An Energy-Based Nonlinear Dynamic Model of Dielectric Elastomer Minimum Energy Structures with Stiffeners: Equilibrium Configuration and the Electromechanical Response

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An Energy-based Nonlinear Dynamic Model of Dielectric Elastomer Minimum Energy Structures with Stiffeners: Equilibrium Configuration and the Electromechanical Response

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Abstract The Dielectric Elastomer-based Minimum Energy Structures (DEMES) pertain to an equilibrium configuration attained by the assembly of a pre-stretched electroactive polymer film and a compliant boundary frame. Because of their unique characteristics, such as fast response and a large reversible stroke; DEMES have been used widely in the development of soft robotic transducers. However, their utility is typically restricted because of the warping resulting from anticlastic curvature. The present investigation examines the effectiveness of stiffeners in controlling this warping, as well as their effect on the resulting electromechanical response when the DEMES is driven electrically. To this end, we devise an energy-based analytical model that predicts the initial equilibrium configuration of an elementary rectangular DEMES with a finite number of stiffeners adhered to the boundary frame. The proposed framework uses the neo-Hookean hyperelasticity model for the polymer film and the linear elastic constitutive model for both the frame and the stiffeners. Predictive capability of the proposed analytical model is established through comparisons with 3D finite element simulations and experimental observations. The analytical model is then extended in the setting of the least-action principle to investigate the complex nonlinear dynamic behavior of the DEMES emanating from the interplay between material and geometric nonlinearities. The proposed dynamic model provides crucial insights into the role of varying levels of geometric and material parameters on the attainment of the initial equilibrium configuration of DEMES and its DC dynamic response when driven by a Heaviside electric load. In particular, we highlight the favorable impact of adding stiffeners in enhancing the stroke of the DEMES and an amplification in the attained equilibrium angle with an increasing spacing between the stiffeners. The analytical model and the results reported in this

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investigation can be of potential use in pre-designing the geometrical and material properties of DEMES for enhancing its electromechanical performance.

Keywords Dielectric elastomer minimum energy structure (DEMES) · Stiffeners · Equilibrium configuration · Nonlinear dynamics · Phase portraits

1 Introduction

Due to their characteristics of undergoing large reversible areal deformations (more than 100 %) [Pelrine et al. \(2000\)](#), the Dielectric elastomers (DEs) have been incorporated constructively in minimum energy structures [Kofod et al. \(2006\)](#). DE based minimum energy structures resemble a 3-D equilibrium configuration delineated by the minimum energy state of the system [Kofod et al. \(2007\)](#). The DEMES is formed by the combination of an inextensible frame and a pre-stretched DE membrane. As the structure is released from external mechanical constraint, the restoring force of the membrane bends the elastic frame into a minimum energy configuration. Further, due to the applied electric voltage across the electrodes of the membrane, the frame flattens out. The electrically induced bending of the compliant frame, acquirement of the primary configuration after removal of the voltage, and the electric voltage-driven bi-stability are certain essential deformation modes of the DEMES actuator which is exploited constructively to develop grippers [Kofod et al. \(2007\)](#), flapping-wing actuators [Liu et al. \(2013\)](#); [Zhao et al. \(2015a\)](#), compliant mechanisms [Gu et al. \(2017\)](#); [Ma et al. \(2013\)](#); [Zhao et al. \(2015b\)](#), etc.

In most of the aforementioned DEMES applications, the dynamic motion strongly affects their performance. Hence, modeling the nonlinear dynamic responses of the DEMES actuator is an important task for the effective design of such devices. Specifically, for the dynamic behavior of the planar DEAs, several investigations considering the effect of prestressing or pre-stretch [Dai and Wang \(2015\)](#); [Joglekar \(2014\)](#); [Sharma et al. \(2017\)](#), viscoelasticity [Kollosche et al. \(2015\)](#); [Wang et al. \(2014\)](#); [Zhao et al. \(2020\)](#), humidity [Zhang et al. \(2021\)](#), the initial stretch ratio [Dai et al. \(2016\)](#), combined DC and AC voltage [Khurana et al. \(2021a\)](#); [Sasikala et al. \(2017\)](#); [Sheng et al. \(2014\)](#) on the electromechanical behavior of the DEAs are available. Realistically, the inherent material nonlinearity of the DE membrane is scrutinized in all the above-stated work of DEAs. However, the DE-based minimum energy structures exhibit very complex and rich behavior due to the interplay of geometric and constitutive nonlinearities. Among available few research related to the dynamic behavior of DEMES actuators, [O'Brien et al. \(2009\)](#), developed a finite element model to interrogate the quasi-static behaviors of the DEMES. [Zhao et al. \(2015b\)](#) presented an experimental investigation depicting the behavior of a DEMES rotary joint actuated by a square form voltage signal. Further, [Zhao et al. \(2016b\)](#) presented a semi-analytical model based on the linear equation of motion to study the behavior of rotary joint DEMES. Recently, few researchers presented a lumped parameter modeling of DE-based minimum energy structures to provide insights on the nonlinear dynamic behavior of the DEMES actuators [Khurana et al. \(2020, 2021b\)](#); [Vatanjou et al. \(2019\)](#).

Apart from the dynamic behavior, the DEMES needs enough deformation range in the desired direction to achieve the vibrational spectrum of the actuation devices, specifically for soft robotic applications. However, the desired in-plane deformation of the DEMES is typically restricted by warping (out-of-plane deformation) of the structure. Several techniques in the recent past have been developed to circumvent such problems, e.g., providing pure shear pre-stretching condition to the membrane Rosset et al. (2014), incorporating anisotropy to the material behavior of the DE membrane to promote bending of the DEMES in required direction Khurana et al. (2021b), Implementing a stiffening frame along with the compliant frame to provide rigidity and avoid the unnecessary out-of-plane bending of the DEMES Zhao et al. (2016a), etc. In context of DEMES, providing initial pure shear prestretching conditions instead of equal bi-axial stretching to the membrane promotes the bending of the DEMES predominantly along the desired length direction and the parasitic bending along the width direction can be controlled easily by strengthening the frame extremities portion of the rectangular shaped bending actuator Rosset et al. (2014). However, achieving the pure shear prestretching of the DE membrane involves a tedious fabrication process Araromi et al. (2014). For planar DE actuator, Lai et. al Lai et al. (2012) presented a multi-layer dielectric elastomer actuator design, reinforced with periodic stiffeners to achieve the complex desired deformation of the DE actuator. To the best of our knowledge the effect of anisotropy, imparted by finitely spaced reinforcements on controlling the warping, and subsequently actuation performance of DEMES has not been investigated so far.

This work first provides an experimentally and numerically validated static analytical framework for assessing the equilibrium configuration of the proposed DEMES. The developed static analytical framework effectively predicts the attained equilibrium angle of the DEMES. Next, we extend the proposed analytical framework to investigate the complex nonlinear dynamic behavior of the DEMES. In the dynamic analysis, a detailed study for extracting the influence of various material and geometrical parameters of the DEMES is presented. Further, we analyse the effect of spacing between the consecutive stiffeners implemented on the surface of compliant frame on the attained initial and final configuration of the DEMES actuator. The proposed investigation implicates that imparting reinforcements on the surface of inextensible frame significantly controls the warping and actuation behavior of the DEMES actuator. The analytical framework developed herein can assist the designing of practical devices for energy harvesting and actuation purposes.

The remaining paper is structured as follows: Section 2, discusses the outlook of the proposed dielectric elastomer based minimum energy structures. Static analysis pertains to the development of analytical, experimental and numerical model to investigate the attained equilibrium state of the proposed DEMES is reported in Section 3. The analytical framework based on the deduced nonlinear governing equation of motion to investigate the complex nonlinear dynamic behavior of the DEMES actuator is presented in Section 4. A comprehensive parametric investigation revealing the impact of frame geometrical parameters and incorporated stiffeners on the performance of DEMES is presented in Section 5. Finally, Section 6 summarizes the salient findings of the present study.

2 Problem Definition

This article presents the incorporation of 3-D reinforcements on the compliant frame to reduce the warping effect and obtain the desired initial as well as final configuration of the DEMES. Fig. 1 elucidates the notion of warping in DEMES actuators using the initial equilibrium configurations predicted numerically using 3-D FE simulations performed in ABAQUS. As evident from the Figure, the DEMES with reinforcements (Fig. 1 b) shows a considerable reduction in warping when compared with the one without reinforcements (Fig. 1 a). The considered DEMES consists of a compliant frame of initial length L_1 , width W_1 , and thickness h , respectively, with a rectangular hole as shown schematically in Fig. 2. The characterizing dimensions of the rectangular cut section of the compliant frame are length C , width W , and thickness h , respectively. An equally bi-axially pre-stretched membrane (Fig. 2) having the same dimensions (length L_1 , and width W_1) as that of the compliant frame is affixed to the frame to develop the proposed minimum energy structure.

