

Probabilistic Modelling of Sustainable Urban Drainage Systems

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Probabilistic modelling of Sustainable Urban Drainage Systems

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Abstract

Sustainable Urban Drainage Systems (SuDS) gatherer effective strategies and control systems for stormwater management especially in highly urbanized areas characterized by large impervious surfaces that increase runoff peak flow and volume. The main goal is to restore the natural water balance by increasing infiltration, evapotranspiration and promoting rainwater reuse. This paper proposes an analytical probabilistic approach for the modelling SuDS applicable to different structures and goals. Developed equations allow to estimate the probability of overflow and the probability of pre-filling at the end of dry periods, to evaluate the efficiency of the storage in rainwater management and its ability to empty between consecutive events. A great advantage of the proposed method is that it allows to consider a chain of rainfall events; this aspect is particularly important for control systems SuDS characterized by low outflow rates which storage capacity is often not completely available at the end of a dry period because pre-filled by previous events. Suggested formulas were tested to two cases studies in Milan and Genoa, Italy.

Keywords: Stormwater; Sustainable Urban Drainage Systems; Infiltration; Evapotranspiration; Rainwater Harvesting Systems; Probability; Overflow; Storage capacity; Pre-filling.

1 Introduction

The combination of more frequent extreme rainfall events and increase on impervious surfaces in highly urbanized areas produced a strong alteration of the water balance from natural conditions. This involves strong upheavals with heavy consequences especially in the most fragile areas; the increase of rainfall intensity causes more frequent floods while the decrease of the number of events entails water scarcity in many areas. In natural water balance, precipitation is largely evapotranspired from vegetation and soil or infiltrated into underlying layers and only a limited component produces runoff. In high urbanized contexts this balance is upset and only a little amount of precipitation is infiltrated and

evapotranspirated; moreover, there are numerous wastewater discharges and large volumes of poor-quality runoffs also for effect of imported potable waters to satisfy urban water needs. SuDS try to restore the natural water balance, promoting infiltration, evapotranspiration and rainwater harvesting; the creation of green spaces into the cities promotes the infiltration and evapotranspiration of part of the rainfall volume; the reuse of stormwater and wastewater reduces both runoff and wastewater discharges, as well as potable water consumption.. In accordance with the SuDS principles, peak discharges and flood volumes must be limited to pre-development values. Traditional urban drainage systems collected runoff is discharged into sewers with consequent high peak runoffs. Runoff peak flow reduction can be achieved by mean of detention tanks or oversize pipes, that store runoff during rainfalls, reducing and deleting peak flows. Runoff volume reduction can be achieved by evapotranspiration, infiltration, and rainwater reuse (Fig 1). SuDS promote these processes reducing both peak flows and runoff volumes; they involve several other benefits since they enhance the quality of urban spaces, improve health and wellbeing, maximise network capacity, improve water quality, provide amenity and recreation, improve thermal comfort, provide education and enhance biodiversity. Systems promoting infiltration are permeable pavements, swales, rain gardens, soakaways, infiltration trenches and infiltration basins; systems promoting evapotranspiration are green roofs, retention basins and wetlands; reuse can be achieved by Rainwater Harvesting Systems.

