Cosmic Gas Thermodynamics at z = 1089

Martin Reid Johnson (mrj@usa.net)
Trinapco, Inc. https://orcid.org/0000-0002-5096-3206

Short Report

Keywords: dark energy, intergalactic medium, gas thermodynamics

Posted Date: October 13th, 2021

DOI: https://doi.org/10.21203/rs.3.rs-966087/v1

Cosmic Gas Thermodynamics at *z* = 1089

By Martin Reid Johnson

https://orcid.org/0000-0002-5096-3206

Abstract – The Universe at $z = 1089$ is treated as an expanding ideal gas. Its internal kinetic energy loss exceeds the amount absorbed by gravity and drives further expansion. A Hubble relation (H_g) is derived and compared to the value found by the ΛCDM model (H_Λ) over the range $z = 1089$ to 0. The results suggest that the adiabatic release of energy from cosmic gas accounts for about half of present-day Universal expansion.

The presently preferred description of Universal expansion is the flat-universe ΛCDM model, given in simple form by (1):

$$H^2(a) = H_0^2[\Omega_{rad}a^{-4} + \Omega_M a^{-3} + \Omega_{CDM}a^{-3} + \Omega_\Lambda] \qquad (1)$$

Where $H(a)$ is the Hubble parameter at scale factor $a = 1/(1+z)$, z is the cosmic redshift $(\lambda_e - \lambda_0)/\lambda_e$ of an emitted photon of known λ_0, and H_0 is the present-day Hubble constant, 67.4 Km/sec/Mpc or 2.184×10^{-18} sec^{-1}. The Ω values or *density parameters* add up to one and relate their energy density to the present-day *critical density* $\varepsilon_{crit} = 3c^2(H_0)^2/8\pi G$:

$$1 = \sum \Omega_x = \sum \frac{\varepsilon_x}{\varepsilon_{crit}} \qquad (2)$$

In eq. (1), $\Omega_m = 0.0486$ is baryonic matter, $\Omega_{cdm} = 0.259$ is cold dark matter, $\Omega_{rad} = 9.00 \times 10^{-5}$ is relativistic energy (photons and neutrinos), and $\Omega_\Lambda \approx 0.69$ is the "dark energy" parameter. The ΛCDM model treats Universal expansion as a function of the sum of the mass-energy components Ω, three of which (Ω_{rad}, Ω_M, and Ω_{CDM}) have constant comoving energy densities ε_{rad}, ε_M, and ε_{CDM}. The fourth parameter Ω_Λ has a density ε_Λ which is not comoving, but rather is the same for any volume of space at any time. The critical density ε_{crit} is the total mass-energy density which gives an exact balance to the energy loss from gravity over time. The empirical accuracy of the ΛCDM model is high and we use it to calibrate our model.

This paper explores the energy release associated with density reduction ("expansion") of cosmic gas and plasma, by treating those portions the Universe not bound by gravity as an ideal gas.

First, we select a time: $z = 1089$, just after recombination. Baryonic matter was almost all neutral gas and acoustic oscillation was minimal so the Universe had constant density. Baryons were then present as a mixture of 75% monatomic hydrogen (H_1) : 25% helium (He) by weight, or about 92 mole % H_1 : 8 mole % He. This gives a mean molecular weight Ж = 1.2475×10^{-3}

Kg/mol. Monatomic gas thermodynamic laws can be reasonably applied to this time period.[1] The 2018 Planck survey[2] gives a ΛCDM-based value for today's mean baryon density: ρ_0 = 4.21 x 10^{-28} Kg/m^3. The baryon density ρ_{1089} was thus (ρ_0/a^3) = (4.21 x 10^{-28})(1090)^3 = 5.45 x 10^{-19} Kg/m^3, or 2.6 x 10^8 atoms per cubic meter. The background radiation had just decoupled so the baryon temperature was ≈ 2971 K (CMB = 2.7255 K)(1/*a* = 1090).[3]

The Universe as a whole is an adiabatic system. In a classic setting, there are two kinds of adiabatic gas expansion: reversible and free. Reversible expansion is isoentropic by definition. When a gas expands reversibly, its internal kinetic energy U_i decreases, the gas performs work, and the temperature and pressure drop. When a gas expands freely, U_i does not decrease and only the pressure drops. The temperature stays the same:

$$\partial V = \partial V_S + \partial V_T \tag{3}$$

Both happen cosmically, but with differences. In the first, *cosmic isoentropic ("reversible") expansion*, the internal kinetic energy (U_i) lost isn't equal to the energy stored by gravity. The excess becomes vectored kinetic energy. The second, *cosmic free expansion*, derives from the fact that the entropy of the Universe always increases over time.[4] On a cosmic scale, gas expansion can't be exclusively isoentropic as no time would have elapsed. There must also be an entropic volume increase. These two forms of gas expansion are linked: one can't happen without the other. We develop our model isoentropically (∂V_S only) and examine ∂V_T later on.

