The observed discordances in cosmology signaling new Physics

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Title: The observed discordances in cosmology signaling new Physics

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Abstract:

The measured values for the cosmic expansion rate, the cosmic radius, the cosmic age, etc. vary with a direct or an indirect methodology. These discrepancies known as the cosmological crisis imply the existence of a new physical field. The coupling of matter to the field causes the ratio between a being measured mass of matter and a reference mass to vary with the field. Any experiment can only measure the relative ratio rather than the absolute mass of matter. Apparently, there are two representations in describing the field dependence of the ratio: the reference (being measured) mass varies with the field while the being measured (reference) mass does not. Therefore, the measured value of every quantity depends on the choice of the representations. A representation is selected based on the conscious or unconscious assumptions in an experiment. This new field can resolve the discrepancies as well as drive the late-time cosmic acceleration. The new closed cosmic model here can remove the tensions in the standard Λ cold dark matter model with Λ being the cosmological constant.

Text:

Edwin Hubble found that our Universe is expanding¹. The expansion rate of the Universe is often presented by the Hubble constant. Since the expansion rate changes with time on cosmic timescales, the Hubble constant now corresponds specifically to the expansion rate at the present time. The value of Hubble constant is measured more and more accurately. Interestingly, the measured values for the same physical quantity are different for direct and indirect measurements^{2,3}. Due to the sufficient precision for each measurement this so-called Hubble tension cannot be attributed to a statistical fluctuation and then challenges the standard cosmological model^{4,5}. By analyzing the essential difference between the two types of measurements in this paper, I demonstrate that the Hubble tension and the related issue, e.g. whether the Universe being closed or open⁵, the problem of the cosmic age etc., signal a generalization of general relativity in which a scalar field is inserted.

This is reminiscent of Brans-Dicke theory^{6,7}. In order to incorporate Mach's principle (MP) into Einstein's general relativity (GR), Brans and Dicke introduced a scalar field to develop their modified relativistic theory of gravitation. MP states that the inertial forces observed in an accelerated laboratory exist only in relation to the distribution of distant matter in the Universe. The evidence for dark matter⁸ of the current cosmology hints that inertial forces may also result from the effects of dark matter accelerated relative to the laboratory. When all the space of the Universe is permeated with fuzzy dark matter^{9,10}, observation of its effects of the inertial forces can be immediate.

Brans and Dicke concluded that an extra degree of freedom must be involved to modify GR due to the partial implementation of MP in GR. When the scalar field is introduced, the same physical situation would be described by two different representations: the Jordan frame and the Einstein frame. The two frames are linked by a mathematical transformation. I will show in this paper that the direct and indirect measures for the Hubble constant correspond to the Jordan and Einstein frames, respectively. Since any nontrivial transformation will change the value of a physical observable, the Hubble tension indicates the existence of an observable scalar field.

Due to the lack of a self-interaction potential, Brans-Dicke's scalar field does not have the property of quintessence and then their model cannot predict the observed cosmic acceleration. The new physical field here with the self-interaction potential can drive the late-time acceleration as well as the inflation of the Universe¹¹.

Two representative frames

There are at least three possible reasons for introducing scalar fields into GR: to improve GR completely compatible with MP^{6,7}, to drive inflation of the early Universe¹²⁻¹⁵, and to drive the cosmic acceleration at the late-time^{16,17}. When the scalar field is introduced to meet MP, we have to choose either the physical quantities themselves varying with the scalar or the reference units varying with it. This so-called frame issue does not appear in GR since both the rest masses of particles and the gravitational constant are believed to be independent of time and spatial location. But the scalar field that is required in MP would interact with matter, and then the rest mass of matter would vary with position and time.

However, no absolute value of any dimensionful quantity can be measured experimentally without the specific definition of reference units^{6,7,18}. Only

the dimensionless ratio between a physical quantity to be measured and the corresponding reference unit is physical.

Take measurement of mass as an example. To depict the mass ratio, a natural reference mass is the reduced Planck mass M_{Pl} , which is defined by the reciprocal of the root of the gravitational constant G , i.e., $M_{\text{Pl}} \equiv (\hbar c / 8\pi G)^{1/2}$

with the reduced Planck constant $\hbar$ and the speed of light c , respectively. Then the mass ratio is defined as m/M_{Pl} with m being the rest mass of matter. The requirement of MP means that the mass ratio is a function of a scalar field. There are infinitely possible choices for G and m to satisfy the requirement of the mass ratio $m(8\pi G/\hbar c)^{1/2}$ being a definite function of the scalar. However, for easy comparison with experiments, it is more convenient to choose either constant G or constant mass m . This discussion for mass is equally valid for length and time. Therefore, there are two typical frames in theoretical description, the Jordan frame and the Einstein frame.

