Crack Fault Diagnosis and Location Method for a Dual-disks Hollow Shaft Rotor System Based on the Radial Basis Function Network and Pattern Recognition Neural Network

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Title page

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ORIGINAL ARTICLE

Crack fault diagnosis and location method for a dual-disks hollow shaft rotor system based on the Radial basis function network and Pattern recognition neural network

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Abstract: In recent years, the crack fault is one of the most common faults in the rotor system, and its fault diagnosis has been paid close attention by researchers. However, the traditional fault diagnosis methods based on various signal processing algorithms can only be adopted to determine whether there is a crack fault in the rotor system, but the dynamic response of the rotor system can hardly be used to calculate the depth and position of the crack. In this paper, a new crack fault diagnosis and location method for a dual-disks hollow shaft rotor system based on the Radial basis function (RBF) network and Pattern recognition neural network (PRNN) is presented. Firstly, a rotor system model with a breathing crack suitable for a short-thick hollow shaft rotor is established based on the finite element method and Timoshenko beam theory. Then the dynamic response is calculated by the harmonic balance method and the analysis results show that the first critical whirl speed, the first subcritical speed, the first critical speed amplitude, and the super-harmonic resonance peak at 1/2 first critical whirl speed of the rotor system are closely related to the depth and position of the crack, which can be used for crack fault diagnosis. Finally, the RBF network and PRNN are adopted to determine the depth and approximate location of the crack by taking the above dynamic response characteristics as input, respectively. The test results show that this method has high fault diagnosis accuracy.

Keywords: Hollow shaft rotor • Breathing crack • Radial basis function network • Pattern recognition neural network

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1 Introduction

Rotating machinery is an important part of the modern industry, which has many applications in gas turbines, aero-engine, wind turbines, and other important machinery equipment. Because the working environment of rotating machinery is harsh, it is easy to generate mechanical failure. The crack fault is one of the most common failure forms in the rotor system, which can pose a serious threat to the reliability of rotating machinery operating. Therefore, timely and accurate fault diagnosis to the crack fault is of great significance to improve the operation reliability of rotating machinery. Since the 1970s, many researchers have studied the crack fault diagnosis of the rotor system in many aspects. Mayes et al. [1] first used a more accurate cosine breathing function to describe the breath behavior of the crack, and studied the dynamic response of the rotor system containing a breathing crack. Then Al-Shudeifat et al. [2][3] proposed a new type of breathing function by using the Fourier series and compared it with Mayes' results. The comparative results show that this respiration function could more accurately represent the breathing process of crack, but its form is more complex. Jun et al. [4] established the equation of motion for a simple rotor with a breathing crack. It is found that the vibration characteristics of a cracked rotor are best identified from the second harmonic components near to the second harmonic resonant speed. Babu et al. [5] used Hilbert–Huang transform (HHT) for transient response of a cracked rotor and the HHT spectrum showed that the sub-harmonics is a typical characteristic of a crack. Chasalevris et al. [6] considered the coupling problem of longitudinal vibration and flexural vibration of the rotor system with two transverse breathing cracks, and the

phenomenon of the natural frequency of the rotor system change with the increase of the crack depth for transverse cracks is discovered. Darpe et al. [7] analyzed the influence of the interaction of the two cracks on the breathing behavior and the dynamic response of the Jeffcott rotor based on the concepts of fracture mechanics. It is found that significant transformations in the dynamic response of the rotor when the angle between the two crack directions changes. Xiang et al. [8] considered an asymmetric rotor-bearing system with coupling faults of crack and rub-impact under oil-film forces and study the effects of crack depth on the onset of instability and nonlinear responses of the rotor-bearing system. Hou et al. [9][10] found the super-harmonic resonance phenomenon of the cracked rotor due to maneuver load. Lu et al. [11] studied the dynamic response of a hollow shaft dual-rotor system with breathing crack and unbalanced excitation. In the spectrum, it is found that the peak value of the super-harmonic resonance is related to the dimensionless depth of the crack. Fu et al. [12] gives similar results in the dynamic response of a normally cracked rotor system with uncertain crack parameters. Most of these studies focus on the dynamic response characteristics of the cracked rotor system or only qualitative analysis of the influence of the crack parameters (such as the depth and location of the crack). However, in many cases, crack fault diagnosis requires quantitative relationships between the dynamic response characteristics and the crack parameters.

In recent years, various machine learning methods, such as the artificial neural network (ANN), support vector machine (SVM), and decision-making tree (DT) have been widely applied in many fields such as fault diagnosis and parameter identification due to strong self-adaptability. Many researchers also use these methods to study the fault diagnosis of the rotor system. Jia et al. [13] made use of spectrum characteristics, based on deep learning network and a large number of experimental data to make fault diagnosis of common faults in rotating machinery including rolling bearing faults and planetary gearbox faults, and achieved good results. Yan et al. [14] researched the remaining useful life prediction problem of the Jeffcott rotor system with crack by using a simple Background propagation (BP) neural network. Lai et al. [15] introduced the concept of fault cumulant and established a fault classifier using the Radial basis function (RBF) neural network, which achieved relatively high identification accuracy. Guo et al. [16][17] proposed a fault diagnosis method for a Jeffcott rotor with breathing crack at the early stage of crack propagation based on empirical mode decomposition (EMD) combined with BP neural network,

and carried out experimental verification. Vashisht et al. [18] investigated the effect of crack on a complex rotor system with ball bearing and advanced a crack detection method using switching control strategy and Short Time Fourier Transform. Pavan et al. [19] used some statistical methods to reduce the fault vibration features, then use the BP neural network and SVM to study the fault diagnosis problem of a dual-disk rotor system. Failure types include a crack in the outer race, inner race with spall and pitting in balls, and comparative test results show that neural network is better than SVM. Bin et al. [20] proposed a new approach for rotating machinery fault diagnosis based on wavelet packet decomposition (WPD)-EMD fault feature extraction and the neural network. Ma et al. [21] presented a diagnosis method for rotor and bearing faults of rotating machinery based on ensemble learning. In this study, the method by weighting and integrating the Convolution Residual Network (CRN), Deep Belief Network (DBN), and Deep AutoEncoder (DAE) obtains great effect on the problem of multi-fault classification. Wang et al. [22] determined the crack parameters including crack location, depth, and angle for a solid shaft by using Kriging Surrogate Model and improved nondominated sorting genetic algorithm-III (NSGA-III), which has high parameter identification accuracy. Zhang et al. [23] studied the application of the fuzzy neural network to the fault diagnosis system for rotary machines and experimental results verified that fuzzy neural network has good identification ability. Wang et al. [24] used K-means clustering method to classify crack fault for a planetary gear box. Li et al. [25] studied multiple crack identification based on the three steps meshing, and experimental verification was also carried out. Most researchers' objects are relatively simple Jeffcott rotors or solid shaft rotors, nevertheless, to improve the rotor operating efficiency in practical problems, most large complex rotor systems adopt the hollow shaft. At present, there are relatively little researches on crack fault diagnosis of hollow shaft rotor systems.

