Tunneling between parallel one-dimensional Wigner crystals

Reyna Mendez-Camacho

Autonomous University of San Luis Potosí

Esteban Cruz-Hernandez (✉ esteban.cruz@uaslp.mx)

Autonomous University of San Luis Potosí

Research Article

Keywords: parallel nanowires, localized electronic distribution, diluted electronic density, Wigner crystal

Posted Date: November 9th, 2021

DOI: <https://doi.org/10.21203/rs.3.rs-986740/v1>

License: © ⓘ This work is licensed under a Creative Commons Attribution 4.0 International License.

[Read Full License](#)

Version of Record: A version of this preprint was published at Scientific Reports on March 16th, 2022. See the published version at <https://doi.org/10.1038/s41598-022-08367-x>.

Tunneling between parallel one-dimensional Wigner crystals

R. Méndez-Camacho and E. Cruz-Hernández

Coordinación para la Innovación y Aplicación de la Ciencia y la
Tecnología, Universidad Autónoma de San Luis Potosí, Sierra
Leona 550, S. L. P., 78210, S. L. P., México.

*Corresponding author(s). E-mail(s): esteban.cruz@uaslp.mx;

Abstract

Vertically aligned arrays are a frequent outcome in the nanowires synthesis by self-assembly techniques or in its subsequent processing. When these nanowires are close enough, quantum electron tunneling is expected between them. Then, because extended or localized electronic states can be established in the wires by tuning its electron density, the tunneling configuration between adjacent wires could be conveniently adjusted by an external gate. In this contribution, by considering the collective nature of electrons using a Yukawa-like effective potential, we explore the electron interaction between closely spaced, parallel nanowires while varying the electron density and geometrical parameters. We find that, at a low-density Wigner crystal regime, the tunneling can take place between adjacent localized states along and transversal to the wires axis, which in turn allows to create two- and three-dimensional electronic distributions with valuable potential applications.

Keywords: parallel nanowires, localized electronic distribution, diluted electronic density, Wigner crystal

1 Introduction

Semiconductor nanowires (NWs) are exciting components to study unique one-dimensional (1D) physics such as the Coulombic strength-dependent electronic charge fractionalization [1], quantized conductance [2, 3]; or the formation of a periodic charge distribution along the wires, known as a Wigner crystal, first predicted by E. Wigner in 1934 [4] and recently observed [5, 6]. In addition to be remarkable systems for basic research, NWs are also key building blocks in the fabrication of nanoscale electronic and optoelectronic devices [7, 8, 9, 10]. Some NW-based devices that has been widely explored includes field-effect transistors (FETs) [11, 12, 13, 14], diodes [15], nano-logic gates [16], and nanoprocessors [17].

When NWs are synthesized by self-assembly, usually pillar or in-plane arrays of parallel NWs (PNWs) are produced. In such arrays, or in the integration into the existing planar electronic platforms [10], the NWs separation can be short enough to permit electron tunneling between them. Furthermore, with the continuous size reduction in the design of smaller and more powerful devices; for example in advanced sub-5 nm nodes in NW-based vertically or laterally stacked gate-all-around FETs [18, 19, 20], the consequences of the closeness of adjacent NWs must be thoroughly analyzed to both prevent undesirable effects and to look for new architecture strategies [21].

On the theoretical side, even when there exist a number of advanced approaches to investigate the electron-electron (e-e) interaction in nanostructures, usually such formalisms are limited to one or few charge carriers and to the nanometric scale [22, 23, 24, 25, 26]. Modeling of micron-long semiconductor NWs and the e-e interaction for electronic densities (n) obtained from usual doping levels (10^{16} - 10^{19} electrons/cm³) is a very cumbersome task, impracticable to carry out in a direct way. For such reason, despite the importance

of this subject, the interaction of adjacent PNWs with realistic characteristics remains largely unexplored.

In this contribution, we theoretically investigate the electron tunneling between closely spaced micron-long semiconductor PNWs. The resultant electronic distributions are analyzed as a function of the NWs separation (from 1.2 nm to 10 nm), for two representative NW cross sections (15 nm and 50 nm), as well as three n -doping levels in the range from 10^{17} to 10^{20} electrons/cm³. Following previous works [27, 28], we consider a Yukawa-like e-e interaction to deduce a real-space effective potential, which allows us to calculate the NWs electronic distribution. We focus on the cases where a discontinuous charge distribution along the NWs is produced as well as in the selective tunneling between these charged regions.

The rest of the paper is organized as follows. In Sec. 2, we present the ground and first excited electronic distributions for two and six 1-micron-long GaAs/AlGaAs PNWs and its dependence on the main geometric parameters. We also discuss some implications and possible applications of the emerging electronic patterns in the PNWs arrays. In Sec. 3 a detailed derivation and some remarks of the model used to generate the electronic distributions are given. Finally, the conclusions are presented in Sec. 4.