Further, the 3-D reinforcements are attached to the surface of the frame to avoid the warping of the structure, as depicted in Fig. 2. Finally, a stiffening frame towards the fixed end of the compliant frame is attached to provide rigidity towards the end portion of the frame. The associated dimensions characterizing the developed DEMES are shown schematically in Fig. 2 and corresponding values are indexed in

Fig. 1 Numerically (ABAQUS) obtained initial configuration of the DEMES (a) without reinforcement (showing warping), and (b) with reinforcement (no warping).

Table 1 Geometrical dimensions of the parts incorporated to form the DEMES.

Part	Length (cm)	Width (cm)	Thickness (mm)
Compliant Frame	$L_1 = 5.50, C = 2.70$	$W_1 = 3.50, W = 2.00$	$h = 0.40$
Elastomer Membrane	$L_1 = 5.50$	$W_1 = 3.50$	$t_1 = f(\lambda_p)$
3D Stiffeners	$a_1 = 0.20$	$W_1 = 3.50$	$a_2 = 2.50$
Stiffening Frame	$L_0 = 2.0$	$W_1 = 3.50$	$h = 0.40$

Fig. 2 Problem definition: Schematic of the proposed DEMES.

Table 1 for a ready reference. The bending strips width (b) and the thickness (h) of the compliant frame are the critical dimensions that can be tuned to alter the stiffness of the compliant frame. The spacing (S) between the adjacent stiffeners also affects the initial and the final state of the developed DEMES actuator.

The upcoming section presents the static analysis of the proposed DEMES to investigate the attained equilibrium configuration of the structure.

3 Static analysis of DE based minimum energy structures

One of the crucial states of the DEMES is referred to as the initial configuration which is obtained as a consequence of energy redistribution between the elastomer film and the deformable frame when the DEMES disengages from external mechanical constraint. This initial or equilibrium configuration of the DEMES is characterized by equilibrium angle denoted as θ_{eq} .

In the following subsections, we present the analytical, experimental and numerical methodology implemented for the measurement of equilibrium angle of the proposed DE based minimum energy structure.

3.1 Analytical modeling of DEMES

This subsection provides a static analytical model to analyse the equilibrium configuration of the proposed DEMES utilizing the principle of minimization of the total potential energy of the system.

The proposed analytical model of the DEMES is shown schematically in Fig. 3. To evaluate the associated strain energy of the stretched membrane, it is assumed that the membrane spans across the two extremities edge of the frame as illustrated in

Fig. 3 Schematic of the DEMES representing the geometrical dimensions in (a) Front view, (b) Side view, and (c) Enlarged side view.

Fig. 3 a,b. However, the membrane also deforms with the compliant frame in actual practice and forms a complex saddle shape. This hypothesis facilitates the lumped parameter modeling of the DE-based minimum energy structures and has been applied constructively in the recent past [Khurana et al. \(2021b\)](#); [Rosset et al. \(2014\)](#); [Vatanjou et al. \(2019\)](#). To evaluate the associated energies of the developed DEMES, we discretized the entire DEMES based on the implemented stiffeners as shown in Fig. 3 c. The expressions for the different curved lengths and angles corresponding to the embedded stiffeners are obtained geometrically from Fig. 3 c and provided in the Appendix A for a ready reference.

3.1.1 Bending energy of frame with embedded stiffeners

In the developed DEMES, the frame is an important part characterized by two bending strips having width $b/2$ each and the thickness h as revealed in Fig. 3 a. The DE membrane enforces a constant bending moment to the compliant frame and thereby causes the frame to bend along length C of the rectangular hole. Therefore, the energy required to bend the frame is expressed as

$$U_1 = \int_0^C \frac{M^2 dx}{2E_f I} = \frac{E_f b h^3}{24C} (\theta)^2 = \frac{K_{bf}}{2} (\theta)^2, \quad (1)$$

in which K_{bf} represents the associated compliant frame bending stiffness which can be defined in terms of mechanical and geometrical parameters of the frame as

$$K_{bf} = \frac{E_f b h^3}{12C}, \text{ where } E_f \text{ represents the Young's modulus of the frame.}$$

To avoid the warping of the DEMES, the stiffeners are also attached to the surface of the compliant frame (Fig. 3). However, on the downside, it enhances the bending stiffness of the frame by the expression:

$$U_2 = \frac{K_{bs}}{2} \left(\frac{W_s}{C^2} \right) \left[(2C - W_s) + \sum_{n=2}^N (2C - W_s - nS) \right] (\theta)^2 \quad (2)$$

in which K_{bs} is the bending stiffness of the stiffener, W_s represents the width of the stiffener, n is the number of stiffeners, N represents the maximum number of stiffeners, and S denotes the spacing between the consecutive stiffeners, respectively. In the present study, we keep the number of stiffeners $N = 4$ for all the analysis. Hence, upon simplification, the Eq. (2) for $N = 4$ stiffeners is rewritten as

$$U_2 = \frac{K_{bs}}{2} \left(\frac{W_s}{C^2} \right) \underbrace{\left((2C - W_s) + (2C - W_s - 2S) + (2C - W_s - 3S) + (2C - W_s - 4S) \right)}_{\text{Stiffeners}} (\theta)^2 \quad (3)$$

The total energy required to bend the compliant frame having stiffeners embedded on the surface is obtained by adding Eq. (1) and Eq. (3) as

$$U_{total} = U_1 + U_2,$$

$$U_{total} = \underbrace{\frac{K_{bf}}{2}(\theta)^2}_{frame} + \underbrace{\frac{K_{bs}}{2}\left(\frac{W_s}{C^2}\right)\left((2C - W_s) + (2C - W_s - 2S) + (2C - W_s - 3S) + (2C - W_s - 4S)\right)}_{Stiffeners}(\theta)^2 \quad (4)$$

Fig. 4 Schematic of DE membrane in (a) Initial state, (b) Prestretched state, and (c) actuated state.

3.1.2 Strain energy of membrane

The schematic of the DE membrane attached to the inextensible frame to develop the required DEMES is shown in Fig. 4. In the initial configuration, the membrane has the initial dimensions of $L_0 \times W_0 \times t_0$ in the X_1 , X_2 , and X_3 directions (Fig. 4 a), respectively. Due to equal biaxial pre-stretching of the membrane in X_1 and X_2 directions, the dimensions of the membrane become $L_1 \times W_1 \times t_1$, respectively. By invoking the incompressibility condition of the DE membrane, the developed pre-stretches in the principal directions of the DE membrane are drafted as

$$\lambda_{p1} = \lambda_p = \frac{L_1}{L_0} = \frac{C}{L_0}; \quad \lambda_{p2} = \lambda_p = \frac{W_1}{W_0}; \quad \lambda_{p3} = \frac{1}{\lambda_{p1}\lambda_{p2}}, \quad (5)$$

where λ_{p1} , λ_{p2} , and λ_{p3} denote the developed pre-stretch in X_1 , X_2 , and X_3 directions, respectively, L_1 represents the length of the pre-stretched membrane equal to the length C of a rectangular hole in the frame. The obtained stretches in Eq. (5) correspond to the developed initial stretch state in which the compliant frame remains flat. Due to applied potential difference $\phi(t)$ between the two electrodes of the membrane, the expansion of the DE membrane along X_1 and X_2 directions, and concomitant reduction in thickness, i.e., along X_3 direction occurs. Due to the implication of the incompressible nature of the DE membrane ($\lambda_1\lambda_2\lambda_3 = 1$), the developed true stretches in the principal directions of the membrane are expressed as

$$\lambda_1 = \frac{L}{L_0}; \quad \lambda_2 = \frac{W}{W_0}; \quad \lambda_3 = \frac{1}{\lambda_1 \lambda_2}, \quad (6)$$

in which λ_1 , λ_2 , and λ_3 denote the developed true pre-stretches in X_1 , X_2 , and X_3 directions, respectively.