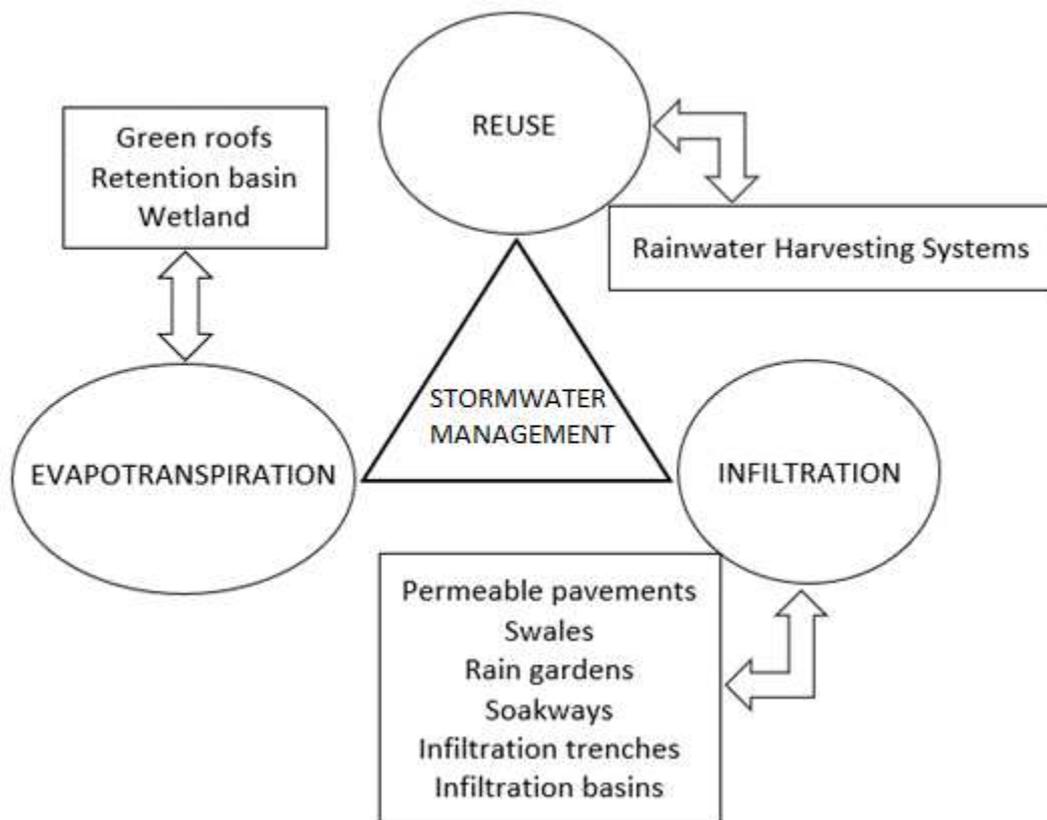


Fig. 1 Conceptual scheme to SuDS principles on maintain pre-development characteristic.

Depending on the kind of structures different parameters can be of interest in the analysis. For example, SuDS based on infiltration must mainly accomplish two different targets: to limit overflow, guaranteeing a sufficient retention capacity and to empty the storage capacity in a sufficient time to make it available to collect subsequent events. On the contrary, reuse systems and structures based on evapotranspiration benefit from partial pre-filling of storage capacity: the first to satisfy water requirement and the second for vegetation survival. This aspect must coexist with the necessity to limit overflows to drainage system. In literature there are several studies on SuDS, to analyse their performance and to define their characteristic parameters (Berndtsson, 2010; Carter and Rasmussen, 2007; Hakimdavar et al., 2014; Herrera et al., 2018; Lee, 2019; Li et al., 2017; Marchioni and Becciu, 2015; Marchioni and Becciu, 2018; Newman et al., 2013; Palermo et al., 2019; Palla et al., 2012). They are often based on continuous simulation, design approach or experimental formulas. In last decades analytical probabilistic approaches were often proposed as a good trade off to couple the simplicity of design storm methods to the accuracy of continuous simulations. They were applied to stormwater detention storages (Becciu and Raimondi, 2014; Becciu and Raimondi, 2015a; Becciu and Raimondi, 2015b; Raimondi and Becciu, 2015; Raimondi and Becciu, 2017), green roofs (Guo and Zhang, 2014; Guo, 2016; Raimondi and Becciu, 2020; Raimondi et al., 2020; Raimondi et al., 2021; Zhang and Guo, 2013), rainwater harvesting systems (Becciu et al., 2018; Raimondi and Becciu, 2011; Raimondi and Becciu, 2014a, Raimondi and Becciu, 2014b), permeable pavements (Raimondi et al., 2020; Zhang and Guo, 2013), infiltration facilities (Guo and Gao, 2016; Wang and Guo, 2020). The aim of this paper is to propose an analytical probabilistic approach for a complete analysis of SuDS suitable to structures with different configuration and functions. Developed equations allow to estimate both the probability of overflow and the probability pre-filling from previous events, typical of low outflow rates structures as SuDS. The combination of these two targets allows a reliable approach to design these kinds of facilities often characterized by the double need to avoid overflow and empty quickly to maximize their storage capacity. In the analysis, to be more adherent to the real process chained rainfalls were considered. Relate the probability to the Average Return Interval (ARI) makes the method useful to be used in practice for the definition of the main design parameters. Developed equations to estimate overflow and pre-filling PDF were tested by their application to two case studies in Genoa and Milan. Obtained results were compared with those obtained by the continuous simulations of observed data, to confirm the suitability of proposed approach.

2 Methodology

Runoff volume reduction can be achieved acting mainly through three main principles: infiltration, evapotranspiration, and rainwater reuse. The structure of the selected SuDS characterizes the output variable q considered in the modeling: for infiltration systems it is the infiltration rate f , for evapotranspiration systems it is the evapotranspiration rate Et and for reuse systems it is the required flow rate y (Fig 2).