2 Results and Discussion

2.1 The effective potential

Figure 1 provides a schematic view of the system studied in this work. Such system consists of GaAs PNWs of cross-sectional area $L_x \times L_y$ and length L_z , which are embedded in an AlGaAs matrix that acts as a finite potential barrier of width L_b . As described in detail in Sec. 3, we solve the time-independent Schrödinger equation for a spinless two-electron wave function,

4 Tunneling between parallel one-dimensional Wigner crystals

Fig. 1 Schematic representation of the parallel quantum wire system studied in this work: square GaAs wires, with the axis directed along the z -direction, are embedded into an AlGaAs matrix and separated by a distance L_b .

$\Psi(r)$, considering a Yukawa-like e-e interaction (Equation 2). Then, by using Fourier transforms, we derive for the z component a real-space effective potential (Equations 25 and 26) able to manage long L_z and high n values in an easy way. For the x and y components we consider the usual solutions for two and one finite quantum wells, respectively. The electronic density distributions shown hereafter are then obtained from the square of the calculated $\Psi(r)$.

2.2 Two parallel nanowires

In Figure 2a we plot the ground state distributions for two PNWs of size $L_{x,y} \equiv L_x = L_y = 50$ nm and $L_z = 1\mu\text{m}$, kept apart by a variable L_b , for $n = 10^{17}$ electrons/cm³ (left) and $n = 10^{20}$ electrons/cm³ (right). The ground and first excited state for a similar system, with a smaller cross-section $L_{x,y} = 15$ nm, are presented in Figures 2b and 2c, respectively. Each row in Figure 2 corresponds to a different L_b separation. We can observe from the upper row in Figure 2 (where there is not tunneling between NWs), that in agreement with previous reports on individual NWs [27, 28], the lower n concentration produces for the ground state well defined individual distributions along the NWs while for the higher concentration the electrons merge in a single distribution. This concordance is also observed in the first excited state.

Fig. 2 $z - x$ projections of the charge distribution probability for two $1\mu\text{m}$ -long GaAs/AlGaAs PNWs. The plots correspond to: a) ground state for $L_{x,y} = 50\text{nm}$, b) ground state for $L_{x,y} = 15\text{nm}$, and c) first excited state for $L_{x,y} = 15\text{nm}$. Two different n -doping concentration are plotted: $n = 10^{17}$ electrons/ cm^3 and $n = 10^{20}$ electrons/ cm^3 . Each row corresponds to a different L_b .

For the ground state, we can observe from Figure 2 that tunneling is strongly dependent on the electron density. Such dependence is mainly due to the localized and extended states induced for the relative low and high densities, respectively. For the excited states, a localized distribution is always found. Given the energy difference between the confined levels (meV for x - y , and μeV for z), the excited states considered here are only relative to the z -component, while the x and y components remains in the ground state.

As shown in Figure 2, the variation on the NWs cross-section affects the way the tunneling take place. For $L_{x,y} = 15\text{ nm}$, electrons are less contained by the AlGaAs barriers, so the tunneling takes place for a larger L_b separation as compared to the PNWs of cross-section $L_{x,y} = 50\text{ nm}$. Even when the L_b variation do not modify the electron distribution along the NWs z -axis, there exist a gradual shift along the x -axis with L_b . This dependence can be seen in Figure 3, where typical profiles taken along the x direction are presented for different values of L_b .

Fig. 3 Electron density profiles obtained from Figure 2a, which are taken along the x -axis where the two PNW distributions are maximal. These profiles do not depend on the value of n . Vertical dotted lines indicates the NW walls position for $L_b = 1.2$ nm (red) and $L_b = 6.4$ nm (blue). Black vertical dotted lines indicates the maximal $\Delta L_b = 5.2$ nm, which shows that the distribution shift by tunneling is larger than the L_b variation.

2.3 Multiple parallel nanowires

Additional interesting properties comes out in systems composed of more than two PNWs. As an example, the electronic ground state distribution in six PNWs array, as the one depicted in Figure 4(b), is plotted in Figure 4(a). The relative low n concentration (1×10^{18} electrons/cm³) is used to induce the formation of localized electronic states along the NWs z -axis (a Wigner molecule). We set a small cross-section ($L_{x,y} = 20$ nm) and a short distance between NWs ($L_b = 5$ nm) to observe a strong tunneling between them. Profiles taken along the x -axis presents a notorious intensity modulation from wire to wire, as the shown in Figure 4c (taken along the horizontal yellow line drawn in Figure 4a). Even when this modulation is not observable in the two PNWs system (see Figure 3), such kind of distributions where the intensity is maximal at the center of a finite number of quantum wells is usually found in common 1D superlattices (see for example reference [29]).

