One-way flow over uniformly heated U-shaped bodies driven by thermal edge effects

Satoshi Taguchi (✉ taguchi.satoshi.5a@kyoto-u.ac.jp)

Kyoto University

Tetsuro Tsuji

Kyoto University

Research Article

Keywords: flow, edge, shaped, thermal, induced

Posted Date: October 29th, 2021

DOI: <https://doi.org/10.21203/rs.3.rs-970317/v1>

License: © ⓘ This work is licensed under a Creative Commons Attribution 4.0 International License.

[Read Full License](#)

Version of Record: A version of this preprint was published at Scientific Reports on February 4th, 2022.
See the published version at <https://doi.org/10.1038/s41598-022-05534-y>.

1 **One-way flow over uniformly heated U-shaped bodies driven by**
2 **thermal edge effects**

3 Satoshi Taguchi^{1,2*} and Tetsuro Tsuji^{1,2}

4 ¹*Department of Advanced Mathematical Sciences,*
5 *Graduate School of Informatics, Kyoto University, Kyoto 606-8501, Japan and*

6 ²*Research Project of Fluid Science and Engineering,*

7 *Advanced Engineering Research Center,*
8 *Kyoto University, Kyoto 606-8501, Japan*

9 (Dated: October 26, 2021)

Abstract

The thermal edge flow is a gas flow typically induced near a sharp edge (or a tip) of a uniformly heated flat plate. This flow has potential applicability as a nonmechanical flow controller in microelectromechanical systems (MEMS). However, it has a shortcoming: the thermal edge flows from each edge cancel out, resulting in no net flow. In this study, to circumvent this difficulty, the use of a U-shaped body is proposed and is examined numerically. More specifically, a rarefied gas flow over an array of U-shaped bodies, periodically arranged in a straight channel, is investigated using the direct simulation Monte-Carlo (DSMC) method. The U-shaped bodies are kept at a uniform temperature different from that of the channel. Two types of U-shaped bodies are considered, namely, a square-U shape and a round-U shape. It is demonstrated that a steady one-way flow is induced in the channel for both types. The mass flow rate is obtained for a wide range of the Knudsen numbers, i.e., the ratio of the molecular mean free path to the characteristic size of the U-shape body. For the square-U type, the direction of the overall mass flow is in the same direction for the entire range of the Knudsen numbers investigated. For the round-U type, the direction of the total mass flux is reversed when the Knudsen number is moderate or larger. This reversal of the mass flow rate is attributed to a kind of thermal edge flow induced over the curved part of the round-U-shaped body, which overwhelms the thermal edge flow induced near the tip. The force acting on each of the bodies is also investigated.

I. INTRODUCTION

In a gas in which the molecular mean free path is not vanishingly small compared to the system characteristic length, that is, in a rarefied gas, a steady flow is induced thermally in the absence of external forces (e.g., gravity). Examples of thermally induced flow, peculiar to a rarefied gas, include the thermal stress slip flow [1, 2], the nonlinear thermal stress flow [2, 3], and the thermal edge flow [4–6] in addition to the classical thermal creep flow (or the thermal transpiration) [7–11]. In this paper, we are concerned with the application of the thermal edge flow with an emphasis on the possibility to induce a one-way flow, which is potentially significant for, e.g., microelectromechanical systems (MEMS) [12, 13].

* taguchi.satoshi.5a@kyoto-u.ac.jp

The thermal edge flow is, typically, induced near the edges of a uniformly heated (or cooled) flat plate. Though localized near the edge, the magnitude of the flow velocity is proportional to the square-root of the mean free path when the Knudsen number is small [5, 6], which is stronger than the thermal creep flow. Here, the Knudsen number is the ratio between the molecular mean free path and the system characteristic length, namely, the size of the plate. Moreover, unlike the thermal creep flow, there is no need to maintain the temperature gradient of the plate, making it easy to use in practice [13, 14]. A prototypical thermal edge pump has been fabricated in [15], in which heated and unheated arrays of plates are arranged cleverly so that a net flow is produced. Later, the use of structured channels with ratchet-like topography was proposed with numerical verification, e.g., [16–18].

In the original thermal edge flow, a single flat plate is insufficient to induce a net mass flow in a particular direction because forward and backward thermal edge flows from each end cancel out each other. Now, suppose that a flat plate is bent so that it forms a U-shaped body. Then, the two tips are aligned, and the other side of the object is of a round (or curved) shape. In this situation, each tip will induce a thermal edge flow toward the curved side when heated, but the curved face is unlikely to induce a counter flow. Thus, we expect a net flow to occur from the tips toward the curved side, or a one-way flow to occur if the U-shaped bodies are arranged repeatedly in a channel. Motivated by this consideration, in this study, we investigate a rarefied gas flow over periodically arranged U-shaped bodies using the direct simulation Monte-Carlo (DSMC) method.

As we will see, each U-shaped body in the array is subject to a force. The situation is similar to the radiometric force exerted on a heated plate [6, 19, 20]. Thus, if the array is freely movable along the channel, a motion of the array will be induced upon being heated. In this connection, it would be worth mentioning a Crookes radiometer with cup-shaped vanes [7]. The present analysis can be viewed as a model of a cup-shaped-vane radiometer.

II. FORMULATION

A. Problem

Let us consider a two-dimensional straight channel between two parallel plates at $x_2 = L$ and $x_2 = -L$ kept at a uniform temperature T_0 filled with a rarefied gas (x_i , $i = 1, 2, 3$,

are Cartesian coordinates). The parallel plates are separated by the distance $2L$ (the width of the channel is $2L$). Inside the channel, infinitely many U-shaped bodies are arranged periodically along the x_1 -axis with period H , as shown in Fig. 1. In this paper, we consider two types of (two-dimensional) U-shaped bodies.

- Type I (square-U type): Type I (also called the square-U type) consists of two horizontal line segments with length D joined by a vertical line segment (length D) (see Fig. 1(a)).
- Type II (round-U type): Type II (also called the round-U type) consists of two horizontal line segments with length $D/2$ joined by a semicircle with radius $D/2$ (see Fig. 1(b)).

The U-shaped bodies are kept at a uniform temperature $T_1(> T_0)$. There is no external force acting on the gas. In this situation, a flow is induced thermally around each body. We investigate the steady behavior of the gas in the channel under the following assumptions:

1. The behavior of the gas is described by the Boltzmann equation for a hard sphere gas.
2. The gas molecules impinging on the plates or the bodies leave the boundary according to the corresponding part of the stationary Maxwellian distribution with temperature and density given by the temperature of the wall and by the impermeability condition across the surface, respectively. In other words, we impose the diffuse reflection boundary conditions [14] on the plates and on the bodies.

In the present study, we assume that the state of the gas is uniform in the x_3 direction (i.e., $\frac{\partial}{\partial x_3} = 0$).