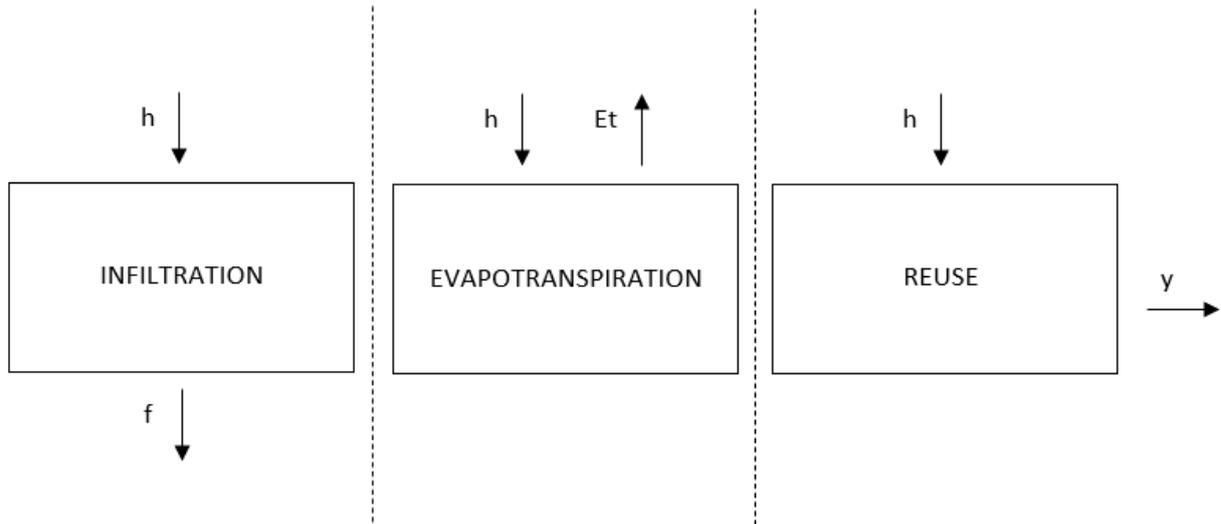


Fig. 2 Scheme of modeling for different structures

Outflow rate q was assumed constant. Infiltration rate was considered equal to infiltration rate at saturation; this condition is in favor of safety to estimate the overflow and the degree of emptying of these systems. Evapotranspiration rate was considered equal to potential evapotranspiration; this condition is again in favor of safety when considering for example green roofs, to assess vegetation survival without irrigation during dry periods. In case of reuse systems to consider a constant outflow means to assume a constant water demand, as often happens in practice on small time scale. For each scheme of Fig. 2, the maximum capacity assumes a different notation. For infiltration systems, as like permeable pavements, it results equal to $w = p \cdot z \cdot A_I$, where p is the porosity, z is the thickness of the storage volume and A_I is the infiltration area. For systems based on evapotranspiration, as like green roofs, it results $w = \phi_f \cdot z_g$, where ϕ_f is the soil moisture content at saturation and z_g is the growing medium thickness. For reuse systems the maximum storage capacity corresponds to the tank volume w .

Meteorological variables involved in the analysis, rainfall depth h , rainfall duration θ and interevent time d were assumed independent and exponentially distributed. These two hypotheses were deepened and defined acceptable by different studies in literature (Eagleson, 1978; Adam et al., 1986; Adam and Papa, 2020; Bedient and Huber, 1992). The bias due to their use is negligible in comparison to the reduction of complexity of the analytical derivation. To identify independent events from a continuous record of rainfalls, a minimum interevent time, the so-called Inter Event Time Definition (IETD) (USEPA 1986), was defined. If the interevent time between two consecutive rainfall events is smaller than IETD, the two rainfalls are joined together into a single event, otherwise they are considered distinct. With reference to the assumption

of exponential distribution of considered rainfall variables, Bacchi et al. (2008) tested that for most Italian basins the Weibull PDF fits the FDF of meteorological input variables better than the exponential PDF but its use involves a considerable complication in the equation's integration. Becciu and Raimondi (2014) verified that the double-exponential PDF complies with the FDF of observed data for the main rainfall characteristic parameters; this PDF has not integration problems, but derived expressions are more complex. Moreover, an application to a case study highlighted that the use of the double-exponential PDF only little improves the accuracy of results. The PDF of rainfall depth, rainfall duration and interevent time, considered exponentially distributed, results:

$$f_h = \xi \cdot e^{-\xi \cdot h} \quad (1)$$

$$f_\theta = \lambda \cdot e^{-\lambda \cdot \theta} \quad (2)$$

$$f_d = \psi \cdot e^{-\psi \cdot (d - IETD)} \quad (3)$$

where: $\xi = 1/\mu_h$; $\lambda = 1/\mu_\theta$; $\psi = 1/(\mu_d - IETD)$. μ_h , μ_θ and μ_d are the mean values of rainfall depth, rainfall duration and interevent time, respectively.

To estimate the overflow and the pre-filling PDF, a chain of rainfall events was considered (Fig. 3).