If, in the parameters used in Figure 4a, n is increased to 2×10^{18} electrons/cm³, a connection between the individual electronic distributions along the NWRs axis can be also established (see Figure 4d). As the lateral tunneling depopulate the more external NWs, the distribution along the z axis is different from wire to wire. This is clear from Figure 4e, where profiles taken along the z -axis from three consecutive NWs in Figure 4d are displayed.

Fig. 4 a) Ground state charge distribution for an array of six 1-micron long PNWs of cross section $L_{x,y} = 20$ nm, $n = 1 \times 10^{18}$ electrons/cm³, and mutual separation $L_b = 5$ nm. b) Illustrative representation of the six GaAs/AlGaAs PNWs array. c) Profile of the electronic density taken along the horizontal yellow line indicated in a). In d) n is increased to 2×10^{18} electrons/cm³ in order to trigger the connection between the charge distributions along the z -axis. In e) three profiles taken along the z direction from d) are plotted; the vertical yellow line in d) indicates where the profile number (1) was taken.

Usually, quantum tunneling is considered only as a 1D problem. The 2D and 3D tunneling cases are addressed as 1D independent tunneling along each spatial direction, without accounting for alterations in the 2D or 3D electron distributions. However, as described before, we found that the electronic tunneling along the transversal direction of the PNWs is actually able to significantly change the electronic distribution along the NWs axis. These effects are more noticeable for higher excited states, where a greater number of localized distributions along the wires emerges.

8 *Tunneling between parallel one-dimensional Wigner crystals*

Fig. 5 a) Fourth excited state charge distribution for an array of six 1-micron long PNWs of cross section $L_{x,y} = 20$ nm, $n = 1 \times 10^{18}$ electrons/cm³, and mutual separation $L_b = 5$ nm. b) Profile of the electronic density taken along the x direction. c) Profiles taken along the z direction at the x positions where the electron density is maximal; each number indicates the place where the profiles were taken from a).

Because confinement along the z axis is very weak and the energy separation between quantized energy levels is very small, of the order of μeV , these states can be easily populated and then are particularly important to analyze. As an example, in Figure 5a we plot the fourth excited state corresponding to a system with the same parameters that the plotted in Figure 4a. A typical profile, taken along the x direction where the density is maximal, is shown in Figure 5b. By the nature of our model, this transversal profile is practically the same as the shown in Figure 4c, with the only difference that its absolute intensity is smaller (as the electron density is distributed in more zones along the wires). In Figure 5c the profiles taken along the z axis show a sharp distribution of the well separated electronic regions.

2.4 2D and 3D interconnected distributions

As discussed before, three main electronic distributions appears in the arrays of PNWs by modifying n , L_b , and $L_{x,y}$. When L_b is large enough to block tunneling and n is low enough to trigger the Wigner crystallization, then a disconnected but ordered 2D charge distribution is established. However, if L_b is small enough to allow lateral tunneling, then a 2D array of superlattices interconnected along the x axis (as in Figures 4a and 5a) is produced. Furthermore, if the n density value allows to connect the charge distributions along the z axis (as in Figures 4d), then a fully connected 2D arrangement can be assembled. At present, we are extending the model to consider 3D PNWs arrays. However, because the NWs y -component is mathematically equivalent to the x -component considered in this work, it is plausible to conclude that the 3D distributions will be a straightforward generalization of the 2D distributions previously discussed.

We would like to highlight that such 2D and 3D distributions, together with the possibility to switch between them by means of an external gate, could be of great interest in practical issues as well as in the investigation of new physical phenomena. For example, these NW arrays could be a valuable alternative to usual approaches, such as the 3D arrays of quantum dots to build 3D superlattices [30] or to substitute the use of different materials along NWs to produce a charge distribution control along the NWs axis [31]. Such well separated charge distributions, at the nanometric scale, could also have applications in the design of alternative 3D NW-based logic architectures [32].

3 Theoretical model

We focus on direct wide bandgap semiconductors, so the small interaction between the conduction and valence bands can be neglected. The model can

be easily applicable to others wide bandgap compounds by changing the material parameters. As we use a Yukawa approach, a minimal in the electronic concentration must be fulfilled to guarantee that the e-e average separation is not larger than the screening length [27]. In this contribution, only suitable nanostructure sizes and n -doping levels that satisfy such condition are considered.

3.1 Solution in the z -direction

The time-independent Schrödinger equation for a spinless two-electron wave function, $\Psi(r) \equiv \Psi((x_1, y_1, z_1), (x_2, y_2, z_2))$, is

$$\left[-\frac{\hbar^2}{2m_e^*} \nabla^2 + V_{\text{eff}}(r) \right] \Psi(r) = E\Psi(r), \quad (1)$$

where $\hbar \equiv h/2\pi$, h the Planck constant, m_e^* the electron effective mass, and V_{eff} is the effective potential. V_{eff} , which is derived further below, includes the finite confinement potential in the transversal x - y plane, the infinite barrier potential at the NW z -edges, and the many-body e-e Yukawa-like interaction.