The dimensions of the membrane in the actuated configuration becomes $L \times W \times t_2$, respectively as shown in Fig. 4 c. From Fig. 3 b, the DE membrane length (L) is expressed in terms of bending angle ($\theta_{X1} = \theta$) of the frame in the X_1 direction as

$$L = \frac{2L_1}{\theta_{X1}} \sin\left(\frac{\theta_{X1}}{2}\right) = \frac{2C}{\theta} \sin\left(\frac{\theta}{2}\right). \quad (7)$$

Similarly, the DE membrane width (W) is expressed in terms of bending angle (θ_{X2}) of the frame in the X_2 direction as

$$W = \frac{2W_1}{\theta_{X2}} \sin\left(\frac{\theta_{X2}}{2}\right). \quad (8)$$

The bending of the frame along width direction is negligible, hence invoking the assumption of $\sin\left(\frac{\theta_{X2}}{2}\right) \approx \frac{\theta_{X2}}{2}$. The expression for width (W) of the membrane (Eq. 8) becomes

$$W = \frac{2W_1}{\theta_{X2}} \sin\left(\frac{\theta_{X2}}{2}\right) = \frac{2W_1}{\theta_{X2}} \times \left(\frac{\theta_{X2}}{2}\right) = W_1 \quad (9)$$

On inserting the expressions of length L (Eq. (7)), and width W (Eq. (9)) in Eq. (6), the developed true stretches in the membrane is expressed as a function of bending angle (θ) of the compliant frame as

$$\lambda_1 = \frac{L}{L_0} = \frac{2C}{\theta L_0} \sin\left(\frac{\theta}{2}\right); \quad \lambda_2 = \frac{W}{W_0} = \frac{W_1}{W_0}; \quad \lambda_3 = \frac{1}{\lambda_1 \lambda_2}. \quad (10)$$

Using the expressions of λ_{p1} and λ_{p2} from Eq. 5 in Eq. 10, the final expressions for the developed true stretch state in the DE membrane is articulated as the function of the prestretch (λ_p) of the membrane and bending angle (θ) of the frame as

$$\lambda_1 = \frac{2\lambda_p}{\theta} \sin\left(\frac{\theta}{2}\right); \quad \lambda_2 = \lambda_p; \quad \lambda_3 = \frac{1}{\lambda_1 \lambda_2}. \quad (11)$$

In this investigation, the generalized neo-Hookean material model [Colonnelli et al. \(2015\)](#); [Luo et al. \(2020\)](#) is used to describe the hyperelastic response of the membrane. However, this choice is not restrictive and the analysis presented here can easily be extended for assessing the effect of other models of hyperelasticity, such as Mooney-Rivlin [Joglekar \(2014\)](#); [Soares et al. \(2020\)](#), [YeohChen et al. \(2018\)](#); [Wissler and Mazza \(2005\)](#), [Gent Zhang and Chen \(2020\)](#); [Zhang et al. \(2018\)](#); [Zhou et al. \(2018\)](#) and [Arruda-Boyce Park et al. \(2012\)](#), etc. Based on the neo-Hookean hyperelastic material model, the expression for the strain energy density of the DE membrane is written as

$$\psi_{membrane} = \frac{\mu}{2} (\lambda_1^2 + \lambda_2^2 + \lambda_3^2 - 3) \quad (12)$$

in which μ is the shear modulus of the DE membrane. Upon inheriting the expressions of λ_1 , λ_2 , and λ_3 from Eq. 11 and substituted in Eq. 12, the expression for the strain energy density of the membrane is drafted as

$$\psi_{membrane} = \frac{\mu}{2} \left(\frac{4\lambda_p^2}{\theta^2} \sin^2 \left(\frac{\theta}{2} \right) + \lambda_p^2 + \frac{\theta^2}{4\lambda_p^4 \sin^2 \left(\frac{\theta}{2} \right)} - 3 \right) \quad (13)$$

In the present study, an ideal dielectric model is adopted Tang et al. (2017), i.e., the permittivity (ϵ) of the DE membrane is considered constant and independent of mechanical deformation and electric displacement. The electrical free energy of the DE membrane using the ideal dielectric model is expressed as Rosset et al. (2014)

$$\psi_{electrical} = -\frac{1}{2} \epsilon E^2 \lambda_1^2 \lambda_2^2 = -\frac{\epsilon E^2}{2} \frac{4\lambda_p^4}{\theta^2} \sin^2 \left(\frac{\theta}{2} \right), \quad (14)$$

Here, ϵ represents the membrane permittivity, E signifies the developed nominal electric field in the membrane obtained as $E = \frac{\phi(t)}{t_1}$, in which $\phi(t)$ denotes the potential difference applied across the membrane, and t_1 represent the pre-stretched membrane thickness. The total free energy density expression of the DE membrane is articulated as

$$\psi = \psi_{membrane} + \psi_{electrical}$$

$$\psi = \left[\underbrace{\frac{\mu}{2} \left(\frac{4\lambda_p^2}{\theta^2} \sin^2 \left(\frac{\theta}{2} \right) + \lambda_p^2 + \frac{\theta^2}{4\lambda_p^4 \sin^2 \left(\frac{\theta}{2} \right)} - 3 \right)}_{membrane} - \underbrace{\frac{\epsilon E^2}{2} \frac{4\lambda_p^4}{\theta^2} \sin^2 \left(\frac{\theta}{2} \right)}_{electrical} \right] \quad (15)$$

The expression for the total free energy associated with the membrane in the actuated configuration is evaluated by taking the volume integral of the total strain energy density (ψ) from Eq. (15) as

$$U_{membrane} = \int_V \psi dV = W_0 L_0 t_0 \left[\underbrace{\frac{\mu}{2} \left(\frac{4\lambda_p^2}{\theta^2} \sin^2 \left(\frac{\theta}{2} \right) + \lambda_p^2 + \frac{\theta^2}{4\lambda_p^4 \sin^2 \left(\frac{\theta}{2} \right)} - 3 \right)}_{membrane} - \underbrace{\frac{\epsilon E^2}{2} \frac{4\lambda_p^4}{\theta^2} \sin^2 \left(\frac{\theta}{2} \right)}_{electrical} \right] \quad (16)$$

The total potential energy of the considered DEMES is expressed as the summation of total bending energy of frame with embedded stiffeners (Eq. (4)) and the total free energy of the DE membrane (Eq. (16)) and is written as

$$U = U_{total} + U_{membrane}$$

$$U = \left[\underbrace{\left(\frac{K_{bf}}{2} (\theta)^2 \right)}_{Frame} + \underbrace{\frac{K_{bs}}{2} \left(\frac{W_s}{C^2} \right) ((2C - W_s) + (2C - W_s - 2S) + (2C - W_s - 3S) + (2C - W_s - 4S)) (\theta)^2}_{Stiffeners} + \underbrace{W_0 L_0 t_0 \left(\frac{\mu}{2} \left(\frac{4\lambda_p^2}{\theta^2} \sin^2 \left(\frac{\theta}{2} \right) + \lambda_p^2 + \frac{\theta^2}{4\lambda_p^4 \sin^2 \left(\frac{\theta}{2} \right)} - 3 \right) - \frac{\epsilon E^2}{2} \frac{4\lambda_p^4}{\theta^2} \sin^2 \left(\frac{\theta}{2} \right)} \right)}_{membrane} \right] \quad (17)$$

The static behavior of the DEMES is obtained by equating the first derivative of the total potential energy (U) with bending angle θ equal to zero, i.e., $\frac{dU}{d\theta} = 0$.

3.2 Experimental analysis of DEMES

In the experimental study, a commercially available acrylic elastomer (VHB-4910) with an initial thickness of 1 mm, PET (polyethylene terephthalate) compliant frame of thickness 0.4 mm, and 3D printed PLA (polylactic acid) reinforcements are used to form the required DE based minimum energy structures. The working mechanism implemented experimentally to develop the required DEMES is shown in Fig. 5. First of all, a DE membrane of initial dimensions $10cm \times 10cm \times 1mm$, is attached to the mechanism as shown in Fig. 5 a. An equal biaxial prestretch is provided to the membrane by utilizing the stretching mechanism (Fig. 5 b). Next, a compliant PET frame having a rectangular hole at its center is attached to the stretched membrane. The rectangular-shaped compliant electrodes are produced at the center portion of the membrane using carbon grease (MG chemicals, 846-80G). Copper strips are used to make the connection of the electrodes with the input voltage terminals. The 3-D PLA reinforcements are attached on the surface of the compliant frame using an adhesive (Arclear 92892). Finally, a stiffening frame made up of PET material is attached towards the end portion of the compliant frame to develop the proposed DEMES, as shown in Fig. 5 d. The mechanically constrained DEMES in undeformed configuration is shown in Fig. 5 e. An exploded view of the stretching mechanism used for providing equal bi-axial prestretch to the membrane is revealed in Fig. 6 a. To experimentally investigate the effect of spacing (S) between the adjacent stiffeners on the performance of DEMES, we implemented four different DEMES with varying

Fig. 5 Experimental working mechanism implemented to obtain the proposed DEMES.

spacing (S) together, as shown in Fig. 6 a. A partition tape is implemented for easy removal of all the DEMES. Finally, the DEMES is removed from the constrained mechanism with the help of a razor blade. Due to redistribution of the energy among the structure, a newly deformed structure in the bend configuration is obtained. Figure 6 b presents the experimentally obtained bend configurations of the four different DEMES having varying levels of spacing (S) between the adjacent stiffeners. From Fig. 6 b, it is evident that the bending of the structure increases with the increase in spacing (S) between the stiffeners.