The Einstein frame is one of the representations of the gravitational theory with the scalar field, in which the gravitational constant G is defined as constant. Thus, the rest masses of particles must vary with the scalar (correspondingly, the energy levels of all atoms vary with the scalar in the same way), and then particles do not move along geodesics of the geometry. However, Einstein's field equation is satisfied in the Einstein frame. Mathematically, the gravitational constant G and the rest mass m in the Einstein frame are required to be expressed as

$$G_{\text{E}} = G_{\text{O}}, \quad m_{\text{E}} = m_{\text{O}} A(\phi) \quad (1)$$

where the label “E” stands for the Einstein frame, G_{O} and m_{O} are constant with the label “O” marking the scalar-independence, $A(\phi)$ is a definite function of the scalar field ϕ , called the coupling function of matter to the scalar.

The Jordan frame is another representation in which the masses of particles are defined to be constant (correspondingly, the energy levels of all atoms are also independent of the scalar field). Thus, the gravitational “constant” G must be a function of the scalar field. The constant m ensures that the equation of motion of particle is the geodesic equation. Thus, every galaxy freely falling in the Universe, like a small particle on the cosmic scale, does

move along a geodesic of the Jordan frame geometry. Mathematically, G and m in the Jordan frame are required to be expressed as

$$G_J = G_O A^2(\phi), \quad m_J = m_O \quad (2)$$

where the label “J” stands for the Jordan frame. All three fundamental physical quantities, length, time interval, reciprocal mass in the Jordan frame, whether being served as references or to be measured, can be obtained from the Einstein frame by the same mathematical transformation, i.e.,

$$x_J = x_E A(\phi) \quad (3)$$

where x stands for any of the three physical quantities mentioned above. For example, the rest mass: $1/m_J = A(\phi)/m_E$, etc.. The translations for a rational function of the three quantities can be deduced by their definition. For example, the translation of the matter density is $\rho_J = \rho_E/A^4(\phi)$, the translation of G is $G_J = G_E A^2(\phi)$, etc..

It is needed to emphasize that, either Eqs. 1 or 2 results from the physical requirement of MP, while Eq. 3 is only the universal mathematic transformation. From the standpoint of physics, the object to be measured and the reference unit should be distinguished explicitly. Their definite functional relations to the scalar field depend on the choice of frames. The two frames are connected by mathematic transformations. From the standpoint of mathematics, all the objects to be measured and all the objects served as references should be transformed in the same fashion.

Frame-dependent Hubble parameter

If the Universe is homogeneous and isotropic, the line element in the Einstein frame is

$$ds_E^2 = -dt_E^2 + a_E^2(t_E) \left[\frac{dr^2}{1 - Kr^2} + r^2 (d\theta^2 + \sin^2 \theta d\varphi^2) \right] \quad (4)$$

where K is the spatial curvature parameter and $K = 1, 0$, or -1 corresponds respectively to closed, flat, or open spaces, a_E is the scale factor of the Universe, t_E is the cosmic time in the Einstein frame. Since the length scale a_E is scalar-independent and the mass of matter depends on the scalar as $m_E = m_O A(\phi)$, the matter density of the Universe in the Einstein frame is

deduced as $\rho_E = \rho_O A(\phi)$ with ρ_O being the scalar-independent density of matter.

According to Eq. 3, the line element in the Jordan frame can be obtained as follows⁶

$$ds_J = A(\phi) ds_E \quad (5)$$

Hence, the cosmic time interval in the Jordan frame is $dt_J = A(\phi) dt_E$, and the scale factor is $a_J = a_E A(\phi)$. Considering $m_J = m_O$, the matter density of the Universe is $\rho_J = \rho_O / A^3(\phi)$ in the Jordan frame. Eq. (5) shows that the transformation between the two frames is geometrically conformal. The conformal factor $A(\phi)$ is indeed the coupling function of matter to the scalar.