The Yukawa-like potential is given by

$$V_Y(r) = \frac{e^2}{4\pi\epsilon} \frac{e^{-\kappa r}}{r}, \quad (2)$$

where $r = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$ is the e-e separation; $\epsilon = \epsilon_0\epsilon_r$ is the absolute permittivity, with ϵ_0 the vacuum permittivity and ϵ_r the relative permittivity of the material ($\epsilon_r = 12.9$ for GaAs and $\epsilon_r = 12.247$ for $\text{Al}_x\text{Ga}_{1-x}\text{As}$ for the Al concentration $x = 0.23$ considered in the calculations).

The screening parameter κ is given by $\sqrt{\frac{2e^2n}{\epsilon K_B T}}$, with n the electronic density, K_B the Boltzmann constant, and T ($= 300K$) the temperature.

For $L_x, L_y \leq 55$ nm, $l_b \leq 40$ nm, and $L_z \approx 1$ μ m, $V_Y(r)$ can be considered as a small perturbation in the x and y directions. Then, we can make an approximation in the left side of Eq. 1 by splitting the transversal ($\perp$) and parallel ($\parallel$) contributions as

$$H_{\perp}(x, y) = -\frac{\hbar^2}{2m_e^*} \left(\frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2} + \frac{\partial^2}{\partial y_1^2} + \frac{\partial^2}{\partial y_2^2} \right) + V_{x-y}(x, y) \quad (3)$$

$$H_{\parallel}(z) = -\frac{\hbar^2}{2m_e^*} \left(\frac{\partial^2}{\partial z_1^2} + \frac{\partial^2}{\partial z_2^2} \right) + \frac{e^2}{4\pi\epsilon\epsilon_0} \frac{e^{-\kappa r}}{r} \quad (4)$$

Considering that the wave function is separable as $\Psi(r) = \psi_x(x)\psi_y(y)\psi_z(z)$, the y component can be directly calculated from Equations 1 and 3 as

$$\psi_y(y) = N_y \text{Cos}(l_y y) \quad (5)$$

where $l_y = \sqrt{\frac{2m_{\text{nw}}^*}{\hbar^2} E_y}$, $m_{\text{nw}}^* = 0.0665m_e$ for GaAs (m_e the electron mass), and N_y is a normalization constant. In the x -direction we must solve a system of two-coupled quantum wells, composed of the wave functions in the AlGaAs barriers (ψ_b) and the GaAs NW (ψ_{nw}):

$$\psi_{b_i}(x) = A_i e^{l_b x} + B_i e^{-l_b x} \quad (6)$$

$$\psi_{\text{nw}_j}(x) = C_j \cos(l_{\text{nw}}x) + D_j \sin(l_{\text{nw}}x) \quad (7)$$

in which $i = 1, 2, 3$ (the three barriers), $j = 1, 2$ (the two NW), $l_b = \sqrt{\frac{2m_b^*}{\hbar^2}(V_0 - E_x)}$, $l_{\text{nw}} = \sqrt{\frac{2m_{\text{nw}}^*}{\hbar^2}E_x}$, $V_0 = 187\text{meV}$, and $m_b^* = 0.0857m_e$ for $\text{Al}_{0.23}\text{Ga}_{0.77}\text{As}$. A_i , B_i , C_j and D_j are additional normalization constants.

In order to derive the effective potential, we consider the Fourier transform of each of the cross-sectional wave functions, given by

$$\psi_{b_i}^2(x) = \frac{1}{2\pi} \int dq_x G_{b_i}(q_x) e^{-iq_x x} \quad (8)$$

$$\psi_{\text{nw}_j}^2(x) = \frac{1}{2\pi} \int dq_x G_{\text{nw}_j}(q_x) e^{-iq_x x} \quad (9)$$

$$\psi^2(y) = \frac{1}{2\pi} \int dq_y G_y(q_y) e^{-iq_y y} \quad (10)$$

where,

$$\begin{aligned} G_{b_i}(q_x) = & \sqrt{2\pi} (A_i^2 \delta(q_x - 2il_b) + B_i^2 \delta(q_x + 2il_b)) \\ & + 2\sqrt{2\pi} A_i B_i \delta(q_x) \end{aligned} \quad (11)$$

$$\begin{aligned} G_{\text{nw}_j}(q_x) = & \sqrt{\frac{\pi}{2}} [A_j^2 (\delta(q_x) + \frac{\delta(q_x - 2il_{\text{nw}})}{2} + \frac{\delta(q_x + 2il_{\text{nw}})}{2}) \\ & + B_j^2 (\delta(q_x) - \frac{\delta(q_x - 2il_{\text{nw}})}{2} - \frac{\delta(q_x + 2il_{\text{nw}})}{2}) \\ & + iA_j B_j (\delta(q_x - 2il_{\text{nw}}) - \delta(q_x + 2il_{\text{nw}}))] \end{aligned} \quad (12)$$

$$G_y(q_y) = \sqrt{\frac{\pi}{2}} N_y^2 [\delta(q_y) + \delta(q_y - 2l_y) + \delta(q_y + 2l_y)] \quad (13)$$

