Sociocentric SNA on Fuzzy Graph Social Network Model

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Abstract

The paper presents a scalable and generalized approach to social network analysis using fuzzy graph theory. In this, we propose an intelligent sociocentric approach that calculates the degree of potential relationship of a social network of finite size by proposing a fuzzy graph social network model. It takes into account social entity functional and relational attributes simultaneously. In this, the degree of potential relationship of a social network is computed by using two steps. In the first step, the fuzzy pairwise relationship between all social nodes or entities is computed using the proposed fuzzy node activeness index parameter with their online and offline communication relationship parameters. In the second step, all fuzzy pairwise relationships calculated in the first step are further employed to calculate the degree of potential relationship of a social network using an astute function utilizing both weighted arithmetic and geometric means. Here two weights - betweenness and closeness centrality of an entity are assigned to the social entities. The paper performs the experimental work on a small size WhatsApp social network of undergraduate students in the university. The proposed degree of potential relationship can further be used as a global parameter to compare social networks by incorporating both the functional and relational attributes of social entities.

Keywords Betweenness centrality, Closeness centrality, Fuzzy graphs, Social network analysis, WhatsApp social network, Fuzzy graph social network model.

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1. Introduction

Nowadays, Web2.0[1] power aids like Blogs, Micro Blogs, Podcasts, Wikis, ePortfolios, Social Networks, and Social Bookmarking allow people to work together and express their ideas or experiences. People gain, get power, and get better chances in all fields by using these tools easily. Out of these tools, Social Networks are one of the most powerful, user-friendly, easy-to-use, most explored, all-purpose and robust tools for all classes of people. Social networks efficiently use structured, unstructured, multi-lingual, audio, video, and other types of data or information. A Social Network[2]–[4] is a collection of social nodes linked together with one or more kinds of relations. The different dimensions of social networks with possible issues and challenges are explored in our previous work[5]. Social network analysis (SNA) [2], [3] is "a different aspect of research that mainly aims at *relationships* among the social nodes, on the structure and applications of these relationships, primarily in social & behavioral sciences." Broadly SNA is divided into two main categories – egocentric and sociocentric SNA. The egocentric analysis focuses on an individual nodes in the network and studies its effect on the network, whereas sociocentric SNA focus on a group of nodes or the whole social network.

SNA is the main branch of network analysis that aims only at relationships parameter rather than the attributes of individual entities. The study, experiment analysis, and visualization of how and why the social network is useful and how they benefit our society, all these kinds of questions can be addressed using SNA. The regular or irregular pattern of relationships in a social network is the base that helps in spreading information, ideas, news, and rumors among entities. Currently, several SNA software tools help greatly in analyzing different social networks. A short review of 10 different SNA software tools is given in our work[5]. The detailed survey of 12 different SNA tool with 3 programming languages for SNA is explored in our recent work[6]. In these above papers, we exlore one such tool Gephi[7], to visualize the relationship structures of social networks. The relationship structures in SNA are the quintessential parameters of social networks. These structures have greatly reduced the dimensions from a larger world to a smaller world by including valuables links. These relationship structures give rise to the concept of communication structures between social entities. These communication structures or patterns need to be explored and quantified fully at every stage to study their fruitful applications. This is one of the major potential tasks of SNA as discussed in our previous paper[5]. Also, the characteristics or parameters of existing social networks are complex, uncertain, and dynamic. So, it is difficult to understand, retrieve, and address these communication structures. This type of structural analysis can be further utilized as a global parameter in comparing diverse social networks. The direct approaches for the comparison task are already explored in our earlier papers[8][9][10][11]. by employing the Ordered Weighted Aggregation (OWA) operator[8] and fuzzy-approach[9][10][11].. So, in this paper, we propose one indirect methodology to compare different social networks depending on social nodes functional and relational attributes simultaneously.

The paper uses the survey report from our previous paper[12]. From this survey report, we have collected possible parameters that can affect the relationship directly in social networks. So, this helps us to find out which parameter is required to quantify and compare social networks. That survey report has considered the parameters, namely trust factor, reach, demographics, communication factor, content analysis, centrality measures, etc. From that survey report, we have explored that communication whether done in any form - online or offline mode, is the topmost parameter. It is also justified by the fact that - if there is no communication or usage of the links between entities in social networks, then there is no meaning and use of forming social networks. As social networks are fruitful only when nodes have some relationship between them, and the communication relationship between nodes makes them more social entity. The quantification of these relationships or communication structures remains a challenging aspect of social networks as discussed in our earlier work[5]. So, the paper proposes a sociocentric SNA approach to address this issue. The proposed method is applied to a WhatsApp social network group of university undergraduate students with finite small size dimensions.

Contribution: This paper explores an Intelligent Social network modeling by incorporating the fuzzy concepts at both the node and relational attributes. It calculates the potential sociocentric relationship degree of the finite size of the social network of social nodes by employing a proposed Fuzzy Graph Social Network model in the following steps-

1. It captures the uncertainty at both node and connection levels. So, it proposes one fuzzy parameter to a node and proposes a fuzzy edge parameter depending on online and offline communication.
2. The fuzzy pairwise relationship between all social nodes is computed with their online and offline communication relationship parameters using the proposed fuzzy node activeness parameter.
3. All fuzzy pairwise relationships are further explored to calculate the sociocentric degree of potential relationship of the social network using the proposed astute function utilizing both weighted arithmetic and geometric means.
4. The experiment is performed on the real dataset using betweenness and closeness centrality

measures.

5. The Closeness centrality provides better results, almost three times in average case than the betweenness.

The rest part of the paper is systematically organized as follows. Section 2 explores the correlated tasks done in this direction. Section 3 introduces the main concept of fuzzy graphs with other preliminary mathematical definitions explored in this work. Section 4 discusses the proposed work. Section 5 experiments on a real dataset with five nodes using the Gephi [7] tool. Section 6 ends the paper by giving a conclusion with its future scope.

