

# A Geometry-Based Deep Learning Feature Extraction Scheme for Airfoils

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## Research

**Keywords:** Airfoils, Manifold, Geometry-feature fusion, GF encoder

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## RESEARCH

# A Geometry-based Deep Learning Feature Extraction Scheme for Airfoils

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## Abstract

The perception of geometry-features of airfoils is the basis in aerodynamic area for performance prediction, parameterization, aircraft inverse design, etc. There are three approaches to percept the geometric shape of an airfoil, namely manual design of airfoil geometry parameter, polynomial definition and deep learning. The first two methods can directly extract geometry-features of airfoils or polynomial equations of airfoil curves, but the number of features extracted is limited. While deep learning algorithms can extract a large number of potential features (called latent features), however, the features extracted by deep learning are lacking of explicit geometrical meaning. Motivated by the advantages of polynomial definition and deep learning, we propose a geometry-based deep learning feature extraction scheme (named Bézier-based feature extraction, BFE) for airfoils, which consists of two parts: manifold metric feature extraction and geometry-feature fusion encoder (GF encoder). Manifold metric feature extraction, with the help of the Bézier curve, captures features from tangent space of airfoil curves, and GF encoder combines airfoil coordinate data and manifold metrics together to form a novel feature representation. A public UIUC airfoil dataset is used to verify the proposed BFE. Compared with classic Auto-Encoder, the mean square error (MSE) of BFE is reduced by 17.97% ~ 29.14%.

**Keywords:** Airfoils; Manifold; Geometry-feature fusion; GF encoder

## Introduction

The geometry shape of airfoils greatly affects the aerodynamic performance, parameterization as well as aircraft inverse design [1, 2]. One of a traditional airfoil design method is to define airfoil geometry parameters manually, which is effective to perceive the variations in airfoil geometry [3]. However, this approach always fails on complex airfoils, that causes limited applications. Polynomial definition is an alternative efficient mathematical approach to approximate airfoil curves, for examples, Bézier curve [4] and B-spline [5], etc. These approaches usually employ linear combinations of high degree polynomials to approximate airfoils, which to some extent describes the geometry variation of airfoil structures. Nevertheless, these polynomial approaches can only give approximate expressions of airfoils and they are not powerful enough to exploit features of airfoils deeply from different aspects. Therefore it is difficult for polynomials to describe airfoils comprehensively.

In recent years, deep learning has achieved great success in feature extraction, for example Auto-Encoder [6], generative adversarial networks (GANs)[7, 8], convolution neural networks (CNNs)[9] and multi-task learning [10], etc. These methods

conventionally take the coordinates  $(X, Y)$  or pictures of airfoils as input. The problems existed in deep learning are that the extracted features are lacking of explicit geometrical meaning and some latent geometry-features cannot be further explored.

Motivated by deep learning and inspired by the Bézier theory, we propose a geometric-based deep learning feature extraction scheme (named Bézier-based feature extraction, BFE) for airfoils. The BFE consists of two parts, namely manifold metric feature extraction and geometry-feature fusion encoder (GF encoder). In manifold metric feature extraction, a 3-degree Bézier curve is employed to build an Bézier manifold from airfoil coordinates [11]. Manifold metric, as a geometry-feature, is then calculated with the inner-product of two vectors from the airfoil manifold tangent space. The GF encoder is composed of three components: an encoder, a novel feature representation and a Decoder. The encoder consists of a CNN which take airfoil coordinate data as input and a fully connected network (FCN) which take manifold metrics as input. Airfoil coordinate data and manifold metrics are encoded into a feature representation that is taken as input of the decoder. The decoder consists of a deconvolution neural network (DeCNN) and another FCN. The output of the DeCNN is generated airfoil coordinate data, and the output of FCN in this decoder is generated manifold metrics. The feature representation is a fusion feature that represents the geometry of an airfoil.