For the longitudinal component, the corresponding Fourier transform is

$$\psi_z^2(z) = \frac{1}{(2\pi)^2} \int dq_z G_z(q_z) e^{-iq_z z} \quad (14)$$

By replacing Equations (8-9), (10), (14), and (16) into the e-e interaction energy, in the integral representation:

$$W = \int d^3r_1 d^3r_2 \left(\frac{e^2}{4\pi\epsilon\epsilon_0} \frac{e^{-\kappa r}}{r} \right) \|\Psi(\vec{r})\|^2, \quad (15)$$

where

$$\frac{e^{-\kappa r}}{r} = \frac{1}{2\pi^2} \int d^3q \frac{e^{-i(q_x x + q_y y + q_z z)}}{\kappa^2 + q_x^2 + q_y^2 + q_z^2}, \quad (16)$$

we obtain

$$W = \frac{e^2}{4\pi\epsilon_{\text{nw/b}} 2^7 \pi^8} \int d^3r \int d^3q \frac{G_y(q_y) G_{\text{nwj/bi}}(q_x) G_z(q_z)}{\kappa^2 + q_x^2 + q_y^2 + q_z^2} \times e^{-i(q_x x + q_y y + q_z z)} \quad (17)$$

14 *Tunneling between parallel one-dimensional Wigner crystals*

where $d^3q \equiv dq_x dq_y dq_z$ and $d^3r \equiv dx dy dz$; $\epsilon_{\text{nw/b}}$ corresponds to the permittivity that is replaced for each corresponding region (NW or the barrier). Integrating over r ,

$$W = \frac{e^2}{4\pi\epsilon_{\text{nw/b}}2\pi^2} \int d^3q \frac{G_y(q_y)G_{\text{nwj/bi}}(q_x)G_z(q_z)}{\kappa^2 + q_x^2 + q_y^2 + q_z^2} \quad (18)$$

where $G_{\text{nw/b}}$ is defined analogous to the permittivity.

Now, as

$$G_z(q_z) = \int dz \psi_z^2(z) e^{\pm i(q_z z)}, \quad (19)$$

Eq. (18) can be expressed as

$$W = \frac{e^2}{4\pi\epsilon_{\text{nw/b}}2\pi^2} \int dz d^3q \frac{G_y(q_y)G_{\text{nwj/bi}}(q_x)\psi^2 z(z) e^{iq_z z}}{\kappa^2 + q_x^2 + q_y^2 + q_z^2} \quad (20)$$

Then, by using the equality

$$\int_{-\infty}^{\infty} dq_z \frac{e^{iq_z z}}{q_x^2 + q_y^2 + q_z^2 + \kappa^2} = \frac{\pi e^{-z\sqrt{q_x^2 + q_y^2 + \kappa^2}}}{\sqrt{q_x^2 + q_y^2 + \kappa^2}}, \quad (21)$$

Eq. (20) can be rewritten as

$$W = \frac{1}{2\pi} \int_{-\infty}^{\infty} dz \psi_z^2(z) V_{\text{eff}}(z), \quad (22)$$

where V_{eff} is defined by

$$V_{\text{eff}}(z) = \frac{e^2}{4\pi\epsilon_{\text{nw/b}}} \int_l^\infty dq_x dq_y G_{\text{nwj/b}_i}^2(q_x) G_y^2(q_y) \frac{e^{-z\sqrt{q_x^2 + q_y^2 + \kappa^2}}}{\sqrt{q_x^2 + q_y^2 + \kappa^2}} \quad (23)$$

By replacing Eqs. (8-9) and (13) in (23), we can obtain the final expression for $V_{\text{eff}}(z, \kappa, l_b, l_{\text{nw}}) \equiv V_{\text{eff}}$, which must be evaluated in each part of the GaAs/Al-GaAs NW along the z -direction. Then, for the extreme left and right AlGaAs barriers ($i = 1$ and 3),

$$V_{\text{eff}_b} = \frac{N_y^2 \pi^2 (A_i B_i)^2}{2} \left[O_b \frac{e^{-z\sqrt{4l_b^2 + 4l_y^2 + \kappa^2}}}{\sqrt{4l_b^2 + 4l_y^2 + \kappa^2}} \right], \quad (24)$$