In this study, a new crack fault diagnosis and location method for a two-disk hollow shaft rotor system is proposed by using the RBF neural network and pattern recognition network. At present, the research object of most researchers about the rotor system crack fault diagnosis is the relatively simple solid shaft rotor or Jeffcott rotor and there are little researches on the hollow shaft rotor. Besides, many researchers focus on the pattern recognition of the crack fault in the various faults of the rotor system or calculating the crack depth with fixing the crack position. In practice, to improve the operating

efficiency of the rotor system, the shaft is often designed to be hollow and the actual crack failure may occur at any position of the shaft, it is a challenge to identify the crack parameters when the depth and location of the crack are both unknown. In this paper, the dynamic response of a cracked dual-disks hollow shaft rotor is studied, and the RBF network and pattern recognition network are used to solve the crack fault diagnosis problem when the depth and location of the crack are both uncertain, which can better meet the requirements of actual rotor system crack fault diagnosis.

The paper is organized as follows, in Section 2, the motion equations of the dual-disk hollow shaft rotor system with a breathing crack are constructed by the finite element method. Secondly in Section 3, the harmonic balance method (HBM) is used to solve the dynamic response of the rotor system, and the effect of crack depth and position is analyzed respectively, then the 4th order Runge-Kutta method is used to verify the results of HBM. Estimation of crack depth is discussed detailedly based on RBF neural network and the problem of crack orientation is solved by using the pattern recognition network in Section 4. Finally, in Section 6, the primary results in this paper are summarized.

2 Dynamic model of a dual-disk hollow shaft rotor with a breathing crack

2.1 Finite element model of the rotor system

The finite element model of the cracked double-disks hollow shaft rotor system is shown in Figure 1, which is divided into 20 elements and 21 nodes. Each end of the rotor (node 1 and node 21) has a bearing and support. There are 2 disks in this rotor system, which respectively located at node 5 and node 17. The shaft of this model is hollow, which has inner radius r and outer radius R . Moreover, to better fit the actual failure status, the crack may locate in any element of the shaft. The physical parameters of the model are shown in Table 1.

Figure 1 Finite element model of a cracked two-disk hollow shaft rotor system

Table 1 The physical parameters of the rotor system

Parameters	Value (units)
Shaft length (L)	0.8562 m
External radius (R)	0.0441 m
Inside radius (r)	0.0110 m
Material density (ρ)	7800 kg/m ³
Young's modulus (E)	2.09 Gpa
Shear modulus (G)	80.38 Gpa
Bearing stiffness (K)	5×10^7 N/m
Disk 1 diameter (d_1)	0.1215 m
Disk 1 mass (m_1)	6.5157 kg
Disk 2 diameter (d_2)	0.1493 m
Disk 2 mass (m_2)	3.9543 kg
Amount of unbalance ($m_e d$)	5×10^{-7} kg·m
Unbalance deflection (θ)	0
Bearing damping (c_{xx})	1000 Ns/m

2.2 Crack model

The crack in the hollow shaft is more complex than that in the solid shaft. Figure 2 shows different crack conditions at disparate crack depth, the dashed area represents the uncracked segment, when the crack is relatively shallow, it is a non-penetrating crack (Figure 2(a)) and when the crack depth is large, it is a passthrough crack (Figure 1(b)). A_1 and A_c represent the cross-sectional area of uncracked and cracked segments respectively. $O-xy$ is the fixed coordinate system and $C-\bar{x}\bar{y}$ is the centroid coordinate system, where C is the centroid of the section A_1 . Ω , h and e represent rotor speed, crack depth, and centroid offset distance. α and α_1 is the angle between the crack edge and O .

Figure 2 Cross-section diagram of hollow shaft crack: (a) Non-penetrating crack. (b) Passthrough crack. The dashed area represents the uncracked segment

From Figure 2, the moment of inertia of the non-penetrating crack across the x and y -axis can be obtained as follows

$$A_1 = R^2 (\pi - \cos^{-1}(1 - \lambda) + (1 - \lambda)\gamma) - \pi r^2, \quad (1)$$

$$I_x^C = \frac{\pi R^2}{8} - \frac{R^4}{4} ((1 - \lambda)(2\lambda^2 - 4\lambda + 1)\gamma + \sin^{-1}(1 - \lambda)), \quad (2a)$$

$$I_y^C = \frac{R^4}{12} ((1-\lambda)(2\lambda^2 - 4\lambda - 3)\gamma + 3\sin^{-1} \gamma), \quad (2b)$$

$$e = \frac{2R^3}{3A_1} \gamma^3, \quad (3)$$

where $\lambda = h/R$ represent the dimensionless crack depth and $\gamma = \sqrt{\lambda(2-\lambda)}$. I_x^C and I_y^C is the moment of inertia of cracked segment A_C across the x and y -axis.

When the crack is deep, the above equation can be rewritten as

$$A_1 = R^2 (\pi - \cos^{-1}(1-\lambda) + (1-\lambda)\gamma - (\pi r^2 - \alpha_1 r^2 / 2 + (R-h)\sqrt{r^2 - (R-h)^2})), \quad (4)$$

$$I_x^C = R^2 (\pi - \cos^{-1}(1-\lambda)(2\lambda^2 - 4\lambda + 1)\gamma + \sin^{-1}(1-\lambda)) - \int_{-r}^{h-R} 2x^2 \sqrt{r^2 - x^2} dx, \quad (5a)$$

$$I_y^C = \frac{R^4}{12} ((1-\lambda)(2\lambda^2 - 4\lambda - 3)\gamma + 3\sin^{-1} \gamma) - \int_{-\sqrt{r^2 - (R-h)^2}}^0 2x^2 (\sqrt{r^2 - x^2} + h - R) dx, \quad (5b)$$

$$e = \frac{2R^3}{3A_1} \gamma^3 - \frac{1}{A_1} \int_{h-R}^r 2x^2 \sqrt{r^2 - x^2} dx, \quad (6)$$

Suppose that $R=1$ and $r=0.5$, Figure 3 shows the curve of I_x^C and I_y^C when λ changes from 0 to 1. It can be seen that both I_x^C and I_y^C increase with the increase of λ , but the change trends are different. Furthermore, note that I_x^C and I_y^C are the same value at dimensionless depth of the crack $\lambda = 1$, this is because the cross section of the crack element is a semicircle in this case which has the equal moment of inertia across the x and y -axis. Similar results are also discussed in Refs. [2][3] and [11].