To summarize, the contributions to our work are:

- 1 We prove that the Bezier curve, as also a smooth manifold, its manifold metric can be used as a proper geometry-feature for representing airfoils;
- 2 we propose a geometry-based deep learning feature extraction scheme named BFE to extract geometry-features from both Euclidean space and manifold space and then fuse these geometry-features together.
- 3 we propose a geometry-feature fusion encoder (GF encoder) to integrate airfoil coordinate data and manifold metric feature together to form a feature presentation;

The structure of the remainder of this paper is as follows. Related work section introduces the research status of polynomial definition and deep learning in the field of airfoil feature extraction. In methodology section, the details of our proposed BFE are elaborated. In experimental results section, a public UIUC airfoil dataset is applied to validate the effectiveness of BFE. The conclusions of our work are shown in conclusion section.

## Related Works

In this section, we introduce current research status on polynomial definition and deep learning in the field of airfoil feature extraction.

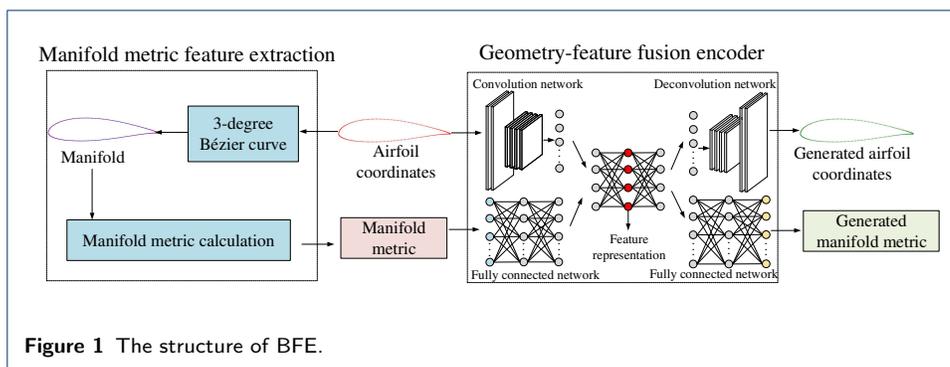
The Bézier curve, B-spline and NURBS are typical polynomials that were used to derive equations of airfoil curves [4, 5, 12]. Among them, the Bézier curve is the most basic and common expression. An airfoil usually consists of multiple control points  $\{x, y\}$  to a polynomial function. Usually, a  $n$ -degree Bézier curve that connects  $n + 1$  control points is chosen as basis to form a smooth curve which are used to approximate the airfoil function. Then the airfoil curve function can be described as a liner combination of the basis. These polynomial expressions mentioned above are flexible and they can be combined with other parameterization methods to describe

airfoil characteristics more accurately [5]. These polynomial approaches that are mathematically interpretable were widely used to parameterize airfoils. However, there have been some frontier studies which show that curves or surfaces of an airfoil exist in manifold space [13, 14]. Hence, existing polynomial approaches can only capture features from Euclidean space, some latent features e.g. geometry-features from manifold space are omitted.

Deep learning, as a data-driven model, has made grate progress in feature extraction of airfoils. Chen et al. [15] adopted the info-GAN to generate massive airfoils. In their work, Chen established corresponding relations between three latent codes in the info-GAN and design parameters of airfoils, and the info-GAN was controlled to generate specific airfoils by adjusting latent codes. Yilmaz et al. [16] explored the relationship between NACA 63(2)-615 airfoil and its pressure distribution. With the help of the local feature perception ability of CNNs, Yilmaz constructed a deep learning model to predict the pressure at each airfoil coordinate. Jing et al. [17] combined an Auto-Encoder and a GAN together to generate target wall Mach distributions for the inverse design that matches locations of suction peak, shock and aft loading. The existing studies mostly take airfoil coordinates or pictures of airfoils as inputs, and the features extracted by them can not be explained clearly.