Figure 7 shows the methodology implemented for measuring the equilibrium angle of the DEMES. The steps implemented in the image processing technique utilizing MATLAB code are sequentially illustrated as follows: First, the experimentally obtained image of the DEMES with green color marks at the end of the stiffeners (Fig. 7 a) is considered as an input image of the DEMES in the developed image processing code. The input image is filter out to obtain the green channel image as depicted in Fig. 7 b. Further, the input image of the DEMES is converted to a grayscale image, as shown in Fig. 7 c. The obtained grayscale image is subtracted from the green channel image to get the end cross-section view of the stiffeners (Fig. 7 d). In the next step, the small objects are filter out and the obtained image depicting the cross-section view of the stiffeners is converted to a binary image, as shown in Fig. 7 e. From the obtained binary image, the contours and their centroids are evaluated, and the obtained

(a) Experimental: Stretching Mechanism

(b) Equilibrium configuration of the DEMES

Fig. 6 Experimental (a) Stretching mechanism, (b) Effect of spacing (S) between the adjacent stiffeners on the attained equilibrium state of the DEMES.

centroids are represented with the help of points in the image (Fig. 7 f). The acquired centroid points are joined by using red color lines, and reference lines represented by green color intersects both the lines as depicted in Fig. 7 g. Finally, the attained equilibrium angle of the DEMES is evaluated as $\theta_{eq.} = 180 - (\theta_1 - \theta_2)$, in which θ_1 , θ_2 represents the obtained top and bottom angle in an anticlockwise direction to the reference lines in Fig. 7 g.

Fig. 7 Flow process representation to measure the attained equilibrium angle of the DEMES: (a) Input image of DEMES, (b) Filter image: Green channel, (c) Convert to a grayscale image, (d) Subtracting grayscale from green channel image, (e) Filter small objects and make image binary, (f) Finding centroid of contours and extracting points from contours, (g) Drawing of lines from points and intersect them with the reference lines.

3.3 Finite element modeling of DEMES

This subsection presents the finite element framework implemented using ABAQUS 6.19 to measure the equilibrium angle of the proposed DEMES.

3.3.1 Kinematics

Consider a continuum rectangular hyperelastic membrane $\mathcal{B}_i$ of thickness t_i in the reference configuration with dimensions L_{xi} and L_{yi} in x and y directions, respectively.

Also, $\frac{t_i}{L_{xi}}, \frac{t_i}{L_{yi}} \ll 1$ such that the deformed membrane can be delineate by the theory of thin hyperelastic membranes under finite deformations. Due to prestretching along x and y directions, the membrane attains the final configuration $\mathcal{B}_f$ of thickness t_f with dimensions L_{xf} and L_{yf} in x and y directions as shown schematically in Fig. 8. A strain energy density function ψ per unit initial volume of membrane can be

Fig. 8 Schematic representation of initial (B_i) and pre-stretched (B_f) continuum rectangular hyperelastic membrane.

expressed as a function of three principal stretches for the unit fibers that are initially oriented along the directions of three axes in the coordinate system, λ_1 , λ_2 , and λ_3 , or in terms of the strain invariant I_1 , I_2 , and I_3 as

$$\psi = \psi(\lambda_1, \lambda_2, \lambda_3) = \psi(I_1, I_2, I_3). \quad (18)$$

3.3.2 Constitutive modeling

In this study, 3M VHB-4910 acrylic tape is the primary material used as the dielectric elastomer. Due to minimal changes in the volume of rubber-type material, incompressible nature is usually assumed, i.e., $I_3 = 1$, and the principal stretch in normal direction becomes $\lambda_3 = \frac{1}{\lambda_1 \lambda_2}$ or, physically, $\lambda_3 = \frac{t_f}{t_i}$.

In the present study, the strain energy of 3M VHB-4910 acrylic tape is described using the neo-Hookean hyperelasticity material model from Eq. 12. In all the simulations, the shear modulus of the membrane is equated as $\mu = 2C_{10}$, in which C_{10} is the material parameter. The induced principal Cauchy stresses due to prestretching of the membrane for the neo-Hookean material model Soares and Gonçalves (2018) is expressed as

Fig. 9 An analytically validated stress-stretch curve of hyperelastic membrane using neo-Hookean model of hyperelasticity.

Table 2 Mechanical properties of the different parts of the DEMES implemented in ABAQUS.

Part	Material	Type	Young's modulus	Poisson's ratio
Elastomer membrane	VHB 3M (4910)	Hyper-elastic	$E(298K) = 0.1537 \text{ MPa}$ Liu et al. (2015)	0.499
Frame	PET	Elastic	5.7 GPa	0.40
Stiffeners	PLA	Elastic	3.5 GPa	0.35

$$\sigma_1 = 2C_{10} \left(\lambda_x^2 - \frac{1}{\lambda_x^2 \lambda_y^2} \right) \quad \text{neo-Hookean} \quad (19)$$

$$\sigma_2 = 2C_{10} \left(\lambda_y^2 - \frac{1}{\lambda_x^2 \lambda_y^2} \right)$$

3.3.3 Numerical implementation

Next, the numerical simulations using the finite element software ABAQUS 6.19, considering a hyperelastic membrane is performed. Convergence of the principal stresses of the membrane considering neo-Hookean material model up to four decimal places is evaluated by implementing the M3D4 mesh of 2500 elements. The obtained finite element results for the principal stresses agree well with the analytical results obtained using Eq. (19) (see Fig. 9).

The flow process technique incorporated to measure the attained equilibrium angle of the DEMES using ABAQUS is revealed in Fig. 10. First of all, an initial hyperelastic membrane with M3D4 mesh element is considered, as depicted in Fig. 10(a). An equal biaxial pre-stretch is provided to the considered membrane (Fig. 10(b)). The obtained principal stresses for different membrane stretches are considered as an initial condition of the membrane. Next, the prestretched DE membrane having the same dimensions as that of the compliant frame (Fig. 10(c)) is affixed on the frame surface to develop the assembly of the membrane and frame, as shown in Fig. 10(d). Finally, the 3-D printed PLA reinforcements and a stiffening frame made up of PET material is attached to the compliant frame to form the required DEMES, as shown in Fig. 10(e). The obtained deformed and undeformed configuration of the DEMES

Fig. 10 Flow process representation implemented in ABAQUS to measure the attained equilibrium angle of the DEMES: (a) Initial meshed model of the hyperelastic membrane, (b) Biaxial pre-stretched membrane, (c) Compliant frame and pre-stretched membrane with same dimensions, (d) Assembly of the compliant frame and pre-stretched membrane, (e) Attached 3-D stiffeners and stiffening frame to the assembly, (f) Deformed and undeformed shape of the DEMES, (g) Measurement of equilibrium angle using a commercially available online protractor.

is depicted in Fig. 10(f). Finally, the equilibrium angle of the DEMES is obtained by using a commercially available online protector (Fig. 10(g)). The material properties of the elastomeric membrane, inextensible PET frame, and 3D printed PLA reinforcements considered for numerical simulations are indexed in Table 2.

3.4 Validation of the proposed model

To affirm the executability of the proposed static analytical framework, we have enclosed a comparison of the obtained analytical model results with performed experimental and numerical studies. Figure 11 presents this comparison with the experimental and numerical observations on a configurationally similar prototype of DEMES for four different prestretch levels $\lambda_p = 1.25, 1.5, 1.75, 2.0$ with $S = 0.5$ cm, and $b = 1.5$ cm, respectively.

The predictions are made for the attained equilibrium angle (θ_{eq}) of the DEMES when it is detached from external mechanical constraints. The examination demonstrates the enlargement in value of the equilibrium angle with increasing prestretch (λ_p) of the membrane. The present static model effectively captures the deviation of the equilibrium angle with applied prestretch as observed in the experimental and numerical observations. To get the accurate experimental predictions of the equilibrium angle of the DEMES, we conducted three sets of experiments for all the specified prestretch levels. Figure 11 shows the upper and lower deviations in θ_{eq} obtained experimentally for all considered prestretch levels. From Fig. 11, it is also observed

Fig. 11 Experimental and numerical validation of the proposed analytical framework.

that the standard deviation in the measurement of θ_{eq} obtained experimentally increases with the increase in the prestretch levels. These large experimental deviations in equilibrium angle measurement can be attributed to fabrication variations. Further, the seemingly slight discrepancy in the attained equilibrium angle in the developed analytical and numerical model with the experimental observations arises due to the inherent viscoelastic nature of the VHB-4910, which is not considered in the present analytical and numerical modeling of the DEMES. However, Fig. 11 shows an excellent corroboration between the upper deviations of the experimental predictions and those extracted using the developed static model, hence ascertaining the legitimacy of the developed model.

As the proposed static analytical framework effectively predicts the attained equilibrium angle of the DEMES; we further extend the analytical framework in the upcoming section to analyze the nonlinear dynamic behavior of the proposed DEMES.

4 Dynamic analysis of DE based minimum energy structures

This section pertains to the development of a nonlinear dynamic model of the DEMES to investigate the effect of various geometrical and mechanical parameters along with incorporated stiffeners on the attained initial and final configuration of the DEMES.