B. Basic equations

Let ρ_{av} be the average density of the gas in the domain $-H/2 < x_1 < H/2$ and $-L < x_2 < L$ and let $p_0 = \rho_{\text{av}}RT_0$ be the reference pressure, where $R = k_B/m$ is the specific gas constant with k_B the Boltzmann constant and m the mass of a gas molecule. Besides the notation introduced in Sec. II A, we denote the molecular velocity by $\boldsymbol{\xi} = (\xi_1, \xi_2, \xi_3)$, the velocity distribution function of the gas molecules by $f(x_1, x_2, \boldsymbol{\xi})$, the mass density of the gas by ρ , the flow velocity by v_i ($i = 1, 2, 3$), the temperature of the gas by T , the pressure

FIG. 1. Schematics of the problem: (a) type I and (b) type II. In panel (a) [or (b)], infinitely many square-U (or round-U) bodies are arranged periodically along the x_1 -axis with period H between two parallel plates. The U-shaped bodies, located at $nH - \frac{D}{2} \leq x_1 \leq nH + \frac{D}{2}$ ($n = 0, \pm 1, \pm 2, \dots$), are uniformly heated at a constant temperature T_1 , while the plates are kept at a uniform temperature T_0 ($T_1 > T_0$). In both type I and type II, the part $-\frac{H}{2} < x_1 < \frac{H}{2}$ and $-L < x_2 < L$ forms a basic unit.

of the gas by p , and the stress tensor by p_{ij} ($i, j = 1, 2, 3$). The steady Boltzmann equation for a hard sphere gas in the present spatially two-dimensional problem is written as

$$\xi_1 \frac{\partial f}{\partial x_1} + \xi_2 \frac{\partial f}{\partial x_2} = \frac{d_m^2}{2m} \int (f' f'_* - f f_*) |\alpha_j (\xi_{*j} - \xi_j)| d\Omega(\boldsymbol{\alpha}) d\xi_{*1} d\xi_{*2} d\xi_{*3}, \quad (1a)$$

$$f = f(\xi_i), \quad f' = f(\xi'_i), \quad f'_* = f(\xi'_{*i}), \quad f_* = f(\xi_{*i}), \quad (1b)$$

$$\xi'_i = \xi_i + (\xi_{*j} - \xi_j) \alpha_j \alpha_i, \quad \xi'_{*i} = \xi_{*i} - (\xi_{*j} - \xi_j) \alpha_j \alpha_i, \quad (1c)$$

where d_m is the molecular diameter, $\boldsymbol{\alpha} = (\alpha_1, \alpha_2, \alpha_3)$ denotes unit vectors, and $d\Omega(\boldsymbol{\alpha})$ denotes the solid angle element in the direction of $\boldsymbol{\alpha}$; the integral in (1a) is carried out over the whole space for $\boldsymbol{\xi}_* = (\xi_{*1}, \xi_{*2}, \xi_{*3})$ and all directions for $\boldsymbol{\alpha}$. Note that the functional

dependency of f on (x_1, x_2) is omitted in (1b).

The diffuse reflection boundary conditions on the two parallel plates are written as

$$f = \frac{\sigma_{w0}}{(2\pi RT_0)^{3/2}} \exp\left(-\frac{|\boldsymbol{\xi}|^2}{2RT_0}\right), \quad \xi_2 \leq 0 \quad (-\infty < x_1 < \infty, x_2 = \pm L), \quad (2a)$$

$$\sigma_{w0} = \begin{cases} \sqrt{\frac{2\pi}{RT_0}} \int_{\xi_2 > 0} \xi_2 f d\boldsymbol{\xi} & (-\infty < x_1 < \infty, x_2 = L), \\ -\sqrt{\frac{2\pi}{RT_0}} \int_{\xi_2 < 0} \xi_2 f d\boldsymbol{\xi} & (-\infty < x_1 < \infty, x_2 = -L), \end{cases} \quad (2b)$$

where $d\boldsymbol{\xi} = d\xi_1 d\xi_2 d\xi_3$. Here and in what follows, the upper and lower signs go together.

The diffuse reflection boundary condition on the U-shaped body in the basic unit $-H/2 < x_1 < H/2$ and $-L < x_2 < L$ is described as follows. Let the U-shaped bodies of type I and II be denoted by D^I and D^{II} , respectively, that is,

$$D^I = \left\{ (x_1, x_2) \mid -\frac{D}{2} < x_1 < \frac{D}{2}, \quad x_2 = \pm \frac{D}{2} \right\} \cup \left\{ (x_1, x_2) \mid x_1 = \frac{D}{2}, \quad -\frac{D}{2} < x_2 < \frac{D}{2} \right\}, \quad (3a)$$

$$D^{II} = \left\{ (x_1, x_2) \mid -\frac{D}{2} < x_1 < 0, \quad x_2 = \pm \frac{D}{2} \right\} \cup \left\{ (x_1, x_2) \mid -\frac{D}{2} < x_2 < \frac{D}{2}, \quad x_1 = \sqrt{\left(\frac{D}{2}\right)^2 - x_2^2} \right\}. \quad (3b)$$

We also denote by D^{J+} and D^{J-} , $J = I$ or II , the convex and concave sides of D^J , respectively (see Fig. 2). Then, the unit normal vector n_i^+ (or n_i^-) on D^{J+} (or D^{J-}) pointing to the gas are given by

(Type I)

$$n_i^\pm(x_1, x_2) = \begin{cases} (0, \pm 1, 0), & -\frac{D}{2} < x_1 < \frac{D}{2}, \quad x_2 = \frac{D}{2} \pm 0, \\ (0, \mp 1, 0), & -\frac{D}{2} < x_1 < \frac{D}{2}, \quad x_2 = -\frac{D}{2} \mp 0, \\ (\pm 1, 0, 0), & x_1 = \frac{D}{2} \pm 0, \quad -\frac{D}{2} < x_2 < \frac{D}{2}, \end{cases} \quad (4a)$$

(Type II)

$$n_i^\pm(x_1, x_2) = \begin{cases} (0, \pm 1, 0), & -\frac{D}{2} < x_1 < 0, \quad x_2 = \frac{D}{2} \pm 0, \\ (0, \mp 1, 0), & -\frac{D}{2} < x_1 < 0, \quad x_2 = -\frac{D}{2} \mp 0, \\ \frac{\pm(x_1, x_2, 0)}{\sqrt{x_1^2 + x_2^2}}, & -\frac{D}{2} < x_2 < \frac{D}{2}, \quad x_1 = \sqrt{\left(\frac{D}{2}\right)^2 - x_2^2} \pm 0, \end{cases} \quad (4b)$$

where the upper (or lower) sign corresponds to D^{J+} (or D^{J-}). In the sequel, n_i represents either n_i^+ of n_i^- . With this preparation, the boundary conditions on the U-shaped bodies in the basic unit are summarized as

$$f(x_1, x_2, \xi_i) = \frac{\sigma_{w1}^\pm(x_1, x_2)}{(2\pi RT_1)^{3/2}} \exp\left(-\frac{|\boldsymbol{\xi}|^2}{2RT_1}\right), \quad \xi_i n_i^\pm > 0, \quad (x_1, x_2) \in D^{J\pm}, \quad (5a)$$

$$\sigma_{w1}^\pm(x_1, x_2) = -\sqrt{\frac{2\pi}{RT_1}} \int_{\xi_i n_i^\pm < 0} \xi_i n_i^\pm f(x_1, x_2, \xi_i) d\boldsymbol{\xi}. \quad (5b)$$