Figure 3 The curve of I_x^C and I_y^C with respect to λ ($R=1$, $r=0.5$)

The moment of inertia of the uncracked intact shaft section relative to x and y axes is noted as I , then according

to the principle of superposition of the moment of inertia, the inertia moment of uncracked segment A_1 can be given as

$$I_1 = I - I_x^C, \quad (7a)$$

$$I_2 = I - I_y^C, \quad (7b)$$

where $I = \pi(R^4 - r^4)/4$, subsequently, the area moment of inertia of cross-section A_1 relative to the fixed coordinate system $O-xy$ need to convert to the centroid coordinate system $C-\bar{x}\bar{y}$, which can be denoted as $\bar{I}_1$ and $\bar{I}_2$ respectively and obtained by the parallel axis shift theorem as follows

$$\bar{I}_1 = I_1 - A_1 e^2, \quad (8a)$$

$$\bar{I}_2 = I_2, \quad (8b)$$

During the rotation of the rotor, the crack will periodically open and close due to the influence of the rotor's weight so it is necessary to introduce a breathing function to approximate this process. In this paper, the cosine breathing function proposed by Mayes[1] is adopted, which is given as

$$f(t) = \frac{1}{2} (1 + \cos(\Omega t)), \quad (9)$$

where Ω is the rotational speed of the shaft. Then the time-varying moment of inertia of the crack element can be calculated as.

$$I_{\bar{x}}(t) = I - (I - \bar{I}_1)f(t) = I - (I_x^C + A_1 e^2)f(t), \quad (10a)$$

$$I_{\bar{y}}(t) = I - (I - \bar{I}_2)f(t) = I - I_y^C f(t), \quad (10b)$$

When a crack appears at element j , the time-varying stiffness matrix of the crack element is calculated as follows.

$$K_{ce}^j = \frac{E}{l^3} \begin{bmatrix} 12I_{\bar{x}} \frac{1}{1+\phi_1} & 0 & 0 & 6I_{\bar{x}} \frac{1}{1+\phi_1} & -12I_{\bar{x}} \frac{1}{1+\phi_1} & 0 & 0 & 6I_{\bar{x}} \frac{1}{1+\phi_1} \\ 0 & 12I_{\bar{y}} \frac{1}{1+\phi_2} & 6I_{\bar{y}} \frac{1}{1+\phi_2} & 0 & 0 & -12I_{\bar{y}} \frac{1}{1+\phi_2} & -6I_{\bar{y}} \frac{1}{1+\phi_2} & 0 \\ 0 & -6I_{\bar{y}} \frac{1}{1+\phi_2} & I_{\bar{y}}^2 \frac{4+\phi_2}{1+\phi_2} & 0 & 0 & 6I_{\bar{y}} \frac{1}{1+\phi_2} & I_{\bar{y}}^2 \frac{2-\phi_2}{1+\phi_2} & 0 \\ 6I_{\bar{x}} \frac{1}{1+\phi_1} & 0 & 0 & I_{\bar{x}}^2 \frac{4+\phi_1}{1+\phi_1} & -6I_{\bar{x}} \frac{1}{1+\phi_1} & 0 & 0 & I_{\bar{x}}^2 \frac{2-\phi_1}{1+\phi_1} \\ -12I_{\bar{x}} \frac{1}{1+\phi_1} & 0 & 0 & -6I_{\bar{x}} \frac{1}{1+\phi_1} & 12I_{\bar{x}} \frac{1}{1+\phi_1} & 0 & 0 & -6I_{\bar{x}} \frac{1}{1+\phi_1} \\ 0 & -12I_{\bar{y}} \frac{1}{1+\phi_2} & 6I_{\bar{y}} \frac{1}{1+\phi_2} & 0 & 0 & 12I_{\bar{y}} \frac{1}{1+\phi_2} & 6I_{\bar{y}} \frac{1}{1+\phi_2} & 0 \\ 0 & -6I_{\bar{y}} \frac{1}{1+\phi_2} & I_{\bar{y}}^2 \frac{2-\phi_2}{1+\phi_2} & 0 & 0 & 6I_{\bar{y}} \frac{1}{1+\phi_2} & I_{\bar{y}}^2 \frac{4+\phi_2}{1+\phi_2} & 0 \\ 6I_{\bar{y}} \frac{1}{1+\phi_2} & 0 & 0 & I_{\bar{y}}^2 \frac{2-\phi_1}{1+\phi_1} & -6I_{\bar{y}} \frac{1}{1+\phi_2} & 0 & 0 & I_{\bar{y}}^2 \frac{4+\phi_1}{1+\phi_1} \end{bmatrix}, \quad (11)$$

1 where $\varphi_1 = \frac{12EI_{\bar{x}}}{\mu A_{ce} Gl^2}$ and $\varphi_2 = \frac{12EI_{\bar{y}}}{\mu A_{ce} Gl^2}$, μ is Poisson's ratio of the material, l is element length.

2 For simplicity, formula (11) can also be given in the following form

$$3 \quad \mathbf{K}_{ce}^j = \mathbf{K}^j - f(t)\mathbf{K}_c^j = (\mathbf{K}^j - \frac{1}{2}\mathbf{K}_c^j) - \frac{1}{2}\cos(\Omega t)\mathbf{K}_c^j, \quad (12)$$

4 where $\mathbf{K}^j$ is the stiffness matrix of the intact shaft element, $\mathbf{K}_c^j$ is the reduced stiffness matrix representing the effect of crack, which can be obtained by replacing the $I_{\bar{x}}$ and $I_{\bar{y}}$ in Equation (11) with and respectively.

2.3 The motion equation of the rotor system

5 According to rotor dynamics finite element method, Timoshenko beam theory and Lagrange equation, the motion equation of a dual-disk hollow shaft rotor with breathing crack can be given as follow

$$6 \quad \mathbf{M}\ddot{\mathbf{q}}(t) + \mathbf{C}\dot{\mathbf{q}}(t) + (\mathbf{K}^0 - \frac{1}{2}\mathbf{K}_c \cos \Omega t)\mathbf{q}(t) = \mathbf{F}_1 \cos \Omega t + \mathbf{F}_2 \sin \Omega t + \mathbf{F}_g, \quad (13)$$

7 where $\mathbf{K}^0 = \mathbf{K} - 1/2\mathbf{K}_c$, $\mathbf{C} = \mathbf{C} + \mathbf{G}$, $\mathbf{M}$, $\mathbf{C}$, $\mathbf{G}$ and $\mathbf{K}$ represent the global mass matrix, damping matrix, rotation matrix and stiffness matrix of the intact rotor system including flexible shaft element, disk, bearing and support all of which are $4(N+1) \times 4(N+1)$ matrix, N is the number of elements. $\mathbf{K}_c$ is the reduced stiffness matrix of the crack element, which is $\mathbf{K}_c^j$ in the element j and others are 0. $\mathbf{q}(t)$ is $4(N+1)$ the displacement vector of nodes, each node has 2 lateral displacements and 2 bending angles. $\mathbf{F}_1$ and $\mathbf{F}_2$ is the component of the unbalanced excitation in x and y direction, $\mathbf{F}_g$ is equivalent gravity force on the node and all of them are $4(N+1)$ vector. The specific forms of each matrix and vector in formula 13 are shown in the appendix.