For nonrelativistic matter, the evolution of the scalar field ϕ in the Einstein frame is determined by an effective potential¹⁹. The effective potential is $V_{\text{eff}}(\phi) = V_E(\phi) + V_{\text{int}}$, where $V_E(\phi)$ is the self-interaction potential of the field and $V_{\text{int}} = \hbar^3 c^5 \rho_E$ is the interaction potential between the scalar field and matter. In the status of matter dominated Universe, the scalar field needs to be damped to the minimum ϕ_{min} of the effective potential during the expansion of the Universe and then sit stably at the minimum²⁰. Since Einstein's field equation is valid in the Einstein frame, the equations of motion for the scale factor a_E can be derived as follows:

$$H_E^2 = \frac{8\pi G_O}{3} (\rho_\phi + \rho_E) - \frac{Kc^2}{a_E^2} \quad (6)$$

$$\frac{\ddot{a}_E}{a_E} = \frac{8\pi G_O}{3} \left(\rho_\phi - \frac{1}{2} \rho_E \right) \quad (7)$$

where $H_E \equiv \dot{a}_E / a_E$ is the Hubble parameter with $\dot{a}_E \equiv da_E / dt_E$ being the expansion rate of the Universe in the Einstein frame, $\rho_\phi \equiv \hbar^{-3} c^{-5} V_E(\phi_{\text{min}})$ is the mass density of the scalar field, $\ddot{a}_E \equiv d^2 a_E / dt_E^2$ is the cosmic acceleration. The formal structures of these equations of motion are identical to that in the standard Λ -cold dark matter (Λ CDM) model with Λ

being the cosmological constant²¹. The value of the self-interaction potential at the minimum plays the role of the cosmological constant, i.e.,

$$\Lambda_E \equiv \frac{8\pi G_0 V_E(\phi_{\min})}{\hbar^3 c^7} \quad (8)$$

In the case of nonrelativistic matter, the minimum $\phi_{\min}$ depends only on the mass density ρ_0 of matter. Thus, the conformal factor $A(\phi_{\min})$ at the minimum is a function of one variable ρ_0 . Eq.6 can be rewritten as

$$\Omega_m + \Omega_\phi + \Omega_K = 1 \text{ with } \Omega_m = \rho_m/[3H_E^2(8\pi G_0)], \Omega_\phi = \rho_\phi/[3H_E^2(8\pi G_0)] \text{ and } \Omega_K = -Kc^2/(a_E H_E)^2. \Omega_\phi \text{ corresponds to the cosmological constant.}$$

Using the conformal translations¹¹, the equations of motion for the scale factor in the Jordan frame can be derived easily as follows:

$$H_J = \frac{H_E}{A} \left(1 - 3 \frac{d \ln A}{d \ln \rho_0} \right) \quad (9)$$

$$\frac{\ddot{a}_J}{a_J} = \frac{\ddot{a}_E}{a_E} \frac{1}{A^2} \left(1 - 3 \frac{d \ln A}{d \ln \rho_0} \right) + \frac{9H_E^2}{A^2} \frac{d^2 \ln A}{(d \ln \rho_0)^2} \quad (10)$$

where $H_J \equiv \dot{a}_J/a_J$ is the Hubble parameter in the Jordan frame with $\dot{a}_J \equiv$

da_J/dt_J being the cosmic expansion rate, and $\ddot{a}_J \equiv d^2 a_J/dt_J^2$ is the cosmic acceleration. The structure of the acceleration equation in the Jordan frame shown by Eq. 10 is no longer identical to that in the Λ CDM model²¹. Thus, it is impossible to define a definite cosmological constant in the Jordan frame, albeit the statement of the cosmic acceleration is still valid.

It has been demonstrated that the scalar field can stably sit at the minimum of the effective potential when the functional forms of the coupling function

$$A(\phi) = \exp[(\phi^2 - M_2^2 c^4)^2 / (4M_1^4 c^8)] \text{ and the self-interaction potential } V_E(\phi) =$$

$\lambda\phi/4$ in the Einstein frame¹¹, where M_1 , M_2 and λ are undetermined

parameters (λ will be naturally chosen as $1/3!$ in calculations). Using Eq. 8 a dynamical cosmological “constant” in the Einstein frame is obtained as

$$\Lambda_E = 2\pi\lambda c\hbar^{-3}G_0M_2^4 \left(\frac{\rho_0}{\lambda c^3\hbar^{-3}M_1^4 + \rho_0} \right)^2 \quad (11)$$

When the density of matter is large enough, the dynamical cosmological “constant” behaves as the fixed cosmological constant in the Einstein frame, i.e., $\Lambda_E \simeq 2\pi\lambda c\hbar^{-3}G_0M_2^4$. The functional forms for both the self-interaction potential and the symmetry-breaking coupling function guarantee the scalar adiabatic tracking the minimum of the effective potential. The symmetry-breaking interaction clamps the field at the minimum, differing from the traditional slow-roll quintessence models. Thus, this model here may be called the trapped quintessence model to facilitate the subsequent writing.