4.1 Dynamic governing equations

The nonlinear dynamic governing equation of the developed DEMES is evaluated by using the Euler-Lagrange's (E-L) equation of motion based on the least action principle and is expressed as

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{\theta}} \right) - \frac{\partial L}{\partial \theta} = 0, \quad (20)$$

in which $\dot{\theta}$ denotes the derivative of the bending angle with time and $\mathcal{L}$ signifies the Lagrangian, which is expressed as the difference between the total kinetic and potential energy of the DEMES. In the following subsection, we devise the expression of the total kinetic energy of the developed DEMES.

4.2 Total kinetic energy (K) of DEMES

The considered structure comprises of a compliant frame with 3-D stiffeners embedded on its surface. This assembly of 3-D stiffeners, and the compliant frame adheres to a prestretched DE membrane whose weight is negligible. Hence, we ignore the effect of DE membrane weight in comparison to the compliant frame and stiffeners. Further, we consider the compliant frame as a rotational joint to evaluate the total kinetic energy of the developed DEMES as

$$K = \underbrace{\frac{m_f C^2}{24} \left(\frac{d\theta}{dt} \right)^2}_{Frame} + \underbrace{\frac{1}{2} m_s \sum_{n=1}^N V_n^2}_{Stiffeners} \quad (21)$$

where, m_f and m_s represent the mass of the compliant frame and 3-D stiffener, respectively.

To obtain the final expression for the kinetic energy (K) of the DEMES, the resultant velocities (V_1 - V_4) associated with the incorporated ($N = 4$) stiffeners need to be evaluated. The methodology implemented to determine the velocities are indexed in Appendix A. Upon invoking the expressions of the velocities from the Appendix A, the final kinetic energy of the developed DEMES ($N = 4$ stiffeners) is written as

$$K = \underbrace{\frac{m_f C^2}{24} \left(\frac{d\theta}{dt} \right)^2}_{Frame} + \frac{m_s}{8} \left[\begin{aligned} & \underbrace{\left((2C - 6S - W_s)^2 \left(\frac{2C - 6S - W_s}{C} \right)^2 \right)}_{Stiffener1} \\ & + \underbrace{\left((2C - 6S - W_s)^2 \left(\frac{2C - 6S - W_s}{C} \right)^2 + S^2 \left(\frac{4C - 10S - 2W_s}{C} \right)^2 \right.}_{Stiffener2} \\ & \quad \left. + 2S(2C - 6S - W_s) \left(\frac{2C - 6S - W_s}{C} \right) \left(\frac{4C - 10S - 2W_s}{2C} \right) \cos \left(\frac{4S + W_s - 2C}{2C} \right) \theta \right) \\ & + \underbrace{\left((2C - 6S - W_s)^2 \left(\frac{2C - 6S - W_s}{2C} \right)^2 + S^2 \left(\frac{10C - 22S - 5W_s}{C} \right)^2 \right.}_{Stiffener3} \\ & \quad \left. + 2S(2C - 6S - W_s) \left(\frac{2C - 6S - W_s}{C} \right) \left(\frac{10C - 22S - 5W_s}{2C} \right) \cos \left(\frac{16S + 4W_s - 8C}{2C} \right) \theta \right) \\ & + \underbrace{\left((2C - 6S - W_s)^2 \left(\frac{2C - 6S - W_s}{2C} \right)^2 + S^2 \left(\frac{18C - 34S - 9W_s}{C} \right)^2 \right.}_{Stiffener4} \\ & \quad \left. + 2S(2C - 6S - W_s) \left(\frac{2C - 6S - W_s}{C} \right) \left(\frac{18C - 34S - 9W_s}{2C} \right) \cos \left(\frac{28S + 8W_s - 16C}{2C} \right) \theta \right) \end{aligned} \right] \left(\frac{d\theta}{dt} \right)^2 \quad (22)$$

By invoking the expressions of total kinetic energy (K) and total potential energy (U) of the DEMES from Eq. (22) and Eq. (17) in Lagrange's equation of motion (Eq. 20), the ensuing nonlinear dynamic equation that governs the motion of developed DEMES is expressed as

$$\frac{m_f C^2}{12} \left(\frac{d^2 \theta}{dt^2} \right) + \frac{m_s}{4} (Z_1) \left(\frac{d^2 \theta}{dt^2} \right) - \frac{m_s}{8} (Z_2) \left(\frac{d\theta}{dt} \right)^2 + (Z_3) \theta + \mu W_0 L_0 t_0 (Z_4) = 0 \quad (23)$$

The required expressions for the terms $Z_1 - Z_4$ are provided in the Appendix B for the ready reference.

Initially, when a pre-stretched membrane is attached to the compliant frame, the structure remains flat. Hence, the associated two initial condition that correspond to the developed DEMES are

$$\theta|_{t=0} = 0; \quad \left. \frac{d\theta}{dt} \right|_{t=0} = 0 \quad (24)$$

The dynamic behavior of the developed DEMES is evaluated by simulating the governing differential Eq. (23) along with initial conditions from Eq. (24) using the built-in MATLAB ODE45 solver. Specifically, ODE45 solver incorporates the 4th order Runge Kutta method to solve the differential equations with truncation error of order dt^5 , in which dt represents the time step. In the next section, the influence of incorporated stiffeners on the attained initial and final configurations of the DEMES are investigated.

5 Results and analysis

The present section investigates the effect of different geometrical parameters on the nonlinear dynamic behavior of the proposed DEMES. To interrogate the effect of stiffeners implemented on the surface of the frame, we vary the spacing $S = 0.25, 0.50, 0.75$, and 1.0 cm between the adjacent stiffeners. The bending stiffness of the frame is also tuned by varying the levels of the width of bending strips, i.e., $b = 1.0, 1.5, 2.0, 2.5$, and 3.0 cm. Further, four different levels of equal bi-axial prestretch $\lambda_p = 1.25, 1.50, 1.75$, and 2.0 of the DE membrane is implemented. The shear modulus (μ) of the DE membrane obtained at room temperature ($T = 298K$) is considered Liu et al. (2015) for all the simulations.

In the following, we exhibit a parametric investigation to examine the influence of prestretch (λ_p) of the membrane, spacing (S) between the stiffeners, width (b) of the bending strips of the frame on the obtained equilibrium angle of the structure when it is disengaged from external mechanical constraints.

5.1 Attainment of the initial equilibrium configuration

The equilibrium state attained by the structure can be tuned by altering the various geometrical and material properties of the membrane, frame, and stiffeners incorporated to develop the proposed DEMES. In this subsection, we carried out a parametric study to assess the impact of prestretch (λ_p) of the membrane, bending strips width (b) of the frame, and spacing (S) between the stiffeners on the attained initial equilibrium state of the DEMES. The DEMES actuator attained an equilibrium configuration

Fig. 12 Effect of prestretch (λ_p) of the DE membrane on the equilibrium angle (θ_{eq}) of the DEMES.

Fig. 13 Effect of spacing (S) between the adjacent stiffeners on the equilibrium angle (θ_{eq}) of the DEMES.

due to redistribution of energy between the elastomer film and the inextensible frame. Thus, no electrical load ($E = 0$) is applied for the cases considered in this subsection.

First of all, we evaluate the effect of prestretch (λ_p) of the membrane on the attained equilibrium angle (θ_{eq}) of the DEMES. For this purpose, we obtain the phase portraits illustrating the alteration of equilibrium angle (θ_{eq}) with angular velocity ($\frac{d\theta_{eq}}{dt}$) for different prestretch level ($\lambda_p = 1.50, 1.75, 2.0, 2.25, \text{ and } 2.50$) at specified conditions of $S = 1.0$ cm, and $b = 1.5$ cm, respectively as shown in Fig. 12. From Fig. 12, it is inferred that the structure for all the considered prestretch levels manifests a supercritical pitchfork bifurcation of equilibrium configuration [Khurana et al. \(2020\)](#); [Strogatz \(2018\)](#). The stated bifurcation of equilibrium configuration represents the attainment of positive as well as negative equilibrium angle of the equal magnitude. This bifurcation of the equilibrium state is referred to as the bi-stability of DEMES [Follador et al. \(2015\)](#); [Zhao et al. \(2016a\)](#).

Next, the phase portraits to investigate the influence of spacing (S) between the adjacent stiffeners on the attained equilibrium angle of the structure for the aforementioned four spacing levels is plotted (see Fig. 13). This figure reveals that the attained θ_{eq} of the DEMES amplifies as the spacing (S) between the adjacent stiffeners increases for the specified conditions. This is because as spacing (S) increases, the bending moment imposed by the stiffeners decreases. Additionally, Fig. 13 also shows that the envelope illustrating the maximum and minimum value of angular velocity ($\frac{d\theta_{eq}}{dt}$) enlarges as spacing (S) between the adjacent stiffeners increases.