Since the U-shaped bodies are periodically arranged with the uniform interval H in the current setting, we look for a periodic solution with period H , that is,

$$f(x_1, x_2, \xi_i) = f(x_1 + H, x_2, \xi_i). \quad (6)$$

In other words, we restrict ourselves to the basic unit $-H/2 < x_1 < H/2$ and $-L < x_2 < L$ and impose the periodic boundary conditions at $x_1 = \pm H/2$ (see Fig. 2). Moreover, since $x_2 = 0$ is the symmetry plane, we shall analyze the problem in the upper half domain by imposing the following specular reflection at $x_2 = 0$:

$$f(x_1, 0, \xi_1, \xi_2, \xi_3) = f(x_1, 0, \xi_1, -\xi_2, \xi_3), \quad \xi_2 > 0. \quad (7)$$

The solution in the lower half domain is obtained by the mirror image of that in the upper half domain, that is, $f(x_1, x_2, \xi_1, \xi_2, \xi_3) = f(x_1, -x_2, \xi_1, -\xi_2, \xi_3)$ for $-L < x_2 < 0$.

Once the velocity distribution function f is obtained by solving the boundary-value problem (1), (2), and (5), the macroscopic quantities are given as the moments of f :

$$\begin{aligned} \rho &= \int f d\boldsymbol{\xi}, \quad v_i = \frac{1}{\rho} \int \xi_i f d\boldsymbol{\xi}, \quad RT = \frac{1}{3\rho} \int (\xi_i - v_i)^2 f d\boldsymbol{\xi}, \\ p &= \rho RT, \quad p_{ij} = \int (\xi_i - v_i)(\xi_j - v_j) f d\boldsymbol{\xi}, \end{aligned} \quad (8)$$

where the range of integration is the whole space of $\boldsymbol{\xi}$. To evaluate the pumping performance, the mass flow rate per unit length in the x_3 direction is introduced as M_f , i.e.,

$$M_f = \int_{-L}^L \rho v_1 dx_2. \quad (9)$$

Furthermore, we introduce the force F_i acting on each of the U-shaped body per unit length in the x_3 direction:

$$F_i = - \int_{\Sigma} p_{ij} n_j dS, \quad (10)$$

FIG. 2. The computational domains in the basic unit for (a) type I and (b) type II.

where dS is the surface element and the integration is carried out on the surface $\Sigma = D^{I+} \cup D^{I-}$ (or $\Sigma = D^{II+} \cup D^{II-}$) for type I (or type II); see the sentence below Eq. (3). Note that the geometrical symmetry leads to $F_2 = F_3 = 0$ for both type I and II.

Finally, we list the physical parameters that characterize the problem. They are summarized as

$$\frac{T_1}{T_0}, \quad \text{Kn} = \frac{\ell_0}{D} \quad \left(\ell_0 = \frac{1}{\sqrt{2}\pi d_m^2 (\rho_{av}/m)} \right), \quad \frac{H}{D}, \quad \frac{L}{D}, \quad (11)$$

where we have introduced the Knudsen number Kn with ℓ_0 being the mean free path of the gas molecules at the reference equilibrium state at rest. The DSMC simulation is carried out for various sets of these parameters. Further details of the simulation are given in Method.

III. RESULTS AND DISCUSSION

In the present paper, the geometric parameters are fixed to $H/D = 2$ and $L/D = 1$. There should be some optimal parameter set $(L/D, H/D)$ for given T_1/T_0 and Kn that maximizes M_f . Nonetheless, in this paper, we focus on the effects of Kn and T_1/T_0 , and, in particular, the impact of the difference between the square and round-U types.

A. Case of a single body

Before presenting the main results, we first consider the case in which a single U-shaped body is confined in a two-dimensional rectangular vessel filled with a rarefied gas and investigate the flow induced thermally in the vessel as a preliminary. To be more specific, we

consider a U-shaped body (type I or II) placed in a vessel enclosed by planes $x_1 = \pm H/2$ and $x_2 = \pm L$ kept at uniform temperature T_0 . The U-shaped body is located in the region $-D/2 \leq x_1 \leq D/2$ and $-D/2 \leq x_2 \leq D/2$ and is kept at a uniform temperature $T_1(> T_0)$ as in Sec. II. We investigate the steady behavior of the gas induced in the vessel under the same assumptions as in Sec. II, that is, the molecules make diffuse reflections on the sidewalls of the vessel as well as on the upper and lower walls. This situation is appropriate for investigating the qualitative behavior of the gas flow induced by the uniformly heated U-shaped body because there is no interaction between neighboring bodies inherent in the original problem. The parameters are the same as those shown in Eq. (11), and we will show the results for the case $H/D = 2$, $L/D = 1$, and $T_1/T_0 = 2$.

Figure 3 shows the flow velocity vector (v_1, v_2) and the contour curves of the temperature T for the square-U body case, in which (a) $\text{Kn} = 0.01$, (b) 0.05, (c) 0.1, (d) 0.5, (e) 1, and (f) 5. Note that the magnitude of the reference vector is smaller in panels (a), (e), and (f). It is seen from the figure that a significant flow is induced near the edge $(x_1/D, x_2/D) = (-0.5, 0.5)$ in the rightward direction when $\text{Kn} \lesssim 0.5$. The magnitude of this flow becomes relatively large when $\text{Kn} \approx 0.1$ and decreases as Kn is further increased. There is another relatively strong flow near the right corner $(x_1/D, x_2/D) = (0.5, 0.5)$ along the top and side plates when $\text{Kn} \gtrsim 0.1$. The corner flow is attributed to the same mechanism as the thermal edge flow, and its flow direction over the top side is opposite to the edge flow near the edge $(x_1/D, x_2/D) = (-0.5, 0.5)$. This leftward corner flow is weaker than the thermal edge flow when Kn is small, but it competes well against the rightward edge flow when $\text{Kn} \gtrsim 0.5$. Consequently, we observe four circulating flow patterns when $0.1 \lesssim \text{Kn} \lesssim 1$, whose centers are located near $(x_1/D, x_2/D) \approx (-0.5, 0.25)$ (clockwise), $(x_1/D, x_2/D) \approx (-0.5, 0.75)$ (counter clockwise), $(x_1/D, x_2/D) \approx (0.5, 0.75)$ (clockwise), and $(x_1/D, x_2/D) \approx (0.75, 0.25)$ (counter clockwise). These rolls are most significant for (c) $\text{Kn} = 0.1$ and (d) $\text{Kn} = 0.5$ and tend to vanish with a further increase of Kn .