Different from existing studies, the proposed BFE addresses the problems mentioned above. On one hand, with the help of Bézier manifold, manifold metric feature extracted from airfoil tangent space is applied to reflect latent geometry-feature of an airfoil. On the other hand, with the help of deep learning, airfoil coordinate data and manifold metrics are fused to form a feature representation. Because that airfoil coordinates are physically interpretable and manifold metric is mathematically interpretable, BFE, to some extent, improves the geometrical meaning of feature representation.

### Methodology



#### Overview

Fig.1 describes the structure of BFE which consists of two parts: manifold metric feature extraction and GF encoder. In the manifold metric feature extraction part, a 3-degree Bézier curve is chosen as the basis, then an airfoil shape can be described as a Bézier manifold that are composed of multiple basis. A manifold metric calculation module is responsible for calculating the inner-product of two vectors from the

airfoil manifold tangent space. In the GF encoder part, the airfoil coordinate data and manifold metrics are all encoded into a feature presentation, based on which the generated airfoil coordinate data and the corresponding generated manifold metrics are exported. The feature presentation is the final output that can express geometry-features of an airfoil from both Euclidean and manifold space. In this section, we will introduce manifold metric feature extraction and GF encoder separately.

### The Manifold Metric Extraction

Given an 2D airfoil coordinate set  $D = \{P_i = (x_i, y_i) | i = 1, 2, \dots, M\}$ , where  $M$  denotes the number of coordinate points. A 3-degree Bézier curve can be built:

$$r(D; t) = \sum_{i=0}^n P_i B_{i,n}(t), t \in [0, 1], n = 3 \quad (1)$$

where  $r(D; t)$  is the function of Bézier curve,  $D$  denotes the sample space where the airfoil coordinates located,  $t$  is the parameter of this Bézier curve,  $n$  is the degree of Bézier curve and  $B_{i,n}(t)$  denotes the coefficient which satisfies:

$$B_{i,n}(t) = C_n^i t^i (1-t)^{n-i} = \frac{n!}{i!(n-i)!} t^i (1-t)^{n-i} [i = 0, 1, \dots, n] \quad (2)$$

First, we prove that the Bézier curve constructed in Formula.(1) forms a smooth Bézier manifold. We introduce the lemma of manifold [18, 19]:

**Lemma 1** *Let  $\mathcal{M}$  be a non-empty Hausdorff space.  $\mathcal{M}$  is called an  $m$ -dimensional topological manifold if, for every point  $P \in \mathcal{M}$ , there exists an open neighborhood  $U$  of the point  $P$  that satisfied  $U \subset \mathcal{M}$  and a homeomorphic  $\phi : U \rightarrow \mathcal{R}^m$  from  $U$  to an open set of the  $m$  dimensional Euclidean space  $\mathcal{R}^m$ .*

*Proof* Because that the space  $\mathcal{M}$  where a Bézier curve  $r(D; t)$  located is a subset of  $\mathcal{R}$ , i.e.  $\mathcal{M} \subset \mathcal{R}$ , then  $\exists \mathcal{T}$ , s.t.  $(r, \mathcal{T})$  constitutes a topological space (i.e. Hausdorff space). This is the prerequisite of Lemma 1.

Let the Euclidean space where  $D = \{P_i = (x_i, y_i) | i = 1, 2, \dots, M\}$  located be  $\mathcal{R}^2$ . Because of the smoothness of  $r(D; t)$ , for  $\forall t_0 \in r(D; t)$ , there must exists an open neighborhood  $U$  s.t.  $t_0 \in U$ . For  $\forall t_1, t_2 \in U$ , and  $t_1 \neq t_2$ ,  $\exists P_1 \in \mathcal{R}^2$  s.t.  $r : t_1 \rightarrow P_1$  and  $\exists P_2 \in \mathcal{R}^2$  s.t.  $r : t_2 \rightarrow P_2$ .

Suppose that  $P_1 = P_2$ , then  $P_1 = r(D; t_1)$  and  $P_2 = r(D; t_2)$ , therefore  $r(D; t_1) = r(D; t_2)$  which contradicts the functional properties of  $r(D; t)$ , i.e.  $P_1 \neq P_2$ .  $r(D; t)$  is a homeomorphic.