2. Related Work

Papers[8]–[11][13][14][15][16] have compared different social networks from different angles. Faust[13] has compared different social networks depending upon nodes and edges in a network. This comparison is performed on the triad census, involving local properties, dyad distribution, and network density. Perkins *et al.*[14] have done "the comparison of social networks derived from ecological data". They have studied the disease transmission in rodents networks formed by rodents contact. They have explored two methodologies, - "Capture–Mark–Recapture" and "Radio tracking" simultaneously on a diverse group of rodents. They have explored that the capture–mark-recapture is performing better when rodent density is high, and on the other hand, Radio-tracking is performing better when the rodent density is low. In this, researchers have explored a very specific kind of Rodent social networks. They have explored metrics - average contact rate, betweenness and closeness centrality measures, and connectedness score in the comparison task. This methodology cannot be generalized, so a generic methodology is needed for the social network comparisons task. Burns and Lippold [15] have explored "a comparison between social networks of adults with intellectual disability and those with the physical disability". They have directly contrasted the relational atributes of people in two specific types of Social networks formed due to their Intellectual Disabilities (IDs) and Physical Disabilities (PDs) respectively. They have explored the parameters viz. - life experiences and functional and social support, based on some statistical measurements. They have concluded that the PDs social networks have more nodes than IDs social network. So, they have researched a very specific case where functional, social support, and life experiences are incorporated. Johnson *et al.*[16] have done "the comparison of the email networks and the survey-based social networks in a bank". The authors have done a detailed bank study to match the formation of email networks with communication, advice-seeking, and friendship networks. The authors have used an egocentric analysis also. They have done an offline survey to map to their email networks.

We have also proposed four direct methodologies for the comparison task of diverse social networks. In the first methodology, we have proposed a "Quantitative solution using OWA operator"[8]. We have proposed one generic quantitative approach for the comparison task incorporating six vital metrics at all levels of networks by using the Gephi tool. Here, we have also provided weightage to each metric according to their role in a social network and computed one global quantitative value for every social network for the comparison task. Moreover, we have used diverse social network data sets for the comparison task. This generic methodology is better than the previous ones because we have assigned weightage to all attributes based on their importance factor in network. As social networks are dynamic, so they have uncertainty factors. We have captured this uncertainty factor in our second proposed methodology[9] - "qualitative solution using fuzzy membership function" by using fuzzy sets[17]. This methodology is more intuitive and human-readable as here parameters are clearly outlined by fuzzy concepts and experimented on the diverse data sets. So, we have explored a fuzzy conept based methodlogy for the comparison task of diverse social networks that works better than any traditional approach. All above papers in the past have not considered the user perception view towards the social networks that we have tackled in our third proposed methodology. This user perception view is not incorporated in our earlier research work also. So, in our third methodology [10] user perception viz. user point of view towards particular social network has been focused. Here, we have introduced single global idealness variable computed using fuzzy k-means clustering for the comparison operation. The limitation about this is that it can not address or tackle outliers. To tackle outliers, we have explored fuzzy-K-mediods [18], [19] in our recent work[11]. We have compared multiple social networks with static and dynamic parameters. We have explored static parameters viz. Safety, Connectivity, Instantaneity, Ease of use, Friendliness for New user, and benefits. These static parmameters have used a constant numbers or score from one to ten. They have been given score accordingly user opinion based on their experience on that parameter. We have explored the dynamic parameters - time spent and number of times opened, with variable score. They have been rated according to the user experience on that particular social network. These have beed normalized in order to map with static parameters. We have explored linear scales for these parameters, that determine the inclination of node towards the respective social network as well as the influence of that social network in their growth. We have visualized our findings on a 3-Dimesional conical model to show why one kind of social network is superior than other social network. The above all papers have not taken into account content or degree of relationship or communication, that we take into account into this paper.

Brunelli and collaborators[20]–[22] have proposed indirect methodology to compare social networks. They have computed m-size relationships by employing pairwise fuzzy relations between m nodes by applying the OWA operator, where m is a positive integer. These papers have boosted the analysis dimensions from two nodes to m nodes. They have taken the fuzzy adjacency matrix as the input and captured uncertainty only in the relationships of nodes or entities in the network. They have employed fuzzy membership mapping to express the degree of relationship within nodes of the network and OWA-based aggregation to calculate the relationships between m entities. Their approach is better over the traditional binary relationship as it conserve more details or statistics, although entities personal attributes have not been incorporated. However, they have not explained how and from where they have obtained these fuzzy pairwise relationship values. Neither, they have provided any quantification aspect for the relationships in social networks. This paper has taken one such aspect or parameter– the communication parameter upon which relationship can be directly quantified, as discussed in[12]. Communication structures prevailing among social nodes in the network are the chief parameters that make everyone's life easy and comfortable. The mode and frequency factors of communication can directly quantify the relationships among the social entities. The broad types of online and offline communication modes in the social network are the conditions for quantifying relationships between social entities. So, the communication structure always reflects the relationship between entities. Earlier works[20]–[22] have considered the symmetrical relationship between two entities in social networks. But in real life, it is not possible as no two relations are perfectly symmetric. So, it is always suitable and advisable to model the relationships between social entities as asymmetrical. The earlier work [20]–[22] has employed only one type of relationship. But in actual life, there always exist simultaneous multiple types of relationships between the entities in a network. This paper employs two types of asymmetrical relationships depending on the online and offline communication parameters. Besides this, the earlier work has presented a recursive approach, but discovering a recursive answer is not always practical in the presence of complexity, dynamism, and uncertainty dimensions. In earlier works [20]–[22], the OWA operator is explored for aggregation of the relationships between all entities in networks. The weights explored in their work are selected randomly and must have some physical significance. These weights should depend on both nodes and the connection or relational attributes of a network.The previous work has explored the fuzzy set theory to tackle the uncertainty only in the relationship between entities. As real social networks are full of dynamism, so there is also uncertainty in node or entity parameters. Yager in [23] has presented the idea of "Intelligent Social Network Modeling", where he has incorporated both entities attributes and relationship characteristics of social entities in the vector-valued form. He has provided a way to capture uncertainty simultaneously in both node and relationship attributes. He has explored a hybrid theory using the graph and fuzzy set theory. This unique combination may provide a good analysis of social networks. Today, it may be used to study, analyze, and visualize complexity in the relationship among social entities. So, we employ this "Intelligent Social Network Modeling", in this paper to make it an intelligent approach by using fuzzy graph theory.

