}}=\{a_4^1\}$, then, add a_4 into $Red, Red_{Bel,}Red_{Pl}$.

From the analysis of Table 1 from Example 1, we can obtain that $A^1=\{a_2^1\ ,a_4^1\}$ is the reduction of AT^1 in the first level of granulations and $A^2=\{a_2^2\ ,a_4^2\}$ is the reduction of AT^2 in the second level of granulations where for every $x\epsilon U$, there exists $[X]^{*\geq}_{A^1}=[X]^{*\geq}_{A^2}$. However, $A^3=\{a_2^3\ ,a_4^3\}$ is not the reduction of AT^3 in the third level of granulations. This result does prove the idea that the positive reduction and belief reduction are equivalent, and these two reductions is unnecessary and sufficient condition for plausibility reduction in the same level, and if cover structure order of different level are the same, the corresponding equivalent positive region reduction. In this example, data redundancy reduced by 50% by attribute reduction based on belief structure, which largely improves the information processing capability of sensor nodes.

4 Conclusion

Considering the issue with respect to the high data redundancy and high cost of information collection in wireless sensor nodes, this paper proposes a data fusion method based on belief structure to reduce attribution in multi-granulation rough set. In this paper, the relationship between multi-granulation rough set based on multi-scale and evidence theory is explored to analyze that the former has its corresponding belief structure when certain conditions are satisfied. Then, by introducing evidence structure, the positive region reduction, belief reduction and plausibility reduction are put forward, and analyze the consistency in the same level and transitivity in the different level by means of information system. Meanwhile, the algorithm proposed in this paper to perform three reductions reduces the time complexity largely, improving the computing efficiency. In essence, this paper not only successfully solves the problem of sensor node data fusion, but also provides new ideas and methods for the research in incomplete multi-granulation ordered rough set theory.

However, one of the drawbacks of the proposed algorithm is that it requires attribute discretization and it will lose efficacy when the reducible attributes are few. And the future work will concentrate on the synthesis of multi-granulation ordered information system evidence under the belief space with respect to data fusion in wireless sensor networks.

Abbreviations

WSN: Wireless sensor network

Eqs: Equally spaced

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Author's information

[1]School of Information and Intelligence Science & Technology, Hunan Agricultural University, Changsha 410128, China

[2]School of Computer and Communications Engineering, Changsha University of Science & Technology, Changsha 410114, China

[3]Department of Computer Science Hunan City University, Yiyang 413000, China

Authors' Contributions

Chenfeng Long is main author of the current paper. Chenfeng Long contributed to the development of the idea, design of the study, theory, result analysis, and article writing. Xingxin Liu was responsible for the reducing attribute, processing the algorithm and drawing figures. Yakun Yang and Tao Zhang were responsible for collecting data. Siqiao Tan participated in the design of the study. Xiaoyong Tang helped to draft the manuscript. Gelan Yang participated in the result analysis. The authors read and approved the final manuscript.

Availability of data and materials

The datasets used and/or analyzed during the current study are available from the corresponding author on reasonable request.

Ethics approval and consent to participate

This article does not contain any studies with human participants or animals performed by any of the authors.

Consent for publication

All authors agree to submit this version and claim that no part of this manuscript has been published or submitted elsewhere.

Competing interests

The authors declare that they have no competing interests.

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Figures Legend

Fig.1 shows the hierarchical structure of WSN. With localized principle, sensor nodes only interact with their neighbors within a limited range, which is called the cluster, and locally integrate the data to simplify knowledge representation to a certain extent. The cluster header can relay the data from the others. In addition, one of the subsets of all the cluster headers is responsible for interacting with the sink node, sending aggregate data and fusing global data to realize the overall optimization of sensor network load, energy consumption and information processing.