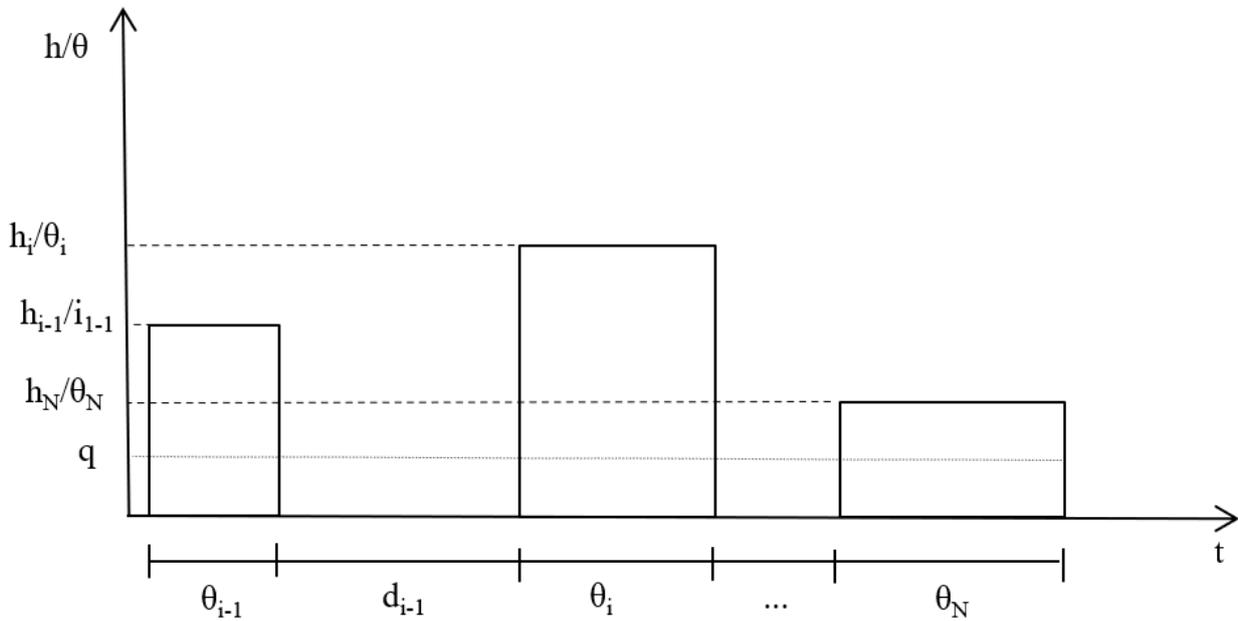


Fig. 3 Chain of rainfall events

To assess overflow and pre-filling probability, the water content $w_{e,i-1}$ in the system at the end of the (i-1)-th generic rainfall event was first defined:

$$w_{u,i-1} = \begin{cases} w_{e,i-1} + h_{i-1} - q \cdot \theta_{i-1} & 0 \leq w_{e,i-1} + h_{i-1} - q \cdot \theta_{i-1} < w \\ w & w_{e,i-1} + h_{i-1} - q \cdot \theta_{i-1} \geq w \\ 0 & \text{otherwise} \end{cases} \quad (4)$$

The subscripts u and e are referred, respectively, to the end and the beginning of the rainfall event. The subscripts i and $i-1$ identify the position of the event in the stochastic series of rainfalls, that is the $(i-1)$ -th and i -th events.

For $i = 1$, it results:

$$w_{u,0} = \begin{cases} w_{e,0} + h_0 - q \cdot \theta_0 & 0 < h_i - q \cdot \theta_i < w \\ w & h_i - q \cdot \theta_i \geq w \\ 0 & h_i - q \cdot \theta_i \leq 0 \end{cases}$$

$$w_{e,i} = \begin{cases} w_{u,i-1} - q \cdot d_{i-1} & w_{u,i-1} - q \cdot d_{i-1} > 0 \\ 0 & \text{otherwise} \end{cases} \quad (5)$$

For $i = 0$, it was assumed that storage capacity is empty ($w_{e,0} = 0$).

The overflow at the end of the i -th generic event v_i can be calculated by the following expression:

$$v_i = \begin{cases} w_{u,i-1} - q \cdot d_i + h_i - q \cdot \theta_i - w & \text{Condition}_1 \\ h_i - q \cdot \theta_i - w & \text{Condition}_2; \text{Condition}_3 \\ w - q \cdot d_i + h_i - q \cdot \theta_i - w & \text{Condition}_4 \\ 0 & \text{Otherwise} \end{cases} \quad (6)$$

*Condition*₁: $w_{u,i-1} \leq w$; $w_{u,i-1} > q \cdot d_i$; $w_{u,i-1} - q \cdot d_i + h_i - q \cdot \theta_i > w$

*Condition*₂: $w_{u,i-1} \leq w$; $w_{u,i-1} \leq q \cdot d_i$; $h_i - q \cdot \theta_i > w$

*Condition*₃: $w_{u,i-1} > w$; $w \leq q \cdot d_i$; $h_i - q \cdot \theta_i > w$

*Condition*₄: $w_{u,i-1} > w$; $w > q \cdot d_i$; $w - q \cdot d_i + h_i - q \cdot \theta_i > w$

event

*Condition*₁ expresses the case for which there is no overflow at the end of $(i-1)$ -th event, there is a pre-filling from previous $(i-1)$ -th event at the beginning and an overflow at the end of the i -th event. *Condition*₂ expresses the case for which there is no overflow at the end of $(i-1)$ -th event, there is no pre-filling from $(i-1)$ -th event at the beginning of i -th event and there is overflow from the system at the end of i -th event. *Condition*₃ expresses the case there is overflow at the end of $(i-1)$ -th event, there is no pre-filling from the $(i-1)$ -th event the beginning of i -th event and there is overflow from the green roof at the end of i -th event. *Condition*₄ expresses the case there is overflow at the end of $(i-1)$ th-event, there is pre-filling from $(i-1)$ -th event at the beginning of i -th event and there is overflow from the green roof at the end of event. The overflow for $i = 0$, that is v_0 , results:

$$v_0 = \begin{cases} h_0 - q \cdot \theta_0 - w & h_i - q \cdot \theta_i > w \\ 0 & \text{Otherwise} \end{cases} \quad (7)$$

The application of the probabilistic approach to analytical equations (6) and (7), allows to estimate the overflow PDF.