at the central barrier ($i = 2$),

$$V_{\text{eff}_b} = \frac{N_y^2 \pi^2 (A_2 B_2)^2}{2} \left[\frac{A_2^4 + B_2^4}{A_2^2 B_2^2} \right] \times \left[O_b \frac{e^{-z\sqrt{4l_b^2 + 4l_y^2 + \kappa^2}}}{\sqrt{4l_b^2 + 4l_y^2 + \kappa^2}} \right], \quad (25)$$

and, in the GaAs NW ($j = 1$ and 2),

$$V_{\text{eff}_{\text{nw}}} = \frac{N_y^2 \pi^2 (C_j D_j)^2}{16} \left[\frac{C_j^2 + D_j^2}{(C_j D_j)^2} + \frac{2}{A_j^2 B_j^2} \right] \times \left[O_{\text{nw}} \frac{e^{-z\sqrt{4l_{\text{nw}}^2 + 4l_y^2 + \kappa^2}}}{\sqrt{4l_{\text{nw}}^2 + 4l_y^2 + \kappa^2}} \right],$$

(26)

where $B_1 = A_3 = 0$, $O_b = \frac{e^4}{4\pi\epsilon_b}$ outside the NW and $O_{nw} = \frac{e^4}{4\pi\epsilon_{nw}}$ inside the NW. To solve the Schrödinger equation by using Eqs. (24), (25), and (26), we use the finite difference method. The solution in the x -direction is explained further below.

3.2 Solution in the x -direction

Along the x -direction the solution corresponds to the well-known problem of two coupled quantum wells, with the wave functions given by Eqns. (6) and (7). To find the eigen-energies in this direction, we use the boundary conditions,

$$\begin{aligned}
 \psi_{b_1}(x)|_{x=b_1} &= \psi_{nw_1}(x)|_{x=b_1}, \\
 \psi_{nw_1}(x)|_{x=p_1} &= \psi_{b_2}(x)|_{x=p_1}, \\
 \psi_{b_2}(x)|_{x=p_2} &= \psi_{nw_2}(x)|_{x=p_2}, \\
 \psi_{nw_2}(x)|_{x=p_3} &= \psi_{b_3}(x)|_{x=p_3},
 \end{aligned}
 \tag{27}$$

$$\begin{aligned}
 \frac{d\psi_{b_1}}{dx}|_{x=b_1} &= \frac{d\psi_{nw_1}}{dx}|_{x=b_1}, \\
 \frac{d\psi_{nw_1}}{dx}|_{x=p_1} &= \frac{d\psi_{b_2}}{dx}|_{x=p_1}, \\
 \frac{d\psi_{b_2}}{dx}|_{x=p_2} &= \frac{d\psi_{nw_2}}{dx}|_{x=p_2}, \\
 \frac{d\psi_{nw_2}}{dx}|_{x=p_3} &= \frac{d\psi_{b_3}}{dx}|_{x=p_3},
 \end{aligned}$$

(28)

with $p_1 = L_{b_1} + L_{nw}$, $p_2 = L_{b_1} + L_{nw} + L_{b_2}$ and $p_3 = L_{b_1} + 2L_{nw} + L_{b_2}$, where we consider the same cross-section size for both NWs. By using the boundary conditions [Eqns. (27) and (28)], one can find the eigen-value equation:

$$\begin{aligned}
 & F_1 e^{l_b(b_1-p_1)} [e^{2l_b p_1} (F_2^2 + F_3^2)^2 \\
 & - e^{2l_b p_2} (F_2^4 - 6L^2 M^2 + F_3^4)] F_1 \\
 & + \left(e^{l_b(b_1+p_1)} + e^{l_b(b_1-p_1+2p_2)} \right) (F_2^2 + F_3^2)^2 F_4 \\
 & + 4e^{l_b(b_1-p_1+2p_2)} LM(F_2 - F_3)(F_2 + F_3)F_5 = 0,
 \end{aligned} \tag{29}$$

with

$$L = l_{nw} l_b,$$

$$M = m_{nw} m_b,$$

$$F_1 = \cos(l_{nw}[b_1 - p_1 + p_2 - p_3]),$$

$$F_2 = l_{nw} m_b, \quad F_3 = l_b m_{nw},$$

$$F_4 = \cos(l_{nw}[b_1 - p_1 - p_2 + p_3]) \text{ and}$$

$$F_5 = \sin(l_{nw}[b_1 - p_1 + p_2 - p_3]).$$

So, by solving the last equations, the values of E_x can be found by considering the l_{nw} and l_b values obtained in the earlier subsection.

Finally, for the well-known and straightforward solution along the y direction, see for example Reference [28].

4 Conclusion

We present theoretical calculations of many-electron states in semiconductor parallel NWs, considering the effect of the system size, the n -doping level and the NWs separation. When a low-density Wigner crystal regime is considered, localized effects in the tunneling between adjacent quantum wires are observed. By modifying NWs parameters such as the cross-section, the n concentration and the NWs separation, the charge distribution pattern in a 2D PNWs array can form 2D interconnected distributions. A 3D generalization of adjacent 2D PNWs arrays allows to conceive the formation of 3D interconnected charge distributions. Such nanoscale localized charge distribution could be valuable in the design of new architectures for photonics and electronics applications.