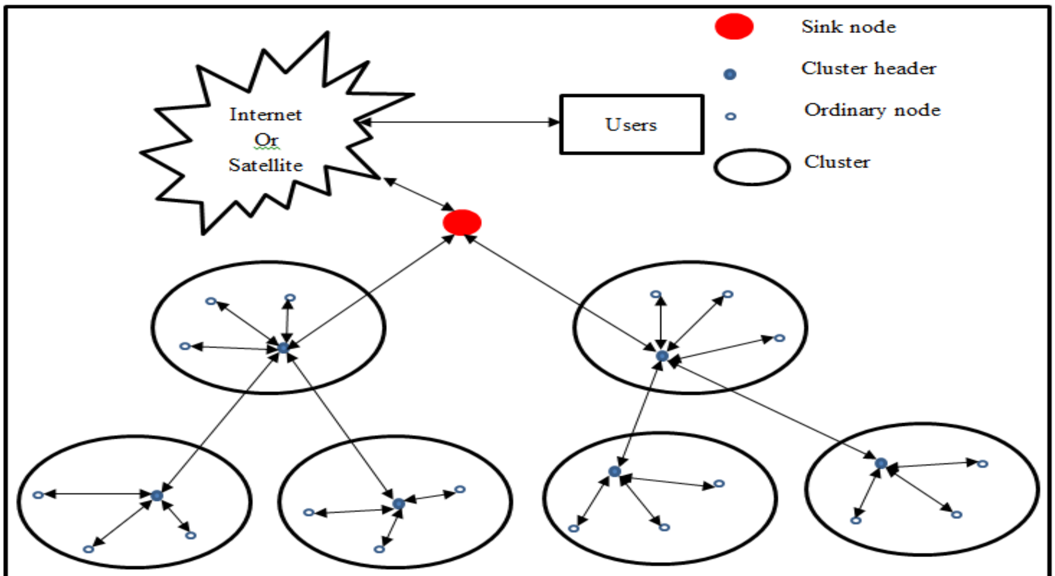

Fig.1 The hierarchical structure of WSN

The Fig.2 is the consistency of several reductions in the same level of granulations. Each of nodes represents a kind of reduction and the unidirectional arrow is granted as a reduction of the end point from the starting point. Fig.2 shows the positive region reduction and belief reduction are equivalent, and the positive region reduction and belief reduction is unnecessary and sufficient condition to the plausibility reduction in the same level.

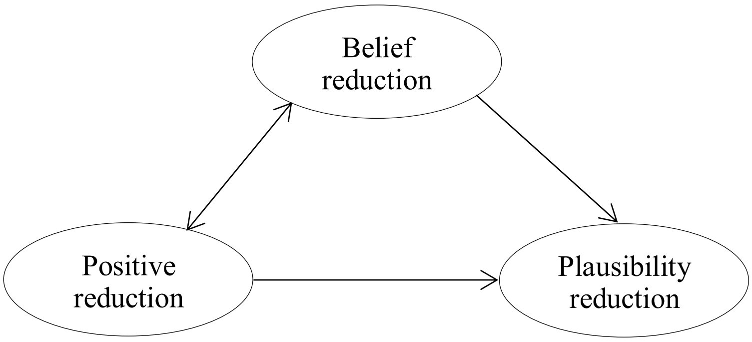

Fig.2 The consistency of several reductions in the same levels of granulations

The Fig.3 represents the transitivity of several reductions between k and k+1 level of granulations. The nodes and arrows are the same as Fig.2. Fig.3 shows if the cover structure order of different levels are the same, the corresponding equivalent positive region reduction.

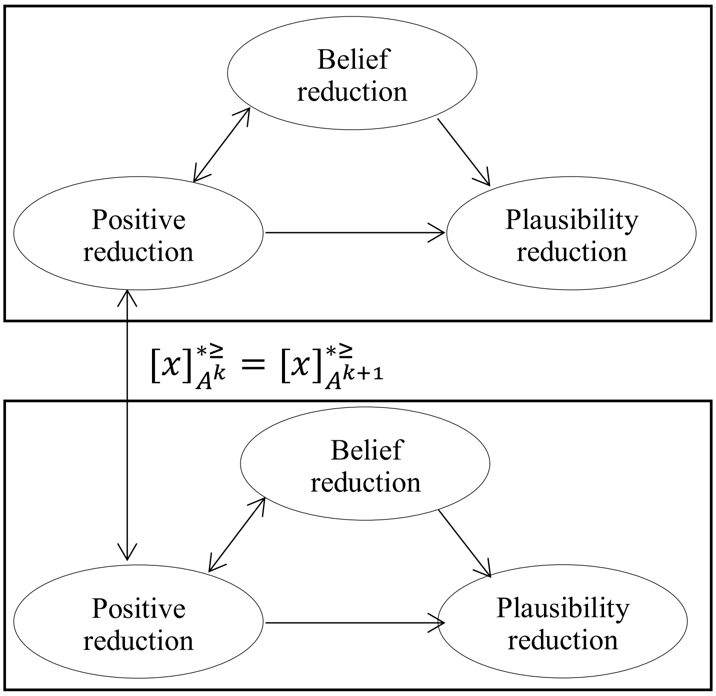

Fig.3 The transitivity of several reductions between k and k+1 level of granulations

