Rabinovich-Fabrikant Chaotic System and its application to Secure Communication

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Research Article

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Abstract: This paper aims for the implementation and analysis of Rabinovich-Fabrikant System and its properties where it is found that for the value of parameters having $\alpha=0.1$, $\gamma=0.077$ the system was stable and it was able to Mask any kind of digital signals properly and it showed non-linearity properties which paved the path to secure communication.

Introduction: In the year 1979, two theoretical physicist Mikhail Rabinovich and Anatoly Fabrikant, invented a new type of chaotic system, named Rabinovich-Fabrikant. [1], it is continuous in time domain and real in space domain, and it has three space dimensions and the other familiar chaotic systems contains only second-order nonlinearities (such as the Lorenz system), but the Rabinovich-Fabrikant chaotic attractor system with third-order nonlinearities presents some unusual dynamics, namely some complex dynamics that were rarely or never seen from other systems in the past, This system have shapes of waveforms like virtual “saddles” and even like “double vortex tornado” based on the varying parameters in the Rabinovich-Fabrikant equations [2].

Mathematical notation of Rabinovich-Fabrikant:

$$x^* = y(z-1+x^2) + \gamma x \quad \text{-----}(1)$$

$$y^* = x(3z+1-x^2) + \gamma y \quad \text{-----}(2)$$

$$z^* = -2z(\alpha+xy) \quad \text{-----}(3)$$

Where α , γ are the constants,[3] but the system dynamics depend sensibly on α but less on γ , therefore α is considered to be bifurcation parameter. Based on the above equations, the

simulink model was made using MATLAB and the system was analyzed with various values of α and γ , then the message signal was modulated with the Rabinovich-Fabrikant system on the transmitter side, and the same system is again used in the receiver side for the purpose of demodulation, then it is noted that the system is more sensitive with the message signal as, the size of the pulse varies in the message signal, the waveform is also varied, and it is seen that many values of has repetition in the signals, but it is found that for some certain values of α and γ , the waveform was exponentially increasing or decreasing.

In this particular we took $\alpha=0.1$, $\gamma=0.077$ because, in this value, we found that the chance of repetition of the signals was less for any kind of digital pulse signals, unlike others values of α and γ . Here the value of α is greater when compared to γ , therefore the system is said to be dissipative chaotic system, and then the synchronization technique is necessary for the system. But here the chaotic system used in the transmitter and the receiver side are the same and therefore complete synchronization takes place, as the set of initial conditions of the systems eventually evolve identical in time .[5]

It is necessary to find whether the system is hyper chaotic system or not, so in order to check it we need to find the number of positive Lyapunov components present in the system, if it exceeds two then it is hyper chaotic system, to find Lyapunov components Equilibrium analysis is made.

Calculation of Equilibrium Points:

Rabinovich-Fabrikant system has five equilibrium points and it is given as

$$x_0 = (0, 0, 0) \quad \text{-----}(4)$$

$$x_{1,2} = (\pm q_-, \frac{\alpha}{q_-}, 1 - (1 - \frac{\gamma}{\alpha})q_-^2) \quad \text{-----}(5)$$

$$x_{3,4} = (\pm q_+, -\frac{\alpha}{q_+}, 1 - (1 - \frac{\gamma}{\alpha})q_+^2) \quad \text{-----}(6)$$

Where,

$$q_{\pm} = \frac{\sqrt{\left(1 \pm \sqrt{\left(1 - \alpha \gamma \left(1 - \frac{3\gamma}{4\alpha}\right)\right)}\right)}}{2\left(1 - \frac{3\gamma}{4\alpha}\right)} \text{--- (7)}$$

Substituting the value of α and γ as 0.1 and 0.077 respectively in equation (7) we get,

$$q_+ = 2.1747 \text{ and } q_- = 9 \times 10^{-4}$$

Substituting the values of q_+ and q_- in equations (4), (5), (6) we get

$$x_0 = (0, 0, 0) \text{-----(8)}$$

$$x_{1,2} = (\pm 9 \times 10^{-4}, 111.111, 0.9999) \text{-----(9)}$$

$$x_{3,4} = (\pm 2.1747, 0.0459, -0.08774) \text{-----(10)}$$

The Jacobian Matrix for the equations (1), (2), (3) is given as, [6]

$$J(E) = \begin{pmatrix} \gamma + 2xy & z - 1 + x^2 & y \\ 3z + 1 - 3x^2 & \gamma & 3x \\ -2yz & -2zx & -2\alpha - 2xy \end{pmatrix} \text{-----(11)}$$

Substituting α and γ as 0.1 and 0.077 respectively in equation (11) and finding the characteristic equation we get.

$$\lambda^3(0.0154 + 0.154xy) + \lambda^2(0.00308 + 0.0616xy + 0.308x^2y^2) - \lambda(5z + 2x^2 - 3z^2 + 3x^2z) + xyz(-4 + 6z) + x^2(0.8 + 8xy - 0.6x^2 - 6x^3y) + z(0.6z - 0.2) - 0.123 = 0 \text{-----(12)}$$

Considering initial conditions (0, 0, 0) in equation (12) we get,

$$0.0154\lambda^3 + 3.08 \times 10^{-3}\lambda^2 = 0 \text{----- (13)}$$

In order to find whether the system is stable or not, we apply Routh–Hurwitz Stability Criterion in equation (13) and we get that the system is stable.