We have predicted the Facebook group relationships using reactions to posts in our previous work[24]. In this, we have predicted one methodology for the aggregation of entities relationships with matching interests and views depending on how they react to common posts on Facebook. The limitation of this methodology is that it is a very specific solution. It is not providing a generalized solution for every social network. The "Fuzzy set network Model with OWA" [20]–[22] relates to the real or physical world issues in social networks, but it has a lot of research gaps, as explained above. Hence, it may not be applied effectively to complex, dynamic, and uncertain real-life social networks systems. Thus modeling social networks using fuzzy sets on the edges is an incomplete solution, as it is not mapping the characteristics or attributes of entities in the real world social network. The "Fuzzy Graph Social Network model" is the only solution, which can tackle uncertainty in both attributes - node and relationship that is missing in all the above papers in the literature. Hence, this paper proposes a Fuzzy Graph Social Network model in place of the fuzzy adjacency matrix model to handle uncertainty in nodes attributes and their relationships attributes. The fuzzy adjacency matrix is no doubt good for small size social networks. But it is very complex for a large network. It is not efficient and suitable in the case of scalable social networks. The fuzzy graph model of the social network is good for both small and large social networks. The other limitation of earlier work is that fuzzy weights are not attached to the nodes. This paper tackles this issue by providing fuzzy vertices in the graph by assigning one fuzzy parameter called a node activeness parameter to each of them. We select the betweenness and closeness centrality index as the weights to nodes. Both betweenness and closeness centrality indexes are helpful while calculating sociocentric relationships. But the experimental results demonstrate that the closeness centrality index is superior to the betweenness centrality index to obtain a better value for the sociocentric relationship degree.

3. Fuzzy Graph and other Preliminary Definitions

In 1975, Rosenfeld[25] introduced the concept of Fuzzy graph theory. He operated fuzzy relations on fuzzy sets[17]. In the fuzzy graph, every node and edge is attached with some uncertainty value that defines the participation value of every member (node and relationships) of a graph. The fuzzy graph is discussed and explored by other researchers[26]–[29] in their work. It has been used in a fuzzy telecommunication network[26]. Concerning the registration process of a new user for preventing fake users using a fuzzy graph is discussed in[27]. Further, social network analysis, especially in clustering and identifying the overlapping communities in a complex social network, have been dealt with in[28]. Also, Laszlo in [29] has explored the concept of the fuzzy graph for the evaluation and optimization of networks. The formal definition of the Fuzzy graph (FG) is given below in Definition 1.

Definition 1. A Fuzzy graph (G_ξ) [25] is denoted by 3-tuple $< V_s, \sigma_V, \mu_E >$. It consists of fixed finite set V_s of n nodes, along with two membership mapping, $\sigma_V\colon V_s \rightarrow [0,1]$ and $\mu_E\colon V_s{\times}V_s \rightarrow [0,1]$, such that $\forall\, v_i, v_j \in V_s$, $\mu_E(v_i, v_j) \leq \sigma_V(v_i) \wedge \sigma_V(v_j)$ and μ_E is a symmetric fuzzy relation on σ_V. Here, $\sigma_V(v_i)$and $\mu_E(v_i, v_j)$ represent the membership mapping values of the vertex v_i and the edge(v_i, v_j) in G_ξ respectively. Where $(1 \leq i, j \leq n)$ and the simplest two-node fuzzy graph model with node $n_1 and\ n_2$is depicted in Figure1.

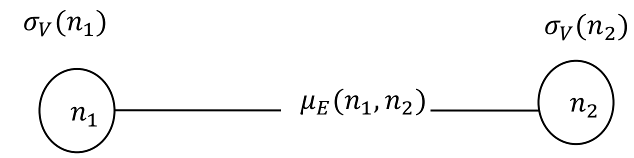

Figure. 1. The simplest 2-node fuzzy graph model

The other three mathematical definitions, which have been explored from literature and used in our proposed work, are also given below:

Definition 2. Degree centrality of an entity[30]**:** It is proportional to the number of direct connections of a social entity to other social entities in the social network G. Mathematically, degree centrality of ith social entity $DC_i(G)$ is defined as follows:

$$DC_i(G) = D_i(G)/(N-1) \tag{1}$$

Where $D_i(G)$ = number of direct connections of ith social entities. There are two types of degrees in the case of directed social network viz. - IN degree and OUT degree. The IN degree is the number of incoming links to the social entities from other entities. Similarly, the OUT degree is the number of outgoing links from social entities to other social entities.

Definition 3. Betweenness centrality index of an entity ($\boldsymbol{\beta}$) [31]**:** Betweenness centrality of a social entity in the social network is proportional to the number of shortest paths that pass through that social entity. Mathematically, the betweenness index of an ith entity in the social network G is denoted by $B_i(G)$ (or $B(i)$) and defined as the ratio of the geodesic shortest distance between kth and jth social entities, on which ith entity lies, to the number of the shortest distance between kth and jth entities. It is formulated as follows:

$$B(i) = \sum_{(i \neq j \neq k)} {P_i(k,j)}\Big/{P(k,j)} \tag{2}$$

where, $P(k,j)$ is the total number of shortest distance from entity k to j entity, and $P_i(k,j)$ is the geodesic shortest path among all k and j on which i entity lies. It defines the importance of a social entity in linking other entities in a social network. For normalization, it is divided by (n-1)(n-2) (for directed) or (n-1)(n-2)/2 (for an undirected social network), where n is the number of social entities in a network.

Definition 4. Closeness centrality index of an entity ($\boldsymbol{\gamma}$) [32]**:** The closeness centrality index of a social entity in the social network is inversely proportional to the sum of the length of the shortest paths between the entity and all other entities in the social network. Mathematically, the closeness centrality index of an ith entity in the social network G with N number of entities is denoted by $C_i(G)$ or C(i) and defined as the reciprocal of the global shortest distance of ith entity to all other jth entities. It is formulated as follows:

$$C(i) = \frac{n-1}{\sum l(i,j)}, \tag{3}$$

where $i \neq j$, l(i,j) = global distance between i and j entities. It is a measure that describes how close a social entity in social network G to all other entities.