The contour curves of the temperature T are concentrated near the edge when Kn is small, as pointed out in [4, 6], indicating a steep temperature variation there. As Kn increases, the deviation of the gas temperature from that of the body becomes large, and it is more prominent outside the body than inside. The gas is hotter inside the U shape than outside because the molecules inside have more chances to hit the heated wall and to encounter faster molecules. For high Kn cases (e.g., $\text{Kn} = 5$), the contour curves are bent near the

edge and the corner. The mechanism of such bending of isolines is discussed in the case of a free molecular gas in [21].

Now, to compare the flow field between the cases of square and round-U bodies, we show in Fig. 4 the corresponding figure for the round-U body. First, we notice that the overall flow structures are similar for the square and round-U types in the left half region, i.e., $x_1 < 0$. A difference in the flow field is observed around the outer face of the curved part, i.e., $x_1^2 + x_2^2 \gtrsim (D/2)^2$ with $x_1 > 0$. When $\text{Kn} \leq 0.1$, we hardly see the flow in this right half region. With the increase of Kn (≥ 0.5), a flow in the negative x_1 direction becomes noticeable along the outer face of the curved part. This flow is caused by the geometry of the round-U shape (called the blunt-edge effect), as discussed later. As we will see later in Sec. III C, the flow in the negative x_1 direction overwhelms the flow near the left edge and results in a negative pumping. The temperature isolines around the round-U type exhibit a similar bending in the left half domain, as seen in Fig. 3, and a milder bending in the right half domain for high Kn ($= 5$) compared with the square-U type [cf. Fig. 3(f) and Fig. 4(f)].

Incidentally, the flow vanishes when $\text{Kn} \rightarrow \infty$ in both square and round-U cases, as proved by a general theory for the free molecular flow [22, 23] (see [14], Chap. 2).

B. Flows over a periodic array of U-shaped bodies

In this section, we examine the behavior of the macroscopic quantities of the main problem (Sec. II). The values of the physical parameters (Kn and T_1/T_0) are the same as those in Sec. III A.

First, we show the flow velocity and the temperature field in Fig. 5 for the square-U body array. For relatively small Knudsen numbers $\text{Kn} \leq 0.1$ (panels (a-c)), we see the onset of a thermal edge flow at $(x_1/D, x_2/D) \approx (-0.5, 0.5)$ as in the case of a closed vessel (see Fig. 3). We also observe a weaker corner flow near $(x_1/D, x_2/D) = (0.5, 0.5)$. This leftward corner flow is overwhelmed by the rightward thermal edge flow in the region $0 < x_1/D < 0.5$ and $0.5 < x_2/D < 1$. Since the mass flow rate M_f across $x_1 = \text{const}$ should be independent of x_1 , this means that a net mass flow in the positive x_1 direction occurs in the channel. Note that $\int_0^{D/2} \rho v_1 dx_2 = 0$ at any cross section $x_1 = \text{const}$ inside the U-shaped body (i.e., $-0.5 < x_1/D < 0.5$ and $0 \leq x_2/D < 0.5$). The magnitude of the flow tends to decrease with the decrease of Kn , as seen from panels (c) $\rightarrow$ (b) $\rightarrow$ (a). On the contrary, for relatively large

FIG. 3. Flow velocity vector (v_1, v_2) and contour map T in the case of a single square-U body in a closed vessel ($T_1/T_0 = 2$, $H/D = 2$, and $L/D = 1$) for typical Knudsen numbers $\text{Kn} =$ (a) 0.01, (b) 0.05, (c) 0.1, (d) 0.5, (e) 1, and (f) 5. Note that the magnitude of the reference vector is different in panels (a), (e) and (f).

Knudsen numbers $\text{Kn} \geq 0.5$ (panels (d–f)), the flow patterns near the corner are similar between Figs. 3 and 5; there are two rolls near the corner. In particular, the appearance of two rolls near the left edge and the right corner in Fig. 5(d–f) suggests that the pumping

FIG. 4. Flow velocity vector (v_1, v_2) and contour map T in the case of a single round-U body in a closed vessel ($T_1/T_0 = 2$, $H/D = 2$, and $L/D = 1$) for typical Knudsen numbers $\text{Kn} =$ (a) 0.01, (b) 0.05, (c) 0.1, (d) 0.5, (e) 1, and (f) 5. See the caption of Fig. 3.

effect, if exists, is weak for these $\text{Kn} (\geq 0.5)$. It is also seen that the magnitude of the flow decreases as Kn becomes large (see, panels (d)→(e)→(f)).

The temperature field exhibits the same feature as in the case of a single body discussed in Sec. III A. However, since the heated bodies repeatedly appear in the current periodic case, the overall temperature inside the channel is higher.

FIG. 5. Flow velocity vector (v_1, v_2) and contour map T in the case of an array of square-U bodies ($T_1/T_0 = 2$, $H/D = 2$, and $L/D = 1$) for typical Knudsen numbers $\text{Kn} =$ (a) 0.01, (b) 0.05, (c) 0.1, (d) 0.5, (e) 1, and (f) 5. See the caption of Fig. 3.

Now, let us move on to the results for the array of round-U bodies, which are shown in Fig. 6. In the region $0 < x_1/D < 0.5$ and $0.5 < x_2/D < 1$, we observe a net mass flow in the positive x_1 direction for relatively small $\text{Kn} \leq 0.1$, as in the case of the array of square-U bodies. However, the case of round-U bodies leads to more x_1 -aligned flow patterns, and the magnitude of the flow in Fig. 6 is more significant than in Fig. 5. For relatively larger Kn

FIG. 6. Flow velocity vector (v_1, v_2) and contour map T in the case of an array of round-U bodies ($T_1/T_0 = 2$, $H/D = 2$, and $L/D = 1$) for typical Knudsen numbers $\text{Kn} =$ (a) 0.01, (b) 0.05, (c) 0.1, (d) 0.5, (e) 1, and (f) 5. See the caption of Fig. 3.

(≥ 0.5), the flow along the curved part, directed in the negative x_1 direction, is apparent. We have also observed such a flow in the closed case (see Fig. 4). If we look at the flow velocity across a particular plane perpendicular to the channel, e.g., $x_1 = H/2 (= D)$, we notice that the mass flow rate across the plane seems to be reduced due to this backward flow. The magnitude of the backflow is the largest around $\text{Kn} \approx 1$ and decreases when Kn

FIG. 7. Mass flow rate M_f in the x_1 direction per unit length in x_3 as a function of the Knudsen number Kn in the cases of $T_1/T_0 = 1.2, 1.5$, and 2 ($H_D = 2$, $L/D = 1$). (a) Square-U type, (b) round-U type. The symbols represent the numerical results, which are joined by the line segments.

TABLE I. Maximum values of M_f with respect to Knudsen number Kn for various T_1/T_0 in the case of $H/D = 2$ and $L/D = 1$.