The simulink block diagram of the Rabinovich-Fabrikant system made by Lazaros Moysis [7] is used and the value of alpha and gamma is changed to 0.1 and 0.077 respectively.

The Fig.1 shows the simulink model of the Rabinovich-Fabrikant System

Fig.1 Simulink Model of Rabinovich-Fabrikant System

The system produces waveform as shown in Fig.2, Fig.3, and Fig.4 in x, y, z axis respectively

Fig.2 (X-Axis)

Fig.3 (Y-Axis)

Fig.4 (Z-Axis)

Fig.5 Block diagram of the chaotic communication

The figure.5 is the systematic block diagram representation of the chaotic communication system. The X-Axis, Y-Axis, Z-Axis is used for masking the signal. In this masking scheme, a low-level message signal is added to the synchronizing driving chaotic signal in order to regenerate a clean driving signal at the receiver. Figure.6 shows the message signal at the transmitter side and the message is masked with

the Rabinovich-Fabrikant System and the modulated signal is shown in the figure.7, and the chaotic signal that is masked with the message signal is subtracted and the information is received in the receiver side without any distortion or change in phase shift and complete Synchronization takes place. Figure.9 represents the Simulink model of the Communication System.

Fig.6 Message Signal

Fig.7 Modulated Wave Form

Fig.8 Received Signal

Fig.9 Communication System using Rabinovich-Fabrikant

Thus, the message has been perfectly recovered by using the signal masking approach through cascading synchronization in the Rabinovich-Fabrikant System [8]. It is seen that unlike other systems the change in the information signal will lead to change in the carrier signal and so the information cannot be predicted based on the modulated signal and it paves the path to the secure communication.

The systematic block diagram is converted into circuit diagram for the practical implementation and the circuit diagram is shown in Fig.10. Conventionally, a pseudo random (PN) sequence was used for direct-sequence code division multiple access (DS-SS) systems, but it does not provide security due to the fact that there are a restricted number of available PN sequences and

they show periodic correlation properties. Chaotic sequences, based on chaotic studies, are non-binary and non-periodic sequences [9]. As the number of available chaotic signals is large it is very difficult for an interceptor to crack the chaotic sequence even if a chaotic function is known. This provides huge advantages over the conventional PN sequences based systems. Chaotic systems provide a rich mechanism for signal design and generation, with potential applications to communications and signal processing. Because chaotic signals are typically broadband, noise-like, and difficult to predict, they can be used in various contexts in communications.

Fig.10 Circuit Diagram of Rabinovich-Fabrikant System

Conclusion: Thus the Rabinovich-Fabrikant system is implemented and the information signal is masked with the System and the difficulty of predicting the context in the communication is increased and for further more complexity of the signal, multi chaotic system can be implemented which consist of different individual algorithms. So far no known method of attack exists for such Systems [10].

References

- [1] Rabinovich, Mikhail I.; Fabrikant, A. L. (1979). "Stochastic Self-Modulation of Waves in No equilibrium Media". Sov. Phys. JETP. 50: 311. Bibcode:1979JETP...50...311R.
- [2] Danca, M.-F.: Hidden transient chaotic attractors of Rabinovich-Fabrikant system. Nonlinear Dyn. 86, 1263–1270 (2016)
- [3] Unusual dynamics and hidden attractors of the Rabinovich–Fabrikant system Marius-F. Danca · Nikolay Kuznetsov . Guanrong Chen Received: 12 September 2016 / Accepted: 7 December 2016
- [4] <https://www.ncbi.nlm.nih.gov/pmc/articles/PMC2465602/>
- [5]. Arenas, Alex; Díaz-Guilera, Albert; Kurths, Jurgen; Moreno, Yamir; Zhou, Changsong (2008-12-01). "Synchronization in complex networks". Physics Reports. 469 (3): 93–153. arXiv:0805.2976 . Bibcode:2008PhR...469...93A. doi:10.1016/j.physrep.2008.09.002.
- [6]. Chaos control of fractional order Rabinovich–Fabrikant system and synchronization between chaotic and chaos controlled fractional order Rabinovich–Fabrikant system Author links open overlay panelM.SrivastavaS.K.AgrawalK.VishalS.Das
- [7].<https://www.mathworks.com/matlabcentral/fileexchange/46602-rabinovich-fabrikant-equations-simulink-model>
- [8]. IOSR Journal of Engineering (IOSRJEN) ISSN (e): 2250-3021, ISSN (p): 2278-8719 Vol. 04, Issue 02 (February. 2014), ||V5|| PP 29-33 Chaotic Signal for Signal Masking in Digital Communications Rahul Ekhande , Sanjay Deshmukh
- [9] Oppenheim, A.V. & Cuomo, K.M. "Chaotic Signals and Signal Processing" Digital Signal Processing Handbook Ed. Vijay K. Madisetti and Douglas B. Williams Boca Raton: CRC Press LLC, 1999
- [10] Methods of Attacking Chaotic Encryption and Countermeasures Mohamed I. Sobhy and Alaa-eldin R. Shehata