Consider a finite sphere around a single atom of H_1, of radius about Earth size (1 au = 6.3781 x 10^6 m), at 2971 K, which at ρ_{1089} has baryon mass *M* = 593 Kg. This sphere is still in thermal equilibrium, a major but necessary departure from reality. Nonequilibrium thermodynamics must be set aside so that the underlying transfer of conserved energy is more clearly described. The sphere's gravitational potential energy (*U*) is: [5]

$$U = \frac{-3GM^2}{5r} \tag{4}$$

Where G is the gravitational constant (6.67408 x 10^{-11} m^{3}kg^{-1}sec^{-2}). The ΛCDM model also contains cold dark matter (CDM), $\Omega_{cdm}/(\Omega_M + \Omega_{cdm})$ = 0.842, ≈ 84% of all cold mass in the

---

[1] Isotopes, heavier elements, and diatomic hydrogen are treated as negligible.

[2] a) Planck Collaboration et. al., *A&A* 641, A1 (2020) doi:10.1051/0004-6361/201833880. b) in Ryden, B. R., Pogge, R.W.; *Interstellar and Intergalactic Medium* (2021), Cambridge University Press, ISBN 978-1-108-74877-3, page 198.

[3] CMB = cosmic microwave background.

[4] This is the Second Law of Thermodynamics. The First and Second Laws are, from Clausius: "The energy of the universe is constant; the entropy of the universe tends to a maximum." Clausius, R. (1865) Annalen der Physik 125: 353–400. See also: Popovic, M. https://arxiv.org/abs/1711.07326.

[5] The subscripts U_1, U_2, and U_r refer to gravitational potential energy. The term U_i is used to denote the internal kinetic energy of a gas or plasma.

Universe, and doesn't act as a gas. Its only influence is gravitational. This is included by dividing the baryon mass by 0.158:

$$U = \frac{-3GM'^2}{5r} = \frac{-3G(\frac{M}{0.158})^2}{5r} \tag{5}$$

The ideal gas law is:

$$PV = nRT = \frac{MRT}{Ж} \tag{6}$$

Where R is the universal gas constant (=8.31446 kg-m^{2}sec^{-2}mole^{-1}K^{-1}). The volume of a sphere is:

$$V = \frac{4}{3}\pi r^3 \tag{7}$$

When (6) and (7) are combined we get the *internal pressure (P_1)*:

$$P_1 = \frac{nRT}{V} = \frac{\rho RT}{Ж} = \frac{MRT}{ЖV} = \frac{3MRT}{4Ж\pi {r_1}^3} \tag{8}$$

Where ρ is the mass density. Entering our values for M, T, and r, we obtain P_1 =1.08 x 10^{-11} Pa. We will also suppose that the sphere isn't getting any bigger over time. It is but for now we'll say it isn't. We increase the sphere's radius by $\sqrt[3]{1.01}$, giving a volume increase of one percent.[6] Work is performed against gravity:

$$U_r = U_1 - U_2 = \frac{-3GM'^2}{5}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) \tag{9}$$

Where U_1 and U_2 are the gravitational potential energies at radii r_1 and r_2 respectively. Entering the values for M, r_1 and r_2 we find that U_r = 2.92 x 10^{-13} J. The internal kinetic energy loss (-E) is, however, much greater than U_r:[7]

$$W = -E = \left(\frac{3}{2}\right) P_1 V_1 \left(\left(\frac{V_2}{V_1}\right)^{-\frac{2}{3}} - 1\right) \tag{10}$$

Where W has the classic meaning of work performed by the gas, P_1 is the internal pressure before expansion, and V_1 and V_2 are the before and after volumes of the sphere respectively. V and P can be calculated from (7) and (8). Entering these into (10) gives E = 1.16 x 10^8 J. This is 10^{20} times as much energy released as absorbed. The excess (E_k) is now outward, *radial kinetic energy*:

---

[6] We define the *increment* as $\frac{r_2 - r_1}{r_1} = \frac{\Delta r_i}{r}$. This is different from the *step* (Δr_s), used for numeric integration.