Two different conditions were distinguished: single event without pre-filling from previous rainfalls (full storage capacity completely available), $w/q \leq IETD$; possibility of pre-filling from previous rainfalls, $w/q > IETD$.

For $w/q \leq IETD$, the overflow PDF results:

$$P_v = P(v > \bar{v}) = \int_{h=w+\bar{v}+q \cdot \theta}^{\infty} f_h \cdot dh \int_{\theta=0}^{\infty} f_{\theta} \cdot d\theta = \gamma \cdot e^{-\xi \cdot (w+\bar{v})} \quad (8)$$

whit $\gamma = \frac{\lambda}{\lambda+q \cdot \xi}$ and \bar{v} the overflow threshold.

For $w/q > IETD$, the overflow PDF can be expressed by:

$$\begin{aligned}
P_v = P(v > \bar{v}) &= \int_{\theta=0}^{\infty} f_{\theta} \cdot d\theta \int_{d=IETD}^{\infty} f_d \cdot dd \int_{h=w+\bar{v}+q \cdot \theta}^{\infty} f_h \cdot dh \\
&+ \sum_{i=2}^N \left[\int_{\theta=0}^{\infty} f_{\theta} \cdot d\theta \int_{d=IETD}^{\frac{w+\bar{v}}{q}} f_d \cdot dd \int_{h=\frac{w+\bar{v}+(i-2) \cdot q \cdot d}{i-1}+q \cdot \theta}^{\frac{w+\bar{v}+(i-1) \cdot q \cdot d}{i-1}+q \cdot \theta} f_h \cdot dh \right] = \\
&= \gamma \cdot \left\{ e^{-\xi \cdot (w+\bar{v})} + \psi \cdot \sum_{i=2}^N \left[-(i-1) \cdot \beta_i \cdot e^{-\xi \cdot q \cdot IETD \cdot \left(\frac{i-2}{i-1}\right) - \frac{\xi}{i-1} \cdot (\bar{v}+w)} - i \cdot \beta_i^* \cdot e^{-\frac{\xi}{i} [q \cdot IETD \cdot (i-1) + (\bar{v}+w)]} - \xi \cdot q \cdot \beta_i \cdot \beta_i^* \cdot \right. \right. \\
&\left. \left. e^{\psi \cdot IETD - (\bar{v}+w) \cdot \left(\frac{\psi}{q} + \xi\right)} \right] \right\} \quad (9)
\end{aligned}$$

With $\gamma = \frac{\lambda}{\lambda+\xi \cdot q}$, $\beta_i = \frac{1}{\xi \cdot q \cdot (i-2) + \psi \cdot (i-1)}$, $\beta_i^* = \frac{1}{\xi \cdot q \cdot (1-i) - i \cdot \psi}$ and N the number of considered chained rainfall events.

Raimondi and Becciu (2014) concluded that to consider two chained rainfall events may be acceptable only for a long IETD and high outflow rates; for SuDS with low outflow rates or when strict limitations on discharges in the downstream drainage system are imposed, the Authors suggested to assume three or more chained rainfall events. In the following the probability of pre-filling, that is the probability to have a residual volume at the end of a dry period was estimated. It comes from the application of the probabilistic approach to the analytical definition of storage volume at the beginning of a rainfall event (equation 5).

For $i=1$:

$$\begin{aligned}
P_w = P(w_e > \bar{w}) &= \int_{h=\bar{w}+q \cdot (d+\theta)}^{\infty} f_h \cdot dh \int_{d=IETD}^{\frac{w-\bar{w}}{q}} f_d \cdot dd \int_{\theta=0}^{\infty} f_{\theta} \cdot d\theta = \\
&= \gamma \cdot \beta \cdot \left[e^{-\xi \cdot (q \cdot IETD + \bar{w})} - e^{\psi \cdot \left(IETD + \frac{\bar{w}}{q}\right) - w \cdot \left(\xi + \frac{\psi}{q}\right)} \right] \quad (10)
\end{aligned}$$