Type	T_1/T_0	Kn	Maximum of $M_f/\rho_{av}D\sqrt{2RT_0}$
Type I (Square-U)	1.2	0.15	0.42×10^{-3}
	1.5	0.1	0.95×10^{-3}
	2.0	0.15	1.67×10^{-3}
Type II (Round-U)	1.2	0.15	0.69×10^{-3}
	1.5	0.15	1.51×10^{-3}
	2.0	0.15	2.70×10^{-3}

C. Mass flow rate

We now show the results for the mass flow rate as a measure of pumping performance. We show in Fig. 7 the mass flow rate M_f in the x_1 direction per unit length in x_3 for various Kn and T_1/T_0 (Kn = 0.1, 0.15, ..., 20; T_1/T_0 = 1.2, 1.5, and 2). The results for even smaller Knudsen numbers, Kn = 0.01, 0.015, ..., 0.07, are also included for the case of T_1/T_0 = 2.

First, let us look at the mass flow rate in the case of type I body, i.e., the square-U shape. When T_1/T_0 = 2, the mass flow rate increases (or decreases) with Kn when the Knudsen number is small (or large). There is a Knudsen number (Kn $\approx$ 0.1) at which M_f reaches its maximum for given T_1/T_0 . Although our results for small Kn are missing, we expect the same tendency for T_1/T_0 = 1.5 and 1.2. Note that M_f takes positive values for all Kn (except possibly noise effects). In other words, the mass flow rate is always in the positive x_1 direction in the case of a square-U type.

In the case of type II body, i.e., a round-U shape, the mass flow rate is also increasing in Kn when Kn is small. This trend is observed in the range Kn $\lesssim$ 0.1, as in type I body. Note that the values of M_f are considerably larger than those for type I body. We can understand the reason from a difference in the flow pattern observed in Fig. 5 (type I) and Fig. 6 (type II). That is, the leftward (blunter) thermal edge flow induced near the corner partly cancels the rightward thermal edge flow in the case of type I, while there is no such a (backward) thermal edge flow in the case of type II body, when Kn is small.

When Kn is large, the behavior of M_f for type II is qualitatively different from that for type I. In the case of type II, M_f decreases first, becomes negative, and then increases again to zero as Kn increases. In other words, a backward one-way flow occurs when Kn is large for type II. This negative M_f is caused by a “blunter” thermal edge flow induced along the round face as seen in Fig. 6 (d-f). More precisely, as Kn becomes large, the round face roughly acts as a (blunt) edge in the scale of the mean free path and induces a leftward flow along the heated surface. Moreover, the numerical result shows that this leftward flow dominates the rightward flow. We note that a similar blunt edge flow is also observed around an oval shape body [24]. We also mention that switching between positive and negative mass flow rates for large Kn has also been reported in the case of a structured channel [18]. In general, the behavior of M_f for large Kn is affected by the geometrical configuration of the boundary, and the flow feature is highly problem-dependent.

FIG. 8. Semi-logarithmic plot of the force $(F_1, 0, 0)$ acting on each of the U-shape bodies as a function of the Knudsen number Kn for $T_1/T_0 = 1.2, 1.5$, and 2 in the case of $H/D = 2$ and $L/D = 1$ ($T_1/T_0 = 1.2, 1.5$, and 2). (a) Type I (square-U shape) and (b) Type II (round-U shape). See the caption of Fig. 7.

To summarize, Table I shows the maximum value of M_f for given T_1/T_0 along with the value of Kn at which the maximum occurs in our computation ($H/D = 2$, $L/D = 1$).

D. Forces acting on U-shaped bodies

Finally, we discuss the force acting on the U-shaped body. Figure 8 summarizes the (normalized) force in the x_1 direction (per unit length in x_3) acting on (a) a square-U type body (type I) and (b) a round-U type body (type II), i.e., F_1 in Eq. (10), for various Kn and T_1/T_0 . The values of Kn and T_1/T_0 are the same as those in Fig. 7.

First, we observe that the force is in the direction of negative x_1 for both square and round-U types, and its magnitude increases with T_1/T_0 . Concerning the dependency on Kn , the magnitude of the force increases with Kn when Kn is small, attains its maximum at $\text{Kn} \approx 0.5$, and then decreases with a further increase of Kn . Table II shows the maximum value of $-F_1$ obtained in our numerical computations for various T_1/T_0 , along with the value

of Kn at which the maximum is attained. The maximum force acting on the round-U-type body is nearly twice larger than that of the square-U-type body. Indeed, it is found that a more significant force acts on the round-U type for all the Knudsen numbers investigated, as seen from Fig. 8. For small Kn, the force likely decreases in proportion to $\text{Kn}^{3/2}$ with the decrease of Kn for both types I and II.

Let us focus our attention on the case of small Kn. The momentum conservation implies that the thermal edge flow near the tip $(x_1/D, x_2/D) = (-0.5, 0.5)$ in the positive x_1 direction pulls the tips (and the body) in the negative x_1 direction. Indeed, as we see below, the distribution of the (local) tangential force on the body is leftward (i.e., the negative x_1 direction) on the whole horizontal part of the type II body (i.e., round-U type) and almost the entire horizontal part of the type I body (i.e., square-U type). Here, “almost” means that the local force is positive near the right corner located at $(x_1/D, x_2/D) = (0.5, 0.5)$ when Kn is small ($\text{Kn} \lesssim 0.1$). Near the right corner, another (weaker) thermal edge flow occurs in the negative x_1 direction. This counter-flow gives a contribution that partly cancels the negative force in the case of a type I shape.

As shown in the previous subsection, the direction of mass flow rate is reversed when Kn becomes large in type II. However, we do not observe such an inversion of the force acting on the array of round-U-shaped bodies (the total force is always negative, irrespective of Kn).

To provide a further insight into the force acting on the U-shaped body, we look at the distribution of the local stress $p_{1j}n_j$ along the body. For this purpose, we introduce a new coordinate (arc length) along the surface of the body, denoted by s , as follows (see also the inset in Fig. 9):

$$\begin{aligned}
 & \text{(Type I)} \\
 & s = s(x_1, x_2) = \begin{cases} x_1 + \frac{D}{2}, & -\frac{D}{2} \leq x_1 \leq \frac{D}{2}, \quad x_2 = \frac{D}{2}, \\ D + \left(\frac{D}{2} - x_2\right), & x_1 = \frac{D}{2}, \quad 0 \leq x_2 \leq \frac{D}{2}, \end{cases} \quad (12a) \\
 & \text{(Type II)}
 \end{aligned}$$

FIG. 9. Profiles of $\langle p_{1j}n_j/p_0 \rangle$ along a U-shape body as a function of the length s measured from the edge $(x_1/D, x_2/D) = (-0.5, -0.5)$ for $Kn = 0.1, 0.4, 2$, and 10 in the case of $T_1/T_0 = 2$, $L/D = 1$, and $H/D = 2$. The symbols represent the numerical results, which are joined by the line segments. The empty (or filled) symbols indicate the results of type I (or type II). The inset shows the definition of the coordinate s .