Acknowledgments. Authors acknowledge financial support from Consejo Nacional de Ciencia y Tecnología (CONACyT).

References

- [1] H. Steinberg, G. Barak, A. Yacoby, L. N. Pfeiffer, K. W. West, B. I. Halperin, and K. L. Hur, Charge Fractionalization in Quantum Wires, *Nature Physics* 4, 116 (2007).
- [2] B. J. Van Wees, H. Van Houten, C. W. J. Beenakker, J. G. Williamson, L. P. Kouwenhoven, D. van der Marel, and C. T. Foxon, Quantized Conductance of Point Contacts in a Two-Dimensional Electron Gas, *Phys. Rev. Lett.* 60, 848 (1988).
- [3] J. M. Krans, J. M. van Ruitenbeek, V. V. Fisun, I. K. Yanson, and L. J. de Jongh, The Signature of Conductance Quantization in Metallic Point Contacts, *Nature* 375, 767 (1995).

- [4] E. Wigner, On the Interaction of Electrons in Metals, *Phys Rev* 46, 1002 (1934).
- [5] I. Shapir, A. Hamo, S. Pecker, C. P. Moca, Legeza, G. Zarand, and S. Ilani, Imaging the Electronic Wigner Crystal in One Dimension, *Science* 364, 870 (2019).
- [6] S. Pecker, F. Kuemmeth, A. Secchi, M. Rontani, D. C. Ralph, P. L. McEuen, and S. Ilani, Observation and Spectroscopy of a Two-Electron Wigner Molecule in an Ultraclean Carbon Nanotube, *Nature Physics* 9, 1 (2013).
- [7] W. Lu and C. M. Lieber, Nanoelectronics from the Bottom Up, *Nat Mater* 6, 841 (2007).
- [8] Y. Zhang, J. Wu, M. Aagesen, and H. Liu, III-V Nanowires and Nanowire Optoelectronic Devices, *J. Phys. D Appl. Phys.* 48, 463001 (2015).
- [9] C. Jia, Z. Lin, Y. Huang, and X. Duan, Nanowire Electronics: From Nanoscale to Macroscale, *Chem. Rev.* 119, 9074 (2019).
- [10] Y. Sun, T. Dong, L. Yu, J. Xu, and K. Chen, Planar Growth, Integration, and Applications of Semiconducting Nanowires, *Adv. Mater.* 32, 1903945 (2020).
- [11] J. Goldberger, A. I. Hochbaum, R. Fan, and P. Yang, Silicon Vertically Integrated Nanowire Field Effect Transistors, *Nano Lett.* 6, 973 (2006).
- [12] A. Heinzig, T. Mikolajick, J. Trommer, D. Grimm, and W. M. Weber, Dually Active Silicon Nanowire Transistors and Circuits with Equal Electron and Hole Transport, *Nano Lett.* 13, 4176 (2013).
- [13] M. Taupin, E. Mannila, P. Krogstrup, V. F. Maisi, H. Nguyen, S. M. Albrecht, J. Nygrd, C. M. Marcus, and J. P. Pekola, InAs Nanowire with Epitaxial Aluminum as a Single-Electron Transistor with Fixed Tunnel Barriers, *Phys. Rev. Appl.* 6, 054017 (2016).