3 Dynamic response analysis

3.1 Equation solving

8 In this paper, the Harmonic Balance Method (HBM) is used to solve equation (13). HBM supposes that the stable solution of the equation has the finite Fourier series solution

$$\mathbf{q}(t) = \mathbf{A}_0 + \sum_{j=1}^n (\mathbf{A}_j \cos(k\Omega t) + \mathbf{B}_j \sin(k\Omega t)), \quad (14)$$

where n is the number of harmonics used in HBM and it can be determined according to the accuracy requirement. This paper takes $n=4$ and inserts solution (14) in equation (13) yields

$$\begin{bmatrix} \hat{\mathbf{K}} & \mathbf{P}_{11} & \mathbf{P}_{21} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ -\mathbf{P}_{11} & \mathbf{P}_{31} & \mathbf{0} & \mathbf{P}_{21} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{P}_{22} & \mathbf{0} & \mathbf{P}_{32} & \mathbf{P}_{12} & \mathbf{P}_{22} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{P}_{22} & -\mathbf{P}_{12} & \mathbf{P}_{32} & \mathbf{0} & \mathbf{P}_{22} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{P}_{23} & \mathbf{0} & \mathbf{P}_{33} & \mathbf{P}_{13} & \mathbf{P}_{23} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{P}_{23} & -\mathbf{P}_{13} & \mathbf{P}_{33} & \mathbf{0} & \mathbf{P}_{23} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{P}_{24} & \mathbf{0} & \mathbf{P}_{34} & \mathbf{P}_{14} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{P}_{24} & -\mathbf{P}_{14} & \mathbf{P}_{34} \end{bmatrix} \begin{bmatrix} \mathbf{A}_1 \\ \mathbf{B}_1 \\ \mathbf{A}_2 \\ \mathbf{B}_2 \\ \mathbf{A}_3 \\ \mathbf{B}_3 \\ \mathbf{A}_4 \\ \mathbf{B}_4 \end{bmatrix} = \begin{bmatrix} \mathbf{F}_1^0 \\ \mathbf{F}_2 \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix}, \quad (15)$$

where $\hat{\mathbf{K}} = \mathbf{K}^0 - \Omega^2 \mathbf{M} - 1/8 \mathbf{K}_c \mathbf{K}^0 \mathbf{K}_c$, $\mathbf{P}_{1i} = i\Omega \mathbf{C}$, $\mathbf{P}_{2i} = 1/4 \mathbf{K}_c$, $\mathbf{P}_{3i} = \mathbf{K}^0 - (i\Omega)^2 \mathbf{M}$, $\mathbf{F}_i^0 = \mathbf{F}_1 - 1/2 \mathbf{K}_c \mathbf{K}^0 \mathbf{F}_g$, $i = 1, 2, 3, 4$. After solving the linear equations of equation (15) to obtain the coefficients of each harmonic component, the direct component $\mathbf{A}_0$ can be calculated as follow

$$\mathbf{A}_0 = \mathbf{K}^0 \left(\mathbf{F}_g - \frac{1}{4} \mathbf{K}_c \mathbf{A}_1 \right), \quad (16)$$

The dynamic response of the cracked rotor can be obtained by equation (14), equation (15), and equation (16). When there is no crack fault in the double-disk rotor system, the first 4 order critical speeds are $\omega_1 = 825.1 \text{ rad/s}$, $\omega_2 = 1359.2 \text{ rad/s}$, $\omega_3 = 1415.6 \text{ rad/s}$, and $\omega_4 = 2183.0 \text{ rad/s}$ according to the physical parameters in Table 1.

3.2 Effects of dimensionless crack depth

The position of the crack c_p is fixed at element 10 and **Error! Reference source not found.** shows the 3D waterfall diagram of rotor speed-dimensionless crack depth-amplitude at node 10 under the low speed (near the first critical speed ω_1) at the different crack depth of the rotor system. Node amplitude is calculated as

$$a_i = \sqrt{u_i^2 + v_i^2}, \quad (17)$$

where a_i is the amplitude of node i , u_i is the horizontal displacement of node i , and v_i is the vertical displacement of node i . Analysis from Figure 4 shows that the rotor system has obvious resonance peaks near the first critical speed, besides, compared to the intact rotor system without crack, the cracked rotor system

will have super-harmonic resonances near $1/2$, $1/3$, or $3/4$ even $1/4$ first-order critical speed due to the occurrence of crack, when the crack depth is relatively shallow, the rotor system will firstly appear obvious super-harmonic resonance peaks at $1/2 \omega_1$ and $1/3 \omega_1$, and as the crack depth gradually increases, there will be super-harmonic resonance peaks at $1/4 \omega_1$, but compared to the peaks at $1/2 \omega_1$ and $1/3 \omega_1$, the subcritical resonance peak at $1/4 \omega_1$ is weak. Furthermore, with the dimensionless crack depth improves, the super-harmonic resonance peaks near $1/n \omega_1$ ($n=2, 3, 4$) tend to be obvious, which indicates that the peak value of super-harmonic resonances near $1/n$ ($n=2, 3, 4$) first-order critical speed is closely related to the dimensionless depth of the crack. Moreover, Figure 5 shows the amplitude-frequency response curves at node 11 with $\lambda=0, 0.3, 0.6, 0.9$, and $c_p = 10$. It can be observed that crack slightly reduces the first-order critical speed of the rotor system, and with the increase of crack depth, the super-harmonic resonance peaks become more and more obvious.

Figure 4 3D waterfall diagram of rotor speed-dimensionless crack depth-amplitude at node 10 ($c_p = 10$)

Figure 5 Amplitude-frequency response curves for different crack depth at node 10 ($c_p = 10$)

Overall, the generation of crack can lead to the phenomenon of super-harmonic resonance near $1/n \omega_1$ ($n=2, 3, 4$), and the first critical speed and the peak

value of super-harmonic resonance are closely related to the dimensionless crack depth. These dynamic characteristics can be used as the basis for fault diagnosis of the depth of the crack.

3.3 Influence of crack position

The dimensionless crack depth λ is fixed at 0.5 and **Error! Reference source not found.** shows the 3D waterfall diagram of rotor speed-crack position-amplitude at node 10 under the low speed (near the first critical speed ω_1) at the different crack position of the rotor system. From **Error! Reference source not found.** it can be seen that when the crack is located at both ends of the rotor system, the effect on the dynamic response of the rotating system is weak, the peak value of super-harmonic resonance in amplitude-frequency response is unobvious. However, when the crack gradually approaches the middle of the rotor system, the super-harmonic resonance peak will increase. When the crack is located in the middle of the rotor system, the super-harmonic resonance of the rotor system is most obvious. At this time, the crack has the greatest impact on the dynamic response of the rotor system and it indicates that the peak value of super-harmonic resonance near $1/n$ ($n=2,3,4$) first critical speed of the rotor system is related to the crack position.