[7] This and the other thermodynamic expressions are found in many textbooks and, *eg*, Wikipedia.

$$E_k = E + U_r = E + (U_1 - U_2) = \left(\frac{3}{2}\right) P_1 V_1 \left(\left(\frac{V_2}{V_1}\right)^{-\frac{2}{3}} - 1\right) - \frac{3GM'^2}{5}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) \tag{11}$$

Gravity loss is negligible and $E_k = E \approx 10^8$ J. The internal pressure drops to a new value, P_2:

$$P_2 = P_1 \left(\frac{V_2}{V_1}\right)^{-\frac{5}{3}} \tag{12}$$

Eq. (12) gives $P_2 = 1.06 \times 10^{-11}$ Pa. Dividing E_k by V_2 gives the increase in *expansion pressure* (ΔP_E):

$$\Delta P_E = \frac{E_{k_2} - E_{k_1}}{V_2} \tag{13}$$

Our sphere was static to start so $E_{k_1} = 0$. Our expanded sphere has $\Delta P_E = 1.06 \times 10^{-13}$ Pa, or 1% of P_2. It's important to emphasize that P_E does not add to P_2,[8] but is instead a vector quantity which results in radial increase only. Each atom is moving in a straight line away from the center, like a bunch of tiny rockets blasting away from their despoiled planet. It helps to ignore U_i to properly visualize this. Expansion pressure already existed in the sphere since the Universe has been expanding all along.

The temperature drop is given as:

$$T_2 = T_1 \left(\frac{P_2}{P_1}\right)^{2/5} \tag{14}$$

The temperature drops from 2971 to 2951 K or 0.7%. This can be compared to the CMB-derived temperature: $(2.7255)\left(1090/\sqrt[3]{1.01}\right) = 2961$ K, or 0.3% for $r = 1 \rightarrow \sqrt[3]{1.01}$ au. The gas is cooling faster than the photons. Eq. (14) is also used to verify the consistency of the calculations. [9]

The linear rate of expansion, or *radial velocity* (v_s) of the sphere is:

$$v_s = \sqrt{\frac{2E_k}{M}} \tag{15}$$

Direct use of (15) ignores the fact that the sphere is already expanding and $E_{k_1} \gg 0$, so v_s is inaccurate and quite low. We can get corrected values for v_s at an instant in time ∂t by modeling (11), using a small increment $\Delta r_i/r = 10^{-9}$ and increasing r independently.

---

[8] Since one of our assumptions is thermal equilibrium, P_2 remains a state function.

[9] The internal energy of an ideal monatomic gas is given as $U_i = \frac{3}{2}\frac{\rho RTV}{Ж}$. The new internal energy $U_{i'}$ can be found with the T_2 value from (14) and the new ρ and V values. The residual error $U_i - (U_{i'} + E)$ is exactly zero to the limit of the spreadsheet.

First we define the *gravity ratio* (X):

$$X = \frac{U_2 - U_1}{E} \tag{16}$$

With our above T, ρ, and $\Delta r_i/r$ held constant, we increase r_1 stepwise. The mass rises and v_s falls until the *adiabatic radius,* or *endpoint* (r_e = 1.2732 x 10^{17} m),[10] is reached, where $X = 1$ and $\frac{\partial E_k}{\partial r} = 0$. This *adiabatic sphere* conserves energy around the central atom.

The *cutoff ratio* (X') is defined as:

$$X' = \frac{-U_r}{E_k} = \frac{U_2 - U_1}{E_k} \leq 0.00001 \tag{17}$$

Below the *cutoff radius* (r_c), gravity can be neglected and (11) simplifies to (10). The choice of $X' < 10^{-5}$ is best seen graphically. Figure 1 shows a semilog map of v_s vs. r from 10^{12} to 10^{16} m at 2971K and ρ_{1089}.[11] Below r_c (= 4 x 10^{14} m) v_s is constant to 2 ppm, giving the same (E/M):[12]

$$\frac{E}{M} = \frac{\partial E}{\partial M} = \frac{\partial V}{\partial M}\frac{\partial E}{\partial V} = \left(\frac{RT}{ЖP}\right)\frac{\partial E}{\partial V} = \frac{RT}{Ж}\left(\frac{\partial E}{P\partial V}\right) = \frac{RT}{Ж} \tag{18}$$

This gives the *initial radial velocity* (v_i):

$$v_i = \sqrt{\frac{2E}{M}} = \sqrt{\frac{2RT}{Ж}} = \sqrt{\frac{2(8.31446)(2971)}{0.0012475}} = 6293 \text{ m/sec} \tag{19}$$