The developed model of DEMES incorporates a compliant frame with two bending strips having a width ($b/2$) each (see Fig. 3 a). The total width (b) of the bending strips alters the bending stiffness of the compliant frame. From Fig. 14 a, it is observed that increase in width (b) of the frame enhances the bending stiffness of the compliant frame. Due to this increase in bending stiffness with the increase in width (b), the frame becomes more restrictive to the bending moment imposed by the DE membrane. Hence, the attained equilibrium angle (θ_{eq}) of the structure decreases with the enhancement in width (b) of the bending strips. It is inferred that tuning of both prestretch (λ_p) of the membrane and bending strips width (b) of the compliant frame alters the equilibrium state of the structures. Therefore, to investigate the combined effect of λ_p and b , we obtained θ_{eq} of the structure for three different prestretch level $\lambda_p = 1.50, 2.0, \text{ and } 2.50$ with aforementioned varying bending strips width b as depicted in Fig. 14 b. As observed from Fig. 14 b, the DEMES with a larger value of λ_p and a smaller value of b attains the maximum value of θ_{eq} . This is because the compliant frame offers less bending stiffness at a lower value of b and membrane possess more strain energy at a higher value of λ_p , which causes the more bending of the structure and hence enhances the equilibrium angle of the structure.

Further, we study the combined influence of spacing (S) between the adjacent stiffeners and prestretch (λ_p) of the membrane. From Fig. 15, it is visualized that the structure with higher spacing (S) between the stiffeners and higher membrane prestretch (λ_p) exhibit the highest value of θ_{eq} . The enhancement in the spacing (S) reduces the bending moment posed by the stiffeners and more prestretching of the membrane enhances the membrane strain energy and thereby increases the attained

(a)

(b)

Fig. 14 Variation of (a) Equilibrium angle (θ_{eq}) and bending stiffness of the frame with width b of the bending strips and (b) Combined effect of width b of the bending strips and prestretch of the membrane on θ_{eq} .

Fig. 15 Combined effect of spacing (S) between the adjacent stiffeners and prestretch (λ_p) of the membrane on θ_{eq} .

Fig. 16 Effect of bending strips width (b) of the compliant frame and spacing (S) between the adjacent stiffeners on the equilibrium angle (θ_{eq}) of the DEMES.

θ_{eq} of the structure. Finally, the combined influence of bending strips width (b) and spacing (S) between the adjacent stiffeners on the attained equilibrium angle (θ_{eq}) of the structure is investigated by contour plots (Fig. 16). From Fig. 16, it is inferred that the DEMES exhibits the maximum value of θ_{eq} at a lower level of bending strips width (b) and at a higher level of spacing (S) between the adjacent stiffeners. The justification for this enhancement in the θ_{eq} of the DEMES is due to the diminution of the compliant frame stiffness at a lower value of b and at higher spacing (S) between the adjacent stiffeners. Due to this reduction in the bending stiffness, less resistance is offered by the frame against the bending moment posed by the membrane and thereby enhances the structure's equilibrium angle (θ_{eq}).

5.2 DC dynamic response

In this subsection, we consider the obtained values of equilibrium angle θ_{eq} of the DEMES in the preceding section as the initial conditions for interrogating the DC dynamic behavior of the structure. A time- independent electric field (e) approximately represented by a constant Heaviside step function is applied. The applied electric field results in the periodic oscillations of the structure, whose time and intensity rely on the magnitude of applied DC voltage signal (V_{dc}).

The DEMES actuator with initial θ_{eq} corresponding to the conditions $\lambda_p = 2.0$, $S = 1.0$, cm and $b = 1.5$ cm is excited by the DC voltage signal up to the threshold state at which the structure almost becomes flat, i.e., the tip angle practically becomes equal to zero as shown in Fig. 17 a. The tip angle of the DEMES is denoted by θ_t ,

and is defined as: $\left\{ \begin{array}{l} \theta_t|_{V_{dc}=0} = \theta_{eq} \\ \theta_t|_{V_{dc} \neq 0} = \theta \end{array} \right\}$. An appreciable reduction in the tip angle of the

DEMES with the increase in applied DC voltage is perceived, as shown in Fig. 17 a. Next, the actuation performance characterized by the actuation range ($\theta_{eq} - \theta$) of the DEMES actuator for different levels of spacing (S) between the adjacent stiffeners corresponding to $V_{dc} = 4000$ V, $\lambda_p = 2.0$, $S = 1.0$ cm and $b = 1.5$ cm, respectively, is plotted in Fig. 17 b. From the figure, it is inferred that the enhancement in the spacing (S) has a favorable impact on the actuation range of the DEMES actuator. This is because the DEMES with higher spacing (S) between the adjacent stiffeners possess the lower value of frame stiffness resulting in more bending of the structure at applied particular DC voltage, thereby enhancing the actuation range of the DEMES actuator.

Next, the phase portraits revealing the alteration of angular velocity ($\frac{d\theta}{dt}$) with bending angle (θ) is plotted for varying levels of applied DC dynamic voltage (Fig. 18 a) and different spacing (S) between the adjacent stiffeners (Fig. 18 b). As anticipated, the bending angle (θ) of the DEMES decreases as applied DC voltage increases, as shown in Fig. 18 a. Further, the envelope of the phase portraits (Fig. 18 a) enhances with the depletion of the DC dynamic voltage. From Fig. 18 b, it is visualized that the bending angle (θ) of the DEMES amplifies as the level of spacing (S) between the adjacent stiffeners increases. Further, the envelope of the phase portraits (Fig. 18 b) enlarges with the increase in spacing (S) between the adjacent stiffeners. Finally, the combined effect of applied DC dynamic voltage and spacing (S) on the tip angle (θ_t) of the structure is evaluated as depicted in Fig. 19. From Fig. 19, the maximum

Fig. 17 DC dynamic behavior of the DEMES actuator showing the (a) Alteration of tip angle with applied DC voltage, (b) Actuation range ($\theta_{eq} - \theta$) for different spacing between the adjacent stiffeners at a specified applied voltage level of $V_{dc} = 4$ kV.

Fig. 18 Phase portraits showing the variation of angular velocity ($\frac{d\theta}{dt}$) with bending angle (θ) for (a) different levels of applied DC voltage, (b) different levels of spacing (S) between the consecutive stiffeners.

Fig. 19 Combined effect of applied DC dynamic voltage and spacing (S) on the tip angle of the DEMES.

reduction in the tip angle of the DEMES is observed at a lower spacing level and at a higher level of applied DC voltage.

6 Concluding remarks

In summary, this work investigated the role of stiffeners in enhancing the electromechanical behavior of dielectric elastomer minimum energy structures. The primary motive of adding stiffeners is to circumvent the warping resulting from anticlastic curvature formed in the elementary rectangular DEMES. The effect of including stiffeners is assessed from two perspectives; they are: (a) impact on the initial equilibrium angle of the DEMES prior to electromechanical actuation, and (b) impact on the electromechanical response of the DEMES when driven by a Heaviside electric load. To this end, an energy-based analytical model is devised to predict the initial equilibrium angle as a function of the underlying geometric and material parameters. The predictive capabilities of the proposed model are corroborated with the 3D finite element simulations and the experimental observations. The analytical model is then extended to accommodate the time-dependent effects for assessing the nonlinear dynamic response of DEMES driven by the electric load.

In particular, we investigated the effect of (i) levels of pre-stretch, (ii) width of the bending strip, (iii) spacing between the stiffeners, and (iv) applied voltage on the initial equilibrium angle and the angular stroke resulting from the electromechanical actuation. The initial equilibrium angle attained by the DEMES increases with

increasing levels of pre-stretch on virtue of the fact that increasing amount of strain energy is available for bending the compliant frame. On parallel lines, for a fixed number of stiffeners, the initial equilibrium angle is found to increase with increasing spacing between the stiffeners; whereas it reduces with increasing frame-width. With electric load used to drive the DEMES actuator, an appreciable diminution in the tip angle of the DEMES with an enhancement in the applied voltage is observed. The actuation range of the DEMES actuator for maximum considered spacing ($S = 1$ cm) between the adjacent stiffeners is enhanced by 28.2 % with respect to the DEMES actuator with no spacing ($S = 0$ cm) at a particular level of applied DC voltage. This clearly indicates the dependency of various geometric and operating parameters on the actuation performance of the DEMES actuator.

The analytical model presented in this work can be of potential use in predesigning a DEMES actuator deprived of warping and can further be refined to accommodate additional effects of practical relevance, such as dissipative material models, (e.g. viscoelasticity) and complex loading conditions (e.g., AC electric loading).

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Data availability

The authors declare that data supporting the findings of this study are available within the article.