According to Lemma 1, there exists a homeomorphic  $r(D; t) : U \rightarrow \mathcal{R}^2$ , s.t. the space  $\mathcal{M}$  is a manifold, we call it Bézier manifold.  $\square$

The construction of Bézier manifold build a map between 2D airfoil coordinates space  $\mathcal{R}^2 = \{x_i \in \mathcal{R}, y_i \in \mathcal{R}\}$  and a new 1D manifold space  $\mathcal{M}$  with parameter  $t$ , which is an efficient approach: on one hand, a Bézier manifold can sense the airfoil

curve by controlling parameter  $t$ ; on the other hand, the Bézier manifold can be used to calculate the manifold metric feature at each  $t$  conveniently.

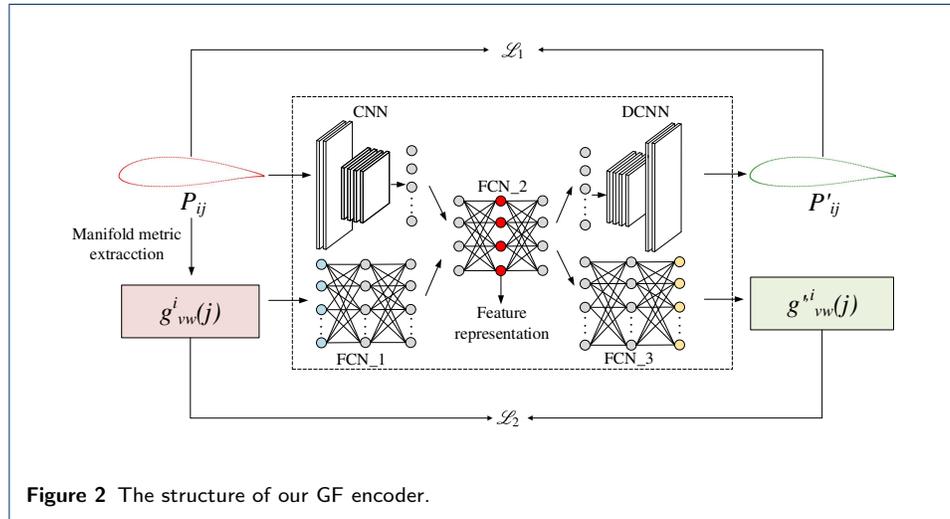
Then, the geometry-feature called manifold metric is extracted by [11]:

$$g_{vw}(t) = \partial_v r(D; t) \partial_w r(D; t) \quad (3)$$

where  $g_{vw}(t)$  is the manifold metric of an airfoil manifold at point  $t$ ,  $\partial_v = \frac{\partial}{\partial t^v}$  denotes directions of partial derivative. Considering that  $r(D; t)$  is an 1D manifold, therefore,  $v = w$ :

$$g_{vw}(t) = g_{vv}(t) = (\partial_v r(D; t))^2 = g_{ww}(t) \quad (4)$$

The manifold metric feature  $g_{vw}(t)$  denotes the inner-product of two vectors in the tangent space  $\frac{\partial}{\partial t^v}$  of  $r(D; t)$ . The two vectors that make up this inner-product can be regarded as the basis of any vector in the tangent space of  $r(D; t)$ . As a result,  $g_{vw}(t)$  is chosen as geometry-feature that represents the geometric characteristics of the tangent space of airfoil curves.