Above r_c, gravity takes its toll, and v_s drops, reaching zero at r_e. The radial velocity (v) of the adiabatic sphere is the sum of the contained shells:

$$v = (v_i)\left(\frac{r_c}{r_e}\right) + \sum_{r_c}^{r_e} v_{s'}\frac{\Delta r_s}{r_e} \rightarrow\rightarrow (v_i)\left(\frac{r_c}{r_e}\right) + \frac{1}{r_e}\int_{r_c}^{r_e} v_{s'}\,\partial r \tag{20}$$

Where Δr_s is the *step*, $v_{s'} = v_i\left(\frac{v_s}{v_{s_0}}\right)$, and v_{s_0} is the constant value of v_s at $r \ll r_c$. For all $r < r_c$, v = 19.8 m/sec. That leaves the remaining 99.7% of v to be found. Integration of (11) is problematic so we resort to a map (Figure 2) whose cumulative value at $r/r_e = 1$ is 4973.2 m/sec. Adding 19.8 to this gives 4993 m/sec, or (0.7934±0.0001) v_i. This proportion K shows little change with input values; v/v_i is constant to nearly the 4th decimal place.

In the special case of atoms separated by $2r_e$, their adiabatic spheres are joined at a tangent point and they are moving apart at $2v$. More generally, for any two atoms separated by a distance r, their recession rate v_r is:

---

[10] The endpoint r_e is found on a spreadsheet by convergence of r around $X = 1$.

[11] Both r_c and r_e are independent of $\Delta r_i/r$ over a wide range.

[12] For isoentropic adiabatic expansion, $\partial E = P\partial V$.

$$v_r = K\frac{r}{r_e}v_i = K\frac{r}{r_e}\sqrt{\frac{2RT}{Ж}} \tag{21}$$

And the comoving gas-derived Hubble value (H_g) is:

$$H_g = \frac{v_r}{r} = \frac{Kv_i}{r_e} = \frac{K}{r_e}\sqrt{\frac{2RT}{Ж}} \tag{22}$$

Which at $z = 1089$ and $K = 0.7934$ gives $H_g = 3.87 \times 10^{-14}$/sec, or $17955H_0$. This is 78% of the value found from (1). If we set $K = 1$, we get $H_g = 4.94 \times 10^{-14}$/sec, or $22630H_0$. This is 99% of the value found from (1). It appears that isoentropic treatment of the gas at v_i is more consistent with the ΛCDM model than treatment at v/v_i:

$$H_g = \frac{v_r}{r} = \frac{v_i}{r_e} = \frac{1}{r_e}\sqrt{\frac{2RT}{Ж}} \tag{23}$$

Use of (23) at varying T from 100 to 4000K at $z = 1089$ gives the same result to five decimal places every time. More extensive input change reveals that (23) has no temperature dependence. The model is also independent of molecular weight. A Universe made of xenon atoms (0.131 Kg/mole) returns the same result as our primordial mix. Other than the CDM adjustment, the mass density ρ is the only remaining variable in the model, and it's a function of the cosmic redshift z. This makes z the sole independent variable.[13]

We now examine entropy, by looking at ∂V. To do this, we use a two-increment model at $r \ll r_c$, where the expansion is first performed reversibly and then freely, giving r_1-r_3 and U_1-U_3. The first increment $r_2 - r_1$ generates v_s. Free expansion of the sphere $r \ll r_c$ has no gravity loss and occurs simultaneously with reversible expansion, so in the second increment $r_3 - r_2$ the atoms coast along at v_s for the same time. The two increments are equal. From (3):

$$\partial V' = \partial V_S + \partial V_T = 4\pi r^2\partial r + 4\pi r^2\partial r = 2\partial V \tag{24}$$

And from (18):

$$\frac{E}{M} = \frac{\partial E}{\partial M} = \frac{\partial V'}{\partial M}\frac{\partial E}{\partial V'} = \frac{2\partial V}{\partial M}\frac{\partial E}{2\partial V} = \left(\frac{RT}{ЖP}\right)\frac{\partial E}{\partial V} = \frac{RT}{Ж}\left(\frac{\partial E}{P\partial V}\right) = \frac{RT}{Ж} \tag{25}$$

Which again gives (18).