4. Proposed Work

People are the social entities that communicate with their friends by forming different social networks on the Internet like Facebook, E-mail, Google+, Hike, Viber, WhatsApp, etc. Broadly, these social entities in their social networks communicate either in offline mode or online mode, or both modes. Their degree of the relationship depends on the amount, frequency and mode of communication between them. The nature and frequency of communications between entities can directly quantify the relationships between social entities as discussed in [12]. The frequency of offline and online data communicated between entities is taken as characteristics of the connection between pairs of entities. Thus, the degree of relationship between the two entities is directly proportional to their degree of communication. More frequency of communication reflects more degree of relationship, whereas less frequency of communication reflects less degree of relationship. As every social entity uses both offline and online communication. So, for the same purpose, the amount of frequency of communication is measured in terms of the number of online and offline communications, both with some weights $\alpha \; and \; (1-\alpha)$ respectively where α is an online fuzzy parameter of a node. Hence, the function for calculating the degree of communication between an ordered pair of social entities s_i and s_j has in its numerators the following two variables viz. $\boldsymbol{On_{ij}} \; \boldsymbol{and} \; \boldsymbol{Of_{ij}}$ are given as follows:

$\boldsymbol{On_{ij}}$ = The number of online communications per day between $s_i \; and \; s_j$

$\boldsymbol{On_{i-sum}}$= The number of total online communications per day between s_i and all entities in a social network.

$\boldsymbol{Of_{ij}}$ = The number of offline communications per day between $s_i \; and \; s_j$

$\boldsymbol{Of_{i-sum}}$ = The number of total offline communications per day between s_i and all entities in a social network.

We propose the fuzzy membership degree of relationship between a pair of social entities $\boldsymbol{s_i}$ and $\boldsymbol{s_j}$ denoted by $R_{ij} \; or \; \deg ree(s_i, s_j)$, including above all four variables, as follows:

$$R_{ij} = \deg ree(s_i, s_j) = \begin{cases} 1, \; if \; maximum \; communication \\ \quad between \; s_i \; and \; s_j \\ \left(\boldsymbol{\alpha_i} * \frac{\boldsymbol{On_{ij}}}{\boldsymbol{On_{i-sum}}} + (1-\alpha_i) * \frac{\boldsymbol{Of_{ij}}}{\boldsymbol{Of_{i-sum}}}\right), \\ if \; some \; communication \; between \\ \quad s_i \; and \; s_j \\ 0, \; if \; no \; communication \; between \\ \quad s_i \; and \; s_j \end{cases} \tag{4}$$

Here, α_i is an online mode index (OLI) of entity $\boldsymbol{s_i}$ per day called the activeness index of entity $\boldsymbol{s_i}$ where $\alpha_i \in [0,1]$. It is the fraction of time when a social entity $\boldsymbol{s_i}$ is in the online mode per day. It measures the activeness of an entity in a network. It depends on the total time in hours or fraction of hours spent by an entity in the social networks per day in online mode ($\boldsymbol{T_{H_{on}}}$) whether communicating or not. Its membership value for i[th] entity $\boldsymbol{s_i}$ per day is calculated as follows:

$$\alpha_i = \begin{cases} 1, & if\ s_i\ in\ the\ online\ mode \\ & for\ the\ whole\ day \\ \left(\frac{T_{H_{on}}}{24}\right), & if\ s_i\ in\ the\ online\ mode \\ & to\ some\ extent \\ 0, & if s_i\ in\ the\ offline\ mode \\ & for\ the\ whole\ day \end{cases} \quad (5)$$

At a particular time, an entity can belong to any two possible states – online and offline. So $(1 - \alpha_i)$ is an offline mode index (OFI) of an entity $\boldsymbol{s_i}$.

In our proposed work, we use the "Intelligent Social Network Modeling"[23], [33] by employing a fuzzy graph social network model (FSN). This intelligent model is more capable and robust for handling uncertainty, as the social entity i.e., node attributes are also added to the parameters of the relationship. This fuzzy graph model approach is scalable, as it may be employed for both small and large social networks. In this model, three types of weights are assigned to the entities i.e nodes. The first is the fuzzy weight or parameter as discussed and defined above in equation (5). It is denoted by α and called the *online activeness index* of the node. Here, α represents the personal attribute of the node that defines the fuzzy presence of a node in the network at any instant. It is used to compute the fuzzy relationship values between each pair of nodes attached directly to the network by their communication parameter. The second one is called the $betweenness\ index\ of\ a\ node.$ It is denoted by β and helpful for calculating the potential relationship of the whole group i.e. Sociocentric analysis. And, the last one is $the\ closeness\ index\ of\ a\ node.$ It is denoted by γ and it also helps us to calculate the potential relationship of the whole group. Formally, the proposed Fuzzy Graph Social Network model (FSN) is defined with six-tuples, $< S_c, E, \alpha_{S_c}, \beta_{S_c}, \gamma_{S_c}, \mu_E >$. It consists of a finite set of social entities S_c forming the social network, a subset of the domain $S = \{s_1, s_2, s_3, \ldots, s_n\}$ with two membership functions sets ($\alpha_{S_c}\ and\ \mu_E$) and two weight-sets ($\beta_{S_c}\ and\ \gamma_{S_c}$). The membership function set, α_{S_c} is a set containing the activeness fuzzy parameter values of each entity in a social network. Its values make the presence of an entity fuzzy in a social network. Its values are evaluated by using equation (5). The weight-sets $\beta_{S_c}\ and\ \gamma_{S_c,}$ are containing all values of the betweenness and closeness index of each entity forming the social network. We use Gephi tool to calculate their values. Mathematically we describe them as follows:

$$\alpha_S : S_c \longrightarrow [0,1]\ where\ \ \alpha_S \epsilon \alpha_{S_c}$$
$$\beta_S : S_c \longrightarrow\ [0,1]\ where\ \beta_S\ \epsilon \beta_{S_c}\ \text{and}$$
$$\gamma_S : S_c \longrightarrow\ [0,1]\ where\ \gamma_S\ \epsilon \gamma_{S_c}$$

The edge set E is the set of an ordered pair of entities, having communication between them and expressed as $E \subseteq S_c \times S_c$. It depicts the relationship between an ordered pair of entities with membership degree set μ_E, between 0 and 1. It is a fuzzy relation defined on set S_c depending on the frequency of their online and offline communications. Formally, we define it as follows

$$\mu_E : S_c \times S_c \longrightarrow [0,1]$$

It is computed by using equations (4) and (5).