Figures

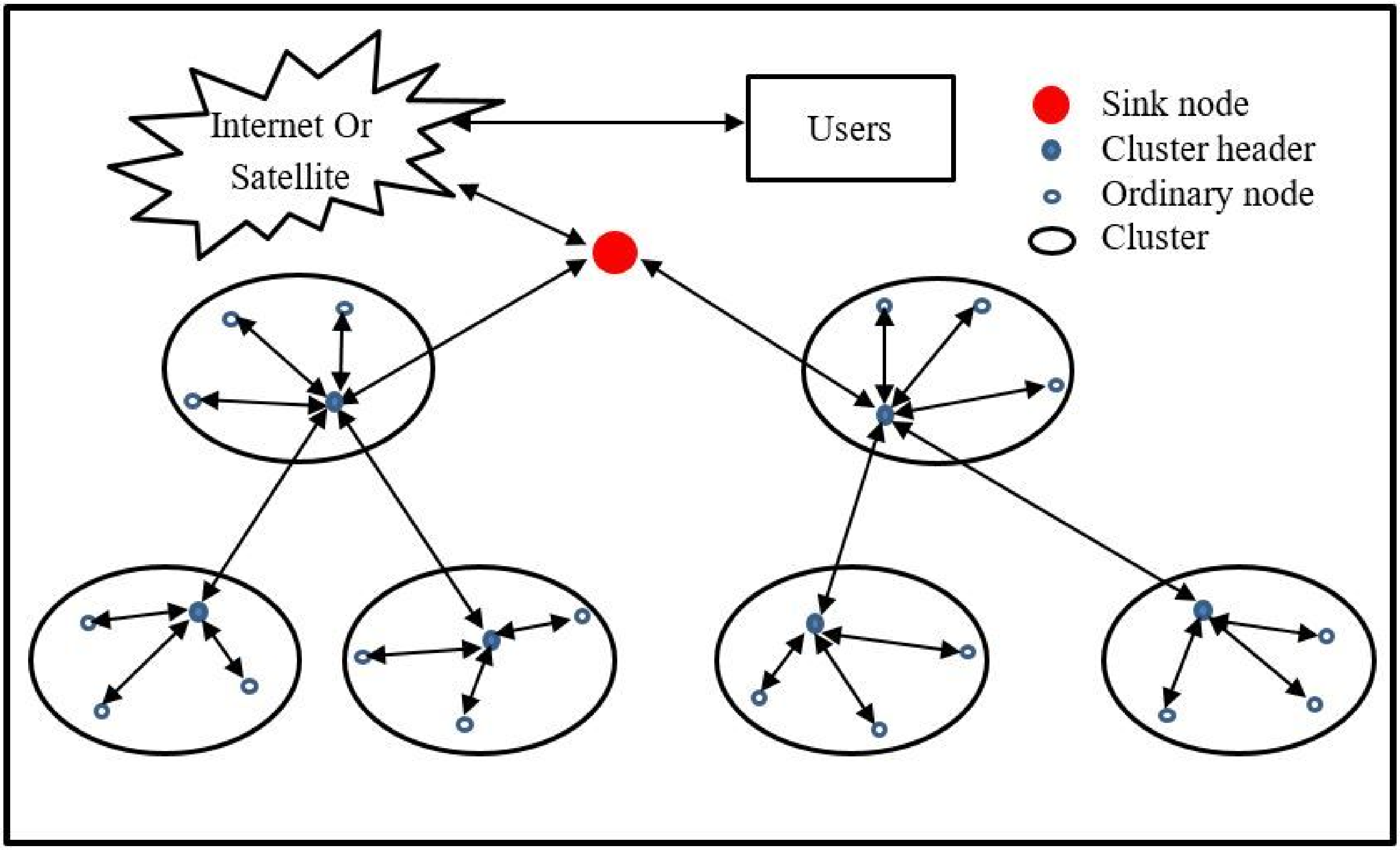

Figure 1

The hierarchical structure of WSN. Fig.1 shows the hierarchical structure of WSN. With localized principle, sensor nodes only interact with their neighbors within a limited range, which is called the cluster, and locally integrate the data to simplify knowledge representation to a certain extent. The cluster header can relay the data from the others. In addition, one of the subsets of all the cluster headers is responsible for interacting with the sink node, sending aggregate data and fusing global data to realize the overall optimization of sensor network load, energy consumption and information processing.