Two typical measurements

There are two basic types of measurements in deriving the expansion rate of the Universe: the indirect method of measuring the cosmic microwave background (CMB)^{3,5,22} and the direct method of measuring the motions of galaxies^{2,24-28}. I will show that indirect determination is performed in the Einstein frame, while direct determination of Hubble constant is performed in the Jordan frame.

I first discuss the indirect methodology. By measuring the CMB using the inflationary flat Λ CDM model, the Planck collaboration³ estimates the Hubble constant to be $H_0 = 67.36 \pm 0.54 \text{ km s}^{-1} \text{ Mpc}^{-1}$ and the cosmological constant $\Lambda_{\text{flat}} = (4.24 \pm 0.11) \times 10^{-66} \text{ eV}^2$.

The inflationary Λ CDM model includes two accelerating eras^{21,23}: the early inflationary era when inflation is driven by a scalar field, and the late-time accelerating era when the expansion of the Universe is sped up by dark energy. The inflation part of the cosmological model is described in the Einstein frame since the gravitational constant is definitely constant. In our trapped quintessence model, the formal structures of the equations of motion for the scale factor in the Einstein frame are identical to that in the dark energy part of the inflationary Λ CDM model. The cosmological constant can be mimicked in the trapped quintessence model if the density of matter is large enough. Consequently, the derivation of the Hubble constant by measuring the CMB can be identified in the Einstein frame.

I now discuss the direct methodologies of measuring the motions of galaxies to obtain the Hubble constant^{2,24-28}. In the direct measurements, there is a self-evident precondition that a freely falling galaxy travels along a geodesic of the geometry. Thus, if the scalar field exists, the direct measurements are implicit in the Jordan frame.

We specify the conclusion using a typical example in the direct measurements. By using Type Ia supernovae (SNe Ia) with standard candle distance measurements based on a Cepheids calibration², $H_0 = 74.03 \pm 1.42$ km s⁻¹ Mpc⁻¹ is estimated. To deduce the Hubble parameter, the equality assumptions are used as follows: The emitted photon wavelength from the distant star is equal to that from a laboratory on our Earth; the emitted power of the distant star is equal to that of the vicinity star, which means that the atom level lifetimes on the two stars are equal. The equality assumptions mean that the energy levels of atoms are independent of the space-time points. Thus, from the standpoint of the scalar for MP, these assumptions mean that the rest masses of matter are invariant, satisfying the feature of the Jordan frame.

Model parameters constrained by Hubble tension

Due to the improvement of astronomical measurement accuracy, the frame-dependent values of the Hubble parameter can be subject to astronomical tests. In fact, the relations of Hubble parameter between the two frames shown as Eq. 9 can offer a way to break the parametric degeneracy in a cosmic model. The parametric degeneracy refers to that for fixed values of astronomical observations, such as the Hubble constant and the cosmological constant, different parameters in the model are more difficult to distinguish.

Based on an enhanced lensing amplitude in CMB power spectra⁵, the Planck result prefers $H_0 = 54.4^{+3.3}_{-4.0}$ km s⁻¹ Mpc⁻¹, the closed Universe with curvature parameter $\Omega_{K0} = -0.0438$ and the current matter fraction $\Omega_{m0} \simeq 0.477$.