The proposed methodology is divided mainly in two steps. In the first step, all the ordered pairs of fuzzy relations are calculated using the proposed equations (4) and (5). The second step evaluates the sociocentric relationship among the group. Sociocentric relationship is the total potential relationship between all entities in the social network. Here it is the relationship degree between all entities participating in social networks. So, it is defined as the relationship of a whole entity group of size m, where m > 2 is any natural number. Formally, the degree of relationship between m entities {1,2,3,.., m} is denoted by membership function $\mu_{S_c} = \ \mu_{(S_1, S_2, \ldots S_m)} =$ deg $(S_1, S_2, \ldots S_m)$. To calculate this, we have taken as input the relationship degree between each pair of entities calculated in the first part using proposed equations (4) and (5).

Let k be the total number of pairs of entities in social networks, excluding the self-communicating edges and the edges that are not communicating. The maximum possible value of k is given by the binomial coefficient ($^{m}C_2$). The sociocentric - degree of relationship among m social entities is found best by employing both the arithmetic mean and geometric mean with some weights factor as explored in our earlier work[34]. It has some limitations when the arithmetic mean (AM), and geometric mean (GM) are taken independently as discussed in detail with an example in the paper[34]. So, we use both AM and GM. in-group analysis. As it is better to use the betweenness or the closeness of a node as an important weight factor, so, weights are assigned to nodes according to their betweenness or closeness centrality measure index. The weight attribute is dependent on the importance of all the entities participating in a network. The importance here is the simple function of each entity's betweenness or closeness index in a network. Further, the weights here use both AM and GM of betweenness or closeness centrality of all entities. So, we propose an astute function, which uses a hybrid approach with the benefits of both the AM method and the GM method with weights function using weights $\beta\ and\ \gamma$ assigned to each entity in a network. The β is the betweenness centrality index of each entity in a social network. The γ is the closeness centrality index of each entity with other entities in a social network. The astute function directly finds the degree of relationships among m entities of FSN, depending on their online and offline communication frequency. The simple function using both arithmetic and geometric mean without using $\beta\ and\ \gamma$ is defined as follows:

$$\mu_{S_c}^{A-G} = \{(\left(\rho * \left(\frac{\sum_{i=1}^{m}\sum_{j=1,j\neq i}^{m}\mu_{S_{(i,j)}}}{k}\right)\right) + \left((1-\rho) * \left(\left(\prod_{i=1}^{i=m}\prod_{j=1,j\neq i}^{j=m}\mu_{S_{(i,j)}}\right)^{\frac{1}{k}}\right)\right))\} \tag{6}$$

Here, ρ is the *or-ness* coefficient ($0 \leq \rho \leq 1$). If no specific purpose is specified, ρ should be taken as 0.5, which means an equal weight of AM and GM. Its value is defined depending upon the type of its application. As a special case, if $\rho = 1$, the proposed function becomes identical to the AM function. Similarly, if $\rho = 0$, the proposed function becomes identical to the GM function. Hence, any intermediate value will result in blending both the AM and GM functions, which makes this function more suitable for any social network. To make it a more astute approach, we use the weighted AM and weighted GM rather than applying simple AM and GM with some ρ parameter. So, the degree of sociocentric relationship among m social entities (μ_{S_c}) is represented by the astute function using both arithmetic and geometric mean with weights $\beta\ or\ \gamma$ is defined by using parameter δ as follows:

$$\mu_{S_c}(\delta) = \frac{(\left(\frac{(((\sum_{i=1}^{m}\delta_i))*(\sum_{i=1}^{m}\sum_{j=1,j\neq i}^{m}R_{ij}))}{m*k}\right)+(\left(\left(\prod_{i=1}^{i=m}\delta_i\right)^{\frac{1}{m}}\right)*\left(\prod_{i=1}^{i=m}\prod_{j=1,j\neq i}^{j=m}\mu_{S_{(i,j)}}\right)^{\frac{1}{k}})}{2} \tag{7}$$

where, $\delta\ can\ be\ \beta\ or\ \gamma$

$$\mu_{S_c}(\delta): (\underbrace{S_c \times S_c \times S_c \times \ldots . \times S_c}_{\text{(m-times)}}) \longrightarrow [0,1]$$

The above-defined degree of relationship among m social entities $\mu_{S_c}(\delta)$ in equation (7) is an astute function as it captures both node and edge attributes of entities in the social network. It gives the potential relationships between m numbers of entities by employing the node as well as the relationship characteristics of the entities. So, fuzzy graph social network block model with four entities $s_1, s_2, s_3, and\ s_4$ with the attributes $\alpha, \beta, \gamma, and\ \mu$ ($i.e\ R_{ij}$) is shown in Figure 2.

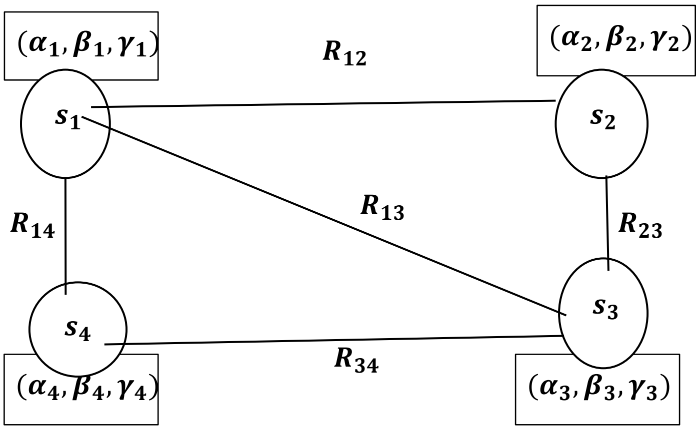

Figure. 2. Fuzzy graph model (4 entities)

The sociocentric relationship analysis model has explored the fuzzy graph model, which incorporated the entities personal attributes and relationship attributes between entities, making it more robust to use it. The proposed algorithm is shown below:

Input: A social network with m social entities.
Output : Degree of sociocentric relationship between total entities m in social network.