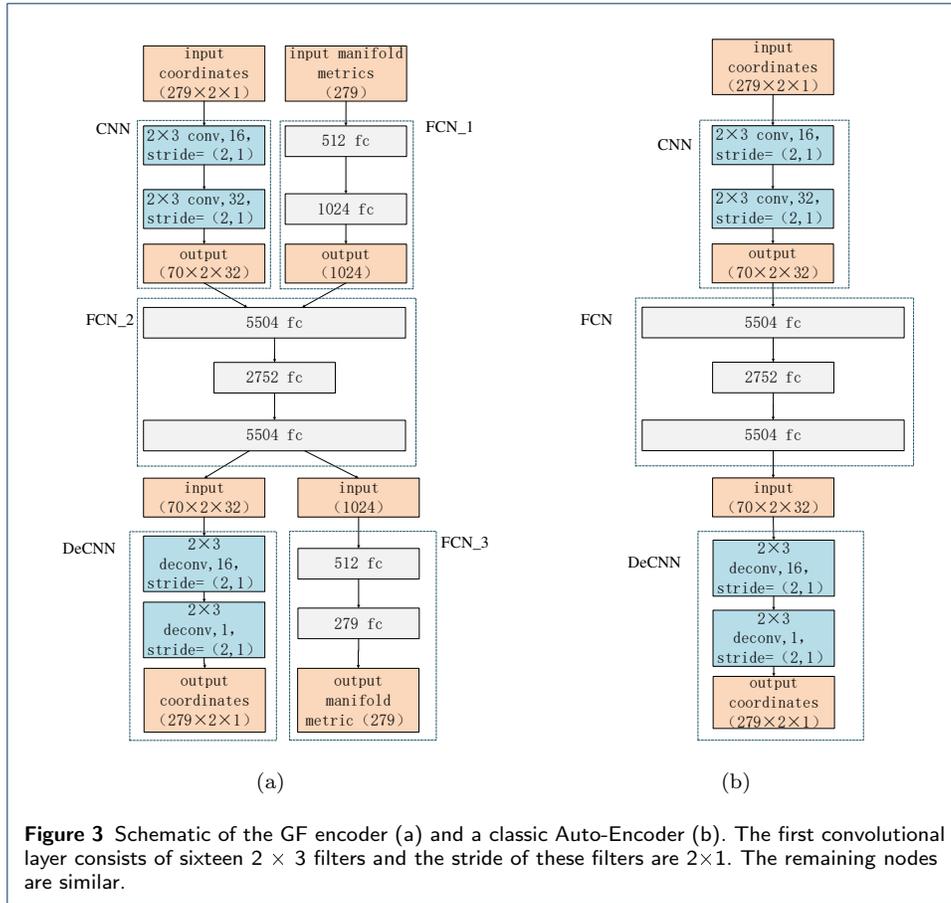


**Figure 2** The structure of our GF encoder.

### The GF Encoder

The aim of our proposed GF encoder is to fuse airfoil coordinate data and manifold metrics together to form a feature presentation. GF encoder (Fig.2) is composed of three components: an Encoder, a feature representation and a Decoder. The Encoder consists of a CNN which take airfoil coordinate data  $P_{ij}$  as input and a FCN (called FCN\_1) which take manifold metric  $g^i_{vw}(j)$  as input. The feature representation (called FCN\_2) combines the output from CNN and FCN\_1 together to express geometry-features. The Decoder consists of a deconvolution neural network (called DeCNN) and another FCN (called FCN\_3). The output of the DeCNN is generated airfoil coordinate data  $P'_{ij}$ , and the output of FCN\_3 is generated manifold metric  $g'^i_{vw}(j)$ .

The loss function of GF encoder is:



**Figure 3** Schematic of the GF encoder (a) and a classic Auto-Encoder (b). The first convolutional layer consists of sixteen  $2 \times 3$  filters and the stride of these filters are  $2 \times 1$ . The remaining nodes are similar.

$$\begin{cases} \mathcal{L} = \mathcal{L}_1 + \mathcal{L}_2 \\ \mathcal{L}_1 = \frac{1}{NM} \sum_{i=1}^N \sum_{j=1}^M (P_{ij} - P'_{ij})^2 \\ \mathcal{L}_2 = \frac{1}{NM'} \sum_{i=1}^N \sum_{j=1}^{M'} (g_{vw}^i(j) - g'^i_{vw}(j))^2 \end{cases} \quad (5)$$

where  $N$  is the number of input data,  $M$  is the number of coordinate points of an airfoil,  $M'$  is the number of manifold metrics of an airfoil,  $P_{ij} = (x_{ij}, y_{ij})$  denotes the  $j$ th coordinate data for the  $i$ th input airfoil,  $P'_{ij}$  denotes the exported coordinate data,  $g_{vw}^i(j)$  is manifold metric of the  $i$ th airfoil at  $t = j$  and  $g'^i_{vw}(j)$  denotes the exported manifold metric.