Declarations

Conflict of interest: The authors declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

Appendix A: Evaluation of velocities

The associated curved lengths and angles based on the embedded stiffeners on the surface of the compliant frame is obtained geometrically by using Fig. 3 c as

$$\begin{aligned}
\psi_1 &= \theta_1 + \left(\frac{\theta_2 - \theta_1}{2} \right) = \left(\frac{\theta_1 + \theta_2}{2} \right), \psi_2 = \psi_1 + \left(\frac{\theta_3 + \theta_4}{2} \right) = \left(\frac{\theta_1 + \theta_2 + \theta_3 + \theta_4}{2} \right) \\
\psi_3 &= \psi_2 + \left(\frac{\theta_5 + \theta_6}{2} \right) = \left(\frac{\theta_1 + \theta_2 + \theta_3 + \theta_4 + \theta_5 + \theta_6}{2} \right) \\
\psi_4 &= \psi_3 + \left(\frac{\theta_7 + \theta_8}{2} \right) = \left(\frac{\theta_1 + \theta_2 + \theta_3 + \theta_4 + \theta_5 + \theta_6 + \theta_7 + \theta_8}{2} \right)
\end{aligned} \tag{A.1}$$

in which,

$$\begin{aligned}
\theta_1 &= \left(\frac{C_1}{C} \right) \theta, \theta_2 = \left(\frac{C_2}{C} \right) \theta, \theta_3 = \left(\frac{C_3}{C} \right) \theta, \theta_4 = \left(\frac{C_4}{C} \right) \theta \\
\theta_5 &= \left(\frac{C_5}{C} \right) \theta, \theta_6 = \left(\frac{C_6}{C} \right) \theta, \theta_7 = \left(\frac{C_7}{C} \right) \theta, \theta_8 = \left(\frac{C_8}{C} \right) \theta = \left(\frac{C}{C} \right) \theta = \theta
\end{aligned} \tag{A.2}$$

The curved lengths associated with the incorporated stiffeners depicts in Fig. 3 c are obtained as

$$\begin{aligned}
C_1 &= C - 3S - W_s, C_2 = C - 3S, C_3 = C - 2S - W_s, C_4 = C - 2S \\
C_5 &= C - S - W_s, C_6 = C - S, C_7 = C - W_s, C_8 \approx C
\end{aligned} \tag{A.3}$$

To estimate the expression for the total kinetic energy of the DEMES (Eq. (21)), we derived the associated expressions for the velocities associated with the incorporated ($N = 4$) stiffeners. The expression for velocity V_1 is obtained as

$$V_1^2 = \left(\frac{dx_1}{dt} \right)^2 + \left(\frac{dy_1}{dt} \right)^2 \tag{A.4}$$

Where,

$$x_1 = \left(C_1 + \frac{W_8}{2} \right) \cos \psi_1, \quad y_1 = \left(C_1 + \frac{W_8}{2} \right) \sin \psi_1 \tag{A.5}$$

On substituting the expression of ψ_1 from Eq. (A.1) in Eq. (A.5), we get

$$x_1 = \left(C - 3S - \frac{W_s}{2} \right) \cos \left(\frac{\theta_1 + \theta_2}{2} \right) = \left(C - 3S - \frac{W_s}{2} \right) \cos \left(\frac{C_1 + C_2}{2C} \right) \theta \tag{A.6}$$

Hence,

$$\frac{dx_1}{dt} = \frac{dx_1}{d\theta} \times \frac{d\theta}{dt} = \left(C - 3S - \frac{W_s}{2} \right) \left(-\sin \left(\frac{C_1 + C_2}{2C} \right) \theta \right) \left(\frac{C_1 + C_2}{2C} \right) \frac{d\theta}{dt} \tag{A.7}$$

Upon rearranging, we obtained

$$\frac{dx_1}{dt} = - \left(C - 3S - \frac{W_s}{2} \right) \left(\sin \left(\frac{2C - 6S - W_s}{2C} \right) \theta \right) \left(\frac{2C - 6S - W_s}{2C} \right) \frac{d\theta}{dt} \quad (\text{A.8})$$

Similarly, we get

$$\frac{dy_1}{dt} = \left(C - 3S - \frac{W_s}{2} \right) \left(\cos \left(\frac{2C - 6S - W_s}{2C} \right) \theta \right) \left(\frac{2C - 6S - W_s}{2C} \right) \frac{d\theta}{dt} \quad (\text{A.9})$$

Finally, on substituting the expressions from Eqs. (A.8 - A.9) in Eq. (A.4), we obtained

$$V_1^2 = \left(\frac{dx_1}{dt} \right)^2 + \left(\frac{dy_1}{dt} \right)^2 = \frac{1}{4} (2C - 6S - W_s)^2 \left(\frac{2C - 6S - W_s}{C} \right)^2 \left(\frac{d\theta}{dt} \right)^2 \quad (\text{A.10})$$

Similarly, the expression for velocity V_2 is evaluated as

$$V_2^2 = \left(\frac{dx_2}{dt} \right)^2 + \left(\frac{dy_2}{dt} \right)^2 \quad (\text{A.11})$$

Where,

$$x_2 = x_1 + S \cos \psi_2, \quad y_2 = y_1 + S \sin \psi_2 \quad (\text{A.12})$$

On substituting the expressions of ψ_2 from Eq. (A.1) in Eq. (A.12), we get

$$\begin{aligned} x_2 &= \left(C - 3S - \frac{W_s}{2} \right) \cos \left(\frac{\theta_1 + \theta_2}{2} \right) + S \cos \left(\frac{\theta_1 + \theta_2 + \theta_3 + \theta_4}{2} \right) \\ &= \left(C - 3S - \frac{W_s}{2} \right) \cos \left(\frac{C_1 + C_2}{2C} \right) \theta + S \cos \left(\frac{C_1 + C_2 + C_3 + C_4}{2C} \right) \theta \end{aligned} \quad (\text{A.13})$$

Hence,

$$\frac{dx_2}{dt} = \frac{dx_2}{d\theta} \times \frac{d\theta}{dt} = - \left(\begin{aligned} &\left(C - 3S - \frac{W_s}{2} \right) \sin \left(\frac{C_1 + C_2}{2C} \right) \theta \left(\frac{C_1 + C_2}{2C} \right) \\ &+ S \sin \left(\frac{C_1 + C_2 + C_3 + C_4}{2C} \right) \theta \left(\frac{C_1 + C_2 + C_3 + C_4}{2C} \right) \end{aligned} \right) \frac{d\theta}{dt} \quad (\text{A.14})$$

Taking the square of Eq. (A.14), we obtained the following expression.

$$\left(\frac{dx_2}{dt}\right)^2 = \begin{pmatrix} \left(C - 3S - \frac{W_s}{2}\right)^2 \sin^2\left(\frac{C_1 + C_2}{2C}\right) \theta \left(\frac{C_1 + C_2}{2C}\right)^2 \\ + S^2 \sin^2\left(\frac{C_1 + C_2 + C_3 + C_4}{2C}\right) \theta \left(\frac{C_1 + C_2 + C_3 + C_4}{2C}\right)^2 \\ + 2\left(C - 3S - \frac{W_s}{2}\right) \sin\left(\frac{C_1 + C_2}{2C}\right) \theta \left(\frac{C_1 + C_2}{2C}\right) \\ \times S \sin\left(\frac{C_1 + C_2 + C_3 + C_4}{2C}\right) \theta \left(\frac{C_1 + C_2 + C_3 + C_4}{2C}\right) \end{pmatrix} \left(\frac{d\theta}{dt}\right)^2 \quad (\text{A.15})$$

556 Similarly we get,

$$\left(\frac{dy_2}{dt}\right)^2 = \begin{pmatrix} \left(C - 3S - \frac{W_s}{2}\right)^2 \cos^2\left(\frac{C_1 + C_2}{2C}\right) \theta \left(\frac{C_1 + C_2}{2C}\right)^2 \\ + S^2 \cos^2\left(\frac{C_1 + C_2 + C_3 + C_4}{2C}\right) \theta \left(\frac{C_1 + C_2 + C_3 + C_4}{2C}\right)^2 \\ + 2\left(C - 3S - \frac{W_s}{2}\right) \cos\left(\frac{C_1 + C_2}{2C}\right) \theta \left(\frac{C_1 + C_2}{2C}\right) \\ \times S \cos\left(\frac{C_1 + C_2 + C_3 + C_4}{2C}\right) \theta \left(\frac{C_1 + C_2 + C_3 + C_4}{2C}\right) \end{pmatrix} \left(\frac{d\theta}{dt}\right)^2 \quad (\text{A.16})$$

557 Hence, the expression for V_2 using Eq. (A.11) is written as

$$V_2^2 = \left(\frac{dx_2}{dt}\right)^2 + \left(\frac{dy_2}{dt}\right)^2$$

$$= \begin{pmatrix} \left(C - 3S - \frac{W_s}{2}\right)^2 \left(\frac{C_1 + C_2}{2C}\right)^2 + S^2 \left(\frac{C_1 + C_2 + C_3 + C_4}{2C}\right)^2 \\ + 2S \left(C - 3S - \frac{W_s}{2}\right) \left(\frac{C_1 + C_2}{2C}\right) \left(\frac{C_1 + C_2 + C_3 + C_4}{2C}\right) \\ \times \begin{pmatrix} \sin\left(\frac{C_1 + C_2}{2C}\right) \theta \sin\left(\frac{C_1 + C_2 + C_3 + C_4}{2C}\right) \theta \\ + \cos\left(\frac{C_1 + C_2}{2C}\right) \theta \cos\left(\frac{C_1 + C_2 + C_3 + C_4}{2C}\right) \theta \end{pmatrix} \end{pmatrix} \left(\frac{d\theta}{dt}\right)^2 \quad (\text{A.17})$$