For $i > 1$:

$$\begin{aligned}
P_w = P(w_e > \bar{w}) &= \int_{\theta=0}^{\infty} f_{\theta} \cdot d\theta \\
&\cdot \left\{ \int_{d=IETD}^{\frac{w-\bar{w}}{q}} f_d \cdot dd \cdot \left[\int_{h=w+q\cdot\theta}^{\infty} f_h \cdot dh \cdot + \int_{h=\frac{\bar{w}}{i-1}+q\cdot(\theta+d)}^{\frac{w+q\cdot d\cdot(i-2)}{i-1}+q\cdot\theta} f_h \cdot dh \right] \right. \\
&+ \left. \int_{\frac{w+q\cdot d\cdot(i-1)}{i}+q\cdot\theta}^{\frac{w+q\cdot d\cdot(i-2)}{i-1}+q\cdot\theta} f_h \cdot dh \int_{d=IETD}^{\frac{d=\frac{w\cdot(i-1)-i\cdot\bar{w}}{q\cdot(i-1)}}}{i-1}} f_d \cdot dd \right\} = \\
&= \gamma \cdot \left\{ e^{-\xi\cdot w} \cdot \left[1 - e^{\psi\cdot(IETD+\frac{\bar{w}}{q}-\frac{w}{q})} \right] + \frac{2\cdot(1-\beta)\cdot\beta_i\cdot\psi\cdot(i-1)}{i-1} \cdot e^{-\left(\frac{\psi}{q}+\xi\right)\cdot(w-\bar{w})+\psi\cdot IETD-\frac{\xi\cdot\bar{w}}{i-1}} - \beta_i \cdot \psi \cdot (i-1) \cdot \left[2 \cdot \right. \right. \\
&e^{-\frac{\xi}{i-1}\cdot[w+q\cdot IETD\cdot(i-2)]} + e^{\frac{\xi\cdot\bar{w}\cdot i\cdot(i-2)}{(i-1)^2}-w\cdot\left(\frac{\psi}{q}+\xi\right)+\psi\cdot IETD+\frac{\psi\cdot i\cdot\bar{w}}{q\cdot(i-1)}} \left. \right] + \beta \cdot e^{-\frac{\xi\cdot\bar{w}}{i-1}-\xi\cdot q\cdot IETD} + \beta_i^* \cdot \psi \cdot i \cdot \left[e^{-w\cdot\left(\frac{\psi}{q}+\xi\right)+\psi\cdot IETD+\frac{\psi\cdot i\cdot\bar{w}}{q\cdot(i-1)}} - \right. \\
&e^{-\frac{\xi}{i}\cdot[w+q\cdot IETD\cdot(i-1)]} \left. \right] \left. \right\} \quad (11)
\end{aligned}$$

\bar{w} is the capacity threshold. The quantities γ , β , β_i and β_i^* are equal to:

$$\gamma = \frac{\lambda}{\lambda+\xi\cdot q}; \beta = \frac{\psi}{\psi+\xi\cdot q}; \beta_i = \frac{1}{(i-1)\cdot\psi+\xi\cdot q\cdot(i-2)}; \beta_i^* = \frac{1}{\xi\cdot q-i\cdot(\xi\cdot q+\psi)}.$$

3 Application

Developed equations were applied to two case studies in Genoa and Milan to estimate the probability of overflow (equations 8 and 9) and the probability of pre-filling at the end of a dry period (equations 10 and 11). A system with limited discharge to downstream network was considered. A constant outflow rate $q = 10 [l/(s \cdot ha_{imp})]$ was assumed, according with the threshold suggested by the Regional Regulation of the Lombardy Region (R.R n.7, 2017). The cities of Milan and Genoa have a similar climate excepting for the distribution of rainfalls along the year. Climate in Milan is warm and temperate. There is significant rainfall throughout the year in Milan, even in the driest months. The average temperature is 13.1 [°C], the average annual rainfall is 1,013 [mm]. Genoa also has a warm and temperate climate, but winter are much more rainfall than summer. The average temperature is 14.7 [°C] and the average annual rainfall is 1,086 [mm].

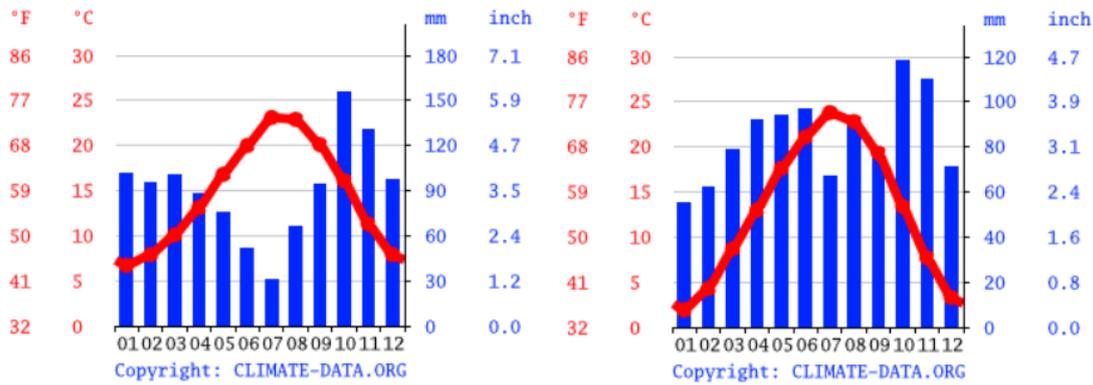


Fig 4. Rainfall and temperature characteristics of Genoa (on the left) and Milan (on the right)

The series recorded at Milano-Monviso gauge station in the period 1971-2005, for a total of 35 [years] and 1,129 [events] and at Genoa-Villa Cambiaso gauge station in the period 1993-2017, for a total of 25 [years] and 1,412 [events], were used in the modeling. Genoa is surely rainier than Milan with a mean of 56 [events] per year in comparison to 32 [events] for year of Milan. An Initial Abstraction, IA= 2 [mm], was considered: rainfall depths lower than this threshold were neglected in calculation. To identify independent rainfall event an IETD=6 [hours] was assumed for both locations. Table 1 reports mean of main rainfall variables involved in the process, rainfall depth h , rainfall duration θ and interevent time d .