The crack position also has a certain influence on the critical speed of the rotor system. **Error! Reference source not found.** shows the amplitude-frequency response curves at node 10 near the main resonance area when the crack is located at element 1, 3, 5, and 10, where the corresponding first critical speed is 824.3rad/s, 822.1rad/s, 819rad/s, and 814.6rad/s separately. It can be found that as the position of the crack moves to the middle of the rotor system, the critical speed of the rotor system will slightly decrease. When the crack is located in the middle of the rotor system, the critical speed of the rotor system is the lowest.

Figure 6 3D waterfall diagram of rotor speed-crack position-amplitude at node 10 ($\lambda=0.5$)

Figure 7 Amplitude-frequency response curves for different crack position at node 10 ($\lambda=0.5$)

In short, the crack position affects the dynamic response of the rotor system. When the depth of the crack is constant, the closer the crack is to the middle of the rotor system, the more obvious the main resonance peak value and the super-harmonic resonance peak at $1/n$ critical speed are, and the smaller the critical speed is. These conclusions have been confirmed in Refs. [26], which can be used as the theoretical basis for identifying crack position.

3.4 Numerical verification

To verify the accuracy of the HBM adopted in this paper, this section uses the fourth-order Runge-Kutta method (R-K) to solve equation (13) and compares the results of HBM. Figure 8 shows time history, whirling orbits, and frequency spectrum solved by HBM and R-K respectively for $\lambda=0.3$, $c_p=10$ and $\Omega=500\text{rad/s}$, it can be seen that the calculation results are similar. At this time, the whirling orbit of the rotor system is an ellipse, and there is no multiple frequency components in the frequency spectrum, which indicates that there is no super-harmonic resonance phenomenon in the rotor system.

Moreover, Figure 9 shows time history, whirling orbits, and frequency spectrum solved by HBM and R-K respectively for $\lambda=0.3$, $c_p=10$ and $\Omega=410\text{rad/s} \approx 1/2\omega_1$ and the 2nd super-harmonic resonance occurs in this operating speed. It can be seen from Figure 9 that in the case of super-harmonic resonance, the calculation results of HBM and the R-K are still close, and the whirling orbits of the rotor system present shape with two closed loops. At the same time, the $2\times$ frequency component can be observed in the spectrum. These results are consistent with Refs. [11].

Figure 8 Comparison of time history, whirling orbits, and frequency spectrum for $\lambda=0.3$, $c_p=10$ and $\Omega=500\text{rad/s}$ at node 10: (a) Time history. (b) Whirling orbits. (c) Frequency spectrum.

Figure 8 Comparison of time history, whirling orbits, and frequency spectrum for $\lambda=0.3$, $c_p=10$ and $\Omega=410\text{rad/s} \approx 1/2\omega_1$ at node 10: (a) Time history. (b) Whirling orbits. (c) Frequency spectrum.

4 The fault diagnosis method of rotor system crack based on neural network

4.1 Radial basis function neural network

Radial Basis Function (RBF) neural network is a feedforward fully connected neural network. Compared with the traditional Backpropagation (BP) neural network, the RBF network generally contains only one hidden layer, then uses the radial basis function as the activation function in the hidden layer and adopts a direct connection between the input layer and hidden layer instead of weight connection, as shown in Figure 10.

Figure 10 Structure of RBF and BP neural network: (a) RBF neural network. (b) BP neural network.

The output of the RBF network can be calculated as

$$y_i = \sum_{j=1}^q W_{ij} \rho(\mathbf{x}, \mathbf{c}_j) + b_i, \quad (18)$$

where $\mathbf{x}$ is the input vector of RBF network, $\rho(\mathbf{x}, \mathbf{c}_j)$ is the activation function of j th hidden layer neuron, $\mathbf{c}_j$ is the center of radial basis function in j th hidden layer neuron, W_{ij} represents the weight between the j th hidden layer neuron and the i th output layer neuron, and b_i is the bias of i th output layer neuron, q is the number of hidden layer neuron and y_i is the output value of i th output layer neuron. There are various forms of radial basis functions in

the RBF network, and the most commonly used is the Gaussian function, which is given as

$$\rho(\mathbf{x}, \mathbf{c}) = \exp\left(-\frac{\|\mathbf{x} - \mathbf{c}\|_2^2}{2\sigma^2}\right), \quad (19)$$

where $\|\mathbf{x} - \mathbf{c}\|_2$ represents the Euclidean norm, which is used to characterize the distance between the input vector and the center of the radial basis function. σ is the spread constant of the RBF and it affects the size of the radial basis function activation region. When the input dimension is 2, take the spread constant is 1 and the center is (0,0), then the Gaussian function image is shown in Figure 11. It can be seen from Figure 11 that the Gaussian function has a relatively sensitive response near the center, while the function value in most areas far away from the center is almost 0, which means inactive. The nature of local activation allows the RBF network to set up more neurons without overfitting.

Figure 11 Amplitude-frequency response curves for different crack position at node 10 ($\lambda=0.5$)

4.1 Calculate the dimensionless crack depth with RBF network

The dimensionless depth and position of the crack are assigned by random numbers and then the dynamic response of the crack rotor system can be obtained by using the method in Section 3. The dynamic characteristics of first critical whirl speed, first subcritical speed, first critical speed amplitude, and the super-harmonic resonance peak at 1/2 first critical whirl speed are taken as the input of the RBF network and the output of the RBF network is dimensionless crack depth. Take 70% of the 188 sets of data as the training set, 15% as the validation set, and 15% as the testing set. Some data sets are shown in Table 2 and Table 3.

Table 2 Part of the data used to train the RBF network

λ	c_p	first critical speed (rad/s)	first critical speed amplitude (log)	first critical speed (rad/s)	first critical speed amplitude (log)
0.4050	4	822	-5.4783	411	-8.0111
0.3086	10	820	-5.4808	410	-8.0141
0.3149	19	824	-5.5058	412	-8.1428
0.3384	20	825	-5.5032	412	-8.0971
0.3507	13	820	-5.5021	410	-7.9737
0.3919	11	817	-5.4271	409	-7.9061
0.4295	1	824	-5.4968	412	-7.9544
0.4328	3	823	-5.4879	411	-7.9721
0.4749	6	818	-5.3933	409	-7.8852
0.4947	17	821	-5.4587	411	-7.9143

Table 3 Part of the data used to validate and test the RBF network

λ	c_p	first critical speed (rad/s)	first critical speed amplitude (log)	first critical speed (rad/s)	first critical speed amplitude (log)
0.5740	4	820	-5.4097	410	-7.8359
0.3204	6	821	-5.4837	411	-8.0734
0.3889	15	820	-5.4652	410	-7.9463
0.5191	10	813	-5.0707	406	-7.7704
0.32279	5	822	-5.4847	411	-8.0861
0.3319	8	820	-5.4774	410	-8.0090
0.3450	14	820	-5.4742	410	-7.9882
0.3488	3	823	-5.4874	412	-8.0571
0.3512	5	822	-5.4925	411	-8.0536
0.3536	9	819	-5.4659	409	-7.9957