$$s = s(x_1, x_2) = \begin{cases} x_1 + \frac{D}{2}, & -\frac{D}{2} \leq x_1 \leq 0, \quad x_2 = \frac{D}{2}, \\ \frac{D}{2} + \frac{D}{2} \left[\frac{\pi}{2} - \arctan \left(\frac{x_2}{x_1} \right) \right], & 0 \leq x_1 \leq \frac{D}{2}, \quad x_2 = \sqrt{\left(\frac{D}{2} \right)^2 - x_1^2}. \end{cases} \quad (12b)$$

Then, the sum of the local stress over two faces D^{J+} and D^{J-} ($J = \text{I, II}$), which we denote by $\langle p_{1j}n_j \rangle$, is defined as a function of s :

$$\langle p_{1j}n_j \rangle(s) = p_{1j}n_j^+|_{(x_1, x_2) \in D^{J+}} + p_{1j}n_j^-|_{(x_1, x_2) \in D^{J-}}. \quad (13)$$

Note that the integration of $\langle p_{1j}n_j \rangle(s)$ over the entire s ($0 \leq s/D \leq 3/2$ for type I and $0 \leq s/D \leq (1 + \pi/2)/2$ for type II) leads to half the force F_1 .

We show in Fig. 9 the distribution of $\langle p_{1j}n_j \rangle(s)$ for types I and II for $Kn = 0.1, 0.4, 2$, and 10 in the case of $T_1/T_0 = 2$. It is seen that the negative component is dominant for the part $0 \leq s/D \leq 0.5$ for both types of I and II and for all Kn . In particular, the open symbols (type I) and closed symbols (type II) almost overlap when Kn is small (cf. $Kn = 0.1$) in

Type	T_1/T_0	Kn	Maximum of $-F_1/(p_0 D)$
Type I (Square-U)	1.2	0.4	1.16×10^{-3}
	1.5	0.4	2.90×10^{-3}
	2.0	0.4	5.58×10^{-3}
Type II (Round-U)	1.2	0.4	2.49×10^{-3}
	1.5	0.5	6.10×10^{-3}
	2.0	0.5	11.47×10^{-3}

TABLE II. Maximum values of $-F_1$ with respect to the Knudsen number Kn for various T_1/T_0 in the case of $H/D = 2$ and $L/D = 1$.

the same range, indicating that the local flow structures near the tips are the same for both types I and II in the near continuum regime: a thermal edge flow prevails there for both cases. A difference between type I and type II becomes apparent in the region $s/D > 0.5$. For type II, $\langle p_{1j} n_j \rangle$ increases with s and changes the sign from negative to positive at some value of s . If we magnify the figure (not shown here), we see that the coordinate s at which $\langle p_{1j} n_j \rangle$ changes the sign is larger for smaller Kn. On the other hand, for type I, $\langle p_{1j} n_j \rangle$ is negative up to $s/D = 1_-$ for Kn = 0.4, 2, and 10, and suddenly changes to positive values at $s/D = 1_+$ and remains positive for $s/D > 1$. In the case of Kn = 0.1, $\langle p_{1j} n_j \rangle$ becomes slightly positive in the region $0.9 \lesssim s/D < 1$, jumps to a negative value at $s/D = 1_+$, and then goes up to positive values as s increases. The decrease of $\langle p_{1j} n_j \rangle$ at $s/D = 1_+$ when Kn = 0.1 is attributed to the thermal stress enhanced by the sharp corner. We will come back to this point below. The area of the positive part in Fig. 9 is larger for type I than for type II, and this excess positive contribution explains smaller $|F_1|$ for type I.

Considering that the body's surface area is smaller for the round-U type than for the square-U type, such an enhancement of the force is helpful in applications for microflows, for which a design for further miniaturization is essential.

Finally, we discuss the cause of the negative jump of $\langle p_{1j} n_j \rangle$ at the right corner observed in the type I body when Kn is small. Figure 10 shows the profiles of the normal stress p_{11} along the outer and inner sides of the vertical segment of the type I body (i.e., $x_1 = D/2 \pm 0$ and $0 \leq x_2 < D/2$) for various Kn ($T_1/T_0 = 2$). It is seen that, with the decrease of Kn, a peak is formed near the corner $x_2 = D/2$ on the outer side of the segment [see Fig. 10(a)].

FIG. 10. Normal stress p_{11} along the vertical part of the type I shape, i.e., $x_1 = D/2 \pm 0$ and $0 \leq x_2 < D/2$, for various Kn in the case of $T_1/T_0 = 2$ ($H/D = 2$, $L/D = 1$). (a,b) Profiles of p_{11} along (a) $x_1 = D/2 + 0$ and (b) $x_1 = D/2 - 0$ for various Kn. Filled symbols are used for $\text{Kn} \geq 0.1$ and empty symbols for $\text{Kn} < 0.1$.

FIG. 11. An infinite expanse of a gas separated by a sharp-cornered two dimensional body with a uniform temperature T_1 . The temperature isolines around the body are schematically shown.

FIG. 12. Double-log plot of $\Delta p_{11} \equiv \sup_{0 < x_2 < D/2} (p_{11}(x_1 = \frac{D}{2} + 0, x_2) - p_{11}(x_1 = \frac{D}{2} + 0, 0))$ versus Kn . The value of $p_{11}(x_1 = \frac{D}{2} + 0, \frac{1}{2}D)$ is approximated by the node nearest to the corner (red circle), the linear extrapolation using two points near the corner (green square), and the 2nd-order extrapolation using 3 points near the corner (blue lower triangle). Extrapolation is based on the Lagrange formula.

On the other hand, we observe no formation of such a peak on the inner side; the profile becomes more flattened as Kn decreases [see Fig. 10(b)]. Therefore, the negative jump in $\langle p_{1j} n_j \rangle$ near $s/D = 1$ (see Fig. 9, open symbols $\circ$) is caused by the peak on the outer side near the corner. Here, we try to explain the origin of the peak using a crude estimate of the thermal stress near the corner, following the discussion of [6]. Consider an infinite expanse of a gas separated by a corner with a right angle as depicted in Fig. 11. Suppose that the temperature of the gas is described by the steady heat conduction equation (with a constant thermal conductivity for simplicity). Then, the temperature of the gas around the corner is given for the convex side by

$$T = T_1 \left[1 - a^+ r^{2/3} \sin \left(\frac{2}{3} \theta + \frac{\pi}{3} \right) \right], \quad -\frac{\pi}{2} < \theta < \pi, \quad (14)$$

and for the concave side by

$$T = T_1 [1 - a^- r^2 \sin(2\theta)], \quad \pi < \theta < \frac{3}{2}\pi, \quad (15)$$

where (r, θ) are polar coordinates with the origin on the corner, T_1 is the uniform temperature of the body, and a^+ and a^- are constants (a^+ and a^- are positive when the body is heated). Now we consider a plane $x_1 = \text{const}$ in the vicinity of the corner and estimate the normal stress τ_{11} on the plane, which is roughly given by [6]

$$\tau_{11} \sim T(x_1 + \ell, x_2) + T(x_1 - \ell, x_2) \sim 2T|_{(x_1, x_2)} + \frac{\partial^2 T}{\partial x_1^2} \Big|_{(x_1, x_2)} \ell^2. \quad (16)$$