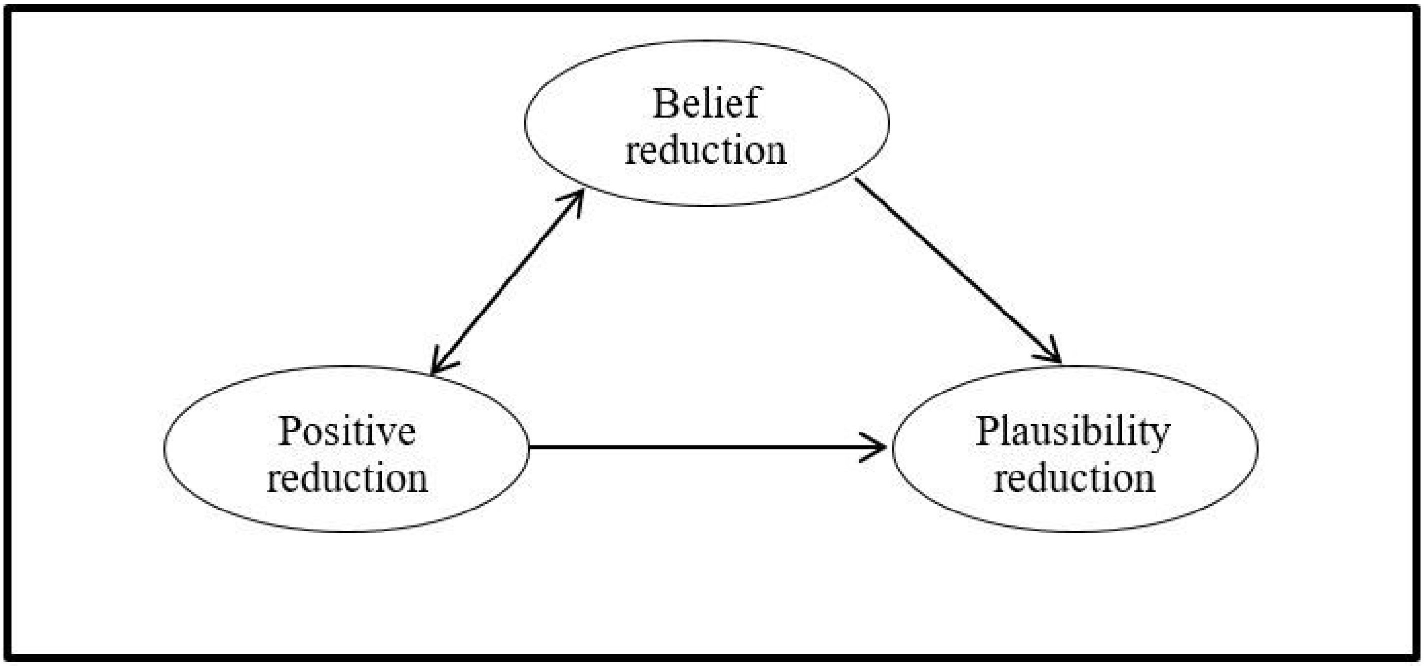

Figure 2

The consistency of several reductions in the same levels of granulations. The Fig.2 is the consistency of several reductions in the same level of granulations. Each of nodes represents a kind of reduction and the unidirectional arrow is granted as a reduction of the end point from the starting point. Fig.2 shows the positive region reduction and belief reduction are equivalent, and the positive region reduction and belief reduction is unnecessary and sufficient condition to the plausibility reduction in the same level.