Thus, the current dark energy fraction is $\Omega_{\phi 0} = 0.567$ which corresponds to the cosmological constant $\Lambda_{\text{closed}} = 2.289 \times 10^{-66}$ eV². Assuming that the above data are generated in the Einstein frame (the model parameter of λ is naturally chosen as 1/3!), the Hubble constant in the Jordan frame can be calculated using Eq. 9. It is found that the value of Hubble constant in the

Jordan frame is very sensitive to the parameter ratio of M_2/M_1 (The sensitivity to the cosmic model's parameters is shown in the *Methods*). If the Hubble constant in the Jordan frame is chosen as $H_0 = 74.03 \text{ km s}^{-1} \text{ Mpc}^{-1}$ from Riess et al. ², that is, $H_{J0}/H_{E0} = 74.03/54.4$, by using Eqs. 6, 9 and 11 the parameter ratio is determined to be $M_2/M_1 = 3$ with $M_2 = 4.40353 \text{ meV}$ and the cosmic matter density $\rho_{00} = 2.41828 \times 10^{-27} \text{ kg m}^{-3}$, respectively.

Interestingly, in the closed Universe, $\Lambda_{\text{closed}}(H_{J0}/H_{E0})^2 = 4.2394 \times 10^{-66} \text{ eV}^2$ is exactly equal to the cosmological constant³ $\Lambda_{\text{flat}} = (4.24 \pm 0.11) \times 10^{-66} \text{ eV}^2$ in a flat universe. Noticing Eqs. 6 and 9, this coincidence may mask the existence of the new physical field ϕ .

In fact, from Eq. 9, it can be seen that the ratio of the predicted values for the Hubble parameter between the two frames varies with time. Figure 1 shows the comparison picture of the predicted values for the Hubble parameter in the closed Universe. Based on Eq. 9, the ratio of the predicted values for the cosmic expansion rate in the closed space between the two frames is also drawn in the figure. The time variable is acted by redshift $z_E = a_{E0}/a_E - 1$ in the Einstein frame, where the subscript '0' is the current time.

Fig. 1. Evolution of the cosmic expansion parameters' ratios of the two frames in the case of nonrelativistic matter for the closed Universe. The solid curve describes the values of the Hubble parameter in the Jordan frame relative to those in the Einstein frame as a function of redshift in the Einstein frame. The dotted curve describes the relative values of the expansion rate as a function of redshift.

It is evident that the spatial curvature parameter K in Eq. 4 is frame independent due to Eq. 5. Choosing that either the mass of matter or the reference unit varies with the new scalar field can only change the numerical

value of physical quantities, but cannot change the fundamental attribute of the space-time geometry. The conformal factor between the two frames is not arbitrary. It must satisfy both Mach's principle and the observed cosmic acceleration.

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Competing interests I declare no competing interests.

Methods:

To clarify that the Hubble constant in the Jordan frame is very sensitive to the parameter ratio of M_2/M_1 in the trapped quintessence model, we use the astronomical observation data from the Planck collaboration³ which are based on the almost flat universe. The well-known tension in the Planck dataset is removed based on the closed space in literature [5]. When the data in [5] and the data in [2] are used, the result of $M_2/M_1 = 3$ has been obtained directly as shown in main text.

The simultaneous equations are obtained by substituting the following astronomical observation data from the Planck collaboration³ into Eqs. 6 and 11: the current matter fraction in the Universe $\Omega_{m0} = 31.58\%$ with the subscript ‘0’ marking the current time, the current cosmological constant $\Lambda_{\phi 0} = \Lambda_{\text{flat}} = 4.24 \times 10^{-66} \text{ eV}^2 = 1.089 \times 10^{-52} \text{ m}^{-2}$, and the current Hubble parameter in the Einstein frame $H_{E0} = H_{\text{flat}} = 67.36 \text{ km s}^{-1} \text{ Mpc}^{-1}$.

Table 1 shows the calculation results of the values of Hubble constant in the Jordan frame versus the parameters’ ratio M_2/M_1 in the trapped quintessence cosmic model in an almost flat space. Since the Universe might be closed^{5,11}, the spatial curvature parameter $K = 1$ is fixed in all the calculations. It is the frame-dependency of the Hubble constant that leads to different measured values for the two well-regarded measurements. Thus, the discrepancy of the values of the Hubble parameter estimated from the two methodologies may be called Hubble distinction rather than Hubble tension. Therefore, the significance of this table is to clarify the sensitivity of the Hubble distinction to the parameters in the cosmic model. It is seen that when M_2/M_1 takes on a value from 5.5 to 6, the Hubble constant in the Jordan frame takes on a value consistent with the direct measurements, i.e., from $74.29 \text{ km s}^{-1} \text{ Mpc}^{-1}$ to $72.25 \text{ km s}^{-1} \text{ Mpc}^{-1}$.