1. Collect the values of the online and offline communications of each social entity s_i in a Social Network.
2. Calculate node activeness value for each element s_i in social network using equation (5).
3. Calculate the fuzzy edge i.e relationship degree value for every pair of entities (s_i, s_j) using equation (4).
4. Calculate betweenness and closeness centrality index for each entity s_i in social network using the Gephi tool.
5. Calculate the sociocentric relationship degree using equation (7) with both betweenness and closeness centrality index calculated in step 4.

5. Experimental Work

Nowadays students are the most influential class who use the maximum benefits of technology and social networks wisely. Clafferty[35] has aimed on fecilating a social networking environment for students. They have revealed that the social networking environment is an effective and valuable strategy for students. Students explore and use these social network tools very interactively and intelligently. So, an analysis is required to perform on their social network for studying parameters like the most influencing or potential group in class depending on their communications. A student usually communicates with his/her friends using different types of social networks like WhatsApp, Facebook, Google+, Viber, Hike, email, etc. The most influential and popular among them is the WhatsApp messenger social network as explored in papers[9][10][12][36]. Recently students are the main class, which are mainly exploring WhatsApp for their academic, personal and professional purpose[9][10][11], [35], [36]. The paper[9][10][12] has also concluded "WhatsApp is the most preferred social media platform among students. It has also found that communication is the topmost parameter on which relationship analysis of social network can be done". The popularity of WhatsApp is also seen from the statistic[38], which provides idea on the most popular global mobile messenger apps as of October 2021, based on the number of monthly active users. As of that month, 1.8 billion users are accessing the WhatsApp messenger monthly. Also, the growth rate achieved by WhatsApp after its launching years is exponential that too in very less time. Its growth rate is much faster than Facebook and Twitter, as clearly presented in Figure 3. This all is due to the features provided by WhatsApp. WhatsApp is the easiest, simplest, userfriendly, and fastest method to transmit SMS, images, video, audio or video calls, etc. It is also explored from the our research[34] that youths are more user-friendly and comfortable with its usage. It is more desirable for the students as it is easier and fastest to use. It is more effective, reliable and robust to use and money efficient also. The students use it to inform, share and discuss all the issues concerning personal information exchanges and educational learning purposes. "The Idealness of WhatsApp is much superior than that of Facebook" as, concluded from our work[9][10].

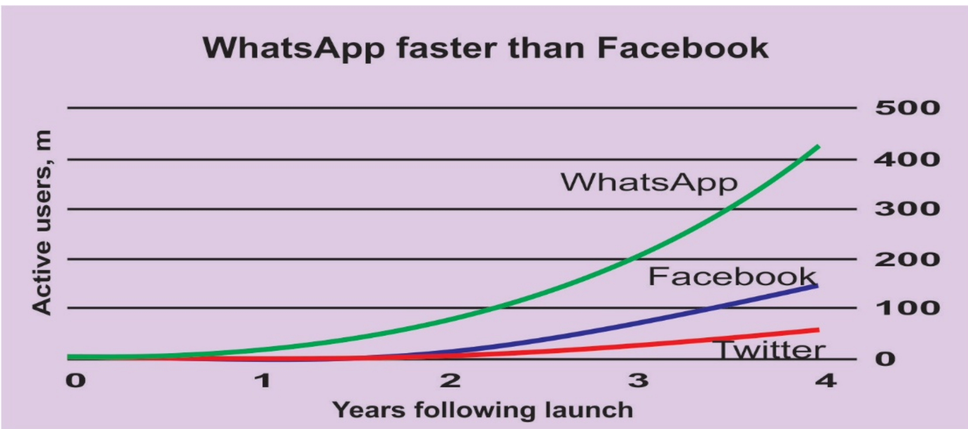

Figure. 3. WhatsApp Faster growth rate

The results of the paper[9][10] depict that students choose "Facebook in terms of time spent, the number of times opened and friendly to new users". This is probably because Facebook's newsfeed is psychologically more addictive than WhatsApp chats. However, when it comes to parameters viz. People Connectivity, Ease of Usage, Educational Purposes, Transfer of Files, and Safety, WhatsApp is superior to Facebook with some significant margin.

So, we experiment on the small size "Student WhatsApp fuzzy graph social network" (SWFSN). In SWFSN, entities or nodes are the students and the edges between any two nodes are the relationships between the students. The communication parameter between nodes defines the relationships between the students. The relationship between students on WhatsApp is directly calculated based on the communication characteristics existing among them. For experimental results, we take the small size SWFSN with 5 students, student1 (s_1), student2 (s_2), student3 (s_3), student4(s_4), and student5 (s_5). The data is collected from students by providing them data collection survey form. We have taken one-week data for analysis. Their online and offline communication relationship values are collected through Google survey form and stored in an excel file. Depending on the data, the directed SWFSN for all seven days is created, visualized, and analyzed using the Gephi tool. The directed SWFSN visualization for all days with their β and γ values is shown in figures number from 4 to 10, respectively. In all these figures, the following color-coding scheme is defined.

1. The degree centrality concept of each student is represented with the different color-coding schemes. The blue color node shows a higher degree of centrality than the green one, which is higher than the red one.
2. The width of the link or edge i.e, relationship degree between students, is encoded with the different colors blue, green and orange. The blue color depicts more weights on edges or more degree of the relationship than the green, which in turn is more than orange.
3. The width of the link or degree of relationship is proportional to the frequency of communication.
4. The higher the width and darkness of one color mean more communication relations than the lesser degree of the same color.

The 1st day online and offline communication frequencies are provided in Table 1 and Table 2 below respectively, showing the number of online and offline frequency of communication between all pairs of students respectively.