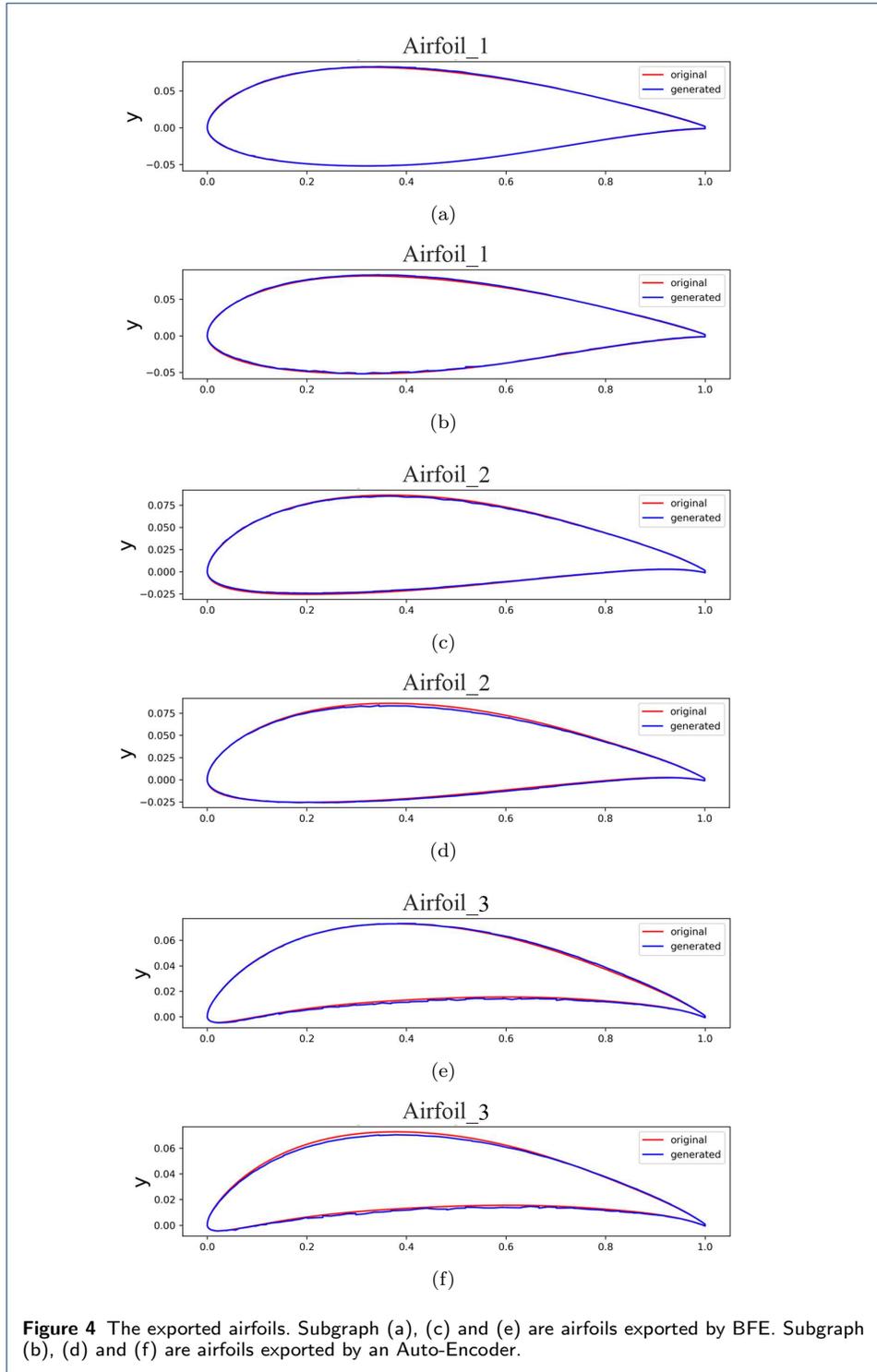
## Experimental Results

### Dataset and model hyper parameters

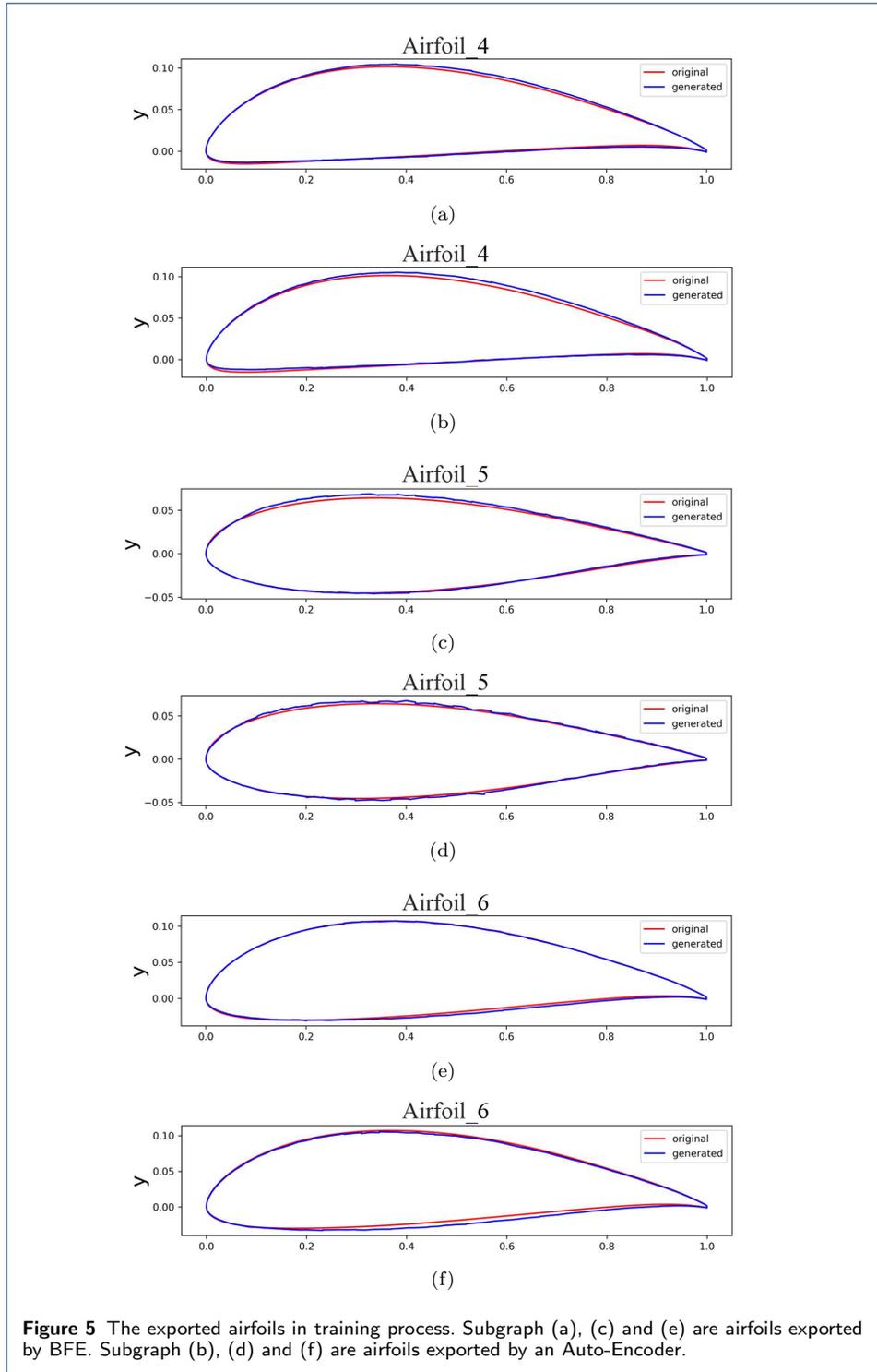
The public UIUC database <sup>[1]</sup> is used to verify the feasibility of proposed BFE. This database provides more than 1500 real airfoils, each of which is discretized by 2D coordinates.