Table1. The sensitivity of the Hubble distinction to the cosmic model’s parameters. The so-called Hubble tension can be regarded as the Hubble distinction of the two frames, which breaks the parametric degeneracy in the cosmic model due to the high sensitivity. Notice that for the matter density ρ_{00} of the universe, the label ‘O’ marks the scalar-independence while the subscript ‘0’ is the current time.

M_2/M_1	M_2 (meV)	ρ_{00} (10^{-27} kg m $^{-3}$)	H_{J0}/H_{E0}	a_{E0} (Gpc)	H_{J0} (km s $^{-1}$ Mpc $^{-1}$)
4	5.05481	2.45873	1.371	204.322	92.34
5	4.99281	2.60667	1.151	204.322	77.53
5.5	4.98079	2.63514	1.103	204.322	74.29
6	4.97342	2.65257	1.073	204.322	72.25
8	4.96168	2.68026	1.023	204.322	68.91

In order to draw a comparison picture of the predicted values for the Hubble parameter between the two frames, the translation of the Hubble parameter shown by Eq. 9 is rewritten as follows:

$$\frac{H_J}{H_E} = \frac{1}{A} \left(1 - 3 \frac{d \ln A}{d \ln \rho_0} \right) \quad (1)$$

The Hubble parameter is often used to describe the expansion rate of the Universe, but the two quantities are not exactly equal due to $H_E \equiv \dot{a}_E/a_E$

($H_J \equiv \dot{a}_J/a_J$). To distinguish explicitly the Hubble parameter and the cosmic expansion rate, the ratio of the predicted values for the cosmic expansion rate between the two frames is also given as follows:

$$\frac{\dot{a}_J}{\dot{a}_E} = 1 - 3 \frac{d \ln A}{d \ln \rho_0} \quad (2)$$

Figure 2 shows the evolution of the above ratios in an almost flat universe as functions of redshift. The redshift acts as the time variable, and is defined by $z_E = a_{E0}/a_E - 1$ in the Einstein frame, where the subscript ‘0’ is the current time (of course, the time variable can also be substituted with redshift $z_J = a_{J0}/a_J - 1$ in the Jordan frame). The picture is drawn under the choice of $M_2/M_1 = 6$ with $M_2 = 4.97342$ meV. The values of the cosmic expansion rate (the Hubble parameter) in the Jordan frame are almost equal to those in the Einstein frame in the past of the flat universe. For the cosmic expansion rate, after becoming larger over time, its values in the Jordan frame eventually converge with those in the Einstein frame in the future and the relation of $\dot{a}_J \geq \dot{a}_E$ remains throughout. For the Hubble parameter, however, after becoming larger with time, its values in the Jordan frame eventually become far less than that in the Einstein frame in the future due to the scale factor increasing faster in the Jordan frame than that in the Einstein frame.

Fig. 2. Evolution of the cosmic expansion parameters' ratios of the two frames in the case of nonrelativistic matter for an almost flat universe. The dotted curve describes the values of the expansion rate of the flat universe in the Jordan frame relative to those in the Einstein frame as a function of redshift in the Einstein frame. The solid curve describes the relative values of the Hubble parameter as a function of redshift. While the evolution of the flat universe can be independently described in either the Einstein frame or the Jordan frame, the measured value for any dimensionful quantity is dependent on a definite frame. The Jordan frame and the Einstein frame will be explicitly distinguishable in the future due to the ratios departing away from 1.

Data Availability: All data is available in the main text and methods.

Figures

Figure 1

Evolution of the cosmic expansion parameters' ratios of the two frames in the case of nonrelativistic matter for the closed Universe. The solid curve describes the values of the Hubble parameter in the Jordan frame relative to those in the Einstein frame as a function of redshift in the Einstein frame. The dotted curve describes the relative values of the expansion rate as a function of redshift.

Figure 2

Evolution of the cosmic expansion parameters' ratios of the two frames in the case of nonrelativistic matter for an almost flat universe. The dotted curve describes the values of the expansion rate of the flat universe in the Jordan frame relative to those in the Einstein frame as a function of redshift in the Einstein frame. The solid curve describes the relative values of the Hubble parameter as a function of redshift. While the evolution of the flat universe can be independently described in either the Einstein frame or the Jordan frame, the measured value for any dimensionful quantity is dependent on a definite frame. The Jordan frame and the Einstein frame will be explicitly distinguishable in the future due to the ratios departing away from 1.