Table1: Online communication data for the 1st day

	s_1	s_2	s_3	s_4	s_5
s_1	-	134	0	6	0
s_2	9	-	52	199	30
s_3	10	31	-	250	0
s_4	9	4	1	-	100
s_5	70	0	10	26	-

Table 2: Offline communication data for the 1st day

	s_1	s_2	s_3	s_4	s_5
s_1	-	120	0	5	7
s_2	30	-	11	205	37
s_3	26	17	-	248	0
s_4	67	71	9	-	90
s_5	100	0	35	66	-

The online fuzzy index (OLI) α is the activeness of students in its online mode and is computed by using equation (5). As social network size is taken as 5 so $i = \{1,2,3,4,5\}$. Here $(1 - \alpha_i)$ is called the offline index (OFI) of the ith student. It signifies the non-activeness of a student in the group. The collected values of online hours spend by all students are mentioned in Table 3. The calculated values of the OLI (α) and OFI $(1 - \alpha)$ are given in Table 3.

Table 3: Activeness and non-activeness values for 1st day

	Online hour spent	OLI (α)	OFI ($1-\alpha$)
s_1	18	0.75	0.25
s_2	8	0.333	0.667
s_3	10	0.4167	0.5833
s_4	12	0.5	0.5
s_5	14	0.583	0.417

Using equations (4) and (5) the pairwise fuzzy relationship between five students is calculated and is provided in Table 4. It uses both online and offline communication values mentioned in Table 1 and Table 2, along with activeness and non-activeness index values simultaneously mentioned in Table 3.

Table 4: Pair-wise fuzzy relationship between students for the 1st day

	s_1	s_2	s_3	s_4	s_5
s_1	1	0.94512987	0	0.04161255	0.01325758
s_2	0.0810412	1	0.08563614	0.711669441	0.121653223
s_3	0.06643574	0.07846667	1	0.85509759	0
s_4	0.180823895	0.167332889	0.023373307	1	0.628469909
s_5	0.59246269	0	0.12761194	0.277992537	1

We use the Gephi tool to calculate the betweenness (β_i) and closeness (γ_i) index value of each ith student in SWSN and their values for all five students are mentioned in Table 5. These are normalized values and lie in the interval [0,1].

Table 5: Betweenness and closeness index values (up to 3 decimal) for the 1st day

	Betweenness (β_i)	Closeness (γ_i)
s_1	0.667	0.875
s_2	0.667	1
s_3	0.333	0.875
s_4	1	1
s_5	0.333	0.875

We employ the formula proposed in the last section in the equation (7) to compute the degree of group relationship. The degree of group relationship value calculated with γ is 0.206436278, which is greater than the value (0.130021957) using β. The directed SWFSN visualization for the first day of five students with their β *and* γ values is shown in Figure 4. Student 4 (s_4) has the highest degree of centrality of communication with the other four students. Students s_3 and s_5 have the lowest degree encoded with red color and, s_1 and s_2 have intermediate degrees encoded with green color. The width of edges is proportional to the intensity of communication frequency as shown in Figure 4. Further their β *and* γ values are shown in the text on the respective student node. The first-day communication shows that s_4 has maximum closeness and betweenness index than others and their more degree is encoded by blue color. The maximum frequency amount of communication is between s_1 *and* s_2, *which* is encoded with the potential color blue with a maximum width of the edge. The s_1 *and* s_2 student nodes have the same degree and that's why encoded by green color in Figure 4.

We encode green as the second potential color after blue. Student 2 (s_2) has a more closeness index value than s_1. The student's s_3 *and* s_5 have a minimum degree with a minimum number of direct communication links. They have minimum betweenness and closeness index values encoded with red color. The red is showing its least potentiality in all three colors. The similar encoding with three colors *viz.* blue, green and orange are used for the edges or degree of communication between edges. Here blue or its version - grey color edges show more degree of communication than the green one, which in turn is more than orange color edges. More width of the same color on edges shows more degree of communications than less width of the same color. The section has employed the same color coding for nodes and edges for all other six days as done for the first day. The section has provided data for the first day but has not provided communication data for the other six days. But their visualization is directly reflecting the communication characteristics that the paper has collected from the survey is shown in figures from Figure.5. to Figure10. The degree of sociocentric relationship values for the SWFSN of students for all seven days, using β and γ are calculated and mentioned in Table 6. The average degree of sociocentric relationship value for a week using the betweenness index and closeness index is 0.077983966 and 0.227699880 respectively.

Table 6: Sociocentric relationship values for the WSN of students using β and γ

	$\mu_{S_c}(\beta)$	$\mu_{S_c}(\gamma)$
Day_1	0.130021957	0.206436278
Day_2	0.0560569472	0.20314579371
Day_3	0.02628947371	0.23335119033
Day_4	0	0.21754278285
Day_5	0.055555554	0.23228935926
Day_6	0.19040257982	0.36275030728
Day_7	0.087561251	0.138383452
Average value	0.077983966	0.227699880

The value in an average case using the closeness centrality index is around three times more than the betweenness centrality index. From this, we conclude that using the closeness centrality index, we get a better relationship between five students than the betweenness index value. Hence, the closeness centrality index is a better parameter than the betweenness index while quantifying the relationship of the group. This fact is better justified by the real theoretical fact that if each student is directly connected then the loss of information while communication between students is minimum and it will be secure and reliable in terms of confidentiality. But if other students connect students, there is always a loss of information at each link during the communication. In the latter case, there is a possibility that the meaning of information may get changed or reversed. Thus, considering the closeness index is a more suitable approach while calculating the sociocentric relationship. For better and accurate results, this process may be repeated for a long period i.e. months or years with a greater number of students.

This work is advantageous for assigning projects to potential fixed size groups of students depending on their degree of sociocentric relationship in that group. It will be used to find the most influential subgroup in a social network. The calculated degree of sociocentric relationship values for the most influential subgroup SWFSN of students using γ is provided in Table 7. From this table, it is shown the most influential subgroup of size two, includes students $s_1 and\ s_2$ (1,2) because this pair has a maximum value (0.94) of the degree of relationship value out of all possible pairs. So, if a project is to be given to a group of two students, then this is the best group for that. The most influential subgroup of size three includes students $s_1, s_2\ and\ s_4$ (1,2,4) because this subgroup has a maximum value (0.34) of the degree of relationship value out of all possible subgroups of three sizes. So if a project is to be given to a group of three students, then this is the potential group for that. Similarly, the most influential subgroup of size four includes students $s_1, s_2, s_4, and\ s_5$ (1,2,4,5) because this subgroup has a maximum value (0.24) of the degree of relationship value out of all possible subgroups of size four. So, if a project is to be given to a group of four students, then this is the potential group for that assignment.