Table 4 Training parameters of the RBF network

Loss function	Max epochs	Spread constant	Steps of validation failure	Clustering algorithm	Training algorithm
MSE	100	0.8	6	k-means	OLS

The mean square error (MSE) function is used as the loss function during the training, which is given as

$$MSE = \frac{1}{m} \sum_{k=1}^m \sum_{j=1}^l (\hat{y}_j^k - y_j^k)^2, \quad (20)$$

where l is the number of output layer neurons, m is the sample size, $\hat{y}_j^k$ is the output of the j th neuron for the k th sample input, and y_j^k represents the actual j th output of the k th sample input. The training parameters during RBF network training are shown in Table 4. The spread constant of the RBF network is 0.8. Steps of validation failure are 6 which means that the training process is terminated when the loss function of the validation set is no longer descending for 6 consecutive times. The selection algorithm of the radial basis function center adopts the k-means clustering algorithm and the training algorithm of the RBF network is the ordinary least square (OLS) with a closed-form solution. RBF network gradually reduces the loss function by adding the number of hidden layer neurons one by one from 0, so there is no need to set the initial number of hidden layer neurons in advance.

Figure 12 to Figure 14 show the training and test results of the RBF network under three conditions: crack is in the middle of the rotor system ($c_p = 10$), crack is at the end of the rotor system ($c_p = 1$), and crack position is uncertain.

When the crack is located in the middle of the rotor system, it can be seen from Figure 12 (a) that the loss function of the training set, validation set, and test set all show an oscillating downward trend with the increase of the number of iterative steps, the loss function value of the validation set is minimized at 48th step and the training process terminates in advance at the 54th epoch. At the end of the training, the loss function value of the training set is 1.21×10^{-5} , the validation set is 3.16×10^{-5} , and the test set is 2.24×10^{-5} . All three values are reduced to 10^{-5} and with little difference, indicating that the RBF network has great generalization ability. Besides, it can be seen from Figure 12 (b) that the scatter points of the neural network

output almost all fall on the ideal regression curve, and the correlation coefficient R between the output results of the neural network and the sample results is 0.9985, the maximum error is 1.8%, which indicates that the RBF neural network output is almost linearly correlated with the sample results and the network has good performance. According to the analysis results in Section 3, when the crack is located in the middle of the rotor system at the same dimensionless depth, the impact on the dynamic response of the rotor system is the greatest, and the change of critical speed and the peak of super-harmonic resonance is the largest. That is to say, the dynamic characteristics related to the crack are the most obvious, so when the crack is located in the middle of the rotor system, the RBF network can get a more accurate result.

Furthermore, when the crack is located at the end of the rotor system, similarly, it can be seen from Figure 12 (a) that the loss function of the training set, test set, and validation set to decrease with the increase of iterative steps, the RBF neural network achieves the best performance at 35th step and training process terminates prematurely at 41st epoch. At the end of the training, the loss function value of the training set is 5.42×10^{-4} , the verification set is 1.08×10^{-3} and the test set is 8.40×10^{-4} . Compared with the case where the crack is located in the middle of the rotor system, the value of the loss function is much larger. In addition, from Figure 13 (b), it can be seen that when the crack is relatively shallow, many of the output results of the RBF network deviate from the ideal regression curve compared with Figure 12 (b), and the maximum error is 9.3% which means that the performance of the network declined slightly compared to the condition that the crack is located in the middle of the shaft. According to the calculation results in Section 3, when the crack is located at the end of the rotor system, because of bearing and support with great rigidity, when

the crack is shallow, the effect on the dynamic response of the rotor is relatively small, it leads to very insignificant dynamic characteristics such as critical speed changes and super-harmonic resonance peaks, which results in a decrease in the performance of the RBF network.

Moreover, Figure 12 and Figure 13 are the results of calculating the dimensionless crack depth when the crack position is fixed. However, in actual engineering problems, the crack may appear in various positions in the rotor system. Figure 14 shows training and test results of the RBF network when the crack position is uncertain. From Figure 14 it can be observed that during the training process, the loss function value of the training set oscillates and decays in a small range as the number of iteration steps increases, the loss function value of the validation set is minimized at the 48th epoch and training process terminates prematurely at 56th epoch. When the training is terminated, the loss function value of the training set is 2.19×10^{-4} , the test set is 2.23×10^{-4} . Both are relatively small and not very different, which indicates that the training effect is great and there is no serious overfitting. The performance of the RBF network can be seen more intuitively from the regression output results in Figure 14 (b), most of the regression output results are around the ideal regression curve and without significant deviation. The correlation coefficient R between the RBF network output and the sample result is 0.9916, indicating that the linear correlation is very high, and the maximum error of the entire output result is 7.6%. The above analysis results show that even when the crack location is uncertain, the RBF network can make good use of the dynamic characteristics such as first-order critical speed, first-order subcritical speed, first-order critical speed amplitude, and first-order super-harmonic resonance amplitude to diagnose the crack depth, and can achieve high precision.

Figure 12 Training and test results of the RBF network when the crack is located in the middle of the rotor system: (a) Loss function curve. (b) Regression output.

Figure 13 Training and test results of the RBF network when the crack is located at the end of the rotor system: (a) Loss function curve. (b) Regression output.

Figure 14 Training and test results of the RBF network when the crack position is uncertain: (a) Loss function curve. (b) Regression output.

4.3 Pattern recognition neural network

Pattern recognition neural network, hereinafter referred to as PRNN, is a kind of neural network which is adopted to solve classification problem and its main structure is shown in Figure 15.

Figure 15 The structure of the pattern recognition network

The input first undergoes an affine transformation to the hidden layer, then it will be activated by the hidden layer activation function. The most common activation function

used in the hidden layer of the PRN is the Tan-sigmoid function, which is given as

$$f(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}, \quad (21)$$

The output after the activation function of the hidden layer will undergo an affine transformation to the Softmax layer again, and the function of the Softmax layer is to convert the output into a discrete probability density, which is obtained as

$$y_i = \text{softmax}(x_i) = \frac{\exp(x_i)}{\sum_{i=1}^n \exp(x_i)}, \quad (22)$$

where $\mathbf{x} = [x_1, x_2, \dots, x_n]^T$ is the input vector. From equation (22) we can see that the output vector $\mathbf{y} = [y_1, y_2, \dots, y_m]^T$ is non-negative and the sum of $\mathbf{y}$ is 1 after the softmax layer and it is the property of the discrete probability density. Generally, we take the location of the maximum

probability density as the result of classification, for example, if the output result of PRNN for input is [0.1, 0.6, 0.3], it is considered that this input should be classified into the second category.

4.4 Identification of crack position with PRNN

Similar to Section 4.2, the dynamic characteristics of first critical whirl speed, first subcritical speed, first critical speed amplitude, and the super-harmonic resonance peak at 1/2 first critical whirl speed are taken as the input of PRNN and the output of PRNN is the discrete probability density of crack location (20 elements). The training set, validation set, and test set are the same as in Section 4.2.