Let's look at the free increment r_3-r_2. Near the $z = 1089$ endpoint (1.27×10^{17}m) the initial radial kinetic energy of the sphere $E_i = ½M(v_i)^2 = 4.7 \times 10^{40}$ J. At $\frac{\Delta r_i}{r} = 10^{-9}$ the loss to gravity U_3-U_2 is 2.8×10^{32} J, or about 6×10^{-9} E_i, giving a new $E_{i'} \approx E_i$ and $\frac{r_{3'}-r_{2'}}{r_3-r_2} = \sqrt{\frac{E_{i'}}{E_i}} \approx 1$. At a much

[13] The density ρ can be changed, within constraints, if the Universe is partitioned into regions of varying density.

higher increment $\frac{\Delta r_i}{r} = 0.001$, U_3-U_2 = 0.006 E_i and $\sqrt{\frac{E_{i'}}{E_i}} = 0.93$, so there is some model-based dependency best addressed by keeping the increment low. We can more accurately see the effect of entropic expansion by examining a linear map of H_g/H_Λ vs. z from z = 999 to 19 using equations (23) and (1) (Figure 3). The entropic expansion at z = 1089 is arbitrarily set to zero. The influence of Ω_Λ is negligible in this z range, and the variance k_z is well fit by (26):

$$k_z = \frac{H_g}{H_\Lambda} = 1.1327 - 1.619 * 10^{-4}z + 2.603 * 10^{-8}z^2 \tag{26}$$

We include k_z in (23), giving (27):

$$H_{g'} = \frac{H_g}{k_z} = \frac{v_i}{k_z r_e} \tag{27}$$

Figure 4 shows a map of both H_g/H_Λ and $H_{g'}/H_\Lambda$ vs. z from z = 10 to 0, where curvature is important. At $z = 0$, $H_{g'}/H_0 = (H_g/H_0)(1.1327)^{-1} = (0.63)(1.1327)^{-1} = 0.56$. We make another correction for the mass proportion of the Universe that today is not gravitationally bound: ≈85%. This new density $\rho'' = 0.85\rho$ gives an adjustment of 0.92 and a new ratio $H_{g''}/H_0 = (0.92)(H_{g'}/H_0) = 0.51$. This is more than the sum of $\Omega_m + \Omega_{cdm} + \Omega_{rad} = 0.31$, which means that $H_{g''}$ is responsible for some of H_0 ascribed to Ω_Λ in (1). The model outlined herein does not further address that issue. It does, however, demonstrate that a known source of energy, U_i, can be used to account for much of Universal expansion. While a dark energy field cannot be excluded as a source, the results of this paper suggest that more mundane sources, e.g., kinetic energy stored in gases or plasma, make a substantial contribution.

The presented model only superficially addresses the issue of entropic expansion. Proper treatment of entropy using classic gas thermodynamic principles, and applied to cosmic conditions in more recent times, may yield meaningful results.

The author declares no competing interest.