Table 1 - Average values of rainfall variables per event.

	Milan	Genoa
μ_h [mm]	17.97	23.03
μ_θ [hour]	11.67	11.84
μ_d [hour]	150.56	140.51

Genoa has an average rainfall depth for event higher than Milan, but lower interevent time; average rainfall duration for the two cities is comparable. Table 2 shows the coefficient of variation of the three variables for the two cities.

Table 2 - Coefficient of variation of rainfall variables.

	Milan	Genoa
V_h [-]	1.16	1.51
V_θ [-]	1.04	1.05
V_d [-]	1.42	1.37

In both cases, the assumption of the exponential PDF of rainfall variables, is suitable only for rainfall duration, while for rainfall depth and interevent time there is deviation from the theoretical condition. This is an expected result. As discussed above, despite other PDFs better fit the FDF of recorded data, exponential PDF is preferred to avoid an increase of complexity in the development of final expressions. Table 3 reports the correlation indexes among rainfall variables.

Table 3 - Correlation indexes among rainfall variables.

	Milan	Genoa
$\rho_{h,d}$ [-]	0.10	0.01
$\rho_{\theta,h}$ [-]	0.69	0.61
$\rho_{d,\theta}$ [-]	0.10	0.01

Correlation between interevent time and rainfall depth and between interevent time and rainfall duration is weak and can be neglected, while correlation between rainfall depth and duration is significant. Despite in last decades, copula functions were introduced in the hydrologic research for broadening the multivariate inference capability and overcome the correlation among rainfall variables (Abdollahi et al., 2019), in this paper they were neglected to simplify the derivation of final expressions. The overflow threshold \bar{v} in equations (8) and (9) was assumed equal to zero, as well as the minimum water content \bar{w} in equations (10) and (11). Outflow rate from the system was assumed equal to $q=0.36$ mm/hr corresponding to 10 l/(s·ha_{imp}) (R.R n.7, 2017). Storage capacity w was varied between 0 [mm] and 250 [mm]. Figure 5 shows the probability of overflow for Milan and Genoa.

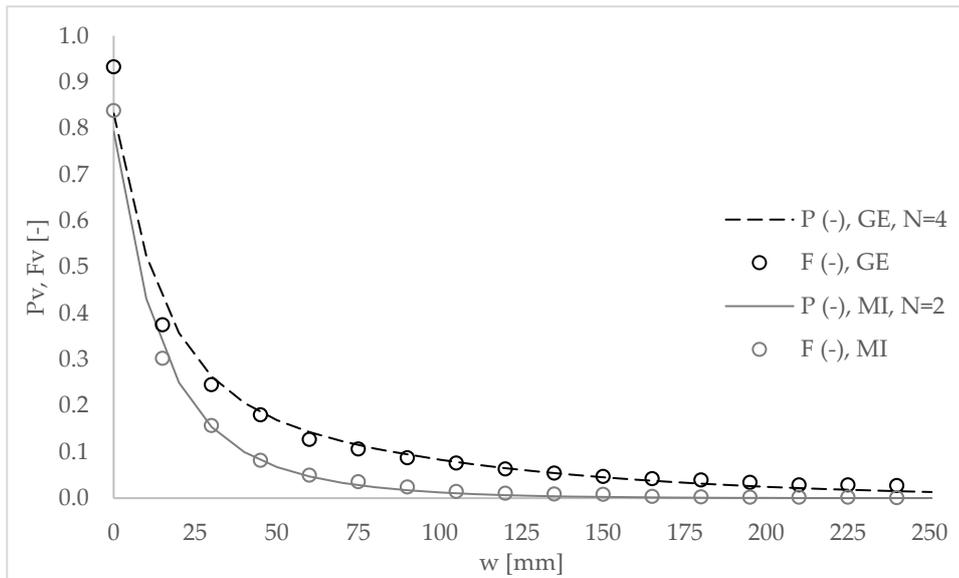


Fig 5. Overflow PDF (P_v) and FDF (F_v) vs storage volume (w).

In both cases, there is a good agreement between overflow PDF and overflow FDF. The probability of overflow decreases as the storage capacity increase. The probability of overflow for the system is higher in Genoa than in Milan. This agrees with the higher average rainfall depth and the lower interevent time in Genoa, that involves a higher number of chained events. The best fitting between overflow PDF and FDF was achieved considering four chained events ($N=4$) for Genoa and two chained events ($N=2$) for Milan. Figure 6 shows the probability of pre-filling at the end of a dry period, obtained by the application of equations (8) and (9). The best fitting between pre-filling PDF and FDF was obtained considering $N=4$ chained events for Genoa and $N=2$ chained events for Milan.