Cross entropy is commonly used as the loss function in the classification problem, which meaning is to characterize the similarity of the probability distribution of two events and can be calculated as

$$Crossentropy(P, Q) = -\sum_{i=1}^n P(x_i) \log(Q(x_i)), \quad (22)$$

where $P(x_i)$ is the actual probability distribution of event x_i , it is a sparse vector in the classification problem where the position of the category is 1 and all the other elements are 0. $Q(x_i)$ is the discrete probability density of event x_i output by PRNN. Some other training parameters are shown in Table 5. The number of hidden neurons is 15, the steps of validation failure are 6 which means that the training process is terminated when the loss function of the validation set is no longer descending for 6 consecutive times, the initial learning rate is 0.001 and the training algorithm adopts Levenberg-Marquardt backpropagation (L-M) which takes up more storage space but is faster.

Table 5 Training parameters of PRNN

Loss function	Number of hidden neurons	Max epochs	Steps of validation failure	Initial learning rate	Training algorithm
Cross entropy	15	100	6	0.001	L-M

Based on these parameters, the loss function curve of training and test are shown in Figure 16. It can be seen from Figure 16 that the loss function of the training set, validation set, and test set oscillates and decreases with the increase in the number of iteration epochs during the training process, the loss function value of the validation set is minimized at 66th step and the training process terminates in advance at the 72nd epoch. At this time, the

loss function value of the training set is 0.0147, the validation set is 0.0390 and the test set is 0.04148. The loss function value of the training set and the test set are close, indicating that the neural network has good generalization ability. The recognition accuracy of PRNN in the test set and validation set at the end of training is 78.6%. The basic requirements of crack location fault diagnosis are achieved.

Figure 16 The loss function curve of PRNN in the training process

Moreover, the problem of classification not only needs to pay attention to the correct situation of classification to achieve relatively high recognition accuracy but also needs to concentrate on the causes of classification error. Figure 17 shows the part of the confusion matrix of PRNN results with the misclassification category. As we can see from Figure 17, most of the elements of the confusion matrix are concentrated on the diagonals (dashed area), which means that the PRNN accurately identifies the location of the crack. And careful analysis of the misclassified elements in the confusion matrix can be found that most of the incorrectly classified PRNN output values are only one unit away from the actual position of the crack, for example, there are 4 cracks in element 15 actually which are identified in element 14 and 3 cracks in element 13 actually which are recognized in element 12 by PRNN. This is because the dynamic characteristics, including the first critical whirl speed, the first subcritical speed, the first critical speed amplitude, and the super-harmonic resonance peak at 1/2 first critical whirl speed, are related to both crack depth and position, which will confuse the PRNN when it comes to classification. But when the shaft is divided into 20 elements, the element length is very small, therefore, the deviation of 1 element length in determining the crack location will not cause a serious problem. Based on the consideration of this factor, the condition for

calculating the recognition accuracy can be extended to the 9
 recognition deviation within 1 element length, which10
 means that if the output of the PRNN is 12 or 14 and the11
 actual crack position is 13, it can also be considered that12
 the output of the PRNN is approximately accurate. In this13
 condition, it can be seen from Figure 17 that almost all the14
 elements in the confusion matrix fall in the approximate15
 accurate area and only 1 element of the 56 samples has a
 16

Figure 17 The part of the confusion matrix of PRNN results with misclassification category

6 Conclusions

In this paper, a dual-disks rotor system model with a breathing crack suitable for short-thick hollow shaft rotors has been established based on the finite element method and Timoshenko beam theory. The dynamic response of the cracked rotor system has been obtained by using the fourth-order harmonic balance method. Then, a new crack fault diagnosis and location method for the dual-disk hollow shaft rotor system based on the RBF network and PRNN has been proposed. The simulation results show that this method has high fault diagnosis accuracy. The main conclusions are as follows.

(1) The crack will cause the rotor system to appear super-harmonic phenomenon near $1/n$ ($n=2,3,4$) first critical speed and the first critical whirl speed to drop

slightly. In addition, the super-harmonic resonance peak values in the amplitude-frequency response and the first-order critical speed of the rotor system are closely related to the dimensionless depth and position of the crack. The greater the crack depth is, the closer it is to the central part of the rotor system, the greater the influence it has on the dynamic response of the rotor system.

(2) Based on the RBF network and PRNN, the fault diagnosis of the crack depth and position is carried out by using the first critical whirl speed, the first subcritical speed, the first critical speed amplitude, and the super-harmonic resonance peak at $1/2$ first critical whirl speed. The results show that when the depth and position of the crack are both uncertain, the maximum error of the dimensionless crack depth calculated by the RBF network is 7.56%, and the approximate

recognition accuracy of the crack position obtained by PRNN can reach 98.2%, the requirements of crack fault diagnosis are satisfied preliminarily.

- (3) The rotor system model established in this study is suitable for short and thick hollow shaft rotor and can be used for accurate fault diagnosis under the circumstance of uncertain crack depth and position, which can provide a reference for rotor crack fault diagnosis in practical engineering problems.

7 Declaration

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Availability of data and materials

The datasets supporting the conclusions of this article are included within the article.

Authors' contributions

The authors' contributions are as follows: Yu-Hong Jin was in charge of the whole analyses and wrote the manuscript; Lei Hou assisted with sample analyses and revised the manuscript; Zhen-Yong Lu assisted with simulation analyses; Yu-Shu Chen revised the final manuscript. All authors read and approved the final manuscript.

Competing interests

The authors declare no competing financial interests.

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Ethics approval and consent to participate

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Appendix

Timoshenko beam finite element matrices in equation (13)

$$\mathbf{K}_s^c = \frac{EI}{(1+\varphi_s^2)l^3} \begin{bmatrix} K_{s1}^c & & & & & & & \\ 0 & K_{s1}^c & & & & & & \\ 0 & -K_{s2}^c & K_{s3}^c & & & & & \\ -K_{s2}^c & 0 & 0 & K_{s3}^c & & & & \\ -K_{s1}^c & 0 & 0 & K_{s2}^c & K_{s1}^c & & & \\ 0 & -K_{s1}^c & K_{s2}^c & 0 & 0 & K_{s1}^c & & \\ 0 & -K_{s2}^c & K_{s4}^c & 0 & 0 & K_{s2}^c & K_{s3}^c & \\ -K_{s2}^c & 0 & 0 & K_{s4}^c & K_{s2}^c & 0 & 0 & K_{s3}^c \end{bmatrix} \quad \text{sym.}$$