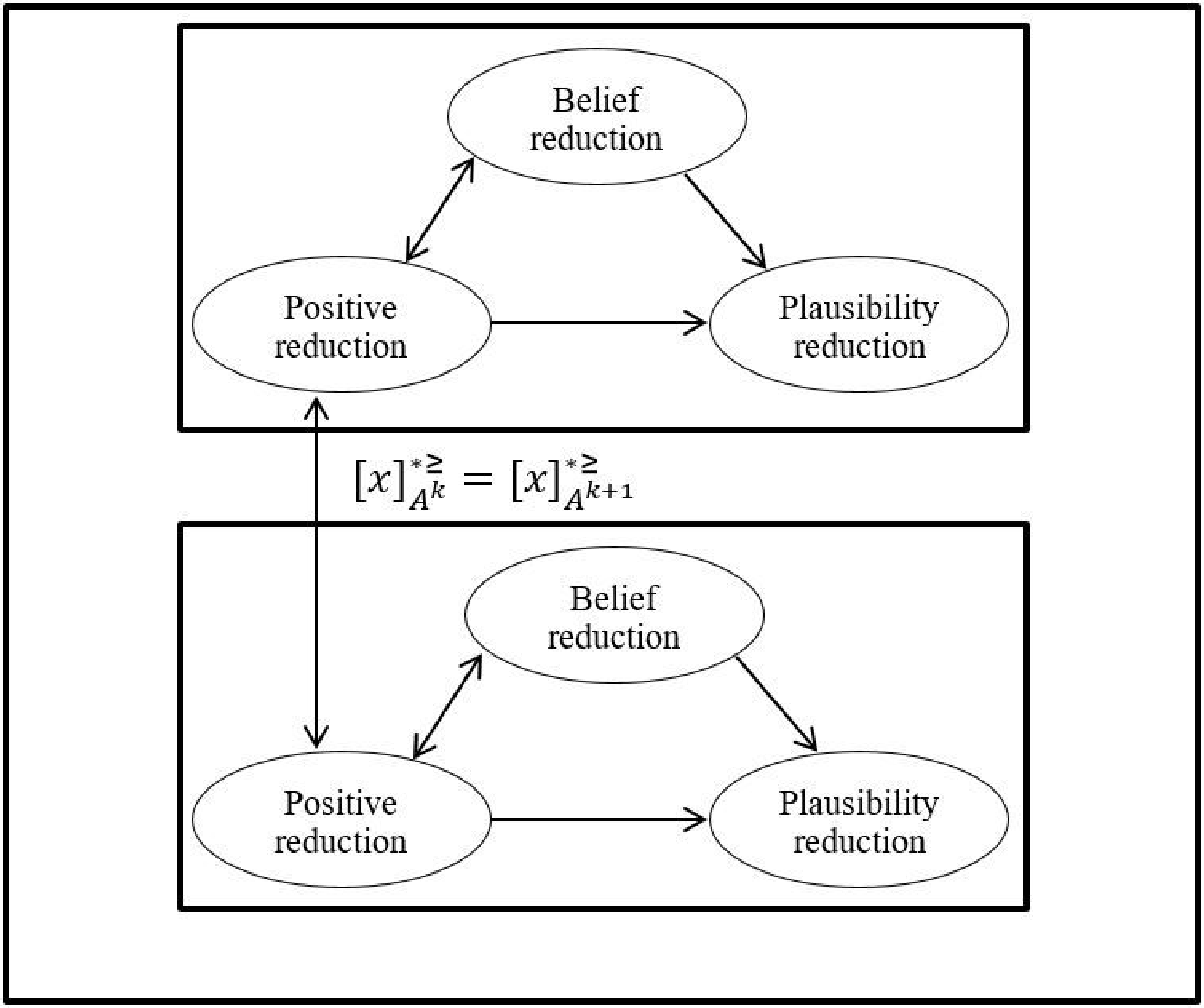

Figure 3

The transitivity of several reductions between k and k+1 level of granulations. The Fig.3 represents the transitivity of several reductions between k and k+1 level of granulations. The nodes and arrows are the same as Fig.2. Fig.3 shows if the cover structure order of different levels are the same, the corresponding equivalent positive region reduction.