Figures

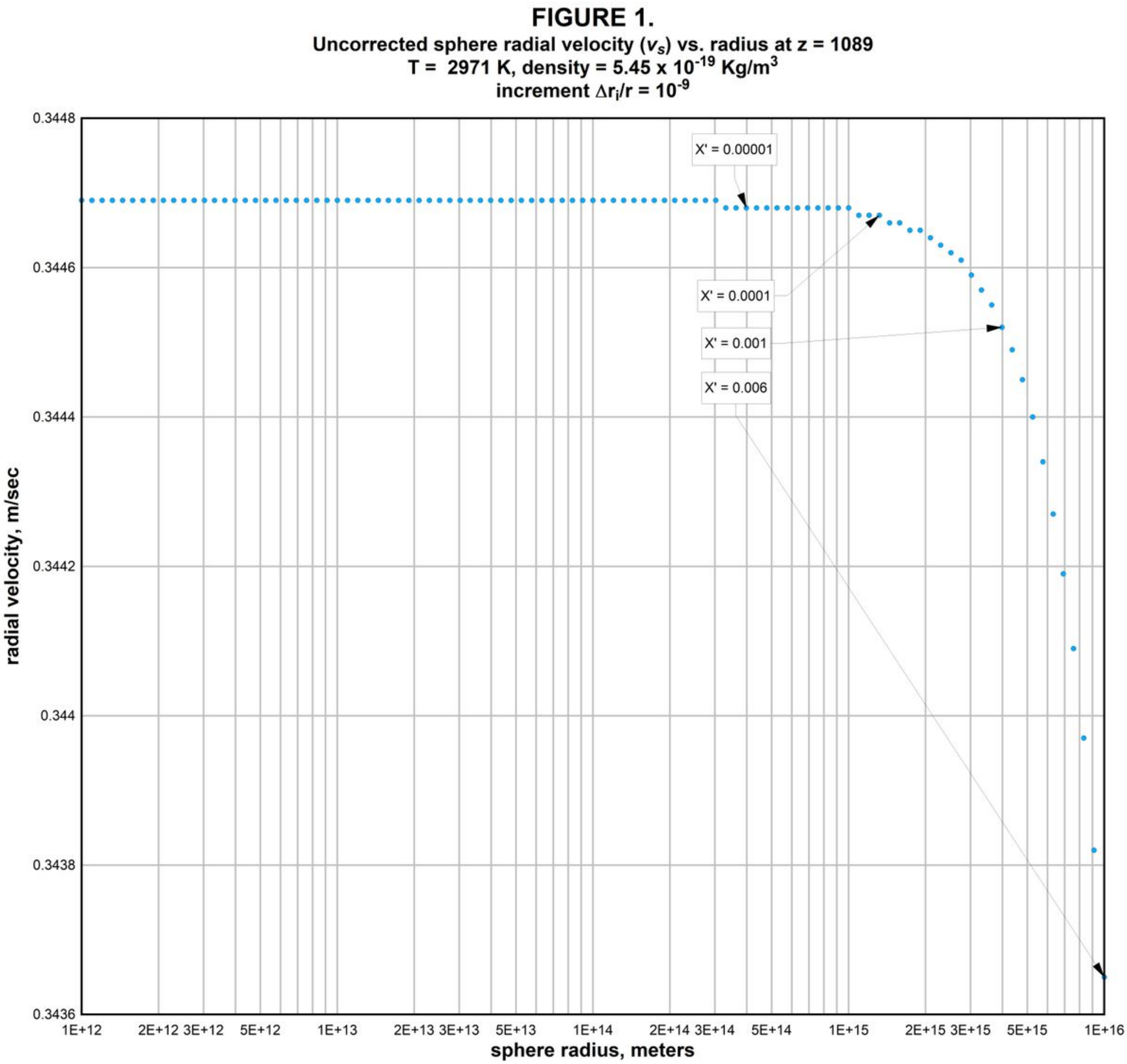

Figure 1

Uncorrected sphere radial velocity vs. radius at z = 1089

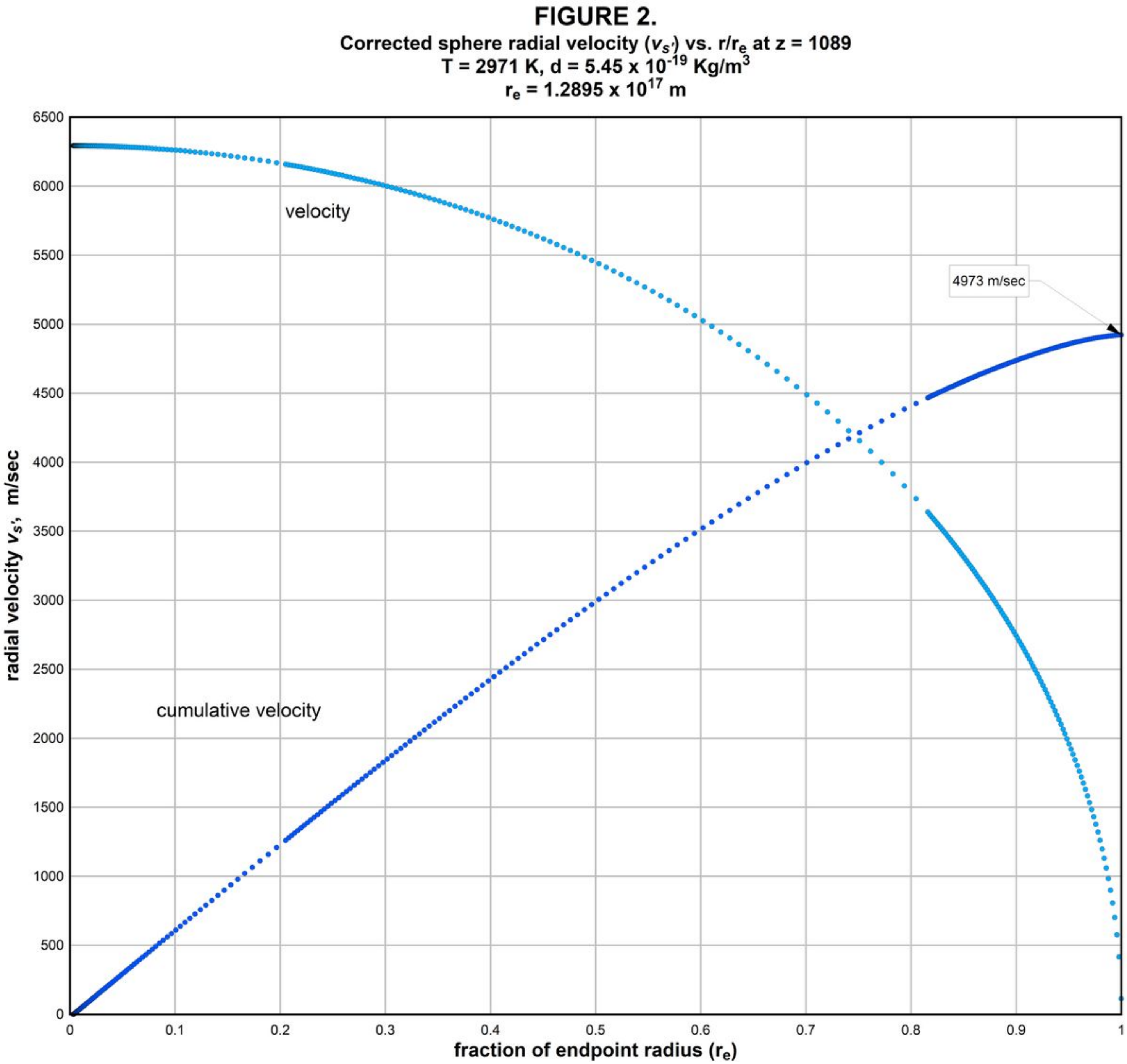

Figure 2