$$K_{s1}^c = 12$$

$$K_{s2}^c = 6l$$

$$K_{s3}^c = (4 + 2\varphi_s + \varphi_s^2)l^2$$

$$K_{s4}^c = (2 - 2\varphi_s - \varphi_s^2)l^2$$

$$\xi = \frac{z}{l}, \varphi_s = \frac{12EI}{\mu A G l^2}$$

$$\mathbf{K}_s^c = \frac{EI\varphi_s}{(1+\varphi_s^2)l^3} \begin{bmatrix} K_{s1}^s & & & & & & & \\ 0 & K_{s1}^s & & & & & & \\ 0 & -K_{s2}^s & K_{s3}^s & & & & & \\ -K_{s2}^s & 0 & 0 & K_{s3}^s & & & & \\ -K_{s1}^s & 0 & 0 & K_{s2}^s & K_{s1}^s & & & \\ 0 & -K_{s1}^s & K_{s2}^s & 0 & 0 & K_{s1}^s & & \\ 0 & -K_{s2}^s & K_{s3}^s & 0 & 0 & K_{s2}^s & K_{s3}^s & \\ -K_{s2}^s & 0 & 0 & K_{s3}^s & K_{s2}^s & 0 & 0 & K_{s3}^s \end{bmatrix} \quad \text{sym.}$$

$$K_{s1} = 12, K_{s2} = 6l, K_{s3} = (2 - \varphi_s)l^2, K_{s4} = (4 + \varphi_s)l^2$$

$$\mathbf{M}_s^c = \frac{\rho_s l}{(1+\varphi_s^2)} \begin{bmatrix} M_{s1}^c & & & & & & & \\ 0 & M_{s1}^c & & & & & & \\ 0 & -M_{s4}^c & M_{s2}^c & & & & & \\ -M_{s4}^c & 0 & 0 & M_{s2}^c & & & & \\ M_{s3}^c & 0 & 0 & -M_{s5}^c & M_{s1}^c & & & \\ 0 & M_{s3}^c & -M_{s5}^c & 0 & 0 & M_{s1}^c & & \\ 0 & M_{s5}^c & -M_{s6}^c & 0 & 0 & M_{s4}^c & M_{s2}^c & \\ M_{s5}^c & 0 & 0 & -M_{s6}^c & M_{s4}^c & 0 & 0 & M_{s2}^c \end{bmatrix} \quad \text{sym.}$$

$$M_{s1}^c = 13/35 + 7/10\varphi_s + \varphi_s^2$$

$$M_{s2}^c = (1/105 + 1/60\varphi_s + 1/120\varphi_s^2)l^2$$

$$M_{s3}^c = 9/70 + 3/10\varphi_s + 1/6\varphi_s^2$$

$$M_{s4}^c = (11/210 + 11/120\varphi_s + 1/24\varphi_s^2)l$$

$$M_{s5}^c = (13/420 + 3/40\varphi_s + 1/24\varphi_s^2)l,$$

$$M_{s6}^c = (1/140 + 1/60\varphi_s + 1/120\varphi_s^2)l^2$$

$$\mathbf{M}_s^s = \frac{j_d}{(1+\varphi_s^2)l} \begin{bmatrix} M_{s1}^s & & & & & & & \\ 0 & M_{s1}^s & & & & & & \\ 0 & -M_{s4}^s & M_{s2}^s & & & & & \\ -M_{s4}^s & 0 & 0 & M_{s2}^s & & & & \\ -M_{s1}^s & 0 & 0 & -M_{s4}^s & M_{s1}^s & & & \\ 0 & -M_{s1}^s & M_{s4}^s & 0 & 0 & M_{s1}^s & & \\ 0 & -M_{s4}^s & M_{s3}^s & 0 & 0 & -M_{s4}^s & M_{s2}^s & \\ -M_{s4}^s & 0 & 0 & M_{s3}^s & -M_{s4}^s & 0 & 0 & M_{s2}^s \end{bmatrix} \quad \text{sym.}$$

$$M_{s1}^s = 6/5, M_{s2}^s = (2/15 + 1/6\varphi + 1/3\varphi^2)l^2,$$

$$M_{s3}^s = (-1/30 - 1/6\varphi + 1/6\varphi^2)l^2, M_{s4}^s = (1/10 - 1/2\varphi)l$$

$$\mathbf{J}_s = \frac{j_p}{30(1+\varphi_s^2)l} \begin{bmatrix} 0 & & & & & & & \\ J_{s1} & 0 & & & & & & \\ -J_{s2} & 0 & 0 & & & & & \\ 0 & -J_{s2} & J_{s4} & 0 & & & & \\ 0 & J_{s1} & -J_{s2} & 0 & 0 & & & \\ -J_{s1} & 0 & 0 & -J_{s2} & J_{s1} & 0 & & \\ -J_{s2} & 0 & 0 & J_{s3} & J_{s2} & 0 & 0 & \\ 0 & -J_{s1} & -J_{s3} & 0 & 0 & J_{s2} & J_{s4} & 0 \end{bmatrix} \quad \text{antisym.}$$

$$J_{s1} = 36$$

$$J_{s2} = 3l - 15l\varphi_s$$

$$J_{s3} = l^2 + 5l^2\varphi_s - 15l^2\varphi_s^2$$

$$J_{s4} = 4l^2 + 5l^2\varphi_s + 10l^2\varphi_s^2$$

Figures

Figure 1

Finite element model of a cracked two-disk hollow shaft rotor system

Figure 2

Cross-section diagram of hollow shaft crack: (a) Non-penetrating crack. (b) Passthrough crack. The dashed area represents the uncracked segment

Figure 3

The curve of I_x^C and I_y^C with respect to λ ($R=1, r=0.5$)

Figure 4

3D waterfall diagram of rotor speed-dimensionless crack depth-amplitude at node 10 ($cp = 10$)

Figure 5

Amplitude-frequency response curves for different crack depth at node 10 ($c_p = 10$)

Figure 6

3D waterfall diagram of rotor speed-crack position-amplitude at node 10 ($\lambda=0.5$)

Figure 7

Amplitude-frequency response curves for different crack position at node 10 ($\lambda=0.5$)

Figure 8

Comparison of time history, whirling orbits, and frequency spectrum for $\lambda=0.3$, $c_p=10$ and $\Omega=500$ rad/s at node 10: (a) Time history. (b) Whirling orbits. (c) Frequency spectrum.

Figure 9

Comparison of time history, whirling orbits, and frequency spectrum for $\lambda=0.3$, $c_p=10$ and $\Omega=410 \text{ rad/s} \approx 1/2 \omega_1$ at node 10: (a) Time history. (b) Whirling orbits. (c) Frequency spectrum.

Figure 10

Structure of RBF and BP neural network: (a) RBF neural network. (b) BP neural network.

Figure 11

Amplitude-frequency response curves for different crack position at node 10 ($\lambda=0.5$)

Figure 12

Training and test results of the RBF network when the crack is located in the middle of the rotor system:
 (a) Loss function curve. (b) Regression output.

Figure 13

Training and test results of the RBF network when the crack is located at the end of the rotor system: (a) Loss function curve. (b) Regression output.

Figure 14

Training and test results of the RBF network when the crack position is uncertain: (a) Loss function curve. (b) Regression output.

Figure 15

The structure of the pattern recognition network

Figure 16

The loss function curve of PRNN in the training process

Figure 17

The part of the confusion matrix of PRNN results with misclassification category