Here, ℓ is the mean free path of the gas molecules. Thus, the normal component of the stress tensor is modulated by the term $(\partial^2 T / \partial x_1^2) \ell^2$ when a temperature inhomogeneity is present in the gas. This contribution, known as the thermal stress [14], is usually a small correction when ℓ is small. However, it can be large around a sharp edge, where the temperature field varies abruptly, as shown in [6]. In the present case with a sharp corner, a similar enhancement may be possible. In fact, if we take three points $(x_1, x_2) = (0, 0)$, $(\ell, 0)$, and $(2\ell, 0)$ in the vicinity of the corner, and estimate $\partial^2 T / \partial x_1^2$ by the central finite difference formula, we have, using (14),

$$\frac{\partial^2 T}{\partial x_1^2} \ell^2 \sim \frac{T(0, 0) - 2T(\ell, 0) + T(2\ell, 0)}{\ell^2} \ell^2 = \frac{\sqrt{3}(2 - 2^{2/3})}{2} a^+ \ell^{2/3}. \quad (17)$$

Therefore, the thermal stress is proportional to $\ell^{2/3}$, which is more substantial than ℓ^2 when ℓ is small. Note that a similar estimate applied to the inner side results in no enhancement of the thermal stress (due to the uniform temperature along the horizontal surface), which is also consistent with the behavior of the normal stress presented in Fig. 10(b).

To support the above crude estimate, Fig. 12 shows a double-log plot of $\Delta p_{11} \equiv \sup_{0 < x_2 < D/2} (p_{11}(x_1 = \frac{D}{2} + 0, x_2) - p_{11}(x_1 = \frac{D}{2} + 0, 0))$ as a measure of the peak on the outer side of the vertical segment, as a function of Kn for type I ($T_1/T_0 = 2$). Since the DSMC method is a cell-based method, it is rather difficult to evaluate the value of p_{11} at $(x_1, x_2) = (\frac{D}{2} + 0, \frac{D}{2})$ precisely. Therefore, we extrapolate p_{11} using the values on the nearest cells. As we can see from the figure, the peak decreases in proportion to $\text{Kn}^{2/3}$ as Kn is reduced, which is consistent with the above estimate (the thermal stress $\sim \ell^{2/3}$). In this way, the peak of the normal stress, and thus the sudden change in the local force at the corner, are explained by the enhancement of the thermal stress near the sharp corner when Kn is small.

IV. CONCLUDING REMARKS

In this paper, we have investigated a rarefied gas flow over an array of uniformly heated U-shaped bodies. With tips aligned in the same direction, such an object is expected to drive a thermally induced flow around the body by the thermal edge effect. A comparison was made between two types of U-shaped bodies: a square-U type and a round-U type.

Our computations using the DSMC method show that a one-way flow is induced in the channel for both types. Locally, the flow direction near the tips is such that the tips are facing the induced flow (i.e., the thermal edge flow). The direction of the overall mass flow rate is in the same direction as the thermal edge flow for type I for the entire range of Kn investigated. In type II shape, on the other hand, the mass flow rate is reversed when the Knudsen number becomes large ($\text{Kn} \gtrsim 1$). A kind of thermal edge flow (or a blunt edge flow) induced along the outer face of the curved part causes this reversal of the mass flow rate, by dominating the rightward thermal edge flow. Thus, there is a threshold of the Knudsen number above which the mass flow rate is reversed for type II. The mass flow rate is generally more significant for type II than type I for the same Knudsen number when they are in the same direction (Fig. 7).

Concerning the force acting on each of the bodies, the force direction is such that it tends to push the body with convex side trailing, irrespective of the value of Kn. The force is, in general, larger for the round-U shape than for the square-U shape for the same parameter set (Fig. 8). For type I body, the normal stress along the vertical side of the body exhibits a negative peak near the corner, which is explained by the thermal stress near the sharp corner.

METHOD

DSMC simulation

The simulation is carried out based on a non-dimensional version of the Boltzmann equation (1), in which all the quantities are scaled by suitable reference values. For instance, we use D and $\sqrt{2RT_0}$ as the reference length and speed, respectively.

The DSMC method solves the time-dependent Boltzmann equation and needs an initial

condition at time $t = 0$. We use the reference equilibrium distribution

$$f = \frac{\rho_{\text{av}}}{(2\pi RT_0)^{3/2}} \exp\left(-\frac{\xi_i^2}{2RT_0}\right), \quad t = 0, \quad (18)$$

as the initial condition. The computational domain $\mathcal{V}^J$

$$\mathcal{V}^J = \left\{ (x_1, x_2) \mid -\frac{H}{2} < x_1 < \frac{H}{2}, 0 < x_2 < L \right\} \setminus D_*^J, \quad J = \text{I, II}, \quad (19)$$

where D_*^J is the set obtained from D^J by restricting the range of x_2 to $x_2 > 0$, is divided into square cells with the area Δx^2 , where $\Delta x/D = 2.5 \times 10^{-2}$, i.e., the number of cells is 3200 for the square-U type. In the case of the round-U type, we need additional non-rectangular cells along the curved part. To be more precise, if a square cell has intersections with D_*^{II} at two points, the cell is divided into two portions, where the curve of D_*^{II} is approximated by a segment that connects these two intersections. These two portions are either (i) two trapezoids or (ii) a triangle and a pentagon, depending on the position of the intersections. The collision process and the computations of moments are carried out using these new cells with areas smaller than Δx^2 . On the other hand, the process of the free transport of the particles is done using the curve of D_*^{II} . The difference between the curve and the segment is so small that it does not affect the discussion of the paper. For $\text{Kn} \leq 0.05$, a smaller size of the basic cell with the length $\Delta x/D = 1.25 \times 10^{-2}$ is used.

At the initial time, N test particles are distributed in each cell, where $N = 10^2$ is used. Time step of the simulation is $\Delta t/t_0 = 10^{-3}$ (or 5×10^{-4}) for $\Delta x/D = 2.5 \times 10^{-2}$ (or 1.25×10^{-2}), where the reference time t_0 is defined as $t_0 = D/\sqrt{2RT_0}$. We judge steady state at $t/t_0 = 2 \times 10^2$ and start sampling at every time step. The number of samples is at least 2×10^7 for all the cases and increased up to 4×10^8 for the cases with small T_1/T_0 . Note that the macroscopic quantities obtained by the DSMC method are cell-based quantities, that is, any function of (x_1, x_2) (e.g., ρ) represents the value in a cell with its center at (x_1, x_2) .