Fig.3 describes the structure of GF encoder and a classic Auto-Encoder. The mean square error (MSE) was chosen to measure the distance between input airfoil

<sup>[1]</sup>[http://m-selig.ae.illinois.edu/adscoord\\_database.html](http://m-selig.ae.illinois.edu/adscoord_database.html)



coordinates and the output ones, see Formula (6). In this Formula,  $P_i$  denotes the input airfoil coordinates and  $P'_i$  denotes the exported airfoil coordinates.



$$MSE = \frac{1}{N} \sum_{i=1}^N (P_i - P'_i)^2 \quad (6)$$

## Results

The experimental MSE are shown in Tab.1. In this table, three typical airfoils are analyzed. We can see that the MSE between input airfoils and airfoils exported by GF encoder is smaller than that of the classic Auto-Encoder. In terms of specific errors, the MSE of GF is reduced by 17.97% ~ 29.14% compared with a classic Auto-Encoder.

**Table 1** The MSE of three typical airfoils decoded by our method and the Auto-Encoder.

airfoils	models	MSE	MSE reduction percentage
airfoil_1	our method	1.91E-3	26.54%
	Auto-Encoder	2.60E-3	
airfoil_2	our method	1.05E-3	17.97%
	Auto-Encoder	1.28E-3	
airfoil_3	our method	1.07E-3	29.14%
	Auto-Encoder	1.51E-3	

Fig.4 illustrates airfoils exported by GF encoder and the classic Auto-Encoder. We can find that the airfoils exported by GF encoder are closer to input airfoils in terms of upper surface and leading edge. We randomly selected other three airfoils in the same epoch during training process, see Fig.5. Comparing subgraph (a) with (b) in Fig.5, we find that the airfoil exported by GF encoder is closer to input airfoil in terms of upper surface and leading edge. Comparing subgraph (c) with (d) in Fig.5, we can see that the airfoil exported by GF encoder is smoother than that exported by the Auto-Encoder. Comparing subgraph (e) with (f) in Fig.5, we can find that the airfoil exported by GF encoder is closer in term of upper and lower surface.

## Conclusion

The conclusions of this paper are as follows:

- 1 the MSE of airfoils exported by GF encoder is reduced by 17.97% ~ 29.14% compared with a classic Auto-Encoder.
- 2 the proposed manifold metric feature indeed capture latent geometry-features of airfoils;
- 3 the proposed BFE is proved to be feasible.

Formula (4) only represent one latent geometry-features of airfoil curves. In the future, more latent geometry-features of airfoil curves will be defined and extracted, and the feature presentation will be enriched and more geometrically significant.

### Availability of data and materials

The airfoil dataset used in our experiments is UIUC Airfoil Coordinates Database. All data exported or analysed during the current study are available from the corresponding author on reasonable request.

### Competing interests

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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### Authors' contributions

Yu Xiang and Liwei Hu contribute equally.

Yu Xiang: Conceptualization, Methodology and Writing.

Liwei Hu: Conceptualization, Methodology and Writing.

Jun Zhang: Software, Validation, Investigation and Formal analysis Review and Editing.

Wenyong Wang: Investigation, Formal analysis, Data Curation and Visualization.

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