20 REFERENCES

- [14] M. Su, X. Zou, Y. Gong, J. Wang, Y. Liu, J. C. Ho, X. Liu, and L. Liao, Sub-KT/q Switching in In₂O₃ Nanowire Negative Capacitance Field-Effect Transistors, *Nanoscale* 10, 19131 (2018).
- [15] J. H. Park, J. S. Kim, Y. J. Choi, W. H. Lee, D. Y. Lee, and J. H. Cho, Gate- and Light-Tunable Pn Heterojunction Microwire Arrays Fabricated via Evaporative Assembly, *Acs Appl. Mater. Inter.* 9, 3857 (2017).
- [16] Y. Huang, X. Duan, Y. Cui, L. J. Lauhon, K.-H. Kim, and C. M. Lieber, Logic Gates and Computation from Assembled Nanowire Building Blocks, *Science* 294, 1313 (2001).
- [17] H. Yan, H. S. Choe, S. Nam, Y. Hu, S. Das, J. F. Klemic, J. C. Ellenbogen, and C. M. Lieber, Programmable Nanowire Circuits for Nanoprocessors, *Nature* 470, 240 (2011).
- [18] H. Mertens, R. Ritzenthaler, V. Penal, G. Santoro, K. Kenis, A. Schulze, E. D. Litta, S. A. Chew, K. Devriendt, T. Chiarella, S. Demuynck, D. Yakimets, D. Jang, A. Spessot, G. Eneman, A. Dango], P. Lagrain, H. Bender, S. Sun, M. Korolik, D. Kioussis, M. Kim, K.-H. Bu, S. C. Chen, M. Cogorno, J. Devraian, J. Machillot, N. Yoshida, N. Kim, K. Barla, D. Mocuta, and N. Horiguchi, Vertically Stacked Gate-All-Around Si Nanowire Transistors: Key Process Optimizations and Ring Oscillator Demonstration, 2017 Ieee Int Electron Devices Meet Iedm 37.4.1 (2017).
- [19] E. Capogreco, L. Witters, H. Arimura, F. Sebaai, C. Porret, A. Hikavy, R. Loo, A. P. Milenin, G. Eneman, P. Favia, H. Bender, K. Wostyn, E. D. Litta, A. Schulze, C. Vrancken, A. Opdebeeck, J. Mitard, R. Langer, F. Holsteyns, N. Waldron, K. Barla, V. D. Heyn, D. Mocuta, and N. Collaert, First Demonstration of Vertically Stacked Gate-All-Around Highly Strained Germanium Nanowire PFETs, *Ieee T Electron Dev* 65, 5145 (2018).

- [20] N. G. Orji, M. Badaroglu, B. M. Barnes, C. Beitia, B. D. Bunday, U. Celano, R. J. Kline, M. Neisser, Y. Obeng, and A. E. Vladar, Metrology for the next Generation of Semiconductor Devices, *Nat Electron* 1, 532 (2018).
- [21] H. H. Radamson, H. Zhu, Z. Wu, X. He, H. Lin, J. Liu, J. Xiang, Z. Kong, W. Xiong, J. Li, H. Cui, J. Gao, H. Yang, Y. Du, B. Xu, B. Li, X. Zhao, J. Yu, Y. Dong, and G. Wang, State of the Art and Future Perspectives in Advanced CMOS Technology, *Nanomaterials-Basel* 10, 1555 (2020).
- [22] P. Harrison and A. Valavanis, Quantum Wells, Wires and Dots: Theoretical and Computational Physics of Semiconductor Nanostructures, Fourth Edition (2016).
- [23] N. Luo, G. Liao, and H. Q. Xu, K.p Theory of Freestanding Narrow Band Gap Semiconductor Nanowires, *Aip. Adv.* 6, 125109 (2016).
- [24] G. Liao, N. Luo, K.-Q. Chen, and H. Q. Xu, Electronic Structures of Free- Standing Nanowires Made from Indirect Bandgap Semiconductor Gallium Phosphide, *Nature Publishing Group* 6, 1 (2016).
- [25] M. Świdorski and M. Zieliński, Atomistic Theory of Excitonic Fine Structure in InAs/InP Nanowire Quantum Dot Molecules, *Phys Rev B* 95, 125407 (2017).
- [26] M. Cygorek, M. Korkusinski, and P. Hawrylak, Atomistic Theory of Electronic and Optical Properties of InAsP/InP Nanowire Quantum Dots, *Phys. Rev. B* 101, 075307 (2020).
- [27] R. Méndez-Camacho, Esteban Cruz-Hernández, and Ramón Castañeda-Priego, Wigner Crystallization in Quantum Wires within the Yukawa Approximation, *Phys. Rev. B* 95, 085437 (2017).
- [28] R. Méndez-Camacho, E. Cruz-Hernández, and R. Castañeda-Priego, Many-electron effective potential in low-dimensional nanostructures:

22 REFERENCES

- Towards understanding the Wigner crystallization, *Phys. Rev. B* 100, 085438 (2019).
- [29] U. Zavala-Moran, M. Bouschet, J. P. Perez, R. Alchaar, S. Bernhardt, I. Ribet-Mohamed, F. de Anda-Salazar, and P. Christol, Structural, Optical and Electrical Characterizations of Midwave Infrared Ga-Free Type-II InAs/InAsSb Superlattice Barrier Photodetector , *Photonics* 7, 76 (2020).
- [30] C.B. Murray, C.R. Kagan, M.G. Bawendi, Self-Organization of CdSe Nanocrystallites into Three-Dimensional Quantum Dot Superlattices, *Science*. 270 (1995) 13351338.
- [31] M.S. Gudiksen, L.J. Lauhon, J. Wang, D.C. Smith, C.M. Lieber, Growth of nanowire superlattice structures for nanoscale photonics and electronics, *Nature*. 415 (2002) 617620.
- [32] B. Gojman, R. Rubin, C. Pilotto, A. DeHon, T. Tanamoto, 3D Nanowire-Based Programmable Logic, 2006 1st International Conference on Nano-Networks and Workshops. (2006) 15.