To compute the mass flow rate M_f and the force F_1 , we use the following procedures instead of the direct integration of (9) and (10). In a steady state, the impulse acting on a body over a period Δt is written as $F_1 \Delta t$. This should be equal to the summation of the impulse given by a gas molecule that hit the body during the same interval Δt . Since the impulse is computed from the momentum change between before and after the impact, we

have

$$F_1 \Delta t = m_* \sum_j (\xi_1^{(j)} - \xi_1'^{(j)}), \quad (20)$$

where m_* is a mass of a test particle, $\xi_1^{(j)}$ and $\xi_1'^{(j)}$ are the x_1 component of the velocity of the j -th particle before and after the impact, respectively, and the summation in Eq. (20) runs over the index j for the molecules that hit the body during the interval Δt . The mass flow rate M_f is obtained by counting the number of test particle that goes across a plane $x_1 = H/2$ during the interval Δt . To be more specific, let $n_+^{(j)}$ and $n_-^{(j)}$ be the number of translocation from $x_1 < H/2$ to $x_1 > H/2$ and $x_1 > H/2$ to $x_1 < H/2$, respectively, of the j -th particle during the interval Δt . Then, M_f is given by

$$M_f \Delta t = m_* \sum_j (n_+^{(j)} - n_-^{(j)}), \quad (21)$$

where j runs over whole particles intersecting the plane $x_1 = H/2$.

ACKNOWLEDGMENTS

This work was supported by JSPS KAKENHI grant No. 17K06146 and No. 20H02067. The authors thank Mr. R. Yoshida for his assistance in numerical computation.

AUTHOR CONTRIBUTIONS

S.T. developed the basic concept, designed the study, and worked out physical interpretation and theoretical consideration. T.T. carried out numerical computations, checked the accuracy and reliability of the data, and worked out physical interpretation. All authors contributed to drafting the paper and approved the final version of the manuscript.

COMPETING INTERESTS

The authors declare no competing interests.

DATA AVAILABILITY

The datasets generated during the current study are available from the corresponding author on reasonable request.

- [1] Y. Sone and S. Tanaka, Thermal stress slip flow induced in rarefied gas between noncoaxial circular cylinders, in *Theoretical and Applied Mechanics*, edited by F. P. J. Rimrott and B. Tabarrok (North-Holland, Amsterdam, 1980) pp. 405–416.
- [2] Y. Sone, Flows induced by temperature fields in a rarefied gas and their ghost effect on the behavior of a gas in the continuum limit, *Annu. Rev. Fluid Mech.* **32**, 779 (2000).
- [3] V. S. Galkin, M. N. Kogan, and O. G. Fridlender, Free convection in a gas in the absence of external forces, *Fluid Dyn.* **6**, 448 (1971).
- [4] K. Aoki, Y. Sone, and N. Masukawa, A rarefied gas flow induced by a temperature field, in *Rarefied Gas Dynamics*, edited by J. Harvey and G. Lord (Oxford Univ. Press, Oxford, 1995) pp. 35–41.
- [5] Y. Sone and M. Yoshimoto, Demonstration of a rarefied gas flow induced near the edge of a uniformly heated plate, *Phys. Fluids* **9**, 3530 (1997), Supplementary Video: Kyoto University Research Information Repository (<http://hdl.handle.net/2433/122357>).
- [6] S. Taguchi and K. Aoki, Rarefied gas flow around a sharp edge induced by a temperature field, *J. Fluid Mech.* **694**, 191 (2012).
- [7] E. H. Kennard, *Kinetic Theory of Gases* (McGraw-Hill, New York, 1938).
- [8] Y. Sone, Thermal creep in rarefied gas, *J. Phys. Soc. Jpn.* **21**, 1836 (1966).
- [9] H. Niimi, Thermal creep flow of rarefied gas between two parallel plates, *J. Phys. Soc. Jpn.* **30**, 572 (1971).
- [10] T. Ohwada, Y. Sone, and K. Aoki, Numerical analysis of the Poiseuille and thermal transpiration flows between two parallel plates on the basis of the Boltzmann equation for hard-sphere molecules, *Phys. Fluids A* **1**, 2042 (1989), erratum: *ibid.* **2**, 639 (1990).
- [11] F. Sharipov, Rarefied gas flow through a long tube at arbitrary pressure and temperature drops, *J. Vac. Sci. Technol. A* **15**, 2434 (1997).

- [12] G. Karniadakis, A. Beskok, and N. Aluru, *Microflows and Nanoflows: Fundamentals and Simulation* (Springer Science+Business Media, New York, 2005).
- [13] X. Wang, T. Su, W. Zhang, Z. Zhang, and S. Zhang, Knudsen pumps: a review, *Microsystems Nanoeng.* **6**, 26 (2020).
- [14] Y. Sone, *Molecular Gas Dynamics: Theory, Techniques, and Applications* (Birkhäuser, Boston, 2007) Supplementary Notes and Errata: Kyoto University Research Information Repository (<http://hdl.handle.net/2433/66098>).
- [15] H. Sugimoto and Y. Sone, Vacuum pump without a moving part by thermal edge flow, in *Rarefied Gas Dynamics*, edited by M. Capitelli (AIP, Melville, 2005) pp. 168–173.
- [16] A. A. Donkov, S. Tiwari, T. Liang, S. Hardt, A. Klar, and W. Ye, Momentum and mass fluxes in a gas confined between periodically structured surfaces at different temperatures, *Phys. Rev. E* **84**, 016304 (2011).
- [17] J. Chen, S. K. Stefanov, L. Baldas, and S. Colin, Analysis of flow induced by temperature fields in ratchet-like microchannels by Direct Simulation Monte Carlo, *Int. J. Heat Mass Transf.* **99**, 672 (2016).
- [18] X. Wang, Z. Zhang, W. Zhang, P. Zhang, and S. Zhang, Numerical simulation of thermal edge flow in ratchet-like periodically patterned micro-channels, *Int. J. Heat Mass Transf.* **135**, 1023 (2019).
- [19] S. Taguchi and K. Aoki, Motion of an array of plates in a rarefied gas caused by radiometric force, *Phys. Rev. E* **91**, 063007 (2015).
- [20] T. Baier, S. Hardt, V. Shahabi, and E. Roohi, Knudsen pump inspired by Crookes radiometer with a specular wall, *Phys. Rev. Fluids* **2**, 033401 (2017).
- [21] S. Takata, T. Yoshida, T. Noguchi, and S. Taguchi, Singular behavior of the macroscopic quantities in the free molecular gas, *Phys. Fluids* **28**, 022002 (2016).
- [22] Y. Sone, Highly rarefied gas around a group of bodies with various temperature distributions I: Small temperature variation, *J. de Mécanique Théorique et Appliquée* **3**, 315 (1984).
- [23] Y. Sone, Highly rarefied gas around a group of bodies with various temperature distributions II: Arbitrary temperature variation, *J. de Mécanique Théorique et Appliquée* **4**, 1 (1985).
- [24] K. Aoki, Y. Sone, and Y. Waniguchi, A rarefied gas flow induced by a temperature field: Numerical analysis of the flow between two coaxial elliptic cylinders with different uniform temperatures, *Computers Math. Applic.* **35**, 15 (1998).