Similarly, this proposed work is advantageous for the indirect comparison of social networks, for making potential groups or committees in an academic/nonacademic organization depending on their communication interactions. This work may be used to recommend a potential group for the formation of Trust or Society. It may be a good methodology for community detection in social networks. It may also apply to the medical field in selecting or recommending an excellent team of doctors with good agreement with their positive communication. Even it can be deployed to find the best potential family or subgroup in a family with a good consensus. In future work, we will use this approach using "new closeness centrality using communication aspect"[39] along with "the type of communication - positive, negative and neutral" [12] that have been proposed in our recent works[12], [39] The proposed fuzzy graph sociocentric SNA model can also be helpful in Big-Data multi-community anomaly detection in dynamic and static social networks[40].

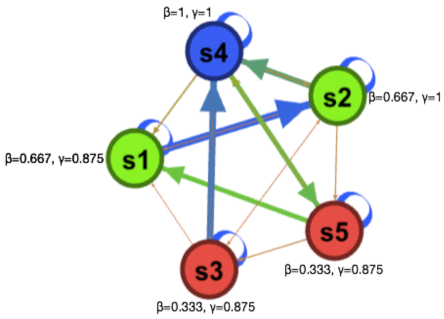

Figure 4. Day1 WhatsApp social network

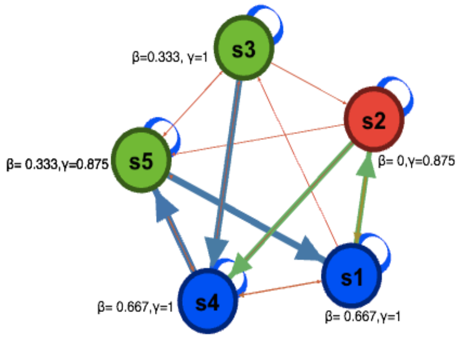

Figure 5. Day2 WhatsApp social network

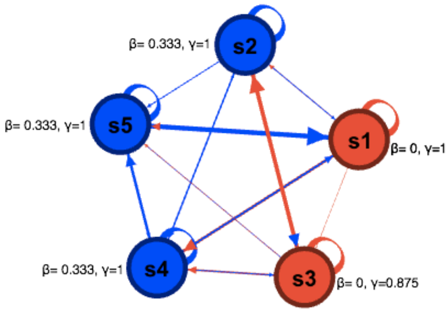

Figure 6. Day3 WhatsApp social network

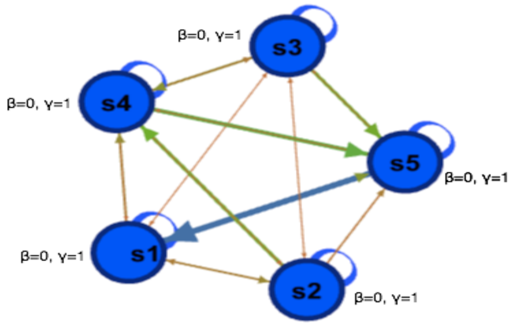

Figure 7. Day4 WhatsApp social network

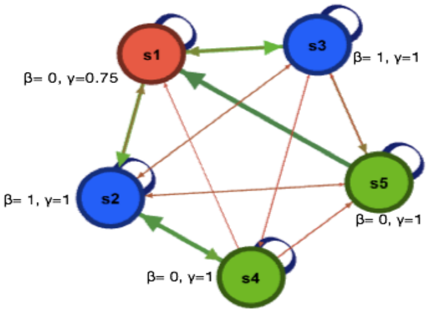

Figure 8. Day5 WhatsApp social network

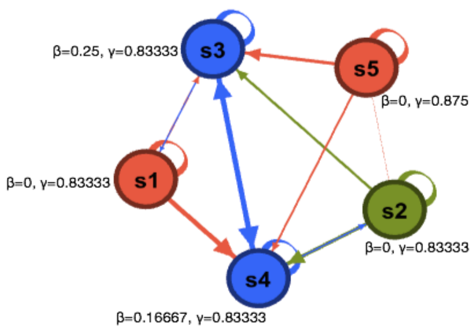

Figure 9. Day6 WhatsApp social network

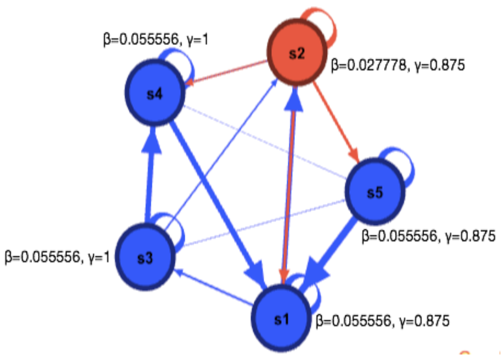

Figure 10. Day7 WhatsApp social network

Table 7: Sociocentric relationship values of most influential subgroup using γ for the first day

Subgroup size	Most influential subgroup	Relationship degree
2	(1,2)	0.94
3	(1,2,4)	0.34
4	(1,2,4,5)	0.24

6. Conclusion

From the experiment results, it is concluded that in sociocentric analysis the closeness centrality gives better value, almost three times in average case than the betweenness. The fuzzy graphs model is suitable for modeling social networks with uncertain parameters. It gives fruitful sociocentric analysis by incorporating node fuzzy activeness and closeness centrality index as the essential parameters. In future work, we plan to focus only on closeness extended centrality index by incorporating the weights of the communication relationship between the entities. Moreover, the paper has taken into account two types of broad communication modes – online and offline parameter. But for better, accurate and stable results, it will be considering all the possible types of communications - online and offline with text, picture, audio, and video communication structure in the next work. Besides this, the study will be for longer periods, i.e. months or years, with more nodes. Also, the trust factor will be taken as another potential parameter along with the communication.

Compliance with Ethical Standards

Source of Funding

The author declares no source of funding.

Conflict of Interest

The author declares no conflict of interest.

Informed Consent

This research is not belongs to any human participants or any welfare of animals

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