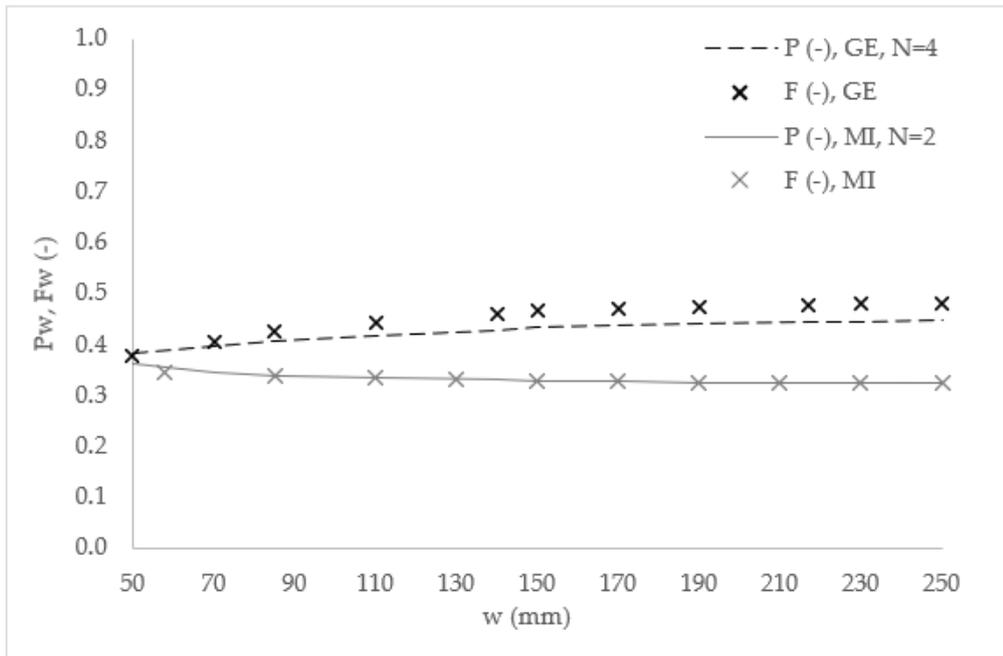


Fig 6. Pre-filling PDF (P_w) and FDF (F_w) vs storage volume (w).

The probability of pre-filling is higher for Genoa as consequence of higher average rainfall depth and lower average interevent time than Milan. There is a good agreement between PFD and FDF, but in case of Genoa equations (10) and (11) slightly underestimate the probability of pre-filling. For Milan, the probability is quite constant and does not increase with the storage capacity. Tables 4 and 5 report the results of the analysis for three different Average Return Interval (ARI), respectively for Genoa (Table 4) and Milan (Table 5). Setting the overflow ARI T_v , linked to the probability of overflow P_v by the relation $T_w = 1/P_v$, the correspondent storage capacity w was estimated by equations (8) and (9); by introducing the resulting value in equation (11) the probability of pre-filling (P_w) and the corresponding ARI (T_w) were calculated.

Table 4 – Summary of results for different overflow ARI (T_v), Genoa.

q =0.36 [mm/hr]	Genoa (N=4)	Genoa (N=4)	Genoa (N=4)
T_v [years]	50	20	10
P_v [-]	0.02	0.05	0.10
w [mm]	217	140	85
P_w [-]	0.45	0.44	0.41
T_w [years]	2.2	2.3	2.4

Table 5 - Summary of results for different overflow ARI (T_v), Milan.

q =0.36 [mm/hr]	Milan (N=2)	Milan (N=2)	Milan (N=2)
T_v [years]	50	20	10
P_v [-]	0.02	0.05	0.10
w [mm]	85	58	40

P_w [-]	0.34	0.35	0.36
T_w [years]	2.9	2.9	2.8

As the overflow ARI grows, the storage volume increases. With the same probability of overflow, the storage capacity for Genoa is higher than that required for Milan. The probability of pre-filling remains constant and is little influenced by the storage capacity.

4 Conclusions

The paper proposes an analytical probabilistic approach for the estimation of the overflow and pre-filling PDF of systems for stormwater management. To consider both the probability of overflow and the probability of pre-filling can be meaningful for the projects of structures as SuDS. An innovative element in the modeling consists in considering a chain of rainfall events, that is an important aspect for the analysis of low release structures. The application of developed formulas to two case studies and the comparison of results with those obtained from the continuous simulation of recorded data confirmed the goodness of suggested method. It allows an optimum design of systems for rainwater control relating storage capacity to overflow and pre-filling ARI. Once set the level of acceptable risk in term of overflow, it is possible to assess if design volume obtained by overflow PDF also ensures an acceptable level of systems pre-filling. The proposed approach can be applied to different systems of rainwater control and to different climate all over the world.

Conflict of Interest: The authors declare that they have no conflict of interest.

Declarations

Ethical approval: not applicable.

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