By using the trigonometric identity: $\sin A \sin B + \cos A \cos B = \cos(A - B)$, we get

$$V_2^2 = \left(\begin{aligned} &\left(C - 3S - \frac{W_s}{2}\right)^2 \left(\frac{C_1 + C_2}{2C}\right)^2 \\ &+ S^2 \left(\frac{C_1 + C_2 + C_3 + C_4}{2C}\right)^2 \\ &+ 2S \left(C - 3S - \frac{W_s}{2}\right) \left(\frac{C_1 + C_2}{2C}\right) \\ &\times \left(\frac{C_1 + C_2 + C_3 + C_4}{2C}\right) \left(\cos\left(\frac{-C_3 - C_4}{2C}\right) \theta\right) \end{aligned} \right) \left(\frac{d\theta}{dt}\right)^2 \quad (\text{A.18})$$

On inheriting the expressions of C_1, C_2, C_3 , and C_4 from Eq. (A.3) and substituting in Eq. (A.18), we get

$$V_2^2 = \frac{1}{4} \left(\begin{aligned} &(2C - 6S - W_s)^2 \left(\frac{2C - 6S - W_s}{C}\right)^2 \\ &+ S^2 \left(\frac{4C - 10S - 2W_s}{C}\right)^2 \\ &+ 2S(2C - 6S - W_s) \left(\frac{2C - 6S - W_s}{C}\right) \\ &\times \left(\frac{4C - 10S - 2W_s}{2C}\right) \left(\cos\left(\frac{4S + W_s - 2C}{2C}\right) \theta\right) \end{aligned} \right) \left(\frac{d\theta}{dt}\right)^2 \quad (\text{A.19})$$

Similarly, by using the following expressions for V_3 , and V_4

$$V_3^2 = \left(\frac{dx_3}{dt}\right)^2 + \left(\frac{dy_3}{dt}\right)^2, \quad V_4^2 = \left(\frac{dx_4}{dt}\right)^2 + \left(\frac{dy_4}{dt}\right)^2 \quad (\text{A.20})$$

Where,

$$x_3 = x_2 + S \cos \psi_3, \quad y_3 = y_2 + S \sin \psi_3$$

$$x_4 = x_3 + S \cos \psi_4, \quad y_4 = y_3 + S \sin \psi_4$$

We get,

$$V_3^2 = \frac{1}{4} \left(\begin{aligned} &(2C - 6S - W_s)^2 \left(\frac{2C - 6S - W_s}{2C} \right)^2 \\ &+ S^2 \left(\frac{10C - 22S - 5W_s}{C} \right)^2 \\ &+ 2S(2C - 6S - W_s) \left(\frac{2C - 6S - W_s}{C} \right) \\ &\times \left(\frac{10C - 22S - 5W_s}{2C} \right) \left(\cos \left(\frac{16S + 4W_s - 8C}{2C} \right) \theta \right) \end{aligned} \right) \left(\frac{d\theta}{dt} \right)^2 \quad (\text{A.21})$$

and,

$$V_4^2 = \frac{1}{4} \left(\begin{aligned} &(2C - 6S - W)^2 \left(\frac{2C - 6S - W_s}{2C} \right)^2 \\ &+ S^2 \left(\frac{18C - 34S - 9W_s}{C} \right)^2 \\ &+ 2S(2C - 6S - W) \left(\frac{2C - 6S - W_s}{C} \right) \\ &\times \left(\frac{18C - 34S - 9W_s}{2C} \right) \left(\cos \left(\frac{28S + 8W_s - 16C}{2C} \right) \theta \right) \end{aligned} \right) \left(\frac{d\theta}{dt} \right)^2 \quad (\text{A.22})$$

The deduced expressions of velocities V_1 , V_2 , V_3 , and V_4 are substituted in Eq. (21) to evaluate the final expression for total kinetic energy of the DEMES.

Appendix B: Expressions of terms (Z_1 - Z_4)

The required expressions of the terms (Z_1 - Z_4) appeared in the nonlinear dynamic governing equation (Eq. 23) are as follows:

$$\begin{aligned}
Z_1 = & \left[\begin{aligned} & \left[(2C - 6S - W_s)^2 \left(\frac{2C - 6S - W_s}{C} \right)^2 \right] \\ & + \left[\begin{aligned} & (2C - 6S - W_s)^2 \left(\frac{2C - 6S - W_s}{C} \right)^2 + S^2 \left(\frac{4C - 10S - 2W_s}{C} \right)^2 \\ & + 2S(2C - 6S - W_s) \left(\frac{2C - 6S - W_s}{C} \right) \left(\frac{4C - 10S - 2W_s}{2C} \right) \cos \left(\frac{4S + W_s - 2C}{2C} \right) \theta \end{aligned} \right] \\ & + \left[\begin{aligned} & (2C - 6S - W_s)^2 \left(\frac{2C - 6S - W_s}{C} \right)^2 + S^2 \left(\frac{10C - 22S - 5W_s}{C} \right)^2 \\ & + 2S(2C - 6S - W_s) \left(\frac{2C - 6S - W_s}{C} \right) \left(\frac{10C - 22S - 5W_s}{2C} \right) \cos \left(\frac{16S + 4W_s - 8C}{2C} \right) \theta \end{aligned} \right] \\ & + \left[\begin{aligned} & (2C - 6S - W_s)^2 \left(\frac{2C - 6S - W_s}{C} \right)^2 + S^2 \left(\frac{18C - 34S - 2W_s}{C} \right)^2 \\ & + 2S(2C - 6S - W_s) \left(\frac{2C - 6S - W_s}{C} \right) \left(\frac{18C - 34S - 2W_s}{2C} \right) \cos \left(\frac{28S + 8W_s - 16C}{2C} \right) \theta \end{aligned} \right] \end{aligned} \right] \\
Z_2 = & - \left[\begin{aligned} & \left[\begin{aligned} & 2S(2C - 6S - W_s) \left(\frac{2C - 6S - W_s}{C} \right) \left(\frac{4C - 10S - 2W_s}{2C} \right) \\ & \sin \left(\frac{4S + W_s - 2C}{2C} \right) \theta \left(\frac{4S + W_s - 2C}{2C} \right) \end{aligned} \right] \\ & \left[\begin{aligned} & 2S(2C - 6S - W_s) \left(\frac{2C - 6S - W_s}{C} \right) \left(\frac{10C - 22S - 5W_s}{2C} \right) \\ & \sin \left(\frac{16S + 4W_s - 8C}{2C} \right) \theta \left(\frac{16S + 4W_s - 8C}{2C} \right) \end{aligned} \right] \\ & \left[\begin{aligned} & 2S(2C - 6S - W_s) \left(\frac{2C - 6S - W_s}{C} \right) \left(\frac{18C - 34S - 2W_s}{2C} \right) \\ & \sin \left(\frac{28S + 8W_s - 16C}{2C} \right) \theta \left(\frac{28S + 8W_s - 16C}{2C} \right) \end{aligned} \right] \end{aligned} \right]
\end{aligned}$$

$$\begin{aligned}
Z_3 &= \begin{bmatrix} K_{bf} + K_{bs} \left(\frac{W_s}{C^2} \right) (2C - W_s) + K_{bs} \left(\frac{W_s}{C^2} \right) (2C - W_s - 2S) \\ + K_{bs} \left(\frac{W_s}{C^2} \right) (2C - W_s - 3S) + K_{bs} \left(\frac{W_s}{C^2} \right) (2C - W_s - 4S) \end{bmatrix} \\
Z_4 &= \begin{bmatrix} 2\lambda_p^2 \sin\left(\frac{\theta}{2}\right) \left[\frac{\cos\left(\frac{\theta}{2}\right) \theta - 2 \sin\left(\frac{\theta}{2}\right)}{\theta^3} \right] + \frac{1}{8\lambda_p^4} \left[\frac{2 \sin\left(\frac{\theta}{2}\right) \theta - \theta^2 \cos\left(\frac{\theta}{2}\right)}{\sin^3\left(\frac{\theta}{2}\right)} \right] \\ - 2\lambda_p^4 \sin\left(\frac{\theta}{2}\right) \left[\frac{\cos\left(\frac{\theta}{2}\right) \theta - 2 \sin\left(\frac{\theta}{2}\right)}{\theta^3} \right] E^2 \frac{\varepsilon}{\mu} \end{bmatrix}
\end{aligned}$$

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