Corrected sphere radial velocity vs. r/re at z = 1089

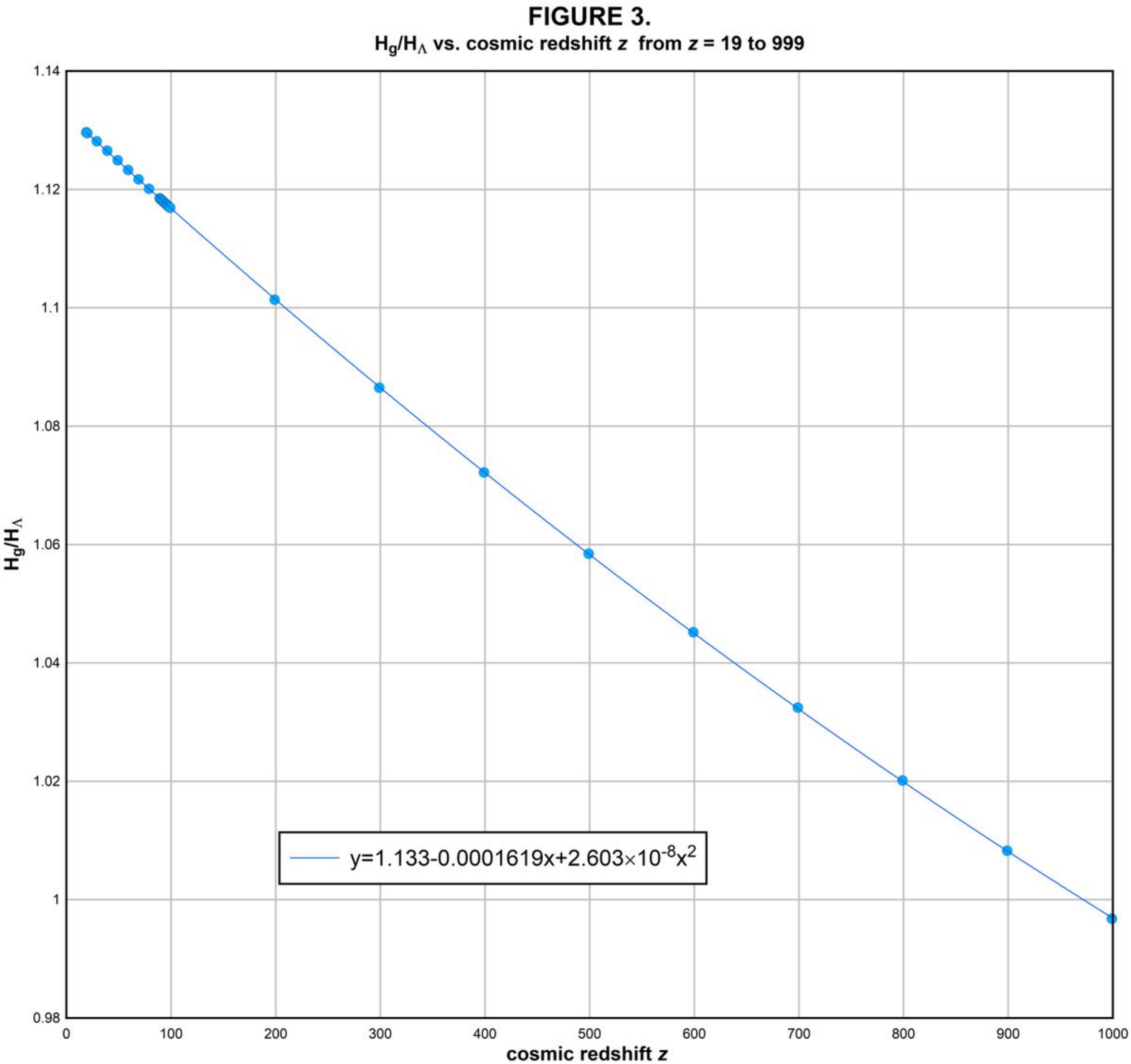

Figure 3

H(g)/H(lambda) vs. cosmic redshift z from z = 19 to 999

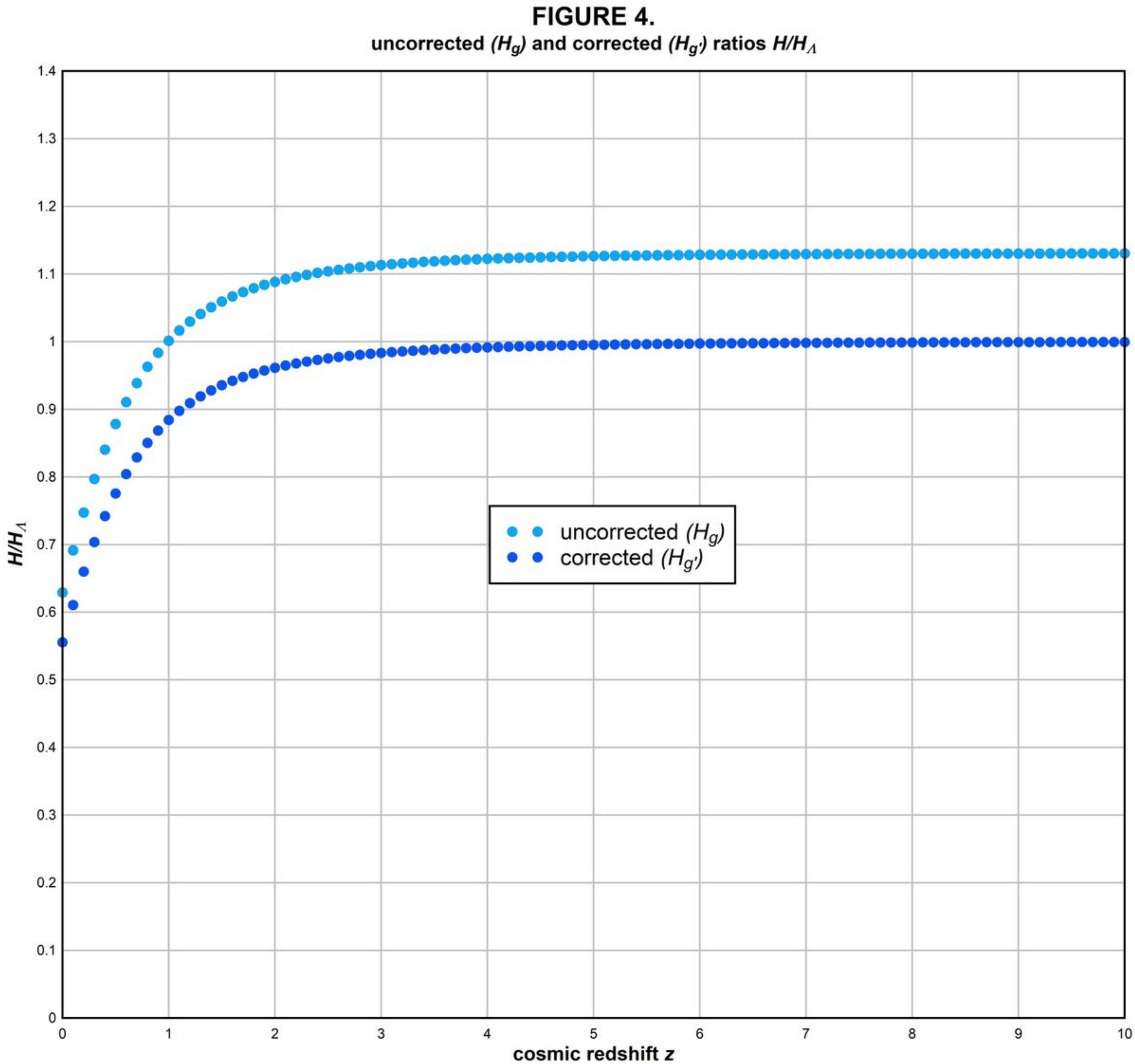

Figure 4

Uncorrected and corrected Hubble ratios at z = 0 to 10

Supplementary Files

This is a list of supplementary files associated with this preprint. Click to download.

- cosmicgasworkbook20211012.xlsx